Overconfidence in Tullock Contests

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Contest Success Functions - some context

- Contests are a tool first introduced by Gordon Tullock (1980) to study the "lobbying" process potentially at play to extract monopoly rents
- In its simplest form,

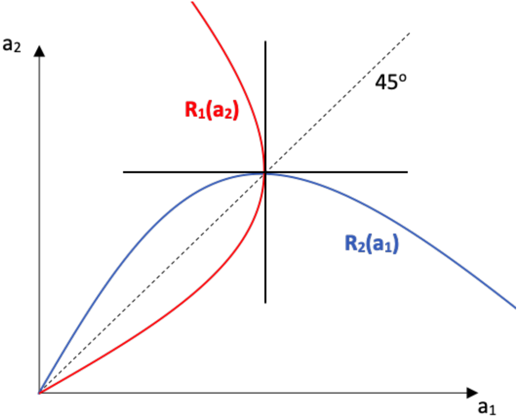
$$P_i(a_i, \mathrm{a_{-i}}) = \frac{a_i}{\sum_{j \in n} a_j}$$

- Nice properties:
  - always sums to 1
  - increasing at a decreasing rate in own efforts
  - interpreted as probability *or* share

Contest Success Functions - some context

- The most basic game involving a contest involves 2 players (labelled 1 & 2) contesting v and optimizing:

$$\max_{a_1} U_1(a_1, a_2) = \max_{a_1} \left\{ \frac{a_1}{a_1 + a_2} v - a_1 \right\}$$



Contest Success Functions - extensive work

- The above specification is quite restrictive; several extensions:
  - heterogeneity
  - number of players & entry
  - risk aversion
  - more general "impact functions" than $f(a_i) = a_i$
- Skaperdas (1996) axiomatized the following function:

$$P_i(a_i, \mathrm{a_{-i}}) = \frac{f(a_i)}{\sum_{j \in n} f(a_j)}$$

  - where $f(a_i) > 0$, for any $a_i > 0$,
  - $f'(a_i) > 0$
  - $f''(a_i) \leq 0$

Contest Success Functions - **Dissipation ratio**

- The *dissipation ratio* D is defined as sum of expenditures over value of the prize:

$$D = \frac{\sum_{i \in n} a_i}{v}$$

Two "puzzles" in the literature:

- Tullock's paradox: In some contexts (lobbying) we observe very low $D << D^{Nash}$
- Overspending: In the lab we typically observe:
  - overspending $D > D^{Nash}$,
  - and even over dissipation $D > 1 > D^{Nash}$

Tullock's paradox

- Milyo, Primo, and Groseclose (2000) studied 15 large corporations in 1998. They gave
  - \$1611 million to charities
  - \$16 million to political campaigns.
- Tullock (1989) gives a personal example from being on board of a firm "manufacturing [...] moderately dangerous product"
  - Estimated benefit of keeping product on the market (forever): \$500K
  - Lobbying expenditures: \$10K

$\Rightarrow$ **How can we explain such minor expenditures?**

Explaining Tullock's paradox

- Risk aversion (Treich 2010)
- Heterogeneity in valuations (Hillman and Riley (1998) and heterogeneity generally speaking
- uncertain number of contestants (Kahana and Klunover 2015)
- Group rent-seeking (Ursprung 1990)

Over-spending and over-dissipation in the lab

- Participants in lab experiments invest systematically more resources in a contest than the Nash prediction (e.g. Sheremeta 2018), why?
  - (overspending); risk lovers (Jindapon and Whaley 2015)
  - (overdissipation) Probabilistic contests may admit mixed strategy equilibria; so overspending in realization (Baye et al. 1999)
  - (overdissipation) Contestants may derive higher utility than the value of the prize (Dickson et al. 2022)
  - (overdissipation) Behavioural biases (Hillman and Long 2019, for review)

Overconfidence as an explanation

What is overconfidence?

