Self-enforcing climate coalitions with farsighted countries: integrated analysis of heterogeneous countries

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Our Research Question

- We model the negotiations of countries to form self-enforcing climate coalitions to reduce emissions.
  - Signatories commit to maximising payoffs of all coalition members when choosing their emission reduction levels.
  - Non-signatories maximise their individual payoff
- Countries/ policymakers are farsighted: rationally predict the overall coalition structure
- We allow for heterogeneity across countries and a dynamic game.
- In this way, we bring together two strands of literature: standard IAM and Coalition Formation Theory

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Our Contribution

- We offer a simple algorithm to **fully characterise** the equilibrium number of climate coalitions and their number of signatories and **closed form** solutions for the equilibrium strategies and payoffs.
- The algorithm relies on Tribonacci numbers $\{1, 2, 4, 7, 13, 24, ...\}$
- The problem of coalition formation of heterogeneous countries can be decoupled:
  - number coalitions and number of signatories
  - composition of signatories in each coalition (in progress)
- The policy message:
  - allow multiple climate coalitions
  - large coalitions can be stable: no small coalition paradox
  - efficiency loss might not be that high even when the grand coalition is not stable

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Literature Review

- **Coalition Formation: two strands of literature**
  - **Cooperative Game Theory**: Which transfer scheme or bargaining rule allows sustaining the grand coaltion?
    - solution concepts: Core, Sharpley Value, Nash Bargaining Solution, Stable Set
    - binding agreements without the question of how to reach such an agreement
    - Scarf(1971), Tulkens(1979), Chandler/Tulkens(1991,1992) and many others
  - **Noncooperative Game Theory:** Which coalition structure can be sustained as an equilibrium for a given transfer scheme or bargaining rule?
    - solution concept: internal-external stability (cartel stability)
    - non binding agreements hence negotiations are a noncooperative process
    - small coalition paradox $m^* \leq 3$ unless some remedy is employed: Stackelberg and particular functions
    - Vast literature: Carraro/Siniscalco (1991,1993), Barrett (1991, 1992, 1994), Diamantoudi and Sartzetakis(2006)

- **Coalition Formation: two strands of literature**
  - Critical assumption about coalition formation: How do the rest of the countries/ coalitions react when a country/coalition deviates?
    - cooperative game theory: the whole coalition structure collapses (depending on the particular concept)→punishment not credible, hurts the punishers as well
    - noncooperative game theory: other coalitions do not react to a potential deviation other by adjusting their policies to the size of the remaining coalition
  - More Realistic Approach: **Farsightedness**
    - no a priory assumption about what the remaining coalitions will do
    - a coalition must predict the whole coalition structure: a deviation may trigger further deviations
    - Chatterjee et al. (1993); Chwe(1994); Bloch (1996); Ray and Vohra (1999), Farsightedness + public goods: Ray and Vohra (2001); Diamantoudi and Sartzetakis (2006, 2018); A De Zeeuw(2008)

Literature Review

- **IAMs**
  Nordhaus (1993); Nordhaus and Yang (1996); Nordhaus (2014)
  Closed form solution: Golosov et al. (2014); Hassler and Krusell (2012,); Van den Bremer and Van der Ploeg (2021)
- **Climate coalitions + IAMs**
  Cartel Stability and Numerical Approach: Lessmann et al.(2009, 2015); Bosetti et al (2013)
- **What we do:**

  We combine Ray and Vohra (2001) and a **multi-country** simplified version of Golosov et al. (2014). Our model

  - is dynamic: infinite horizon climate model(game) after the coalition formation stage
  - incorporates heterogeneous countries(players)

Setup

- N countries, each country is indicated by i and $I = \{1, 2, ..., N\}$
- Time is discrete and infinite, $t = 0, 1, 2, ...$
- Each country has a planner who is player in a coalition formation game(climate negotiations): he makes proposals to coalitions and respond to proposals made to him following a negotiation protocol (to be defined)
- The planner can implement any desired policy in the decentralized economy e.g using taxes.

