Undeclared Labour in Unionized Oligopoly

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Abstract

Undeclared labour constitutes a complex phenomenon that has not yet been analyzed within I/O framework. In a unionized duopoly under decentralized wage bargaining context, we reveal the opportunity cost that exists between the taxation and the contributions for social insurance. Comparing to a benchmarking state where no undeclared labour exist, our findings indicate that if the tax rate is low enough, the rate of undeclared labour that maximizes firms' profit will yield greater clearing wages, greater output and thus employment, greater consumer surplus and lower price. Furthermore, in contrast to common knowledge, we showed that under certain circumstances, undeclared labour may increase firms' profits and unions' utility, but it may also increase public revenues and social welfare. Finally, we propose a Pareto optimal tax rate for the case that firms practice undeclared labour. The proposed tax rate will render greater values in all market's magnitudes (wages, profits, quantities, consumer surplus, and social welfare). However, this policy proves that this specific policy lacks financing.

Keywords: Undeclared Labour, Cournot Duopoly, Labour Unions, Unionisation, Endogenous Objectives

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Introduction

Undeclared work is defined as "any paid activities that are lawful as regards their nature but not declared to public authorities". It is a complex phenomenon associated with tax evasion and social security fraud. Undeclared labour concerns various types of activities, ranging from informal household services to clandestine work by illegal residents, but excludes criminal activities.

It is a process that may engage both employers and employees voluntarily, because of the potential gain in avoiding taxes and social security contributions, social rights and the cost of complying with regulations.

From a macroeconomic point of view, undeclared labour reduces tax revenues (since employees declare no income and then no taxes are imputed) and undermines the financing of social security systems. To the extent that undeclared work competes with and even crowds out activities that comply with regulations, it is the main source of social dumping. In the case of undeclared work performed by individuals who are receiving benefits compensating their inactivity, there is also a dimension of social fraud.

From a microeconomic perspective, undeclared labour distorts fair competition among firms and causes productive inefficiencies, as informal businesses typically avoid access to formal services and inputs (e.g. credit) and prefer to stay small.

Undeclared labour is a decomposite phenomenon, that is influenced by a great range of economic, social, structural and cultural factors, tending to comprise a constraint to economic, fiscal, and social policies applied for the economic growth of an economy.

The fact that undeclared labour on one hand cannot be observed and on the other hand may be otherwise defined among countries, makes it even more difficult to establish credible evaluations about the growth of this phenomenon. However, a research, conducted on behalf of European Committee at 2004, while it accented important differences among countries regarding the qualitative characteristics as

well as the size of undeclared labour, estimated undeclared labour's maximum values at 20% at some countries of Eastern and South Europe.

Given the complexity and the heterogeneity of the phenomenon, there is no simple solution to confront it. Nevertheless, the resolution of the European Union's Council of 29 October 2003 on transforming undeclared work into regular employment proposed the following policies:

• Reducing the financial attractiveness of undeclared work stemming from the design of tax and benefit systems, and the permissiveness of the social protection system with regard to the performing of undeclared work;

• Administrative reform and simplification, with a view to reducing the cost of compliance with regulations;

• Strengthening the surveillance and sanction mechanisms, with the involvement of labour inspectorates, tax offices and social partners;

• Trans-national cooperation between Member States, and

• Awareness raising activities.

Regarding the first policy group of meters, European Committee concluded that there is still a great deal of actions to be done in order to balance both the motives and the disincentives offered by the social security systems. In particular, proposed policies concern the reservation of adequate income levels (taking into account the relation between benefits and contributions), the enforcement of exercising control over the labour market and over the persons entitled to social benefits and the imposition of proper economic penalties for tax and contribution evasion.

To gain all the above, policies should emphasize in:

(i)   Proper taxation of overtime work;
(ii)   Maintaining the institutional minimum wages;

(iii)   Regulating tax distortions between tax systems applied in wage earners and those applied to self-employed;

(iv)   Reducing the taxation of low productivity activities.

Even though during the past decades a broad range of methods has been developed to analyze the undeclared labour phenomenon, to understand its dimensions and causes, to formulate an appropriate policy to constrain its spread, neither this phenomenon has been examined with any available method, nor the discussion about which methodology is the most appropriate has still not come to an end. In particular, there has been an extended use of econometrics and applied statistics in the relevant researches. Surveys from international organizations (such as OECD, ILO, EU etc) based mostly on evidence and results of state audits also consist a notable framework. However, undeclared labour has not yet been approached or analyzed using the framework of industrial organization and game theoretic analytical toolkit.

With this research, we aspire to deliver a different approach, using the industrial's organization framework. Moreover, one of the main goals of this work is to propose a different policy for restraining the phenomenon of undeclared labour. As it is shown, the use of proper tax rates relative to those of social insurance could – under certain circumstances – restrain the economic attractiveness of this phenomenon.

1. The Model

Consider a homogeneous good market, where two symmetric firms compete by adjusting their quantities. Production exhibits constant returns to scale and requires only labour input to produce the good. Moreover, each firm possesses a Leontief technology, so the capital stock is always sufficient to produce the good.

The production function of each firm can be defined as $q_i = L_i$ (i = 1, 2), where q (L) denotes output (employment), and the productivity of labour is normalized to unity. Moreover, let the inverse demand function specified of the simple normalized linear form, P(Q) = 1 - Q, where Q is the aggregate output: $Q = q_1 + q_2$.

Firms apply undeclared (declared) labour to $a_i \cdot L_i = a_i \cdot q_i$ $((1 - a_i) \cdot L_i = (1 - a_i) \cdot q_i)$, 0 < a < 1, of their workers. We assume that the unpaid contributions for social insurance are splitted between employer and employee, rate *z (1-z)* for employee (employer), where 0 < z < 1. Thus, the cost for undeclared labour comprises from the wages $a_i \cdot w_i \cdot L_i = a_i \cdot w_i \cdot q_i$ plus the additional amount of splitted contributions paid to employees $z \cdot k \cdot a_i \cdot w_i \cdot L_i = z \cdot k \cdot a_i \cdot w_i \cdot q_i$, where *k* stands for the social insurance contribution rate (0 < k < 1). Adding the two expressions together, the total cost for undeclared labour forms as $(1 + z \cdot k) \cdot a_i \cdot w_i \cdot q_i$.

Respectively to undeclared labour cost, firms' cost for declared labour comprises from the wages $(1 - a_i) \cdot w_i \cdot L_i = (1 - a_i) \cdot w_i \cdot q_i$ plus the contributions for social insurance $k \cdot (1 - a_i) \cdot w_i \cdot L_i = k \cdot (1 - a_i) \cdot w_i \cdot q_i$. Thus, the total cost for declared labour forms as $(1 + k) \cdot (1 - a_i) \cdot w_i \cdot q_i$.