- Rationality bias: you hold wrong beliefs about your (relative) traits and skills (Santos-Pinto and Sobel, 2005)
- evidence among e.g. entrepreneurs, judges, CEOs, fund managers, poker and chess players, marathon runners, ...
  - 93% of drivers believe that they are better than average (Barber and Odean, 2001)
  - I'll study last minute for the exam; it's easy stuff
  - WWI: all leaders were convinced the war would be short (few weeks) and victorious
  - "We'll take Kiev in few days" / "The Ukrainian fighter is superior because he fights with his soul"

Overconfidence

Can we rationalize such wrong beliefs?

- Overconfidence is compatible with bayesian updating (Benoît and Dubra, 2011)
- Imagine gambler believing has more than $1/6$ chances of getting a 6 on the throw of a dice
  - Everyone knows the odds
  - And yet, given the randomness of the process any beliefs can be rationalized

Related literature

Overconfidence in contests

- Ando (2004): Overestimation of the valuation v
- Ludwig et al. (2011): Unilateral underestimation of cost of effort

Behavioural biases in contests

- Baharad and Nitzan (2008) and Keskin (2018): Cumulative Prospect Theory

Overconfidence in conflict

- Menuet and Sekeris (2021): War of Attrition game

Overconfidence in a Tullock contest

$\lambda_i \geqslant 1$

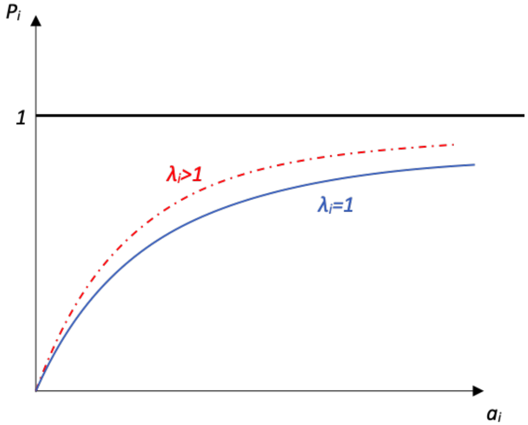
$$P_i(a_i, a_{-i}; \lambda_i) = \begin{cases} \frac{\lambda_i q_i(a_i)}{\lambda_i q_i(a_i) + \sum_{j\neq i} q_j(a_j)} & \text{if } \lambda_i q_i(a_i) + \sum_{j\neq i} q_j(a_j) > 0 \\ 1/n & \text{otherwise} \end{cases},$$

Desirable properties

1. From each player's perspective $\sum_j P_j = 1$
2. Perceived probability of winning for i increases in λ_i
3. Overestimating your ability $\Leftrightarrow$ underestimating the opponents' ability/ies

Overconfidence in a Tullock contest

Overconfidence shifts up expected probability



Symmetric technology, preferences, and overconfidence

- 2 players
- (symmetric) prize: v
- (symmetric) impact function: $q(a_i)$
- $\lambda_i = \lambda$ > 1
- (symmetric) cost function: $c(a_i)$

$$\max_{a_i} \frac{\lambda q(a_i)}{\lambda q(a_i) + q(a_j)} v - c(a_i)$$

Symmetry - results

F.O.C.

$$\frac{\lambda q^{'}(a_i)q(a_j)}{[\lambda q(a_i) + q(a_j)]^2} v - c^{'}(a_i) = 0$$

Lemma 1 *$R_i(a_j)$ is concave in a_j and reaches a maximum for* $q(a_j) = \boxed{\lambda q(a_i)}$

- Apply the IFT to the above FOC and obtain:

> 0

$$sign \left\{ \frac{\partial a_i}{\partial a_j} \right\} = sign \left\{ \lambda q(a_i) - q(a_j) \right\}$$

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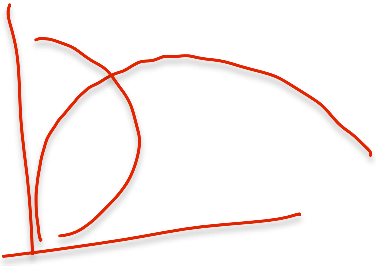
Lemma 1 $R_i(a_j)$ *is concave in* a_j *and reaches a maximum for* $q(a_j) = \lambda q(a_i)$

- Apply the IFT to the above FOC and obtain:
$$sign\left\{\frac{\partial a_i}{\partial a_j}\right\} = sign\left\{\lambda q(a_i) - q(a_j)\right\}$$
- Hence, at symmetric equilibrium, $a_i^* = a_j^* \Rightarrow R_i^{'}(a_j) > 0$

Symmetry - results

Lemma 2 *The contest game with overconfident contestants admits a unique equilibrium*

- Highly intuitive given the concavity of best responses, and the fact both start at $(0, 0)$.



