Accounting for technology heterogeneity in the dynamic stochastic frontier model

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Motivation for efficiency measurement

- In neoclassical economics, the theory of the firm states that producers successfully optimize their production processes

- However, irrespective of the firms' objectives (cost minimization or profit maximization), this assumption rarely holds in practice

- This can be due to governmental regulation, poor management practices or even unforeseen events that are outside the control of producers

- Therefore, empirical studies have focused on quantifying deviations of observed from optimal production

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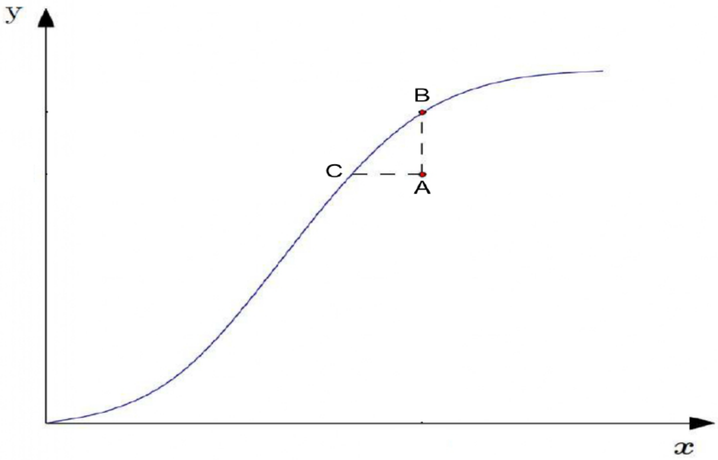
Graphical representation of efficiency measurement



Mathematical formulation for parametric efficiency measurement

Mathematical formulation of production frontier

$$y = f(x) + v - u$$

- $f(x) \rightarrow$ production frontier
- $v \rightarrow$ stochastic disturbance
- $u \rightarrow$ inefficiency

Motivation for dynamic efficiency measurement

- Efficiency measurement has traditionally been based on a static viewpoint of the firm
- However, firms' production decisions are intertemporal in nature
- This is because present choices can affect both today's outcomes and future production possibilities
- This requires the move to the dynamic analog of static efficiency measurement

Aproaches for dynamic efficiency measurement

- The literature on parametric dynamic efficiency measurement is dictated by two approaches
- The first approach is based on reduced-form dynamic efficiency models
- These reduced-form models assume that high adjustment costs result in slugish adjustment of quasi-fixed factors of production, which makes a firm's inefficiency to persist over the short-run. This inefficiency persistence is modelled by allowing the inefficiency component to follow a first-order autoregressive process (AR(1))
- The second approach is based on structural parametric dynamic efficiency models, which specify gross investments in the frontier, which reflect the differences in the values of quasi-fixed assets between two consecutive time periods

Motivation for accounting for technology heterogeneity

- Investments in quasi-fixed assets typically involve adjustments costs which can be either pecuniary (i.e. credit constraints) or related to learning
- In practice though, firms tend to exhibit variations in gross investments given the heterogeneity in their financial conditions and/or in the managers' cognitive capacities
- This in turn implies that firms may face different production potentials
- Hence, firms should be benchmarked not against a common frontier but against their individual frontiers

Objective & contribution

- The objective of this paper is to account for technology heterogeneity whilst measuring firm dynamic efficiency and show how the efficiency scores can be distorted if technology heterogeneity is ignored
- This exercise adds to the existing literature that has completely disregarded the exploration of technology heterogeneity in the structural parametric dynamic efficiency framework
- The study's objective is achieved by specifying a structural parametric dynamic SFA model that includes firms' gross investments whilst allowing for the frontier coefficients to be random, thus assuming different frontiers across firms
- Furthermore, a structural parametric dynamic SFA model with fixed coefficients is estimated and the resulting efficiency scores are compared with those obtained from the random coefficients model

