Downstream cross-holdings and upstream collusion

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Abstract

We examine the effects of (passive) cross-holdings in the downstream market on the sustainability of upstream collusion. We consider two competing vertical chains with downstream Cournot and homogeneous goods. Each downstream firm holds a (symmetric) non-controlling share of its rival. We find that downstream cross-holdings (a) have a negative effect of on the collusive and punishment (competitive) profits (b) can increase profits from deviation. We use the linear demand to show that a higher degree of cross-holdings reduces upstream collusion. Our results are robust for a wide class of demand functions (that exhibit constant elasticity of slope).

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1. Introduction

The effects of cross-holdings receive increasing attention among scholars and policymakers.[1] For instance, European Commission (2014) highlights the possibility of anti-competitive effects caused by minority acquisitions and proposes a system for reviewing the acquisition of minority shareholdings. Heim et al. (2022) report 10,699 cases of minority acquisitions in rival firms across 63 countries between 1990 and 2013.[2]

The related research highlights possible adverse (and desirable) effects of cross-holdings: an acquirer firm internalizes part of the competitive externality imposed on the rival, thus cross-holdings can soften competition and hurt consumers.[3] Cross-holdings (CH) can also produce coordinated effects when firms interact repeatedly in a market.

The present paper deals with cross-holdings in vertical markets. More precisely, we consider a two-tier vertical market where two upstream firms (suppliers) are locked in exclusive relations with two downstream retailers over linear tariffs. Each downstream firm owns part of the equity of its rival with no control rights.

Under a general demand function, we show that downstream CH decrease upstream collusive profits. For a wide family of demand functions that exhibit constant elasticity of slope, we find that CH reduce the profit of an upstream firm in the punishment (competition) phase, while they can increase the profit for a firm that deviates from collusion. Considering stylized cases of demand functions from this wide family, including linear demand, we show that downstream CH reduces upstream collusion.

The present paper is related to the literature on (i) collusion in vertical markets, and (ii) the coordinated effects of cross-ownership. Malueg (1992) and Gilo et al. (2006) study the effect of partial ownership under repeated interaction in oligopolistic industries. The former extends Reynolds and Snapp (1986) and shows that partial ownership (a) facilitates collusion by reducing the short-run gain from cheating because firms internalize some of the cost imposed on rivals (b) hinders collusion because it softens the punishment phase of the trigger strategy. Gilo et al. (2006) show that cross-ownership facilitates collusion in a Bertrand oligopoly with homogeneous goods. Brito et al. (2018) propose an empirical methodology to evaluate the coordinated effects of

[1] In cross-holdings firms possesses (non-controlling) minority shares in rivals.
[2] For examples of cross-holdings in specific markets, see also Hariskos et al. (2022).
[3] See, for instance, Reynolds and Snapp (1986) and Bresnahan and Salop (1986).

partial horizontal acquisitions and de Haas and Paha (2020) examine how competition policy affects the impact of minority shareholdings on the sustainability of collusion.

The present paper is also related to the literature on collusion in vertical markets. This literature studies the effects of vertical mergers (Nocke and White, 2007 and Normann, 2009), retailers' managerial incentives (Bian et al., 2013), exclusive territories (Piccolo and Reisinger, 2011), passive vertical ownership (Charistos et al., 2022) on upstream collusion.[4]

The effect of the structure of the distribution channel on tacit collusion between manufacturers is studied in Reisinger and Thomes (2017). They show that when selling through a common (in contrast to an independent and exclusive) retailer, (a) manufacturers raise the collusive wholesale price above the industry profit maximizing level, in an attempt to mitigate the retailer's threat of rejecting a manufacturer's offer, (b) they realize lower competitive profit along the punishment phase and (c) the ratio between deviation and collusive profits is higher as deviation makes the common retailer to reject the offer of the non-deviating firm leading to the monopolization of the downstream market. Reisinger and Thomes (2017) shows that common retailing implies an unambiguously higher incentive to deviate and hinders collusion between manufacturers compared to independent retailing.

Finally, Hu et al. (2022) stresses the impact of downstream CH on the decision of the upstream supplier to engage in R&D investments. They show that (a) the amount of upstream R&D decreases (b) the total surplus may increase, while the consumer surplus always decreases with the degree of downstream CH.