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Lemma 3 $q(a_j) \lesseqgtr \lambda q(a_i) \Rightarrow \frac{\partial R_i}{\partial \lambda} \lesseqgtr 0$

- If rival expected to exert low effort (of if overconfidence is high), reaction function contracts with λ

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- If rival expected to exert low effort (of if overconfidence is high), reaction function contracts with λ
- If rival expected to exert high effort (of if overconfidence is low), reaction function expands with λ

Intuition: when winning odds are high, you can afford spending little effort; otherwise you attempt maintaining not too low odds

Symmetry - results

Remarkable observation: *The maximum value of a player's best response is independent of his/her degree of overconfidence*

Recall the FOC:

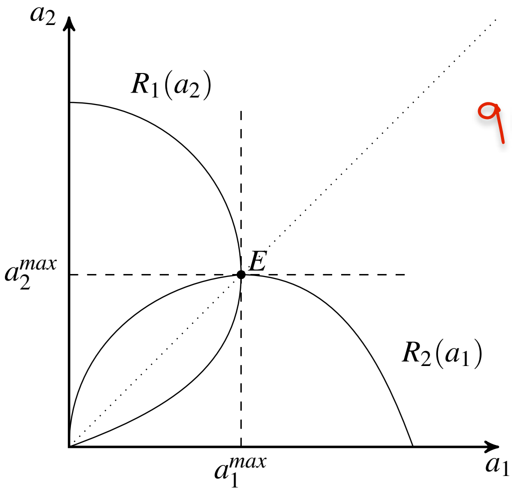
$$\frac{\lambda q^{'}(a_i)q(a_j)}{[\lambda q(a_i)+q(a_j)]^2}v=c^{'}(a_i)$$

Moreover, max of reaction function such that $\lambda q(a_i)=q(a_j)$, hence:

$$\frac{q^{'}(a_i^{max})}{4q(a_i^{max})}v=c^{'}(a_i^{max})$$

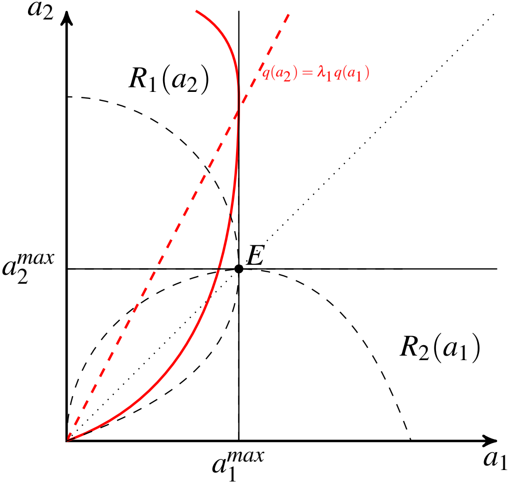
Intuition: No matter how the rival reaches his "contest capacity", the focal player always has the same best response to a given actual strength of the rival

