Testingthe linearity of a time series.

By
Chatzi Dimitra
Phd Student
Economics Department
University of Crete
Phone: 6978222956
d_chatzi@hotmail.com

Supervisor Professor
Dikaios E. Tserkezos
Department of Economics
University of Crete
Gallos,Rethymno 74100
Crete
Phone: 28310 77415
Fax: 28310 77416

Abstract

This letter proposes a simple test for the linearity of a time series. We compare the small and large samples properties of the suggested test via Monte Carlo techniques with well known time domain linearity tests. Our results suggest that the suggested test over performs the power of the other competitive tests in small samples.

Key words: *Testing nonlinearity, Hinich portmanteau bicorrelation test, Keenan, Mcleodi-Li tests,.ARCH and Luukkonen LST Test.*

Rethymno 2014.

I. Introduction

Many time-series encountered in practice exhibit characteristics that cannot be appropriately described by linear Gaussian models. For example, a Gaussian linear process $\{Z_t\}$ is time reversible, i.e., the distribution of $\{Z_{t1},\ldots,Z_{tN}\}$ is the same as that of $\{Z_{tN},\ldots,Z_{t1}\}$. Deviations from this property are suggested by time-series where the number of ascending periods (i.e., periods with increasing observations) is systematically different from the number of descending periods[1].

In this type of situations, it is often necessary to relax the linear assumption, i.e., it is necessary to consider nonlinear models for the analysis of a time-series. Consequently, it is clear that, before modelling any time-series, it may be necessary to test it for linearity.

This letter presents a method to detect nonlinear dependencies in time series. It is a bicorrelation nonparametric test based on a Box-Cox transformation. Using Monte Carlo results we compare its properties with other four well known tests for such cases. These tests are the *The Hinich Portmanteau Bicorrelation Test(1996), McLeodi -Li Test(1983), Test for nonadditivity Keenan (1985), ARCH Test(1982) and the Luukkonen LST 1988 Test.*

According to our simulation results the suggested test is superior to the other tests in small and large samples.

The plan of the paper is as follows. In Section 2 we present the suggested tests. In Section 3, we present some nonlinear models and the Monte Carlo experiments. Conclusions are given in Section 4.

---

[1] The well-known series of the Wolf sunspot numbers is a good example of this pattern. Gaussian linear processes do not exhibit sudden bursts of very large amplitude at irregular time epochs. This sort of behaviour is commonly seen in seismological data, in particular explosion and earthquake data.

2. The Suggested Linearity Test.

The suggested test is based on the Box-Cox[2] transformations of the original series[3] using the following relations:

$$Q(\lambda) = n(n+2)\sum_{k=1}^{K}(n-k)^{-1}\rho(\lambda)_k^2 \sim \chi^2 \text{ with } (K\text{-}m) \qquad (1)$$

$$\rho(\lambda)_k = \frac{Cov\left(Z(\lambda)_t, Z(\lambda)_{t+k}\right)}{\sqrt{V\left(Z(\lambda)_t\right)}\sqrt{V\left(Z(\lambda)_{t+k}\right)}} = \frac{Cov\left(Z(\lambda)_t, Z(\lambda)_{t-k}\right)}{\sqrt{V\left(Z(\lambda)_t\right)}\sqrt{V\left(Z(\lambda)_{t-k}\right)}} \qquad (2)$$

$$Z(\lambda)_t = \frac{Z^{\lambda}{}_t - 1}{\lambda} \qquad (3)$$

$$-1 \le \lambda \le 1 \qquad (4)$$

According to the relations (1)-(4) in order to apply the suggested test the only we have to do is conduct an iterative procedure for $-1 \le \lambda \le 1$ and to estimate the autocorrelation function (2) and the Box-Pierce[4] statistic (2).We choose the value of λ for the highest significant value of the Box-Pierce statistic.