2. The model

In a two-tier market, two upstream firms, $U_i, i = 1, 2$, exclusively supply an essential input to a Cournot duopoly, D_i, forming *competing vertical chains*. Each D_i uses U_i's input in a one-to-one proportion to produce a homogeneous final good. D_is face a general (inverse) demand $p(Q) = p(q_1 + q_2)$, where p is the final-good price, q_i is D_i's output, and $\partial p/\partial q_1 = \partial p/\partial q_2 = \partial p/\partial Q < 0$. D_i's only cost is the per-unit input price w_i. U_i's constant marginal cost is normalized to zero.

[4] In addition, Biancini and Ettinger (2017) and Shekhar and Thomes (2020) study the effects of full vertical mergers and passive acquisitions on downstream collusion.

Each D_i has an exogenous symmetric minority share $k \in (0, 1/2]$ on its rival D_j, with *no control* over D_j's production decisions, a situation known as CH. Regarding downstream firms, we follow Hu et al. (2022) and decouple *operational* profits $\pi_{D_i} = (p_i - w_i)q_i$ from *accounting* profits $v_i = \pi_{D_i} + k\, v_j$, which implies: $v_i = \left(\pi_{D_i} + k\, \pi_{D_j}\right)/(1 - k^2)$.

We consider an infinitely repeated game with discrete time periods. In each period, a two-stage game with observable actions is played. In stage 1, U_i makes D_i a "*take-it or leave-it*" offer w_i. In stage 2, downstream firms compete in quantities. Since repeated games admit multiple equilibria, we are interested in subgame perfect equilibria over pure strategies which support the fully collusive scheme.

3. Equilibrium analysis

3.1. Downstream market

Each downstream firm chooses q_i to maximize:

$$\max_{q_i} v_i = \frac{\pi_{D_i} + k\, \pi_{D_j}}{1 - k^2} = \frac{[p(Q) - w_i]q_i + k\left[p(Q) - w_j\right]q_j}{1 - k^2}$$

The first-order conditions (henceforth FOCs) are:

$$[p(Q) - w_i] + q_i p' + k q_j p' \ = 0 \tag{1}$$

where $p' = \partial p/\partial q_i = \partial p/\partial Q$. Denoting $p'' = \partial^2 p/\partial Q^2$, we introduce the following Assumption.

Assumption 1.

(i) $\dfrac{\partial^2 v_i}{\partial q_i^2} = \dfrac{\partial^2 \pi_{D_i}}{\partial q_i^2} + k \dfrac{\partial^2 \pi_{D_j}}{\partial q_i^2} = \dfrac{2p' + p''\left(q_i + k q_j\right)}{1 - k^2} < 0,$

(ii) $\dfrac{\partial^2 v_i}{\partial q_i \partial q_j} = \dfrac{\partial^2 \pi_{D_i}}{\partial q_i \partial q_j} + k \dfrac{\partial^2 \pi_{D_j}}{\partial q_i \partial q_j} = \dfrac{(1 + k)p' + p''\left(q_i + k q_j\right)}{1 - k^2} < 0,$

(iii) $\dfrac{\partial^2 v_i}{\partial q_i^2} \dfrac{\partial^2 v_j}{\partial q_j^2} > \dfrac{\partial^2 v_i}{\partial q_i \partial q_j} \dfrac{\partial^2 v_j}{\partial q_j \partial q_i}.$

Assumption 1(i) ensures that the second-order conditions are satisfied, (ii) implies strategic substitutability: D_i's best-responses are downward sloping, (iii) implies that best-responses are well-behaved and there exists a unique and stable equilibrium.

From the FOCs in (1), we obtain final-good quantities as functions of input prices and the degree of cross-holdings, $q_i(w_i, w_j, k), i = 1,2, i \neq j$. Substituting the latter into the FOCs in (1), it is shown that:

$$\frac{\partial q_i(\cdot)}{\partial w_i} = \frac{\dfrac{\partial^2 v_j}{\partial q_j^2}}{\dfrac{\partial^2 v_i}{\partial q_i^2}\dfrac{\partial^2 v_j}{\partial q_j^2} - \dfrac{\partial^2 v_i}{\partial q_i \partial q_j}\dfrac{\partial^2 v_j}{\partial q_j \partial q_i}} = \frac{2p' + p''(kq_i + q_j)}{p'(1-k)[p'(3+k) + p''(1+k)Q]} < 0 \tag{2}$$

$$\frac{\partial q_i(\cdot)}{\partial w_j} = -\frac{\dfrac{\partial^2 v_i}{\partial q_i \partial q_j}}{\dfrac{\partial^2 v_i}{\partial q_i^2}\dfrac{\partial^2 v_j}{\partial q_j^2} - \dfrac{\partial^2 v_i}{\partial q_i \partial q_j}\dfrac{\partial^2 v_j}{\partial q_j \partial q_i}} = -\frac{(1+k)p' + p''(q_i + kq_j)}{p'(1-k)[p'(3+k) + p''(1+k)Q]} > 0 \tag{3}$$