Reaction functions w\o overconfidence



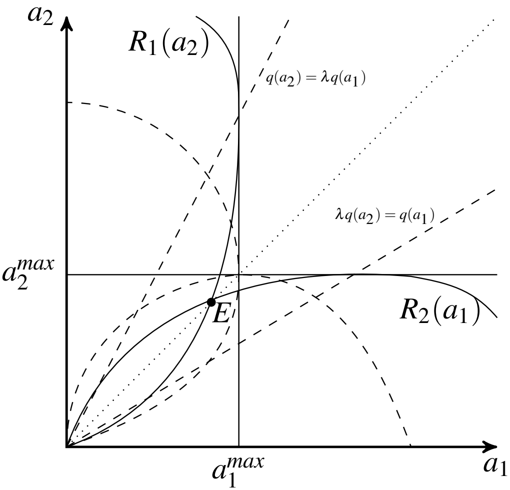
$q(a_i) = q(a_j)$

Reaction functions - overconfidence for player 1



Reaction functions - symmetric equilibrium

$\lambda_1 = \lambda_2$



Extending the reasoning to asymmetric overconfidence

- 2 players
- (symmetric) prize: v
- (symmetric) impact function: $q(a_i)$
- player-specific λ_i
- (symmetric) cost function: $c(a_i)$

$\lambda_1 > \lambda_2 > \lambda$

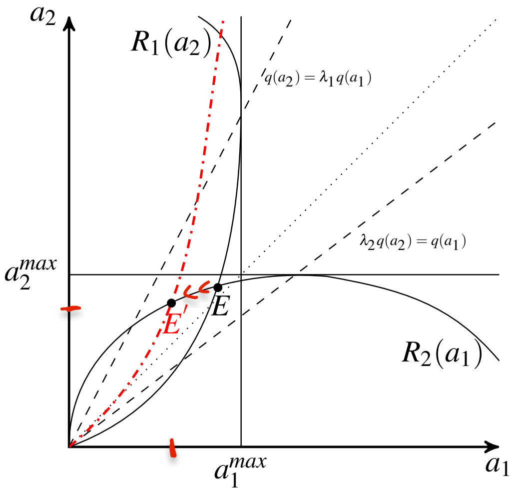
$$\max_{a_i} \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + q(a_j)} v - c(a_i)$$

Extending the reasoning to asymmetric overconfidence

- Previous results extend here (shape of B.R. functions,, effect of overconfidence)

Proposition: *In a two player generalized Tullock contest where both players are overconfident the more overconfident player exerts lower effort. Hence, the more overconfident player is the Nash loser since* $\lambda_i > \lambda_j \Leftrightarrow \boxed{P_i(a_i^*, a_j^*) < 1/2} < P_j(a_i^*, a_j^*)$.

Reaction functions - asymmetric overconfidence



Extending the reasoning to asymmetric overconfidence

- As λ_i increases, contestant i increasingly thinks he is more impactful in the contest, i.e. $\lambda_i q(a_i^*) > q(a_j^*)$

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- Yet for $\lambda_i q(a_i^*) > q(a_j^*)$, the best response function contracts

Extending the reasoning to asymmetric overconfidence

- As λ_i increases, contestant i increasingly thinks he is more impactful in the contest, i.e. $\lambda_i q(a_i^*) > q(a_j^*)$
- Yet for $\lambda_i q(a_i^*) > q(a_j^*)$, the best response function contracts
- At the limit then we obtain:

Corollary: *Both players exert less effort than if both were rational, and as the overconfidence of either player increases, both players' efforts decrease.*

Asymmetry along any dimension (except v)

- 2 players
- (symmetric) prize: $\boxed{v}$
- player-specific impact function: $q_i(a_i)$
- player-specific λ_i
- player-specific cost function: $c_i(a_i)$

$$\max_{a_i} \frac{\lambda_i q_i(a_i)}{\lambda_i q_i(a_i) + q_j(a_j)} v - c_i(a_i)$$

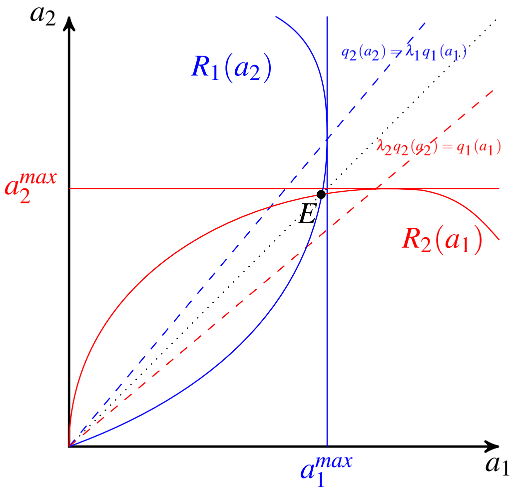
Asymmetry along any dimension (except v)

Lemma: *If the two players are subject to the same overconfidence bias,* $\boxed{a_1^{max}} > a_2^{max} \Leftrightarrow a_1^* > a_2^*$.