Enhanced dynamic hyperbolic distance function

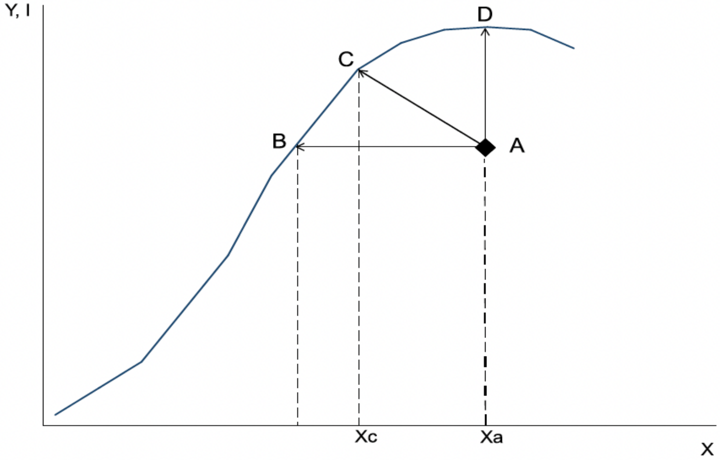
- An enhanced dynamic hyperbolic distance function is used to measure firm dynamic efficiency
- The enhanced dynamic hyperbolic distance function assumes that, unlike fixed and quasi-fixed inputs, outputs, gross investments and variable inputs are decision variables
- I denote the output vector as $\mathrm{y} \in R_+^K$, the vector of gross investments as $\mathrm{I} \in R_+^L$, the vector of variable inputs as $\mathrm{x} \in R_+^Q$, the quasi-fixed inputs vector as $\mathrm{k} \in R_+^L$, and the vector of fixed inputs as $\mathrm{f} \in R_+^Z$

Enhanced dynamic hyperbolic distance function

$$D_{EH}(\mathrm{y}, \mathrm{I}, \mathrm{x}, \mathrm{k}, \mathrm{f}) = min\left\{\theta > 0 : (\mathrm{y}\theta^{-1}, \mathrm{I}\theta^{-1}, \mathrm{x}\theta, \mathrm{k}, \mathrm{f}) \in T\right\}$$

- θ is a positive scalar that allows for the simultaneous equiproportionate expansion of outputs and gross investments and contraction of variable inputs to reach the boundary of the production possibilities set, T

Graphical representation of dynamic efficiency measurement



Estimable form

- Estimation of the enhanced dynamic hyperbolic distance function is based on the almost homogeneity property:

$$D_{EH}(\lambda \mathrm{y}, \lambda \mathrm{I}, \lambda^{-1}\mathrm{x}, \mathrm{k}, \mathrm{f}) = \lambda D_{EH}(\mathrm{y}, \mathrm{I}, \mathrm{x}, \mathrm{k}, \mathrm{f}), \lambda > 0$$

- The almost homogeneity property states that if outputs and gross investments are increased by a given proportion and variable inputs are decreased by the same proportion, then the distance function will increase by the same proportion
- By setting $\lambda = \frac{1}{y_K}$ we have:

$$D_{EH}\left(\frac{\mathrm{y}}{y_K}, \frac{\mathrm{I}}{y_K}, \mathrm{x}y_K, \mathrm{k}, \mathrm{f}\right) = \frac{1}{y_K} D_{EH}(\mathrm{y}, \mathrm{I}, \mathrm{x}, \mathrm{k}, \mathrm{f})$$

- Replacing $D_{EH}(\mathrm{y}, \mathrm{I}, \mathrm{x}, \mathrm{k}, \mathrm{f})$ with EHE, taking logs , rearranging, and appending a noise term, v, we get:

$$-\log y_K = \log D_{EH}\left(\frac{\mathrm{y}}{y_K}, \frac{\mathrm{I}}{y_K}, \mathrm{x}y_K, \mathrm{k}, \mathrm{f}\right) + v - \log EHE$$