---

[2] Box, G.E.P., Cox, D.R., 1964

[3] When negative Z values occur Box-Cox consider the family:

$$Z_t(\lambda) = \begin{cases} [(Z_t + \lambda_1)\lambda_2 - 1]/\lambda_2 & \lambda_2 \ne 0 \\ \log(Z_t + \lambda_1) & \lambda_2 = 0 \end{cases}$$

with $Z_t + \lambda_1 > 0$

[4] Box, G.E.P. & Pierce A.D.A ., 1970

3. Monte Carlo Experiment Results.

Many nonlinear models have been proposed in the literature with the objective of describing different characteristics exhibited by time-series encountered in practice and that cannot be appropriately accounted for by linear processes. The models we will consider in our study represent some of the most important classes of nonlinear processes that have been proposed in the literature. Excellent presentations of these models can be found in Subba Rao and Gabr (1984) and in Tong 1983 and Tong 1990. They are as follows:

For our simulation experiment[5] *we used the specification*[6]*:*

(i) Billinear (BL) model. $Z_t=0.4-0.5Z_{t-1}+0.6Z_{t-1}a_{t-1}+a_t$

(ii) Exponential AR $Z_t=(0.2+(0.3+0.95Z_{t-1})e^{-0.01Z_{t-1}^2})Z_{t-1}+a_t$.

(iii) Nonlinear AR $Z_t=(0.004Z_{t-1}+a_t)(0.55Z_{t-1})+a_t$.

(iv) Nonlinear MA(NLMA)

$$Z_t=a_t-0.3a_{t-1}+0.5a_{t-2}+0.6a_ta_{t-2}-0.2a_{t-1}^2.$$

(v) Threshold AR(TAR).

$$Z_t=\begin{cases} 2.0-0.3Z_{t-1}+0.5Z_{t-2}+a_t & \text{if } Z_{t-3}\le 1.5,\\ 2.5+0.2Z_{t-1}-0.5Z_{t-2}-0.7Z_{t-3}+a_t & \text{if } Z_{t-3}>1.5.\end{cases}$$

The alternative tests used for comparisons are : *The Hinich Portmanteau Bicorrelation Test(1996), McLeodi -Li Test(1983), Test for Nonadditivity Keenan (1985), ARCH Test(1982) and the Luukkonen LST 1988 Test.*

---

[5] The computations were performed in RATS 5.00 with a program written by the author. This RATS code program is available on request.

[6] Wei W.S& Paulo Teles 2000.

Under the null hypothesis, 3,000 replications of the basic nonlinear specifications (i)-(v) are generated. Samples of different sizes from 40 to 1500 data were used in order to evaluate the six nonlinearity tests. Before checking for nonlinearity we remove linear dependencies by fitting an AR(p) fit to the simulated series for the five alternative tests. This way we make sure that the rejection of the null hypothesis of pure noise at the specified threshold level is due only to significant nonlinearity. These results are summarized in Table 1.

Table 1. The Power of the six tests for testing linearity.

The Power of the five tests for the specification : $Z_t=0.4-0.5Z_{t-1}+0.6Z_{t-1}a_{t-1}+a_t$

Tests	Number of Available Observations									
	40	100	200	300	400	500	600	700	1200	1500
1. The Hinich Portmanteau Bicorrelation Test	3,4	50,8	69,8	79,7	86,0	89,8	91,8	93,8	95,2	96,1
2. McLeodi -Li Test	8,6	71,2	93,1	98,3	99,2	99,6	99,9	100,0	100,0	100,0
3. Test for nonadditivity Keenan (1985)	0,5	32,0	56,6	72,7	81,2	86,0	88,1	90,0	91,8	93,3
4. ARCH Test	2,6	88,3	98,9	99,8	100	100	100	100	100	100
5. Luukkonen LST 1988 Test	2,3	86,3	99,0	99,9	100,0	100,0	100,0	100,0	100,0	100,0
6. Suggested Test	21,4	80,9	96,9	99,3	99,9	100,0	100,0	100,0	100,0	100,0

Power of the six tests using the specification : $Z_t=(0.2+(0.3+0.95Z_{t-1})e^{-0.01Z_{t-1}^2})Z_{t-1}+a_t$.