$\Rightarrow$ Highly intuitive; if a player's "contest efficiency" increases or cost structure improves, he can only improve his winning odds

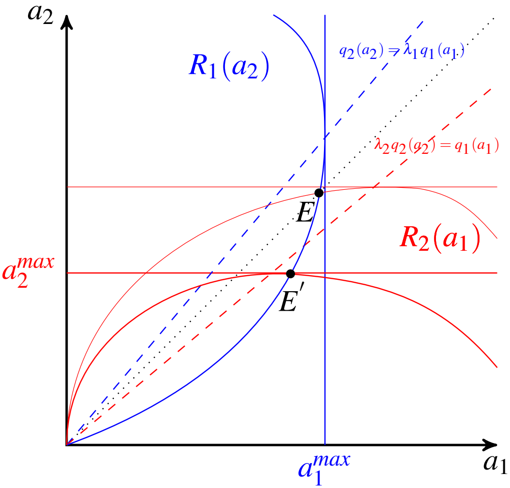
Reaction functions - asymmetric players

Symmetric overconfident players



Reaction functions - asymmetric players

Player 2 less "efficient" [deterioration of cost technology]



Asymmetry along any dimension (except v)

Lemma: *If* $a_1^{max} > a_2^{max}$ *and* $\lambda_2 \geq \lambda_1 > 1$, *then* $a_1^* > a_2^*$.

- We've shown that for symmetric technology, more overconfident produces lower effort
- We've also shown that for symmetric overconfidence, less efficient produces lower effort

$\Rightarrow$ Both "forces" (overconfidence & technology) push in the same direction

Asymmetry along any dimension (except v)

Proposition: *For any a_1^{max}, a_2^{max} and λ_2, there always exist a value $\tilde{\lambda}_1$ such that if $\lambda_1 > \tilde{\lambda}_1$, then $q_1(a_1^*) < q_2(a_2^*)$.*

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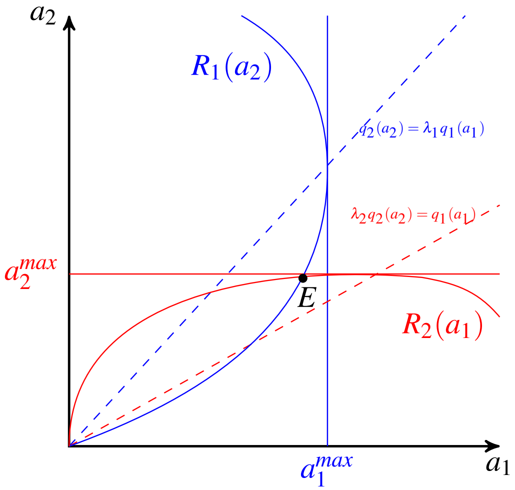
Corollary: *If $\lambda_i \to \infty$, for any $i \in \{1, 2\}$, $a_1^* \to 0$ and $a_2^* \to 0$.*

- Increases in overconfidence lead (beyond some level) to systematic contractions of the best response
- The focal player reduces his/her effort
- Strategic reaction of rival [str. complements] implies both reduce effort

Reaction functions - asymmetric players

Increase in λ_1



Reaction functions - asymmetric players

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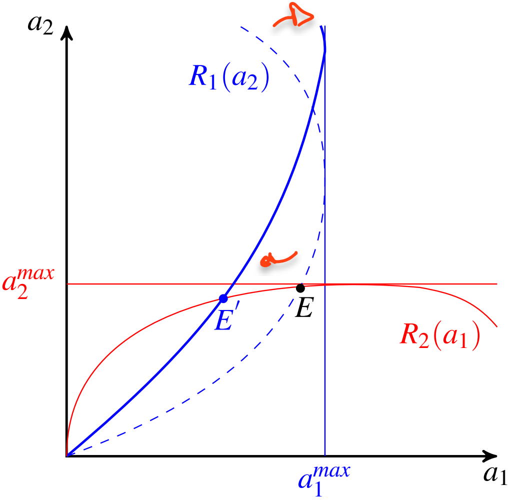