Parametric specification

- The parametric specification of the enhanced dynamic hyperbolic distance function for firm i in time t is:

$$-\log y_{it}^{K} = \log f\left(\alpha_i, \frac{y_{it}^{k}}{y_{it}^{K}}; \gamma_{ik}, \frac{I_{it}^{l}}{y_{it}^{K}}; \delta_{il}, x_{it}^{q} y_{it}^{K}; \zeta_{ir}, k_{it}^{l}; \eta_{il}, f_{it}^{z}; \lambda_{iz}\right) + \mathrm{v}_{it} + u_{it}$$

- α_i is a firm-specific constant term, and $u_{it} \equiv -\log EHE$

- All parameters in the above equation are random as they have an i subscript, thus allowing for technology heterogeneity across firms, and therefore for individual frontiers

Estimation

- Three choices need to be made so as to estimate the model: 1) distributional assumptions for the two-sided error term v_{it} and the one-sided inefficiency component u_{it}, 2) distributional assumptions for the random parameters, and 3) specification of the functional form f

1. Error components: $v_{it} \sim \mathcal{N}(0, 1/\tau)$ and $u_{it} \sim \mathcal{N}^{+}(0, 1/\phi)$.
2. Random parameters: $\boldsymbol{\beta}_i \sim \mathcal{N}(\bar{\boldsymbol{\beta}}, \Omega^{-1})$,
3. Functional form f can be Cobb-Douglas, semi-translog, or fully translog; I let the data decide

Estimation

- Estimation is carried out in a Bayesian framework
- The complete-data likelihood is specified according to the distributional assumptions made for the error terms
- Non-informative priors are imposed on the parameters to be estimated
- The posterior is obtained by multiplying the complete-data likelihood and the priors
- Markov chain monte carlo simulation (MCMC) combined with data augmentation is used to draw samples from the posterior

Model comparison

- Model comparison is used for the specification of the model's functional form f, and to infer which specification of the model's parameters (fixed versus random) fits the data best
- Regarding the functional form, the model is estimated using a Cobb-Douglas form, a translog in investments form, a translog in investments and output form, a translog in investments, output and variable inputs form, and a fully translog form
- In terms of the specification of the model's parameters, the model is once estimated with random parameters and once with fixed parameters
- Bayes factors are used for comparing the above models
- A Bayesian alternative to a t test is used (bayesian estimation supersedes the t test) to test for differences in the efficiency scores

Data & empirical specification

- Farm-level data from Dutch farm accountancy data network (FADN) consisting of 1736 observations:
  - Dutch dairy farms specialized in milk production
  - Period covered: 2009 to 2016
- Specified variables:
  - Two outputs: 1) milk and milk products and 2) meat & other
  - Gross investments in capital
  - Two variable inputs: 1) intermediate inputs and 2) purchased feed
  - One quasi-fixed input: capital (buildings & machinery)
  - Three fixed inputs: 1) labor, 2) land, and 3) animals
  - Time trend
- Data are deflated and normalized by their geometric means

Marginal log-likelihoods, prior and posterior probabilities for each functional form

Functional form	**Marginal log-lik.**	**Prior prob.**	**Posterior prob.**
Cobb-Douglas	1958.750	0.200	0.000
Translog in inv.	1751.670	0.200	0.000
Translog in inv. & outp.	1720.250	0.200	0.000
Translog in inv., outp. & var. inp.	1993.690	0.200	1.000
Fully translog	$-\infty$	0.200	0.000