The first inequality implies that D_i's derived demand for the input is negatively sloped, whereas the second inequality implies that a rise in w_j increases D_i's derived demand. We also assume that $\left|\partial q_i(w_i, w_j)/\partial w_i\right| > \partial q_i(w_i, w_j)/\partial w_j$, that is, own-price effects are larger than cross-price effects.

For given input prices, cross-holdings affect D_i's derived demand as follows:

$$\frac{\partial q_i(\cdot)}{\partial k} = -\frac{p'\left[q_j \dfrac{\partial^2 v_j}{\partial q_j^2} - q_i \dfrac{\partial^2 v_i}{\partial q_i \partial q_j}\right]}{\dfrac{\partial^2 v_i}{\partial q_i^2}\dfrac{\partial^2 v_j}{\partial q_j^2} - \dfrac{\partial^2 v_i}{\partial q_i \partial q_j}\dfrac{\partial^2 v_j}{\partial q_j \partial q_i}} = \frac{p'\left[(1+k)q_i - 2q_j\right] + p''(q_i - q_j)(q_i + kq_j)}{(1-k)[p'(3+k) + p''(1+k)Q]} \tag{4}$$

The sign of the above expression is, in general, ambiguous. For given input prices, a rise in the degree of cross-holdings k relaxes competition in the downstream market and has two effects on D_i's derived demand: a *direct negative* effect as a higher k

reduces q_i and a *positive indirect* effect, as a higher k reduces q_j which in turn raises q_i. With symmetric input prices, $w_i = w$, and therefore $q_i = q$, direct effect dominates:

$$\frac{\partial q(\cdot)}{\partial k} = -\frac{p'q}{[p'(3+k) + p''(1+k)Q]} < 0 \tag{5}$$

With asymmetric input prices, a necessary – but not sufficient – condition for $\partial q_i(\cdot)/\partial k > 0$ to hold is $q_i > q_j$. To see this, from the second expression in (4) note that the sign of the bracketed term determines the sign of $\partial q_i(\cdot)/\partial k$: from the expressions in (2) and (3), and the assumption $\left|\partial q_i(w_i, w_j)/\partial w_i\right| > \partial q_i(w_i, w_j)/\partial w_j$, it is straightforward that $q_i > q_j$ is a necessary condition for D_i's derived demand to increase with k.

3.2. Upstream market

We consider three interactions in the upstream market: *Collusion*, *Deviation*, and *Punishment* (competition), denoted by superscripts $C, D,$ and P respectively.

3.2.1. Collusion

Under collusion, U_is seeks to maximize joint profits $\Pi_U = \pi_{U_1} + \pi_{U_2}$:

$$\max_{w_1, w_2} \Pi_U = \sum_{i=1, i\neq j}^{2} w_i q_i(w_i, w_j, k)$$

The FOCs are

$$q_i(w_i, w_j, k) + w_i \frac{\partial q_i(w_i, w_j, k)}{\partial w_i} + w_j \frac{\partial q_j(w_i, w_j, k)}{\partial w_i} = 0 \tag{6}$$

Denoting optimal collusive input prices by $w_i^C(k)$, upstream total profits are:

$$\Pi_U^C(k) = \sum_{i=1, i\neq j}^{2} w_i^C(k) q_i\left(w_i^C(k), w_j^C(k), k\right)$$

Differentiating with respect to k we get:

$$\frac{d\Pi_U^C(k)}{dk} = \underbrace{\frac{\partial \Pi_U(k)}{\partial k}}_{\substack{direct\\ effect}} + \underbrace{\frac{\partial \Pi_U(k)}{\partial w_1}}_{=0}\frac{\partial w_1^C(k)}{\partial k} + \underbrace{\frac{\partial \Pi_U(k)}{\partial w_2}}_{=0}\frac{\partial w_2^C(k)}{\partial k}$$

or

$$\frac{d\Pi_U^C(k)}{dk} = \frac{\partial \pi_{U_1}(k)}{\partial k} + \frac{\partial \pi_{U_2}(k)}{\partial k} = \sum_{i=1, i\neq j}^{2} w_i^C(k)\frac{\partial q_i(\cdot)}{\partial k}$$