We also assume progressive direct taxation – rate denoted as *t* – for firms' profit formed as $t \cdot \left((p \cdot q_i) - \left((1 + k) \cdot (1 - a_i) \cdot w_i \cdot q_i \right) \right)^2$. Note here that the taxable profits arises abstracting from i's firm revenues only the cost for declared labour. The cost for undeclared labour remains unknown to the authorities.

Summarizing all the above, the firms' net profit function has as follows:

$$\Pi_i = [p \cdot q_i] - [(1 + z \cdot k) \cdot a_i \cdot w_i \cdot q_i] - [(1 + k) \cdot (1 - a_i) \cdot w_i \cdot q_i]$$
$$- \left[t \cdot \left((p \cdot q_i) - \left((1 + k) \cdot (1 - a_i) \cdot w_i \cdot q_i \right) \right)^2 \right]$$

(1)

Firms will choose in the last stage of the game those quantities and that rate of undeclared labour - simultaneously - in order to maximize their profit.

Given risk-neutral fixed membership and immobile labour, according to the utilitarian hypothesis, unions are assumed to maximize rents (for simplicity, we normalize reservation wage to zero, as such a normalization does not qualitatively affect the final state of the equilibrium), reflecting the aggregate labour market preferences of union members. Unions are assumed to be an insider in the labour market, thus having full knowledge of the undeclared labour phenomenon and its size. Assuming proportional taxation for the individuals – employees at the same tax rate t, unions' utility comprises from

-   the income of the undeclared members $(1 + z \cdot k) \cdot a_i \cdot w_i \cdot q_i$
-   the income of the declared members $(1 - a_i) \cdot w_i \cdot q_i$
-   the cost of social insurance of the declared members, valued as a fringe benefit $k \cdot (1 - a_i) \cdot w_i \cdot q_i$
-   minus the taxation of the declared members $t \cdot (1 - a_i) \cdot w_i \cdot q_i$.

Summarizing the above, unions' utility function forms as:

$$U_i = [(1 + z \cdot k) \cdot a_i \cdot w_i \cdot q_i] + [(1 - a_i) \cdot w_i \cdot q_i]$$
$$+ [k \cdot (1 - a_i) \cdot w_i \cdot q_i] - [t \cdot (1 - a_i) \cdot w_i \cdot q_i] \tag{2}$$

Regarding the wage-setting structure, we assume de-facto decentralized wage bargaining regime; each union will negotiate the wage (and thus the employment level) with the relevant firm, considering the maximization of its utility. Unions are moreover assumed to possess a bargaining power of one (monopoly unions) - for simplicity reasons - during labour-management negotiations.

Arising from the above, a two-stage game can be formally addressed as follows:

1. Decentralized wage bargaining takes place, where the wage - and thus the employment – is agreed among firms and unions.
2. Firms determine their quantities in the market (Cournot competition) as well as the optimal level of undeclared labour.

We shall proceed with the further research of the model, using backward induction.

2. Solving the model

Proceeding with the resolution of the model and using backward induction let us consider the second stage of the game first: in the subgame perfect equilibrium (SPE) each firm independently chooses its employment/output level as well as the rate of undeclared labour so as to maximize its profit, given the firm-specific wage contract resulting from Stage 1. Taking first order conditions of the profit functions [1] simultaneously as to quantities and the rates of undeclared labour simultaneously, we derive the optimal output functions, appeared to be as follows:

$$q_1 = \frac{1}{3}(1 - 2(1+k)w_1 + w_2 + kw_2) \tag{3}$$

$$q_2 = \frac{1}{3}(1 - 2(1+k)w_2 + w_1 + kw_1) \tag{4}$$

Furthermore, the derived optimal levels of undeclared labour form as follows:

$$a_1 = \frac{\begin{aligned}2k^2t(6w_1 - 3w_2 - 2)(2w_1 - w_2) + 2k^3t(-2w_1 + w_2)^2 + 2t(1 - 2w_1 + w_2)^2 + \\ k(2t(-1 + 6w_1 - 3w_2)(-1 + 2w_1 - w_2) + 9(z-1))\end{aligned}}{6(1+k)^2 tw_1(-1 + 2(1+k)w_1 - (1+k)w_2)} \tag{5}$$

$$a_2 = \frac{\begin{aligned}(2k^2t(2 + 3w_1 - 6w_2)(w_1 - 2w_2) + 2k^3t(w_1 - 2w_2)^2 + 2t(1 + w_1 - 2w_2)^2 + \\ k(2t(1 + 3w_1 - 6w_2)(1 + w_1 - 2w_2) + 9(z-1)))\end{aligned}}{6(1+k)^2 tw_2(-1 + 2(1+k)w_2 - (1+k)w_1)} \tag{6}$$

Let us therefore proceed to Stage 1 of the game. By virtue of the previous stage and taking first order conditions of unions' utility [2], the following wages are specified:

$$w_1 = \frac{3 + t + k(-1 + 4z)}{(1+k)(9 - 5t + k(5 + 4z))} \tag{7}$$

$$w_2 = \frac{3 + t + k(-1 + 4z)}{(1+k)(9 - 5t + k(5 + 4z))} \tag{8}$$

Replacing expressions [7]-[8] into [1]-[6] and solving the game, we have the following final output:

$$q_1 = \frac{2(1 + k - t)}{9 - 5t + k(5 + 4z)} \tag{9}$$

$$q_2 = \frac{2(1 + k - t)}{9 - 5t + k(5 + 4z)} \tag{10}$$

$$a_1 = \frac{\begin{array}{c}(-8(-1+t)^2t + k(81 - 114t + 57t^2 - 8t^3 - (9-5t)^2z) + k^3(-8t - (-1+z)(5+4z)^2) \\ +2k^2(8t^2 - 9(-5+z+4z^2) + t(-37+5z(1+4z)))) \end{array}}{(4(1+k)(1+k-t)t(3+t+k(-1+4z)))} \tag{11}$$

$$a_2 = \frac{\begin{array}{c}(-8(-1+t)^2t + k(81 - 114t + 57t^2 - 8t^3 - (9-5t)^2z) + k^3(-8t - (-1+z)(5+4z)^2) \\ +2k^2(8t^2 - 9(-5+z+4z^2) + t(-37+5z(1+4z)))) \end{array}}{(4(1+k)(1+k-t)t(3+t+k(-1+4z)))} \tag{12}$$

$$\begin{aligned} U_1 = \frac{1}{2(1+k)^2t(9-5t+k(5+4z))^2}(&-4(-3+t)(-1+t)^2t + kt(121 - 73z + t(-158 + t(57 \\ & -4t - 17z) + 74z)) + k^4(t(4+8z) - (-5+z+4z^2)^2) + k^2(-81(-1+z)^2 \\ & +4t^3(3+2z) + 6t(38 + z(-29+3z)) + t^2(-127 + z(28+15z))) \\ & -k^3(4t^2(3+4z) + 18(-1+z)^2(5+4z) + 3t(-33 + z(7+2(9-4z)z)))) \end{aligned} \tag{13}$$