Tests	40	100	200	300	400	500	600	700	1200	1500
1. The Hinich Portmanteau Bicorrelation Test	0,8	36,5	53,3	59,6	66,3	73,7	78,9	86,4	90,5	93,0
2. McLeodi -Li Test	16,1	19,8	29,2	43,5	58,0	72,8	82,8	89,6	94,7	97,2
3. Test for nonadditivity Keenan (1985)	0,3	9,1	31,2	51,9	71,1	84,0	91,3	95,4	97,6	98,5
4. ARCH Test	2,7	59,4	79,5	91,7	98	99,5	99,9	100	100	100
5. Luukkonen LST 1988 Test	1,0	37,0	52,7	58,6	63,5	72,3	78,1	86,0	90,6	93,0
5. Suggested Test	99,1	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0

Power of the six tests using the specification : $Z_t=(0.004Z_{t-1}+a_t)(0.55Z_{t-1})+a_t$.

Tests	40	100	200	300	400	500	600	700	1200	1500
1. The Hinich Portmanteau Bicorrelation Test	1,4	16,4	25,9	32,0	38,3	42,4	47,1	49,0	53,6	56,1
2. McLeodi -Li Test	2,1	14,9	32,4	47,1	60,2	71,5	79,6	85,7	90,7	94,2
3. Test for nonadditivity Keenan (1985)	0,4	11,0	14,1	16,8	18,0	17,8	20,2	20,4	20,7	20,6
4. ARCH Test	2	34,2	62,4	78,5	87,6	93,5	96,4	98,5	99,1	99,4
5. Luukkonen LST 1988 Test	1,8	34,1	61,6	76,5	87,1	93,1	96,5	98,1	99,0	99,6
6. Suggested Test	6,6	11,5	15,3	19,8	22,0	24,2	28,1	29,9	34,0	36,6

Power of the six tests using the specification: $Z_t = a_t - 0.3a_{t-1} + 0.5a_{t-2} + 0.6a_t a_{t-2} - 0.2a_{t-1}^2$.

1. The Hinich Portmanteau Bicorrelation Test	1,7	15,7	24,4	30,3	35,7	38,2	39,2	42,1	45,1	46,2
2. McLeodi -Li Test	2,4	15,7	32,1	47,5	60,1	72,1	81,4	87,5	92,0	94,9
3. Test for nonadditivity Keenan (1985)	0,3	4,7	6,3	7,5	8,7	9,6	10,9	12,6	13,6	14,9
4. ARCH Test	1,7	17,1	29,8	43,4	55,4	65	72	77	81,5	85,9
5. Luukkonen LST 1988 Test	1,6	17,8	31,6	42,4	53,7	62,5	70,8	77,5	83,1	87,1
6. Suggested Test	24,1	77,2	95,8	99,4	100,0	100,0	100,0	100,0	100,0	100,0

Power of the six tests using the specification:

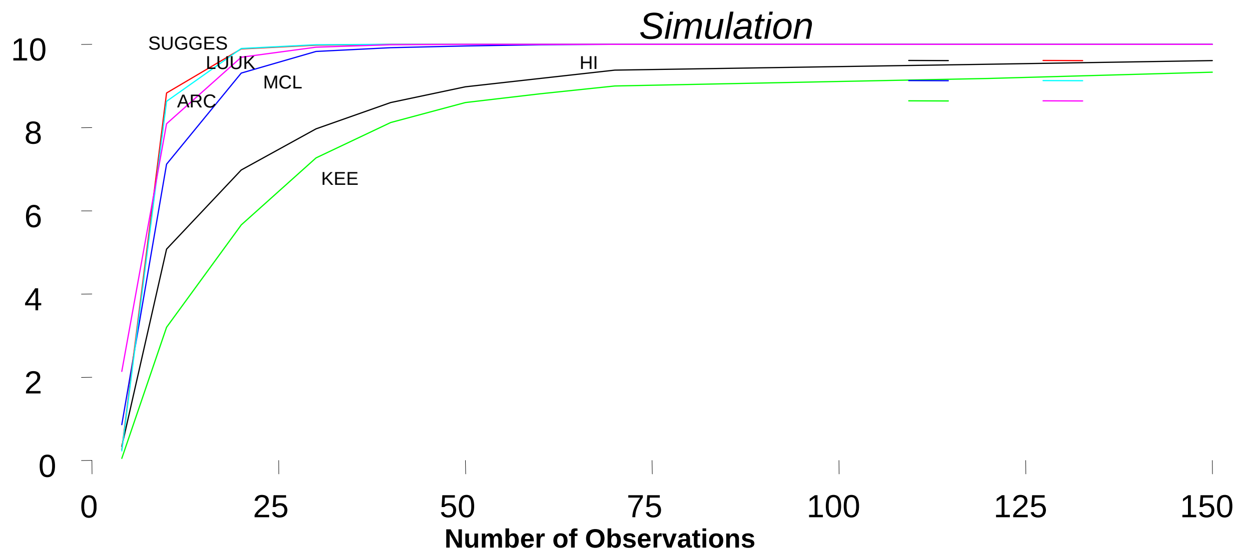
$$Z_t = \begin{cases} 2.0 - 0.3Z_{t-1} + 0.5Z_{t-2} + a_t & \text{if } Z_{t-3} \le 1.5, \\ 2.5 + 0.2Z_{t-1} - 0.5Z_{t-2} - 0.7Z_{t-3} + a_t & \text{if } Z_{t-3} > 1.5. \end{cases}$$