In a symmetric equilibrium,

$$\frac{d\pi_{U_1}(k)}{dk} = \frac{d\pi_{U_2}(k)}{dk} = \frac{d\pi_U(k)}{dk} = w^C(k)\frac{\partial q(\cdot)}{\partial k} < 0$$

Proposition 1. *In a symmetric collusive equilibrium, a higher degree of CH decreases collusive profits for any general demand function.*

Cross-holdings affect equilibrium collusive profits only *directly*: because input prices are set to internalize the externalities that firms exert upon each other, cross-holdings reduces the upstream firms' collusive profits due to the lower derived-demand for the input.

It is necessary for the subsequent analysis to investigate here the behavior of the collusive input prices with respect to the degree of CH. In a symmetric collusive equilibrium, from (2), (3) and (6), we have that:

$$w^C(k) = -q[p'(3+k) + p''Q(1+k)]$$

Differentiating the above with respect to k we obtain:

$$\frac{\partial w^C(k)}{\partial k} = \frac{(1+k)q^2\overbrace{[p'p'' - (p'')^2Q + p'p'''Q]}^{B}}{p'(3+k) + p''Q(3+2k) + 2p'''q^2(1+k)} \tag{7}$$

Lemma 1. *For all demand functions that exhibit constant elasticity of slope, i.e.,* $Qp''/p' = z$, *the symmetric collusive input prices are independent of the degree of CH,* $\partial w^C(k)/\partial k = 0$.

Proof. See Appendix.

The parameter z is the curvature (i.e., relative degree of concavity) of demand: if $z < 0$, demand is convex, whereas if $z > 0$, demand is concave. As shown by López and Vives (2019), the family of demand functions for which the elasticity of slope is constant can be represented by:

$$p(Q) = \begin{cases} \alpha - \beta Q^{1+z} & if \;\; z \neq -1, \\ \alpha - \beta logQ & if \;\; z = -1; \end{cases} \tag{8}$$

with $\alpha \geq 0$ and $\beta > 0$ (resp. $\beta < 0$) for $z \geq -1$ (resp. $z < -1$).[5] The family of demand functions in (8) include the widely used linear (for $z = 0$) and isoelastic demand functions.

3.2.2. Deviation

Suppose U_i unilaterally deviates from collusion. U_i chooses w_i to maximize:

$$\max_{w_i} \pi_{U_i} = w_i q_i\left(w_i, w_j^C, k\right)$$

with the FOCs being

$$q_i\left(w_i, w_j^C, k\right) + w_i \frac{\partial q_i\left(w_i, w_j^C, k\right)}{\partial w_i} = 0 \tag{9}$$

Denoting the deviating input prices by $w_i^D(k)$, it is straightforward that $w_i^D(k) < w_i^C(k)$, as the deviating firm U_i no longer internalizes the negative externality exerted upon its rival U_j.

U_i's deviating profits are:

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[5] See also Tyagi (1999).

$$\pi_{U_i}^D(k) = w_i^D(k)q_i\big(w_i^D(k), w_j^C(k), k\big)$$

Differentiating the above with respect to k:

$$\frac{d\pi_{U_i}^D(k)}{dk} = \underbrace{\frac{\partial\pi_{U_i}(\cdot)}{\partial k}}_{\substack{direct\\ effect}} + \underbrace{\frac{\partial\pi_{U_i}(\cdot)}{\partial w_i}}_{=0}\frac{\partial w_i^D(k)}{\partial k} + \underbrace{\frac{\partial\pi_{U_i}(\cdot)}{\partial w_j}\frac{\partial w_j^C(k)}{\partial k}}_{indirect\ effect}$$

In general, a change in the degree of CH has two effects on U_i's deviating profits: a *direct* effect since, for given input prices, a change in k affects the derived demand, and an *indirect* effect because even though U_j sticks to the collusive input price, a higher k affects that price which in turn affects deviating profits of U_i. Focusing on a symmetric collusive equilibrium:

$$\frac{d\pi_U^D(k)}{dk} = \underbrace{w_i^D(k)\frac{\partial q_i(\cdot)}{\partial k}}_{\substack{direct\\ effect}} + \underbrace{w_i^D(k)\frac{\partial q_i(\cdot)}{\partial w_j}\frac{\partial w^C(k)}{\partial k}}_{indirect\ effect} \tag{10}$$

We know from Lemma 1 that for any demand function that exhibits constant elasticity of slope, $\partial w^C(k)/\partial k = 0$: the indirect effect vanishes leaving at play only the direct effect. Yet, even in that case, one cannot determine the sign of the direct effect (without considering specific demands as in (8)): the deviating input price is lower than the (symmetric) collusive input price, implying that the deviating firm's input quantity is higher than the non-deviating firm's quantity, in which case, as seen in subsection 3.2.1, whether $\partial q_i(\cdot)/\partial k$, and thus the direct effect, is positive or negative is ambiguous.