$$\begin{aligned} U_2 = \frac{1}{2(1+k)^2t(9-5t+k(5+4z))^2}(&-4(-3+t)(-1+t)^2t + kt(121 - 73z + t(-158 + t(57 \\ & -4t - 17z) + 74z)) + k^4(t(4+8z) - (-5+z+4z^2)^2) + k^2(-81(-1+z)^2 \\ & +4t^3(3+2z) + 6t(38 + z(-29+3z)) + t^2(-127 + z(28+15z))) \\ & -k^3(4t^2(3+4z) + 18(-1+z)^2(5+4z) + 3t(-33 + z(7+2(9-4z)z)))) \end{aligned} \tag{14}$$

$$\begin{aligned} \Pi_1 = \frac{1}{4(1+k)^2t(9-5t+k(5+4z))^2}(&16(-1+t)^2t + 16k(-1+t)t(-3+t+(-1+t)z) + k^2(81 \\ & -7t(6+t) - 162z + 2t(114 + t(-57+8t))z + (9-5t)^2z^2) + k^4(25 \\ & +z(16t + (1+4z)(-10+z+4z^2))) - 2k^3(16t^2z - 9(-1+z)^2(5+4z) \\ & +t(17 + z(-54+5z(-3+4z)))))) \end{aligned} \tag{15}$$

$$\begin{aligned} \Pi_2 = \frac{1}{4(1+k)^2t(9-5t+k(5+4z))^2}(&16(-1+t)^2t + 16k(-1+t)t(-3+t+(-1+t)z) + k^2(81 \\ & -7t(6+t) - 162z + 2t(114 + t(-57+8t))z + (9-5t)^2z^2) + k^4(25 \\ & +z(16t + (1+4z)(-10+z+4z^2))) - 2k^3(16t^2z - 9(-1+z)^2(5+4z) \\ & +t(17 + z(-54+5z(-3+4z)))))) \end{aligned} \tag{16}$$

$$p = \frac{5 + k - t + 4kz}{9 + 5k - 5t + 4kz} \tag{17}$$

Continuing our analysis, we further define social revenues and social welfare. Public revenues (R) consist of the contributions for social insurance (R_c) plus the revenues of taxation (R_t), illustrated as below:

$$R_c = \big((1 - a_1) \cdot k \cdot w_1 \cdot q_1\big) + \big((1 - a_2) \cdot k \cdot w_2 \cdot q_2\big) \tag{18}$$

$$\begin{aligned} R_t = &(t \cdot (p \cdot q_1 - (1-a_1) \cdot (1+k) \cdot w_1 \cdot q_1)^2) + (t \cdot (p \cdot q_2 - (1-a_2) \cdot (1+k) \cdot w_2 \cdot q_2)^2) \\ &+ (t \cdot w_1 \cdot (1-a_1) \cdot q_1) + (t \cdot w_2 \cdot (1-a_2) \cdot q_2) \end{aligned} \tag{19}$$

$$R = R_c + R_t \tag{20}$$

The social welfare (SW) results from the aggregation of the unions' utility, the firms' profits and the consumer surplus (CM). Thus, the derived social welfare appears to be as follows:

$$SW = U_1 + U_2 + \Pi_1 + \Pi_2 + CS \tag{21}$$

Substituting the results [9]-[17] to the expressions [18]-[21] and simplifying, we obtain the following results:

$$R_c = \frac{k\left(-\frac{k}{t} + \frac{kz}{t} - \frac{16(1+k)(1+k-t)^2}{(9-5t+k(5+4z))^2} + \frac{4(1+k)(1+k-t)}{9-5t+k(5+4z)}\right)}{(1+k)^2} \tag{22}$$

$$R_t = \frac{\begin{array}{c}(8(-5+t)(-1+t)t^2 + k^4(-5+z+4z^2)^2 + 2k^3(4t^2(1+4z) - t(-10+z)(-1+z)(5+4z) + \\ 9(-1+z)^2(5+4z)) + 2kt(81(-1+z) + t(134-74z+t(-57+4t+9z))) + \\ k^2(81(-1+z)^2 + 54t(-1+z)(5+z) - 16t^3(1+2z) + t^2(181 - z(6+55z)))))\end{array}}{(2(1+k)^2 t(9-5t+k(5+4z))^2)} \tag{23}$$

$$R = \frac{-\frac{k(k+2t)}{t} + 2kz + \frac{k^2z^2}{t} - \frac{32(1+k)(1+k-t)^2(k+t)}{(9-5t+k(5+4z))^2} + \frac{8(1+k)(1+k-t)(k+t)}{9-5t+k(5+4z)}}{2(1+k)^2} \tag{24}$$

$$CS = \frac{8(1+k-t)^2}{(9-5t+k(5+4z))^2} \tag{25}$$

$$SW = \frac{\begin{array}{c}(-8(-7+t)(-1+t)^2t - 2kt(-177 + t(238+4t^2+9t(-9+z)-58z)+65z) + k^4(8t(3+4z) - \\ (-5+z+4z^2)^2) + k^2(-81(-1+z)^2 + 8t^3(5+4z) - 6t(-85+z(20+9z)) + \\ t^2(-357 + z(-58+55z))) + 2k^3(-9(-1+z)^2(5+4z) - 4t^2(7+8z) + \\ t(114+z(33+z(-39+4z)))))\end{array}}{(2(1+k)^2t(9-5t+k(5+4z))^2)} \tag{26}$$

3. Benchmarking Case

Consider a benchmarking state that no undeclared labour exists in the economy. Setting a_1 and a_2 to zero (zero undeclared labour), replacing output functions [3]-[4] and solving the model likewise, we conclude to the following results[1]:

$$w_{1b} = w_{2b} = \frac{1}{3+3k} \tag{27}$$

$$q_{1b} = q_{2b} = \frac{2}{9} \tag{28}$$

$$U_{1b} = U_{2b} = \frac{2(1+k-t)}{27(1+k)} \tag{29}$$

$$\Pi_{1b} = \Pi_{2b} = \frac{4(81-4t)}{6561} \tag{30}$$

$$p_b = \frac{5}{9} \tag{31}$$

$$R_{cb} = \frac{4k}{27(1+k)} \tag{32}$$

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[1] Note that we denote benchmark case with an index *b*.

$$R_{tb} = \frac{4(251 + 8k)t}{6561(1 + k)} \tag{33}$$

$$R_b = \frac{4(251t + k(243 + 8t))}{6561(1 + k)} \tag{34}$$

$$CS_b = \frac{8}{81} \tag{35}$$

$$SW_b = \frac{28}{81} - \frac{4(251 + 8k)t}{6561(1 + k)} \tag{36}$$

The side effects of undeclared labour in goods market as well as in labour market will be revealed by the comparison of the model's results to the corresponding ones of the benchmarking case.