1. The Hinich Portmanteau Bicorrelation Test	2,0	34,9	63,7	78,0	89,0	94,2	97,0	99,1	99,6	99,9
2. McLeodi -Li Test	5,9	19,6	36,8	51,4	64,6	74,5	82,6	88,6	92,8	95,3
3. Test for nonadditivity Keenan (1985)	0,1	23,3	78,6	76,4	64,1	75,7	81,5	87,0	86,3	86,0
4. ARCH Test	1,6	11,5	19,9	25,6	30,9	36,1	41,3	46,4	51,4	55,1
5. Luukkonen LST 1988 Test	1,0	12,3	16,8	24,6	30,4	35,8	39,6	45,5	50,2	52,5
6. Suggested Test	25,9	93,2	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0

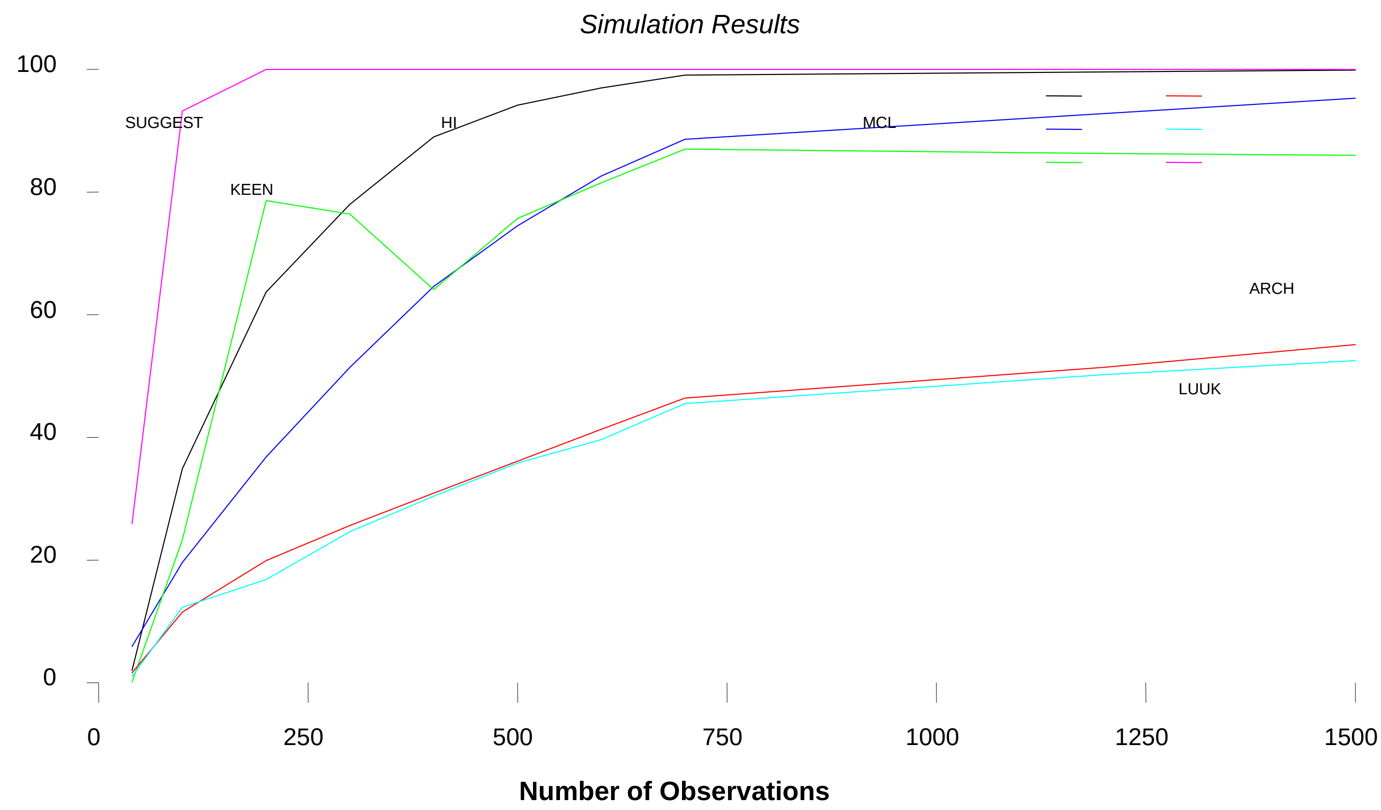
Source: Monte Carlo Experiments.