Timing of the game

Two-stage climate coalition formation

- Beginning of period t: membership stage
- From period t onwards : action(compliance) stage (no renegotiation-irreversible agreements)
  - cooperative decision on emissions reduction (SCC) within each coalition
  - but cross-coalition interaction is non-cooperative
  - country-level decisions on the implementation of the agreed policies (taxation)
- At the end of each period t, emissions are observed and payoffs are realised
- Solve by **backwards induction**: we start with the action stage and move to the membership stage

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The Economy (Golosov et al. (2014))

Representative Household and Production Sectors in country i

- consumers derive utility from the consumption of the final good where $\beta \in (0,1)$ is the discount factor: $\sum_{t=0}^{\infty} \beta^t ln(C_{it})$
- **Energy sector:** $R_{it+1} = R_{it} - E_{it}$ (1)
- **Final output:** $Y_{it} = exp(-\gamma T_t) A_i K_{it}^{1-\nu} E_{it}^{\nu}$

  where R_{it} is the stock of fossil fuel, E_{it} is energy use(and emissions), Y_{it} is final output, K_{it} is capital stock, T_t is global temperature, γ is the damage coefficient, A_i is TFP, ν is output elasticity of energy
- countries are **heterogeneous** with respect to K_{i0}, R_{i0}, A_i
- full capital depreciation and no trade
- Market clearing: fossil fuel eq.(1) and final good

$$C_{it} + K_{it+1} = Y_{it}$$

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- **global temperature(change):**

$$T_t = T_0 + \xi S_t$$

where T_0 is the pre-industrial temperature, S_t is the stock of cumulative emissions of CO_2 and ξ is the transient climate response

- **cumulative emissions:**

$$S_t = S_0 + \sum_{i=1}^{N} \sum_{s=0}^{t} E_{it-s}$$

where S_0 is the pre-industrial stock of cum. emissions.

The Economy

Climate Dynamics (Allen et al.(2009),Matthews et al. (2009))

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Two-Stage climate coalition formation

Second Stage: action stage

- Dynamic Game between different coalitions (also singletons): coalitions act non cooperatively against other coalitions (and cooperatively within)
- Strategies of country $i \in M$:

  $\{E_{it}(M,\Pi), C_{it}(M,\Pi), K_{it+1}(M,\Pi), R_{it+1}(M,\Pi)\}$ from $t = 0$ to infinity given a coalition structure Π to be explained later
- Pure strategy Markov Perfect equilibrium
  $\rightarrow$ **current state**: the formed coalitions (if any); identity (and number) of those negotiating (if any); proposal (if ongoing or signed); S_t; K_{it}; and R_{it}.
- once signed agreements are binding and irreversible: no point in history dependent strategies/punishments

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The problem of the planner of country i in a coalition M with m members is to maximise

$$\sum_{i \in M} \sum_{t=0}^{\infty} lnC_{it}$$

subject to the resource and feasibility constraints.

- the planner chooses the optimal level of emissions taking into account the effect her emissions on **other countries**
- **but** chooses C_{it}, K_{it+1} and R_{it+1} independently
- the FOC's give us the following results

Proposition1

- $C_{it}(M,\Pi) = (1-s)Y_{it}(M,\Pi)$ and $K_{it+1}(M,\Pi) = sY_{it}(M,\Pi)$
- optimal emissions of $i \in M$

$$E_{it}(m) = \nu / [\mu_{it}(1-s) + \hat{\Lambda}(m)]$$

where s is the savings rate, μ_{it} is the per unit scarcity rent and $\hat{\Lambda} = \frac{\xi\gamma m}{1-\beta}$ is the per-unit SCC.

- emission strategies are dominant against what other coalitions choose
- SCC depends **only** on exogenous parameters and the size of the coalition FOC
- optimal emissions can differ among members of the same coalition but the SCC is the **same** for all: this is what coalitions negotiate for

Some Preliminaries

We assume:

- Open membership: no clubs, any country is allowed to negotiate its membership and no country is forced in
- Costless to sign
- Binding: once signed, there is no compliance issue in the action stage
- Irreversible: once signed, countries cannot renegotiate their membership
- No delay equilibria: countries make acceptable offers
- Farsightedness

Some Preliminaries

- **Coalition structure** is a partition of set I into coalitions,
$$\Pi = \{M_1, M_2, ..., M_k\}$$
- m is the number of signatories of M
- **Numerical coalition structure**, $\pi = \{m_1, m_2, ..., m_k\}$
- Coalition formation as a non-cooperative bargaining game
- Coalitions are formed sequentially following the **negotiation protocol**: Deterministic order of the initial proposers (P) and respondents (R) + unanimity rule + first rejector is the next P
- Strategy of P is a proposal: identity of members of M + emission reduct. plan(or SCC) + payoffs of members of M
- Strategy of R: accept or reject

Two-Stage climate coalition formation

First Stage: Membership

- Farsightedness: countries are required to rationally predict the entire coalition structure when considering a deviation, no a priori assumption about the coalitions' behaviour- far more realistic!
    - internal-external stability(cartel stability): upon deviation, the rest of the coalitions remain intact
    - core stability: upon deviation, the rest of the coalitions disintegrate
- the equilibrium coalition structure Π^* is immune to unilateral and multilateral deviations by the deviating group and all the active players in the negotiation room
- So how do we find Π^* ?