4. Undeclared Labour in Unionized Oligopoly

In this section, we shall compare the results of our model vs. the benchmarking case, in order to reveal the role of undeclared labour in the economy and the nature of its influence. Begging with firms' output, abstracting expression [28] from [9]:

$$q_1 - q_{1b} = -\frac{8(t + k(-1 + z))}{9(9 - 5t + k(5 + 4z))}$$

The expression above has one root at $t^* = k - kz$, thus we conclude to:

- If $t > k - kz$, then $q_1 - q_{1b} < 0 \rightarrow q_1 < q_{1b}$ and
- If $t < k - kz$, then $q_1 - q_{1b} > 0 \rightarrow q_1 > q_{1b}$.

It proves that if the tax rate is low enough, lower than $k - kz$, then undeclared labour will increase the firms' output. And since we have made the assumption that the productivity equals to unity, the same results apply for employment proportionally. Reverse criterion applies for the price, though at the same critical value. Subtracting expression [31] from [17]:

$$p - p_b = \frac{16(t + k(-1 + z))}{9(9 - 5t + k(5 + 4z))}$$

The expression above has one root at $t^* = k - kz$, and thus

- If $t > k - kz$, then $p - p_b > 0 \rightarrow p > p_b$ and
- If $t < k - kz$, then $p - p_b < 0 \rightarrow p < p_b$.

Similar effects apply also for the wages in the equilibrium. Subtracting expression [27] from [7], we obtain the following results:

$$w_1 - w_{1b} = \frac{8(t + k(-1 + z))}{3(1 + k)(9 - 5t + k(5 + 4z))}$$

The expression above has one root at $t^* = k - kz$. Therefore,

- If $t > k - kz$, then $w_1 - w_{1b} > 0 \rightarrow w_1 > w_{1b}$ and
- If $t < k - kz$, then $w_1 - w_{1b} < 0 \rightarrow w_1 < w_{1b}$.

Proposition 1 summarizes:

Proposition 1:

If t is low enough, lower than $k - kz$, then undeclared labour will give more output – and therefore employment –, lower price at the final equilibrium and simultaneously lower clearing wages, compared to the full declared labour state. If on the other hand $t > k - kz$, the opposite state apply.

Proceeding with profit analysis, abstracting expression [30] from expression [15], we obtain the following results:

$$\Pi_1 - \Pi_{1b} = \frac{\begin{array}{c}(64t^2(-1377 + t(1044 + 25t)) + k^4(64t^2(5 + 4z)^2 - 1296t(-1 + z)(-25 + 16z) + 6561(-5 + z + 4z^2)^2) + \\ 2k^3(-320t^3(5 + 4z) + 59049(-1 + z)^2(5 + 4z) + 16t^2(2305 + z(-4637 + 64z)) - \\ 81t(-1 + z)(-2497 + z(661 + 1620z))) + 16kt(729(-1 + z) + t(-18(367 + 533z) + \\ t(1591 + 200t + 6401z))) + k^2(1600t^4 + 531441(-1 + z)^2 - 162t(-1 + z)(-3989 + 3773z) + \\ 16t^3(-2785 + 6241z) + t^2(218617 + z(-632498 + 165049z)))))\end{array}}{(26244(1 + k)^2 t(9 - 5t + k(5 + 4z))^2)}$$

The latter expression has no root determined. However, it can be shown that for specific values of t, the difference above turns out positive, meaning that profits under undeclared labour turns out greater than the corresponding ones in benchmarking case (declared labour). Furthermore, it can also be shown that the

derivative of the difference above with respect to t, signs negative; interpreting the latter finding, we argue that as t increases, practicing undeclared labour becomes less attractive from the firms, as regard to their profit. Proposition 2 summarizes:

Proposition 2:

For any z, k $\epsilon (0,1)$, there exist a function of $t_1(z,k)$ such as $\Pi_1 - \Pi_{1b} = 0$, with $\frac{\partial(\Pi_1 - \Pi_{1b})}{\partial t} < 0$, for which:

- if $t < t_1 \rightarrow \Pi_1 > \Pi_{1b}$, then profits under undeclared labour turns greater than profits gained in full declared labour state

- if $t > t_1 \rightarrow \Pi_1 < \Pi_{1b}$, then undeclared labour will grant firms with less profits rather than declared.

The proof of Proposition 2 is illustrated in the Appendix.

As $\frac{\partial(\Pi_1 - \Pi_{1b})}{\partial t} < 0$, we conclude that a low tax rate will strengthen the incentives for firms to practice undeclared labour. As the tax rate increases, firms pay even more taxes. Thus, their strategic choice will alter to declared labour, in order to properly declare their payroll costs and thus claim a tax deduction. Therefore, the lower the tax rate is, the more strengthened incentives are formulated for firms to practice undeclared labour.

Continuing with unions' utility, abstracting expression [29] from [13]:

$$U_1 - U_{1b} = -\frac{\begin{array}{c}(t + k(-1 + z))(8(-9 + t)(-1 + t)t + k^3(27(-1 + z)(5 + 4z)^2 + 8t(1 + 8z)) \\ -2k^2\left(8t^2(1 + 4z) - 243(-5 + z + 4z^2) + t(-719 + z(71 + 540z))\right)) + \\ k(2187(-1 + z) + t(2582 - 2366z + t(-771 + 8t + 611z))))\end{array}}{54(1 + k)^2 t(9 - 5t + k(5 + 4z))^2}$$

The latter expression has no root determined. However, it can be shown that for specific values of *t*, the difference above turns positive, proving that – under certain circumstances – unions' utility under undeclared labour may turn greater rather than the corresponding one in the benchmarking case (declared labour).

Proposition 3:

For any z, k $\epsilon (0,1)$, there exist functions of $t_2(z,k)$ and $t_3(z,k)$ such as $U_1 - U_{1b} = 0$, for which:

- if $t_2 < t < t_3 \rightarrow U_1 > U_{1b}$, then union's utility under undeclared labour turns greater than union's utility in full declared labour state

- if $t < t_2 \, or \, t > t_3 \rightarrow U_1 < U_{1b}$, then undeclared labour will lend unions with less utility rather than declared labour state.

The proof of Proposition 3 is illustrated in the Appendix.

Interpreting the above, firm's optimal undeclared labour rate may increase unions' utility. It can be shown that as the tax rate increases, it is more possible that unions' utility will be greater under undeclared labour. Thus, we can reasonably argue that as the tax rate increases, unions' incentive to collude with firms and practice undeclared labour is even more strengthened.

As regard to the public revenues, those can be categorized into two main categories; revenues from taxation and revenues from contributions for social insurance. Total public revenues result from the aggregation of these two illustrated categories. We shall examine each category discretely.