1. Billinear (BL) model: $Z_t = 0.4 - 0.5Z_{t-1} + 0.6Z_{t-1}a_{t-1} + a_t$

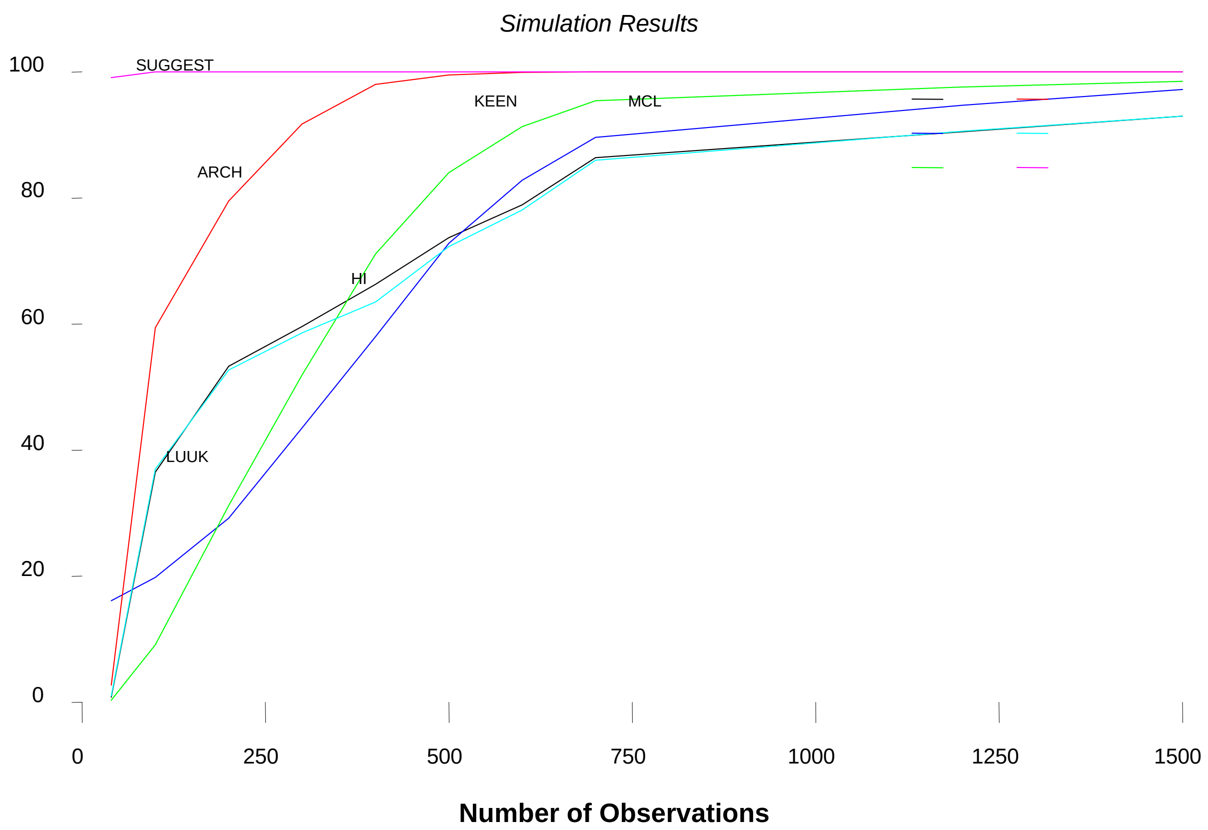
THE POWER OF THE TESTS