3.2.3. Punishment

The upstream firms compete in prices. U_i chooses w_i to maximize its profits:

$$\max_{w_i} \pi_{U_i} = w_i q_i\big(w_i, w_j, k\big)$$

The FOCs are:

$$q_i\left(w_i, w_j, k\right) + w_i \frac{\partial q_i\left(w_i, w_j, k\right)}{\partial w_i} = 0 \tag{11}$$

The solution of (11) gives the optimal input prices, $w_i^P(k)$. Compared to (6), it is straightforward that $w_i^P(k) < w_i^C(k)$ as upstream firms no longer internalize the negative externality that they exert upon each other.

U_i's equilibrium punishment profits are:

$$\pi_{U_i}^P(k) = w_i^P(k) q_i\left(w_i^P(k), w_j^P(k), k\right)$$

Differentiating the above with respect to k we obtain

$$\frac{d\pi_{U_i}^P(k)}{dk} = \underbrace{\frac{\partial \pi_{U_i}(\cdot)}{\partial k}}_{\substack{direct\\effect}} + \underbrace{\frac{\partial \pi_{U_i}(\cdot)}{\partial w_i}}_{=0} \frac{\partial w_i^P(k)}{\partial k} + \underbrace{\frac{\partial \pi_{U_i}(\cdot)}{\partial w_j} \frac{\partial w_j^P(k)}{\partial k}}_{indirect\ effect}$$

In general, a change in the degree of CH has two effects on U_i's punishment profits: a *direct* effect since, for given input prices, a change in k affects the derived demand, and an *indirect* effect because a change in k affects the input price of the rival U_j, which in turn affects equilibrium profits of U_i. In a symmetric equilibrium:

$$\frac{d\pi_U^P(k)}{dk} = \underbrace{w^P(k) \frac{\partial q(\cdot)}{\partial k}}_{\substack{direct\\effect}} + \underbrace{w^P(k) \frac{\partial q_i(\cdot)}{\partial w_j} \frac{\partial w^P(k)}{\partial k}}_{indirect\ effect} \tag{12}$$

The direct effect is negative (see (5)), and the sign of the indirect effect depends on the sign of $w^P(k)/\partial k$. In a symmetric equilibrium, from (2) and (11) we have that:

$$w^P(k) = -\frac{p'q(1-k)(p'(3+k) + 2p''(1+k)q)}{2p' + p''(1+k)q} \tag{13}$$

Determining the sign of $\partial w^P(k)/\partial k$ for any demand function is impossible, but we can obtain the following result when focusing on demand functions with constant elasticity of slope.

Lemma 2. *For all demand functions that exhibit constant elasticity of slope, i.e.,* $Qp''/p' = z$*, the symmetric input prices under punishment (competition) fall with the degree of CH, that is,* $\partial w^P(k)/\partial k < 0$.

Proof. See Appendix.

Therefore, in the punishment phase, both the direct and indirect effect of a higher k on upstream profits are negative, leading to the following Proposition.

Proposition 2. *In a symmetric equilibrium, a higher degree of CH reduces punishment profits if demand function exhibits constant elasticity of slope,* $Qp''/p' = z$.

3.2.4. Sustainability of collusion

We assume that collusion is sustained with *trigger strategies*: a deviation implies that firms revert to competition forever (punishment). Collusion is sustainable if

$$\frac{\pi_U^C(k)}{1-\delta} \geq \pi_U^D(k) + \frac{\delta \pi_U^P(k)}{1-\delta}$$

The critical discount factor, above which upstream collusion can be sustained, is:

$$\delta(k) = \frac{\pi_U^D(k) - \pi_U^C(k)}{\pi_U^D(k) - \pi_U^P(k)} \tag{14}$$

Our analysis thus far has pointed out that the effect of cross-holdings on the sustainability of collusion in the upstream market is ambiguous in a general demand setting. In the next section, we focus on the family of demand functions introduced in (8).