Let us examine revenues from taxation first. Abstracting expression [33] from expression [23], we obtain the following result:

$$R_t - R_{tb} = -\frac{\begin{array}{c} -6561k^2 + 13122kt + 2008t^2 + 2072kt^2 + 64k^2t^2 + 13122k(k-t)z - 6561k^2z^2 \\ + \frac{209952(1+k)(1+k-t)^2t^2}{(9-5t+k(5+4z))^2} - \frac{52488(1+k)(1+k-t)t^2}{9-5t+k(5+4z)} \end{array}}{13122(1+k)^2t}$$

For the expression above, none analytically tractable formula can be obtained. Thus, it can be shown that there exist two different functions of t, $t_4(z,k)$: $R_t - R_{tb} = 0$ and $t_5(z,k)$: $R_t - R_{tb} = 0$, such as:

- If $t_4(z,k) < t < t_5(z,k) \rightarrow R_t - R_{tb} < 0 \rightarrow R_t < R_{tb}$ and
- If $t < t_4(z,k) \, or \, t > t_5(z,k) \rightarrow R_t - R_{tb} > 0 \rightarrow R_t > R_{tb}$.

It reveals that, under $t < t_4(z,k)$ or $t > t_5(z,k)$, firms' optimal rate of undeclared labour may produce more public revenues from taxation, rather than the corresponding ones in benchmarking state (full declared labour).

Continuing our analysis, let us now proceed with public revenues from contributions for social insurance. Abstracting expression [32] from expression [22], we obtain the following results:

$$R_c - R_{cb} = \frac{k(-4 + k(-4 - \frac{27}{t}) + \frac{27kz}{t} - \frac{432(1+k)(1+k-t)^2}{(9-5t+k(5+4z))^2} + \frac{108(1+k)(1+k-t)}{9-5t+k(5+4z)})}{27(1+k)^2}$$

For the expression above, once again, none analytically tractable formula can be obtained. Despite the limitation above, it can be shown that there exists a function of t, such as $t_6(k,z)$: $R_c - R_{cb} = 0$, that applies:

- If $t < t_6(k,z) \to R_c - R_{cb} < 0 \to R_c < R_{cb}$ while
- If $t > t_6(k,z) \to R_c - R_{cb} > 0 \to R_c > R_{cb}$.

Interpreting the above, we observe that if $t > t_6(k,z)$, firms' optimal rate of undeclared labour may produce more public revenues from contributions for social insurance, rather than the corresponding ones in benchmarking state (full declared labour).

Finally, we examine total public revenues. Abstracting expression [34] from expression [24], we obtain the following results:

$$R - R_b = \frac{\begin{array}{c} -486k(31+4k) - \frac{6561k^2}{t} - 8(1+k)(251+8k)t + 13122kz + \\ \frac{6561k^2z^2}{t} - \frac{209952(1+k)(1+k-t)^2(k+t)}{(9-5t+k(5+4z))^2} + \frac{52488(1+k)(1+k-t)(k+t)}{9-5t+k(5+4z)} \end{array}}{13122(1+k)^2}$$

For the expression above, none analytically tractable formula can be obtained. It can be shown that there exists a function of t, such as $t_7(k,z)$: $R - R_b = 0$, that applies:

- If $t < t_7(k,z) \to R - R_b < 0 \to R < R_b$ while
- If $t > t_7(k,z) \to R - R_b > 0 \to R > R_b$.

Therefore, we conclude that if t is high enough, higher than $t_7(k,z)$, then undeclared labour will yield more public revenues than the benchmarking case, where none undeclared labour exists. Proposition 4 summarizes.

Proposition 4:

For any z, k $\epsilon (0,1)$, there exist functions of $t_4(z,k)$: $R_t - R_{tb} = 0$, $t_5(z,k)$: $R_t - R_{tb} = 0$, $t_6(z,k)$: $R_c - R_{cb} = 0$ and $t_7(z,k)$: $R - R_b = 0$, for which:

- If $t_4(z,k) < t < t_5(z,k)$, then public revenues from taxation in the case of undeclared labour will be less than the corresponding ones in the case that no undeclared labour exists. If, on the other hand, $t < t_4(z,k)$ or $t > t_5(z,k)$, then undeclared labour will yield greater revenues from taxation.

- If $t < t_6(k,z)$, then public revenues from contributions for social insurance will be less in the case of undeclared labour, compared to the corresponding ones in the benchmarking case. Contrariwise, if $t > t_6(k,z)$, then undeclared labour will yield greater public revenues derived from contributions compared to the benchmarking case, where no undeclared labour exist.

- If $t < t_7(k,z)$, then total public revenues in the undeclared labour state will be less comparing to the case that no undeclared labour exists (benchmark). Contrary to common knowledge, if $t > t_7(k,z) \rightarrow R - R_b > 0 \rightarrow R > R_b$, then undeclared labour will contribute more to the state budget, comparing to the benchmarking case.

The proof of Proposition 4 is illustrated in the Appendix.

Examining the effect of undeclared labour in consumer surplus, we abstract expression [35] from [25] and we obtain the following results:

$$CS - CS_b = -\frac{64(t + k(-1 + z))(9 - 7t + k(7 + 2z))}{81(9 - 5t + k(5 + 4z))^2}$$

The expression above has two roots,

- $t_1 = \frac{9}{7} + k + \frac{2kz}{7}$, which is rejected as greater than 1 for each and every $k, z \in (0,1)$ and

- $t_2 = k - kz$, which root is accepted.

Consequently,

- If $t > k - kz$, then $CS - CS_b < 0 \rightarrow CS < CS_b$ and
- If $t < k - kz$, then $CS - CS_b > 0 \rightarrow CS > CS_b$

Proposition 5 summarizes.

Proposition 5:

For any z, k $\epsilon(0,1)$, if $t > k - kz$, then undeclared labour will reduce consumer surplus, compared to the non-undeclared labour state. If on the other hand the sufficiently low, lower than $k - kz$, then undeclared labour will yield greater consumer surplus.

Recall Proposition 1; under the same criterion, $t < k - kz$, undeclared labour will modulate lower price and greater product in the market compared to the fully declared labour state. Thus, it results that consumer surplus will be greater too, since it jointly depends from price and the quantities.

Finally, let us now proceed with social welfare. Abstracting expression [36] from [26], we result to the following:

$$SW - SW_b = \frac{1}{13122(1+k)^2}\left(-162(28 + k(-25 + 28k)) - \frac{6561k^2}{t} + 8(1+k)(251 + 8k)t\right.$$
$$+ \frac{13122k(k-t)z}{t} - \frac{6561k^2z^2}{t} - \frac{104976(1+k)(1+k-2t)(1+k-t)^2}{(9 - 5t + k(5+4z))^2}$$
$$\left.+ \frac{52488(1+k)(1+k-t)^2}{9 - 5t + k(5+4z)}\right)$$

For the expression above, none analytically tractable formula can be obtained. It can be shown that there exist two functions of t, such as *t₈(k,z):* $SW - SW_b = 0$ and *t₉(k,z): * $SW - SW_b = 0$, that applies:

- If $t_8(k,z) < t < t_9(k,z)$, then $SW - SW_b > 0 \rightarrow SW > SW_b$ and
- If $t < t_8(k,z)$ or $t > t_9(k,z)$, then $SW - SW_b < 0 \rightarrow SW < SW_b$.