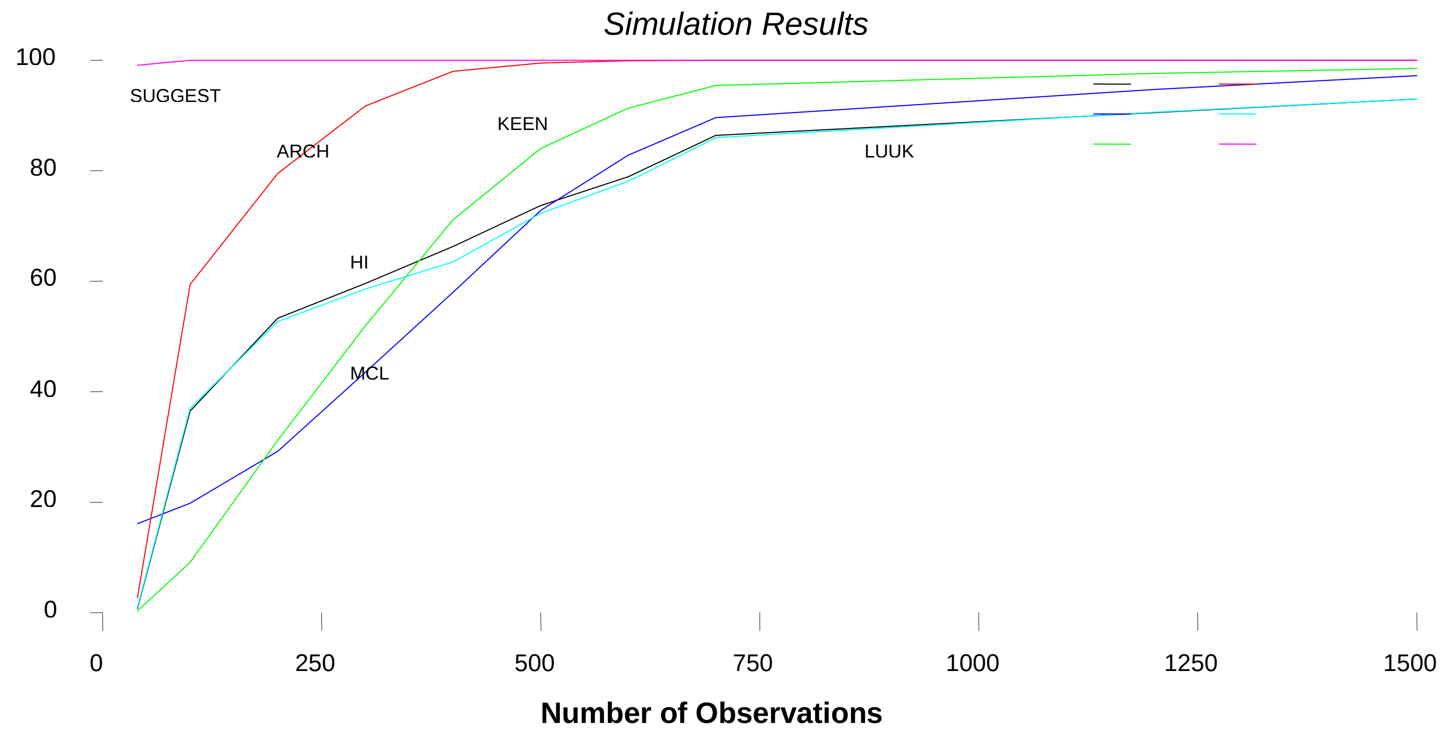
2. Exponential AR: $Z_t=(0.2+(0.3+0.95Z_{t-1})e^{-0.01Z_{t-1}^2})Z_{t-1}+a_t$.