Reaction functions - asymmetric players

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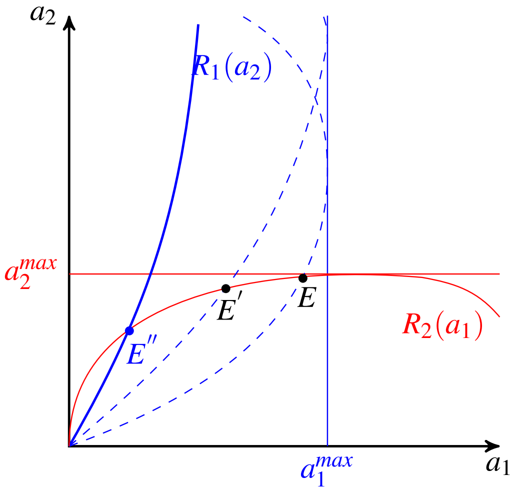


Asymmetry along any dimension (except v)

Proposition: *An increase in player 1's overconfidence implies that*

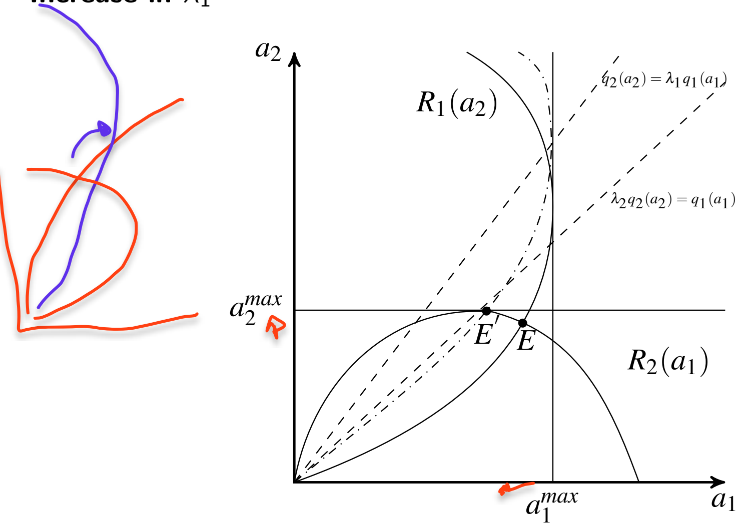
$$\begin{cases} \text{if } \lambda_1 q_1(a_1^*) > q_2(a_2^*) \text{ and } \lambda_2 q_2(a_2^*) > q_1(a_1^*) \text{ then } \partial a_1^*/\partial\lambda_1 < 0 \text{ and } \partial a_2^*/\partial\lambda \\ \text{if } \lambda_2 q_2(a_2^*) < q_1(a_1^*) \text{ then } \partial a_1^*/\partial\lambda_1 < 0 \text{ and } \partial a_2^*/\partial\lambda_1 > 0 \\ \text{otherwise, if } \lambda_1 q_1(a_1^*) < q_2(a_2^*) \text{ then } \partial a_1^*/\partial\lambda_1 > 0 \text{ and } \partial a_2^*/\partial\lambda_1 > 0 \end{cases}$$

Inuition:

- If str. comp. for both, contraction of one B.R. $\Rightarrow$ reduction of efforts
- If str. subst. for $P2$, contraction of R_1 $\Rightarrow \searrow a_1^*, \nearrow a_2^*$
- If str. subst. for $P1$, contraction of R_1 $\Rightarrow$ increase of efforts

Strategic substitutability for Player 2

Increase in λ_1



Extension to n players

- n players
- (symmetric) prize: v
- (symmetric) impact function: $q(a_i)$
- (symmetric) λ
- (symmetric) cost function: $c_(a_i)$

$$\max_{a_i} \frac{\lambda q(a_i)}{\lambda q(a_i) + q(a_j)} v - c(a_i)$$

Extension to n players

Proposition: *Individual and aggregate efforts decrease (increase) with overconfidence if* $\lambda > (<) n - 1$.