Proposition 6 summarizes.

Proposition 6:

For any z, $k \in (0,1)$, if $t_8(k,z) < t < t_9(k,z)$, then undeclared labour will produce greater social welfare in comparison to declared labour case (benchmark). If on the other hand $t < t_8(k,z)$ or $t > t_9(k,z)$, then social welfare will be greater in fully declared labour state, rather than the undeclared one.

The proof of Proposition 6 is illustrated in the Appendix.

5. Pareto Optimal Tax Rate

In this section, we argue that, in the undeclared labour case, there exists such a tax rate that may consist a Pareto optimal compared to the benchmarking case. Interpreting the previous argument, there exists such a tax rate t* that all agents – firms, unions, consumers and community – enjoy equal or even greater payoffs in undeclared labour state rather than in the benchmarking one.

Consider the imposition of a tax rate $t^* = (1-z) \cdot k$. Replacing t^* to expressions [7] to [26] for the undeclared labour case and [27] to [36] for the benchmarking case, we obtain the following results:

Price

As mentioned in proposition 1, $t^* = (1-z) \cdot k$ equates p and p_b.

Quantity (Employment)

As mentioned in proposition 1, $t^* = (1-z) \cdot k$ equates q_i and q_ib and thus the employment, as the production function forms q_i=L_i (i=1,2).

Wages

As mentioned in proposition 1, $t^* = (1-z) \cdot k$ equates w_i and w_ib.

Profits

Substituting t* into the profit expressions of each case, we obtain:

$\Pi_1^* = \frac{16+97k+k(-65+16k)z}{324(1+k)^2}$ and $\Pi_{1b}^* = \frac{4(81+4k(-1+z))}{6561}$, while their subtraction concludes

to $\Pi_1^* - \Pi_{1b}^* = \frac{(73-8k)^2 k(1-z)}{26244(1+k)^2}$. As $z\epsilon(0,1) \to (1-z) > 0$, the mark of all factors of

the quotient remain positive, thus $\Pi_1^* - \Pi_{1b}^* > 0 \to \Pi_1^* > \Pi_{1b}^*$.

Unions' Utility

Substituting t* into the Utility expressions of each case, we obtain:

$U_1^* = U_{1b}^* = \frac{2(1+kz)}{27(1+k)}$

Thus, $t^* = (1-z) \cdot k$ equates U_i and U_{ib}.

Consumer Surplus

As mentioned in proposition 5, $t^* = (1-z) \cdot k$ equates CS and CS_b.

Social Welfare

Substituting t* into the Social Welfare expressions of each case, we obtain:

$SW^* = \frac{56+k(153-41z+8k(2+5z))}{162(1+k)^2}$ and $SW_b^* = \frac{4(567+k(316+8k(-1+z)+251z))}{6561(1+k)}$, while their

subtraction concludes to $SW^* - SW_b^* = \frac{(73-8k)^2 k(1-z)}{13122(1+k)^2}$. As $z\epsilon(0,1) \to (1-z) > 0$,

the mark of all factors of the quotient remain positive, thus $SW^* - SW_b^* > 0 \to$

$SW^* > SW_b^*$.

Proposition 7 summarizes the results.

Proposition 7:

Assume a labour market where firms determine their optimal rate of undeclared labour and a benchmarking case, where no undeclared labour is practiced. For any z, k $\epsilon(0,1)$, the imposition of a direct tax rate $t^* = (1-z) \cdot k$ consists a Pareto optimal for the first case compared to the second, as all agents enjoy equal or even greater

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payoffs; Unions' Utility and Consumer Surplus will remain immutable, while Firms' Profits and Social Welfare will increase.

We should also stress out that this Pareto optimal t* lacks of financing. Substituting t* into the Public Revenues expressions of each case, we obtain:

$R^* = \frac{k(-163-40k(-2+z)+41z)}{162(1+k)^2}$ and $R_b^* = -\frac{4k(-494+8k(-1+z)+251z)}{6561(1+k)}$, while their subtraction concludes to $R^* - R_b^* = \frac{k(-73+8k)(235+8k(-1+z)-73z)}{13122(1+k)^2}$. The mark of the quotient remains negative, thus $R^* - R_b^* < 0 \rightarrow R^* < R_b^*$.

Interpreting the above, if a benevolent social planner implies a policy setting t* in order to handle the undeclared labour phenomenon, then he will have to seek also for additional funding, as the public revenues will thereby suffer losses.

6. Conclusions

Undeclared labour constitutes a complex phenomenon, where tax evasion and social security fraud are involved. Both employers and employees voluntarily collude, because of the potential gain in avoiding taxes and social security contributions, social rights and the cost of complying with regulations. In our research, we highlighted this opportunity cost and revealed the effects that undeclared labour creates respectively to all market's major fundamentals.

As it concerns our present research, we introduced a model that endogenizes undeclared labour and analyzes the phenomenon within I/O framework. We endogenized the selection of the optimal rate of undeclared labour from the firms - simultaneously with the quantities. Furthermore, model's assumptions include progressive taxation for firms and proportional taxation for the rest (e.g. members of the union). We assumed that the extra cost for social insurance is splitted between

employer and employee. Furthermore, the profit/utility functions were properly adjusted to reflect and highlight the opportunity cost between taxation and contributions for social insurance; firms will either declare their personnel and pay contributions - but less taxes - or they will practice undeclared labour and pay less contributions - but more taxes. Unions face relevant dilemma, either they collude with firms to undeclared labour, and thus they are paid more, the pay less taxes but they lack of insurance, or they do not consent to undeclared labour, and thus they earn less, they pay more taxes and they enjoy insurance. Finally, we additionally constructed a benchmarking case with zero undeclared labour and compared those two cases.

The findings of our analysis evince that the side effects of undeclared labour are not clearly visible. Contrary to common knowledge, if *t* is low enough, the rate of undeclared labour that maximizes firms' profit will yield greater clearing wages, greater output and thus employment, greater consumer surplus and lower price. Moreover, we showed that under certain circumstances, undeclared labour may increase firms' profits and unions' utility, but may also increase public revenues and social welfare. Finally, we argue that an imposition of a tax rate $t^* = (1 - z) \cdot k$ consists a Pareto optimal policy for the case of undeclared labour case compared to the benchmarking one; the imposition of such a tax rate, will grant all agents, e.g. firms, unions, consumers and the community, with equal or even greater payoffs.