3. *Nonlinear AR:* $Z_t=(0.004Z_{t-1}+a_t)(0.55Z_{t-1})+a_t$.

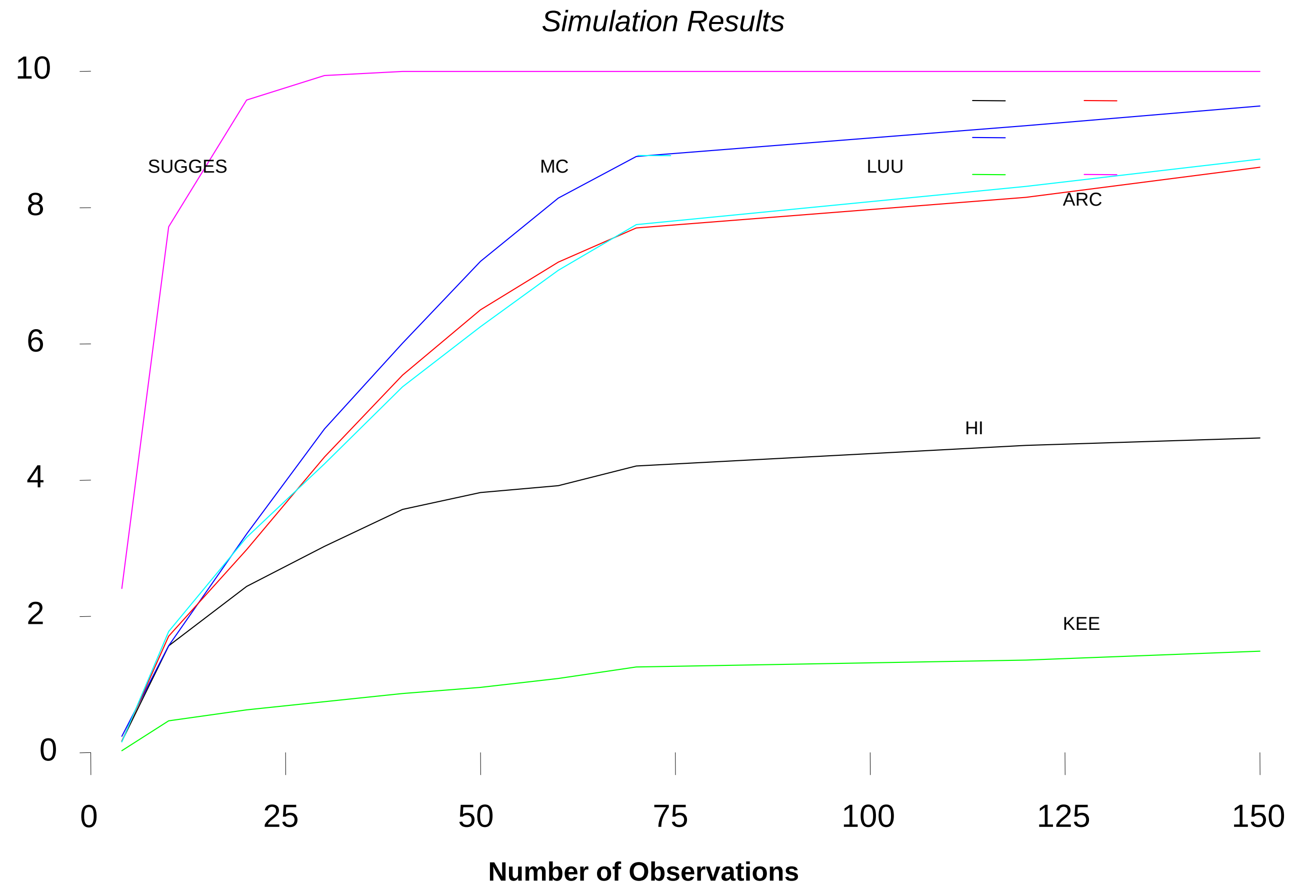


4. ***Nonlinear MA(NLMA):*** $Z_t = a_t - 0.3a_{t-1} + 0.5a_{t-2} + 0.6a_t a_{t-2} - 0.2a_{t-1}^2$



