Auctions with a multi-member bidder

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- Examples:
  1. Spectrum auctions;
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- Economic characteristics:
  1. Public good;
  2. Aggregation problem in a strategic bidding setting.

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- Collusion - a cartel is a "bidding team."
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- Group contests - the group/team wins together or loses together. E.g., Kobayashi and Konishi 2021.

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- Team mechanism=bid aggregation rule (A) and cost sharing rule (s).

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- Theorem
  There does not exist a mechanism that leads to an efficient allocation.

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- Not an equilibrium.

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- Theorem

  Suppose that $M \geq 2n$. Then the linear-proportional model has a unique equilibrium. The equilibrium is symmetric: $\beta_1 = \cdots = \beta_n = \beta^{SPA}$, where the bid function β is given by:

$$\beta^{SPA}(\theta) = max\{\theta - a, 0\},$$

  where a is the unique solution to:

$$a = \frac{n-1}{n+1} \cdot \left(\int_a^1 tf(t)dt + aF(a)\right).$$

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- Proposition

  In the linear-proportional model, the equilibrium-expected-utility of a team member with type θ is:

$$\pi^*(\theta) = \frac{1}{2M} \cdot [2\theta - max\{\theta - a, 0\}] \cdot [2a + max\{\theta - a, 0\}].$$

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  - The team size n and type. dist. F only affects the cutoff a.

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  2. $lim_{n\to\infty} a_n = 1$.
  3. $(\frac{n-1}{n+1})\mathbb{E}(\theta) \leq a_n$ *for all* $n \geq 1$.

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  2. *$lim_{n\to\infty} a_n = 1$.*
  3. *$(\frac{n-1}{n+1})\mathbb{E}(\theta) \leq a_n$ for all $n \geq 1$.*

- Proposition

  Consider two copies of the model—one in which the type distribution is F and one in which it is G, where F first-order stochastically dominates G. Let a^z be the cutoff corresponding to $z \in \{F, G\}$. Then $a^F \geq a^G$.

- Proposition

  Consider the linear-proportional model under the second-price format, and let the regular bidder's type be uniform on $[0, M]$*, where* $M \geq 2n$*. Then the team's equilibrium expected bid,* $n \times \mathbb{E}(\beta^{SPA})$*, is increasing in* n*.*

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  If F and G are both uniform over $[0, 1]$ *and the auction-format is all-pay, then the linear-proportional model has equilibria with complete free riding.*

- Proposition

  If F and G are both uniform over $[0, 1]$ *and the auction-format is first-price, then the linear-proportional model has no equilibrium with complete free riding. Therefore, it has no equilibrium.*

Future research

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- Competition between multiple teams.