4. Specific demand functions

In this section, we focus on the wide family of demand functions in (8), and, in particular, in the case of $z \neq -1$: $p(Q) = \alpha - \beta Q^{1+z}$. We assume that $z > -1$ (so, $\beta > 0$) to satisfy second-order conditions.

In the downstream market, the FOCs are:

$$\max_{q_1} \pi_{D_1}(q_1, q_2) \Rightarrow \alpha - \beta \left((2+z)q_1 + (1+k+kzq_2\right) Q^z - w_1 = 0 \tag{14}$$

and

$$\max_{q_2} \pi_{D_2}(q_1, q_2) \Rightarrow \alpha - \beta \left((1+k+kz)q_1 + (2+z)\, q_2\right) Q^z - w_2 = 0 \tag{15}$$

Summing (14) and (15) we get:

$$2\alpha - \beta\,(3+z+k+kz)\, Q^{1+z} - w_1 - w_2 = 0$$

From the above, we obtain total output as a function of (the sum of) input prices:

$$Q(w_1, w_2) = \left(\frac{2\alpha - (w_1 + w_2)}{\beta(3+z+k+kz)}\right)^{\frac{1}{1+\eta}} \tag{16}$$

Substituting (16) back to (14) and (15), and solving together, we obtain final-good outputs as a function of input prices:

$$q_i\left(w_i, w_j\right) = \frac{(2\alpha - w_1 - w_2)^{\frac{-z}{1+z}}\left(\alpha(1-k)(1+z) - (2+z)w_i + (1+k+kz)\, w_j\right)}{[\beta(3+k+z+kz)]^{\frac{1}{1+z}}(1+z)(1-k)} \tag{17}$$

4.1 Collusion

The symmetric equilibrium *collusive* input prices are:

$$w^C = \frac{\alpha(1+z)}{2(2+z)}$$

The above expression verifies Lemma 1: since the demand function exhibits constant elasticity of slope, the symmetric collusive input prices do *not* depend on the degree of CH. We also have that

$$\pi_U^C(k) = \frac{2\alpha^2(1+z)\left[2^{\frac{1}{1+z}}\left(\frac{\alpha}{\beta(2+z)(3+k+z+kz)}\right)^{\frac{1}{1+z}}\right]^{-z}}{\beta(2+z)^2(3+k+z+kz)} \tag{18}$$

with

$$\frac{d\pi_U^C(k)}{dk} = -\frac{2\alpha^2(1+z)\left[2^{\frac{1}{1+z}}\left(\frac{\alpha}{\beta(2+z)(3+k+z+kz)}\right)^{\frac{1}{1+z}}\right]^{-z}}{\beta(2+z)^2(3+k+z+kz)^2} < 0 \tag{19}$$

A higher degree of CH decreases collusive profits of each upstream firm.

4.2 Deviation

The deviating input price is given by

$$w^D(k) = \frac{\alpha(1+z)(15-k+z(11+2z)-L)}{2(2+z)^3}$$

with $L = \sqrt{k^2+(3+z)^2(9+4z)+2k(3+z)\big(3+2z(3+z)\big)}$. We have that

$$\pi_U^D(k) = \frac{\alpha^2(1+z)\left[2^{\frac{-1}{1+z}}\left(\frac{\alpha\big(9+L+k(1+z)+z(6+L+z)\big)}{\beta(2+z)^3(3+k+z+kz)}\right)^{\frac{1}{1+z}}\right]^{-z} M}{4\beta(1-k)(2+z)^5(3+k+z+kz)} \tag{20}$$

with $M = \big(3-L+z+k(3+2z)\big)\big(-15+L+k-z(11+2z)\big)$.

As noted in the previous section, with demands that exhibit constant elasticity of slope, a change in k has only a direct effect on deviating profits by affecting the derived demand, the sign of which is ambiguous without considering specific demands.

The derivative of $\pi_U^D(k)$ in (20) with respect to k results in a large and unattractive formula. The following Proposition restricts attention to the case of linear demand.

Proposition 3. *Suppose* $z = 0$ *so that demand is linear,* $p = \alpha - \beta Q$*. Then, the profits from deviation are*

$$\pi_U^D(k) = \frac{\alpha^2(3-k)^2}{32(1-k)(3+k)}$$

with

$$\frac{\partial \pi_U^D(k)}{\partial k} = \frac{\alpha^2(3-k)k}{4(1-k)^2(3+k)^2} > 0.$$

Under a linear demand, a higher k increases the profits from deviation. In the next graph, assuming $\alpha = 1$ and $\beta = 1$, we plot $\partial \pi_U^D(k)/\partial k$ for values of z between -1 and 5 (vertical axis) and values of k between 0 and 0.5 (horizontal axis). If final-good demand is concave, $z > 0$, then the profits from deviation increase with k. If demand is convex, $-1 < z < 0$, then there are cases where the profits from deviation decrease with k.