Since the project has not any relative research background, possible extensions of this research may be yet quite more promising. Further research may include different types of competition (e.g. Bertrand Competition), different types of wage bargaining (e.g. centralized bargaining, non-monopoly unions), endogenization of state's interference in labour market (e.g. screening for undeclared labour) and a cost-benefit analysis for the determination of the optimal governmental surveillance's cost or the social's optimal rate of undeclared labour. The forthcoming research will comprise a key role in order for us to acquire a spherical knowledge of the undeclared labour phenomenon and its side effects.

Appendix

Proof of Proposition 2:

Abstracting expression [30] from expression [15], we obtain the following results:

$$\Pi_1 - \Pi_{1b} = \frac{\begin{array}{c}(64t^2(-1377 + t(1044 + 25t)) + k^4(64t^2(5 + 4z)^2 - 1296t(-1 + z)(-25 + 16z) + 6561(-5 + z + 4z^2)^2) + \\ 2k^3(-320t^3(5 + 4z) + 59049(-1 + z)^2(5 + 4z) + 16t^2(2305 + z(-4637 + 64z)) - \\ 81t(-1 + z)(-2497 + z(661 + 1620z))) + 16kt(729(-1 + z) + t(-18(367 + 533z) + \\ t(1591 + 200t + 6401z))) + k^2(1600t^4 + 531441(-1 + z)^2 - 162t(-1 + z)(-3989 + 3773z) + \\ 16t^3(-2785 + 6241z) + t^2(218617 + z(-632498 + 165049z))))\end{array}}{(26244(1 + k)^2 t(9 - 5t + k(5 + 4z))^2)}$$

For the expression above, none analytically tractable formula can be obtained. Nonetheless, we can still check for the sign of $\Pi_1 - \Pi_{1b}$ by contour-plotting the [15]-[30] difference over a fine grid of our critical z and k parameters.
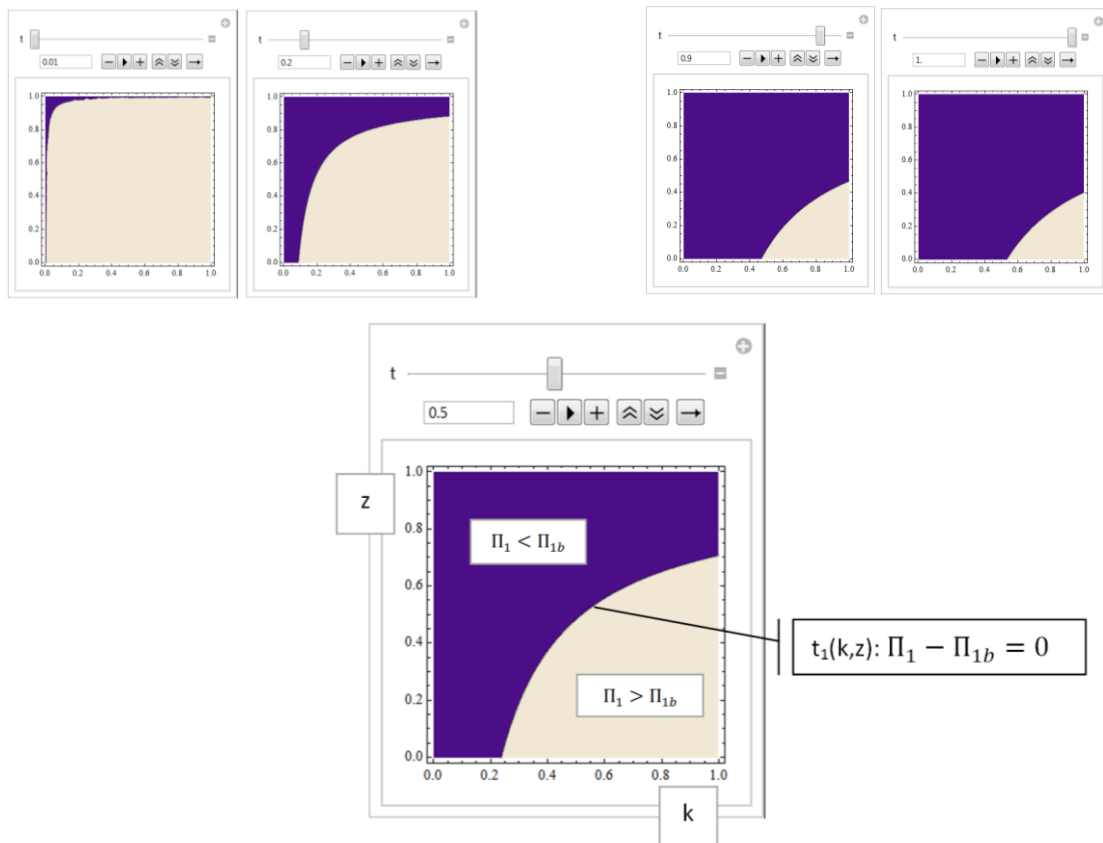
By inspecting the plots above, it can be checked that if $t < t_1 \rightarrow \Pi_1 > \Pi_{1b}$, while if $t > t_1 \rightarrow \Pi_1 < \Pi_{1b}$.

Moreover, to examine the influence of the variation of the tax rate t over the difference $\Pi_1 - \Pi_{1b}$, we take the first differentiate of $\Pi_1 - \Pi_{1b}$ respect to t.

$$\frac{\partial(\Pi_1 - \Pi_{1b})}{\partial t} = \frac{64(1+k)^2 - \frac{6561k^2}{t^2} + \frac{13122k^2z}{t^2} - \frac{6561k^2z^2}{t^2} - \frac{1312200(1+k)(1+k-t)^3}{(9-5t+k(5+4z))^3} + \frac{524880(1+k)(1+k-t)^2}{(9-5t+k(5+4z))^2} - \frac{52488(1+k)(1+k-t)}{9-5t+k(5+4z)}}{26244(1+k)^2}$$