Inuition:

- If λ is low, at symmetric equilibrium his B.R. is downward-slopping [str. subst.]
- Increasing $\lambda \Rightarrow$ expansion of B.R.; all players increase efforts

Extension to n players

Proposition: *Individual and aggregate efforts decrease (increase) with overconfidence if* $\lambda > (<) n - 1$.

Inuition:

- If λ is low, at symmetric equilibrium his B.R. is downward-slopping [str. subst.]
- Increasing $\lambda \Rightarrow$ expansion of B.R.; all players increase efforts
- With low λ B.R. is downward-slopping because players facing a lot of "aggregate contest effort" are pushed to put a lot of effort in the contest.

Extension to n players

Corollary: *With n symmetric players, the maximal rent dissipation is always attained when $\lambda = n - 1$. There always exists a finite n^D such that over-dissipation can be observed at equilibrium for $n > n^D$.*

Inuition:

- For any aggregate effort of rivals there is the same a_i^{max}.
- Max reached when $\lambda_i q(a_i) = \text{agg. effective effort of rivals}$
- If symmetric game: $\lambda q(a) = (n-1)q(a)$
- If that maximal effort is a^{max}, with n players, aggregate effort is na^{max}
- $n \to \infty \Rightarrow \sum na_j^{max} \to \infty$.

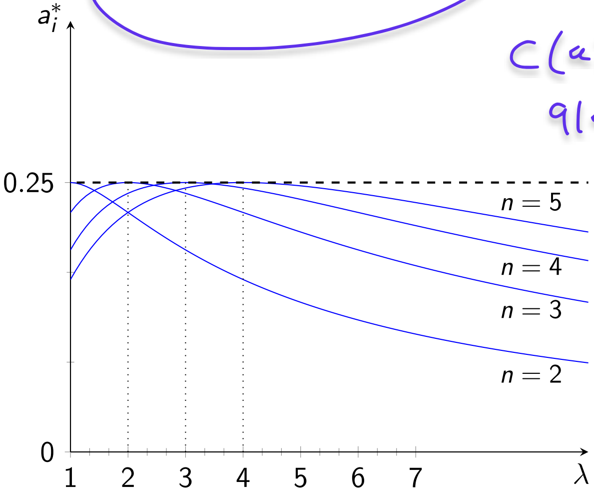
Maximal effort with n players

Consider simplest setup: $U_i = \frac{\lambda a_i}{\lambda a_i + \sum_{j \neq i} a_j} - a_i$.

$V = 1$

$c(a_i) = a_i$

$q(a_i) = a_i$



Implications for the dissipation ratio

Tullock paradox explained

- Tullock's paradox: $D = \frac{\sum a_j}{v}$ is abnormally low in some contexts
- We show that high degrees of overconfidence will push *all* players' efforts to 0 [for any n]
  - Tullock's paradox highlighted in contexts of lobbying (Tullock 1980)
  - Lyons et al. (2020): lobbyists are overconfident!

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Overdissipation explained

- Lab experiments on contests show that over-spending and over-dissipation are rather the rule
- We uncover that there always exist overconfidence parameters such that overdissipation will be observed with numerous enough players

Explaining behaviour differences btw Men *vs* Women

- Men seem to be more overconfident than women (e.g. Niederle and Vesterlund, 2007),
- Recent experimental on contests (Mago and Razzolini, 2019) **all pay auctions** (Chen et al., 2015) reveals that
  - Women bid systematically more than men
  - bids in pairs such that $WW > WM > MW$
- If men are indeed more overconfident than women, then our theory fully explains these differences in behaviour

Conclusion

- We introduce overconfidence in Tullock contests
- We characterize the equilibria in very general setups
- Overconfidence may both increase or decrease players' efforts depending on the circumstances
- Provides explanation for:
  - Tullock paradox
  - Overdissipation
- We propose a very flexible (graphically-inspired) tool for working on contests