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- Second stage of the game (action stage): **Optimum Value function** of $i \in M$ is $V_i(S_t, K_{it}, R_{it}, M, \Pi) \rightarrow$ a country considers a coalition M with the purpose of maximizing its value function value function
- Because of farsightedness, the value function depends not only on the coalition M but also on the whole coalition structure in which M is going to be embedded
- The equilibrium Π^* can be found recursively:
  - if $N = 2$, then $\Pi^*=$?, if $N = 3$, then $\Pi^* =$? ....
  - each stage of the recursion informs the next one
- Extra demanding with heterogeneous countries but not in our case!

Symmetric case: Two Simplifications

- $V_i(S_t, K_{it}, R_{it}, M, \Pi)$ simplifies to $V_i(S_t, K_{it}, R_{it}, \boldsymbol{m}, \boldsymbol{\pi})$: only the size and number of coalitions matters
- We check for which group of countries, a grand coalition forms in equilibrium $\rightarrow \mathcal{T}^*$ is the set of number of countries for which a grand coalition forms in equilibrium
- $D(N) = \{m_1, m_2, ..., m_k\}$ is a decomposition of N, such that m_k is the largest integer in $\mathcal{T}^*$ that is strictly smaller than N. Then any other element is the largest integer that is not greater than $N - \sum_{j=i+1}^{k} m_j$
- Ray and Vohra (1999,2001) show that under low bargaining frictions ($\sigma \rightarrow 1$), $D(N)$ coincides with with the numerical equilibrium structure π^*

First Stage: Membership

Symmetric countries

- **How do we construct $\mathcal{T}^*$?**
- It is easy to show that the first 2 elements of $\mathcal{T}^*$ are 1 and 2 so $\mathcal{T}^* = \{1, 2\}$ Example
- Next, we consider $N = 3$. $\pi = \{1, 1, 1\}$, $\{1, 2\}$, or $\{3\}$ forms in equilibrium?
    - we always have to check whether a country has an incentive to deviate from the grand coalition: Which possible coalitions do we actually have to check?
    - $D(3) = \{1, 2\} \rightarrow$ the only deviation we have to check
    - Why? Only the coalitions in the decomposition are farsighted stable.
    - $\lim_{\beta \rightarrow 1} V_i(\{1\}, \{1, 2\}) > V_i(\{3\}$ so for $N = 3$, $\pi^* = \{1, 2\}$: the grand coalition does not form and $\{3\} \notin \mathcal{T}^*$

- we do the same process for $N = 4$, $N = 5$, ... and check whether a grand coalition forms. If it forms, then we add N to T^*
- this can be very demanding. In our model, it turns out that there is an easy way to generate this set

Lemma 1

Let $D(N) = \{m_1, m_2, ..., m_k\}$, such that m_1 is the smallest element of$D(N)$. If $\beta \to 1$, then, a grand coalition forms in equilibrium if

$$\frac{N}{m_1} < e^{k-1}$$

First Stage: Membership

Symmetric countries

- Using Lemma 1, we show our main result

  Proposition 2

  If $\beta \to 1$, for any number of countries N, a grand coalition occurs in equilibrium if N is an element of

  $$\mathcal{T}^* = \{1, 2, 4, 7, 13, 24, 44, 81, 149, 274, ...\}$$

  which is the Tribonacci sequence.
- **So how do we find the equilibrium numerical coalition structure?**
  - if $N \in \mathcal{T}^*$, then $\pi^* = \{N\}$
  - if $N \notin \mathcal{T}^*$, then $\pi^* = D(N)$
- The equilibrium number of signatories, m^*, in any coalition is always a Tribonacci number.
- Unlike Ray and Vohra(1999,2001), there is **no need** for any recursion as the Tribonacci sequence is a known sequence

First Stage: Membership

Symmetric countries

Our algorithm

$$\mathcal{T}^* = \{1, 2, 4, 7, 13, 24, 44, 81, 149, 274, ...\}$$

Example

If $N = 195$, there will be **three** coalitions with the following sizes $\pi^* = \{2, 44, 149\}$.