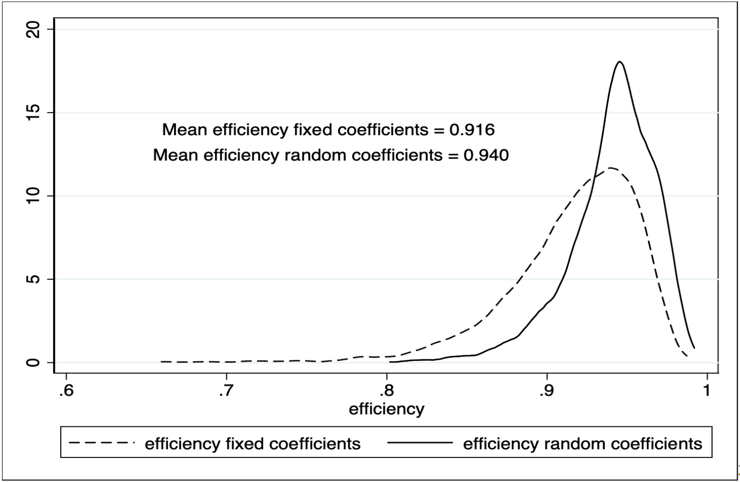
Parameter estimates for first-order terms

	Random coefficients			Fixed coefficients		
Variable	**Mean**	**SD**	**90% CI**	**Mean**	**SD**	**90% CI**
cons.	-0.083	0.005	**[-0.091, -0.076]**	-0.107	0.005	**[-0.116, -0.099]**
log_y2	0.022	0.006	**[0.012, 0.031]**	0.041	0.004	**[0.034, 0.048]**
log_inv.	0.018	0.005	**[0.010, 0.026]**	0.030	0.005	**[0.023, 0.038]**
log_K	-0.037	0.007	**[-0.049, -0.025]**	-0.048	0.006	**[-0.058, -0.038]**
log_L	-0.036	0.013	**[-0.057, -0.016]**	-0.048	0.010	**[-0.064, -0.033]**
log_A	-0.142	0.011	**[-0.161, -0.124]**	-0.148	0.008	**[-0.162, -0.134]**
log_S	-0.025	0.015	**[-0.050, -0.000]**	-0.013	0.010	[-0.030, 0.004]
log_I	-0.118	0.010	**[-0.135, -0.102]**	-0.123	0.008	**[-0.136, -0.110]**
log_F	-0.274	0.010	**[-0.289, -0.257]**	-0.276	0.008	**[-0.290, -0.263]**
trend	-0.001	0.002	[-0.004, 0.002]	0.004	0.001	**[0.002, 0.006]**

Marginal log-likelihoods, prior and posterior probabilities for each model

Model	Marginal log-likelihood	Prior probability	Posterior probability
Random coefficients	1993.690	0.500	1.000
Fixed coefficients	1249.900	0.500	0.000

Efficiencies of fixed and random coefficients models



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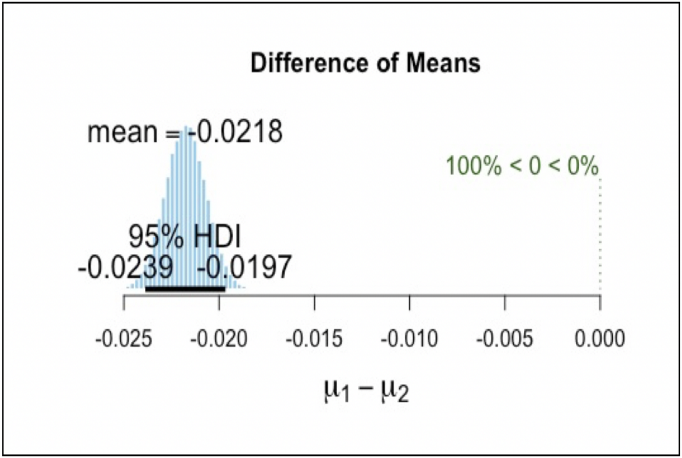
Differences of means from the fixed and the random coefficients models



Synopsis

- A framework that combines the structural parametric dynamic efficiency model with the random coefficients model is presented

- The proposed model is applied to a panel dataset of specialized dairy farms in the Netherlands observed over the period 2009–2016

- The empirical findings suggest that inefficiency is inflated when technology heterogeneity is ignored

- Formal model comparison indicates that the random coefficients model fits the data better than the fixed coefficients model

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