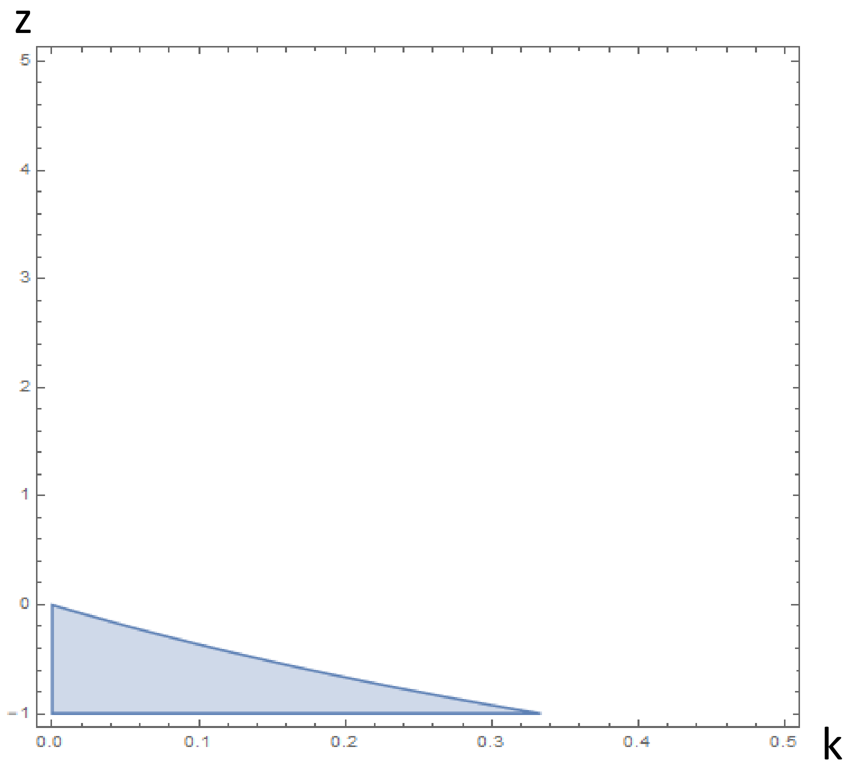

Figure 1. The shaded (white) area shows (k, z) pairs for which $\partial \pi_U^D(k)/\partial k < (>) 0$

4.3 Punishment

The symmetric equilibrium input prices under punishment are

$$w^P(k) = \frac{2\alpha(1-k)(1+z)}{(3-k)(2+z)}$$

with

$$\frac{\partial w^P(k)}{\partial k} = -\frac{4\alpha(1+z)}{(3-k)^2(2+z)} < 0$$

thus, verifying Lemma 2: when the demand function exhibits constant elasticity of slope, then the symmetric equilibrium input prices decrease with the degree of CH.

We also have that

$$\pi_U^P(k) = \frac{2\alpha^2(1-k)(1+z)(4+z+kz)\left[2^{\frac{1}{1+z}}\left(\frac{\alpha(4+z+kz)}{\beta(3-k)(2+z)(3+k+z+kz)}\right)^{\frac{1}{1+z}}\right]^{-z}}{\beta(3-k)^2(2+z)^2(3+k+z+kz)} \quad (21)$$

with

$$\frac{d\pi_U^P(k)}{dk} = -\frac{2\alpha^2(1+z)\left[2^{\frac{1}{1+z}}\left(\frac{\alpha(4+z+kz)}{\beta(3-k)(2+z)(3+k+z+kz)}\right)^{\frac{1}{1+z}}\right]^{-z} N}{\beta(3-k)^3(2+z)^2(3+k+z+kz)^2} < 0 \quad (22)$$

where $N = (8(3+k^2)+(1+k)(13+k(2+k))z+2(1+k)^2z^2)$. A higher degree of CH decreases punishment profits of each upstream firm.

4.4 Sustainability of collusion

Substituting (18), (20) and (21) into (14) gives a very large and rather unattractive formula for the critical discount factor $\delta(k)$. The following Proposition restricts attention to the case of linear demand.