5. <u>Threshold AR(TAR):</u>

$$Z_t = \begin{cases} 2.0 - 0.3Z_{t-1} + 0.5Z_{t-2} + a_t & \text{if } Z_{t-3} \le 1.5, \\ 2.5 + 0.2Z_{t-1} - 0.5Z_{t-2} - 0.7Z_{t-3} + a_t & \text{if } Z_{t-3} > 1.5. \end{cases}$$



4. Conclusions

In this short paper we present a non linearity test for time series. In order to test its properties in small and large samples we conducted a Monte Carlo experiment with five nonlinear specifications and five alternative linearity tests.

According to our Monte Carlo experiments in small samples the suggested test over performs the power of the alternative tests in the majority of the nonlinear specifications used in the experiment. In large samples the six tests converge although we note a limiting over performance of the suggested test in most cases.

Reference.

Ashley, R.A., Patterson, D.M. and Hinich, M.J., 1986. A diagnostic test for nonlinear serial dependence in time-series fitting errors. J. Time Ser. Anal. 7, pp. 165–178.

Box, G.E.P., Cox, D.R., 1964. An analysis of transformations. J. Roy. Statist. Soc. Ser. B 26, pp.211–243.

Box, G.E.P. & Pierce A.D.A ., 1970. Distributions of Residual Autocorrelation in ARMA time Series.,Journal of the American Statistical Assosciation 65.,pp1509-26.

Brock, W., D. Hsieh and B. Le Baron, 1991. Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence, MIT Press, Reading, Mass.

Brooks, C., 1996, Testing for non-linearity in daily sterling exchanges rates, Applied Financial Economics 6, 307 – 317.

Brooks, C. and M.J. Hinich, 1998, Episodic nonstationarity in exchange rates, Applied Economics Letters 5, 719-722.

Brooks, C. and M.J. Hinich, 2001, Bicorrelations and cross-bicorrelations as tests for nonlinearity and as forecasting tools, Journal of Forecasting 20, 181-196.

Hinich, M.J., 1996, Testing for dependence in the input to a linear time series model, Journal of Nonparametric Statistics 6, 205-221.

Hinich, M.J. and D.M. Patterson, 1985, Evidence of nonlinearity in daily stock returns, Journal of Business and Economic Statistics 3, 69-77.

Hsieh, D.A., 1991, Chaos and nonlinear dynamics: application to financial markets, Journal of Finance 46, 1839-1877.

Keenan, D.M., 1985. A Tukey nonadditivity-type test for time-series nonlinearity. Biometrika 72, pp. 39–44.

Luukkonen R,Saikkonen P and Terasvitra T.,(1988)., Testing Linearity against Smooth TransisionAutoregressive Models.,Biometrika 75.,pp-491-500.

Mcleod and Li W.,(1983).,Diagnostic checking ARMA Time Series Models Unsing Squared Residual Autocorrelation.,Journal of Time Series Analysis.,4.,pp.269-273.

Opong, K.K., G. Mulholland, A.F. Fox, and K. Farahmand, 1999, The behavior of some UK equity indices: an application of Hurst and BDS tests, Journal of Empirical Finance 6, 267-282.

Ramsey, J.B., 1996, If nonlinear models cannot forecast, what use are they? Studies in Nonlinear Dynamics and Econometrics 1(2), 65-86.

Scheinkman, J. and B. LeBaron, 1989, Nonlinear dynamics and stock returns, Journal of Business 62, 311-337.

Tong, H., 1983. Threshold Models in Non-linear Time Series Analysis. Springer, New York.

Tong, H., 1990. Non-linear Time Series: A Dynamical System Approach. Oxford University, Oxford.

Tsay, R.S., 1986. Nonlinearity tests for time-series. Biometrika 73, pp. 461–466.

Tsay, R.S., 1989. Testing and modeling threshold autoregressive processes. J. Amer. Statist. Assoc. 84, pp. 231–240.

Wei W.S& Paulo Teles, The effects of temporal aggregation on tests of linearity of a time series

Computational Statistics and Data Analysis, 2000.,pp.91-103.