- Cartel stability: $m^* = 3$
- Very different than the cartel stability predictions: average SCC is 120 times larger!
- Large coalitions are stable $\rightarrow$ efficiency losses might not be that high even when the grand coalition is not stable

First Stage: Membership

Asymmetric countries

- For $\beta \to 1$, any heterogeneity related to K_{i0}, A_{i0}, R_{i0} and μ_{it} vanishes!
- What does this imply?

  Decoupling result

  - algorithm for **symmetric countries** applies in the case of **asymmetric countries** too
  - focus on equilibrium numerical coalition structure but the identity of members is important for questions of **efficiency**

Efficiency

Which coalitions achieve the highest reduction in emissions?

- When the grand coalition is not stable (fully efficient outcome), equilibrium payoffs and global temperature depend on identity of the proposer and the composition of countries across coalitions.
- For $0 < \beta < 1$, global emissions are lower when the high-emitting countries are in larger coalitions
- BUT, for $\beta \to 1$, the case for which we have established the equilibrium, global emissions become asymptotically independent of the identity of the coalitions members

Conclusions

- Capturing various aspects of climate negotiations: farsightedness + heterogeneity + economic growth + general equilibrium + climate dynamics
- Decoupling result: characterising Π^* independent of composition
- A simple algorithm to fully characterise Π^* in climate coalition + IAM
- Climate coalitions with Tribonacci number of signatories in equilibrium
- Suggesting a more ambitious architecture for climate treaties
- Next Steps
  - Relax assumptions: $\beta \to 1$
  - Numerical Analysis

- **Problem of a planner within coalition M (coalition level):**
  F.O.C w.r.t. E_{it}:

$$\frac{\nu Y_{it}}{E_{it}(m)} = \mu_{it} C_{it} + \hat{\Lambda}(m) Y_{it}$$

- **Problem of a planner within each country: (country level)**
  F.O.C w.r.t. C_{it} and K_{it+1}:

$$\frac{s_{it}}{1 - s_{it}} = \beta \frac{1}{1 - s_{it+1}} (1 - \nu)$$

$$\Rightarrow s_{it} = s = \beta(1 - \nu), \quad \text{for all } t \text{ and } i.$$

  F.O.C w.r.t. R_{it+1}:

$$\mu_{it} = \beta \mu_{it+1}$$

back

Value function

$$V_i(S_t, K_{it}, \mu_{it}, M, \mathbb{M}) = ln(C_{it}(M, \mathbb{M})) + \beta ln(C_{it+1}(M, \mathbb{M})) + ...$$
$$= \frac{(1-\nu)ln(K_{it}) + H_1 + H_2 + H_3}{1-s}$$

where

$$H_1 \equiv \frac{sln(s) - sln(1-s) + ln(A_i) - \gamma T_0}{1-\beta}$$

$$H_2 \equiv -\gamma\xi[S_t + \beta S_{t+1} + \beta^2 S_{t+2} + ...]$$

and

$$H_3 \equiv \nu[ln(E_{it}(m)) + \beta ln(E_{it+1}(m)) + \beta^2 ln(E_{it+2}(m)) + ...]$$

back

Example: $N = 2$

For the case $N = 2$, the problem reduces to whether $\{1, 1\}$ or $\{2\}$ forms. It can be shown that this depends on the sign of

$$\begin{aligned} & V_i(1, \{1,1\}) - V_i(\{2\}) = \\ & \frac{1}{1-\beta(1-\nu)} \left\{ \nu \{ ln \left(\frac{E_{it}(1)}{E_{it}(2)} \right) + \beta ln \left(\frac{E_{it+1}(1)}{E_{it+1}(2)} \right) + ... \} \right. \\ & \left. - \frac{2\gamma\xi}{1-\beta} \{ [E_{it}(1) - E_{it}(2)] + \beta [E_{it+1}(1) - E_{it+1}(2)] + ... \} \right\} \end{aligned} \tag{1}$$

- the 2^{nd} line is the discounted infinite sum of a ratio of the benefit of emitting in a singleton coalition relative to the benefit of emitting in a grand coalition, and is positive.
- the 3^{rd} line is the discounted infinite sum of the losses resulting from the damages of emitting in a coalition structure of singleton relative to the damages of emitting in a grand coalition, and is negative.
- $\lim_{\beta \to 1}(V_i(1, \{1,1\}) - V_i(\{2\})) < 0$ so the grand coalition forms

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