Proposition 4. *Suppose* $z = 0$ *so that demand is linear,* $p = \alpha - \beta Q$*. The critical discount factor is*:

$$\delta(k) = \frac{(3-k)^2}{17 - 14k + k^2}$$

with

$$\frac{\partial \delta(k)}{\partial k} = \frac{8(3-k)(1+k)}{(17-14k+k^2)^2} > 0.$$

A higher degree of CH (i) decreases collusive profits and increases the profit from deviation (Proposition 3), implying that upstream firms are less willing to adhere to collusion, (ii) favors collusion by decreasing the profit in the punishment phase. With linear final-good demand, (i) dominates; cross-holdings makes upstream collusion less likely to be sustained.

In the next Figure, assuming $\alpha = 1$ and $\beta = 1$, we plot $\partial\delta(k)/\partial k$ as a function of k for different values of z, that is, for different degrees of demand convexity/concavity. We graphically support that the main finding holds beyond linear demand.

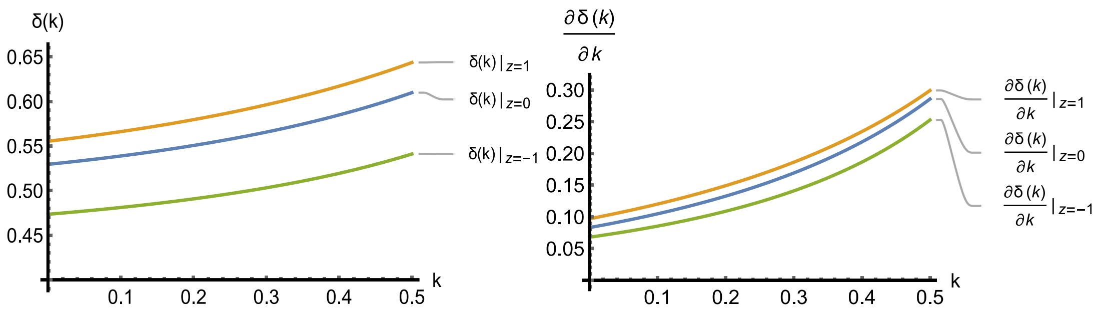

Figure 2. Plots of $\delta(k)$ and $\partial\delta(k)/\partial k$ for different values of z.

5. Conclusions

The present paper investigates the effects of cross-holdings on collusion in two-tier markets. More specifically, we show that cross-holdings between firms in the downstream market affect the sustainability of collusion in the upstream market. We consider two competing vertical chains with downstream Cournot competition and homogeneous final goods. Each downstream firm holds a (symmetric) non-controlling share of its rival.

Under a general demand, we show that downstream cross-holdings reduce the collusive profit of an upstream firm. We also find that cross-holdings between firms in the downstream market a) can increase the profit of the upstream firm that unilaterally deviates from collusion, b) decrease the profits in the punishment phase, if the demand function exhibits constant elasticity of slope. Using the linear demand, we show that the effects of cross-holdings through the lower collusive and the higher defecting profit dominate: upstream collusion becomes less likely when each downstream firm possesses a minority share on its rival. We show robustness of our results for a wide class of demand functions (that exhibit constant elasticity of slope).

Appendix

Proof of Lemma 1. By totally differentiating $Qp''/p' = z$ with respect to Q, we obtain:

$$\frac{p'p'' - (p'')^2Q + p'p'''Q}{p'} = 0 \quad \rightarrow \quad p'p'' - (p'')^2Q + p'p'''Q \equiv B = 0$$

and thus from (7), we get that $\partial w^C(k)/\partial k = 0$.

Proof of Lemma 2. Solving $Qp''/p' = z$ for p'' we obtain

$$p'' = \frac{zp'}{Q} \tag{A1}$$

By totally differentiating $Qp''/p' = z$ with respect to Q, we obtain $p'p'' - (p'')^2Q + p'p'''Q = 0$, which gives:

$$p''' = -\frac{p'p'' - (p'')^2Q}{p'p'''Q} \tag{A2}$$

Using (A1) and (A2), we obtain from (13):

$$\frac{\partial w^P(k)}{\partial k} = -\frac{p'Q(3 + k + (5 + 3k)z + 2(2 - k)(1 + k)z^2)}{-4(1 - k^2)z^2 + p'q(2 + z + kz)^2(3 + k + 2(1 + k)z)} < 0.$$

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