Once again, none analytically tractable formula can be obtained for the derivative above. Thus, we check for its sign by contour-plotting the expression above over a fine grid of our critical z and k parameters.
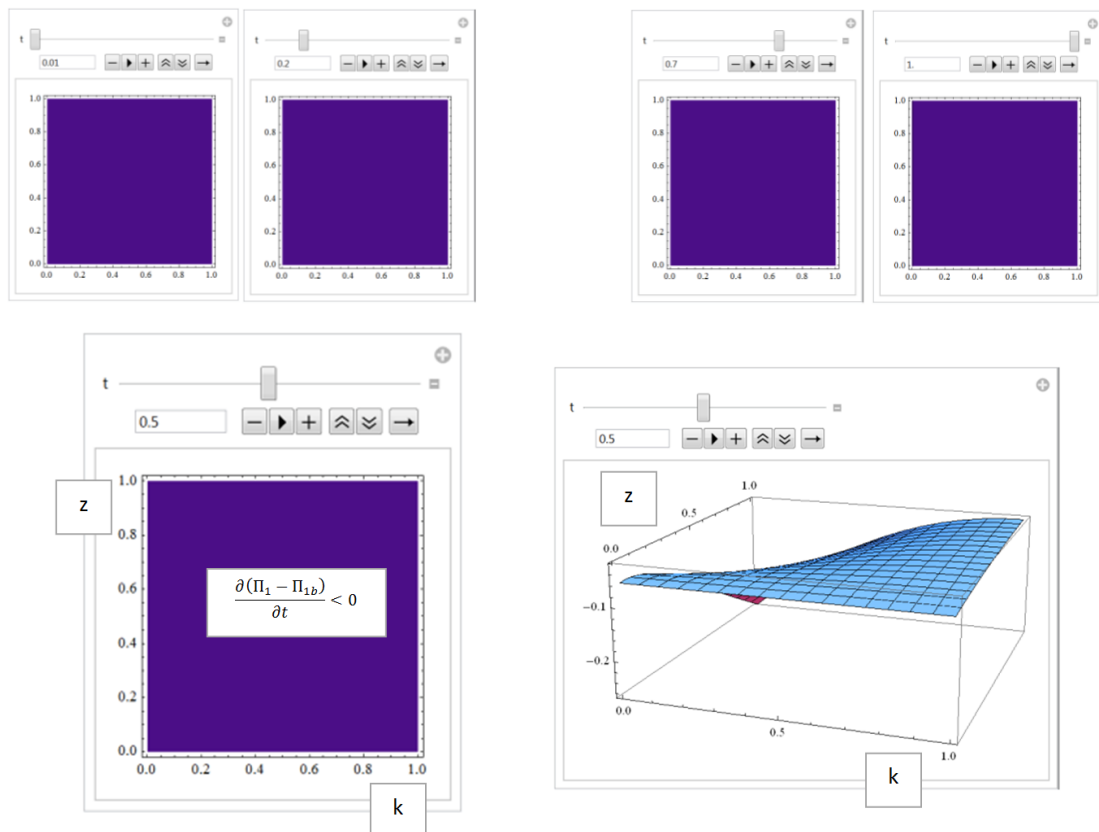
By inspecting these plots it can be checked the negative relationship between tax rate and the difference between the profits under undeclared labour minus the profits in a fully declared labour state; the lower the tax rate is, the more strengthened incentives are formulated for firms to practice undeclared labour.

Proof of Proposition 3:

Examining unions' utility, we abstract expression [29] from [13]:

$$U_1 - U_{1b} = -\frac{\begin{aligned}(t + k(-1+z))(8(-9+t)(-1+t)t + k^3(27(-1+z)(5+4z)^2 + 8t(1+8z)) \\ -2k^2\left(8t^2(1+4z) - 243(-5+z+4z^2) + t(-719+z(71+540z))\right) + \\ k(2187(-1+z) + t(2582 - 2366z + t(-771 + 8t + 611z))))\end{aligned}}{54(1+k)^2 t(9-5t+k(5+4z))^2}$$

Since there cannot be determined any root for the expression above, we shall check for the sign of $U_1 - U_{1b}$ by contour-plotting the [29]-[13] difference over a grid of our critical z and k parameters.
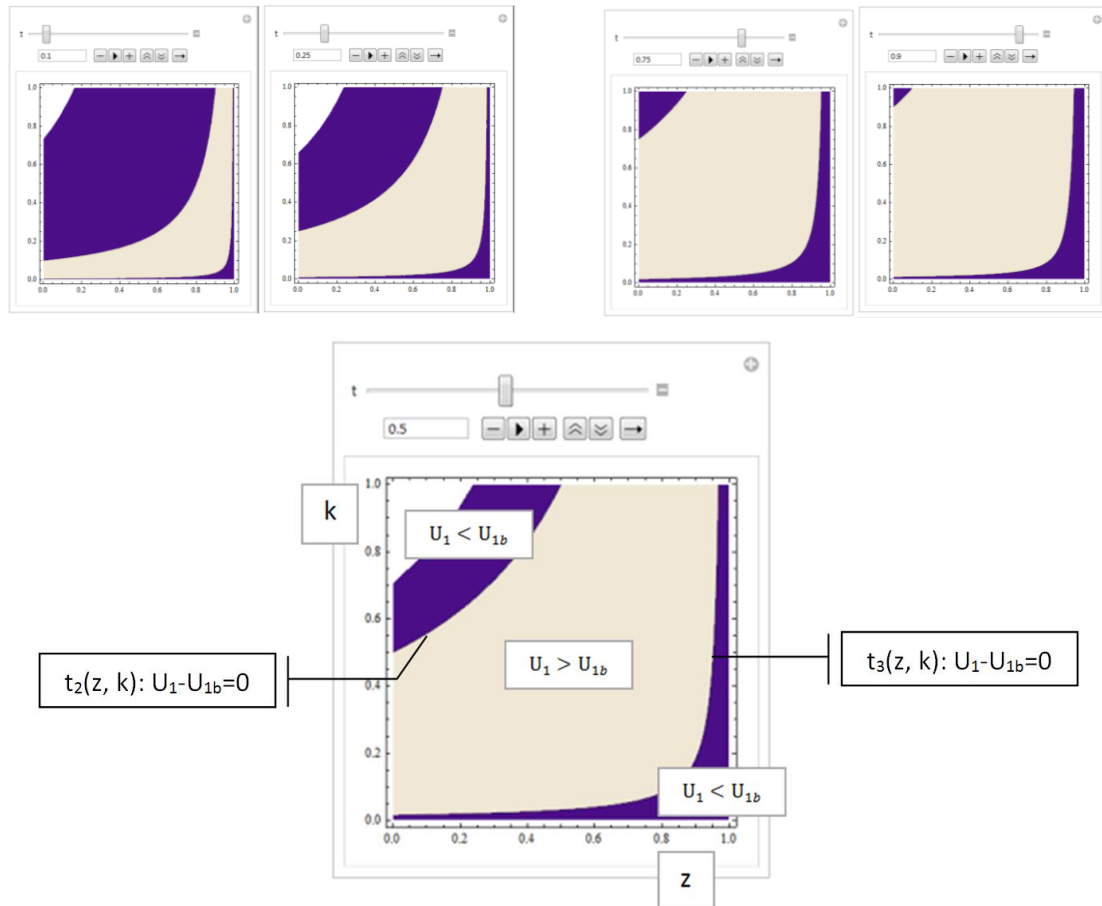
As illustrated above, firm's optimal undeclared labour rate may increase unions' utility. From the examination of the diagrams above (e.g., compare diagram for t=0.1 vs diagram for t=0.9), as the tax rate increases, it is more possilbe that unions' utility will be greater under undeclared labour. Thus, we can resonably argue that as the tax rate increases, unions' incentive to collude with firms and practice undeclared labour is even more strenghtened.

Proof of Proposition 4:

Public Revenues from Taxation:

Abstracting expression [33] from expression [23], we obtain the following results:

$$R_t - R_{tb} = -\frac{-6561k^2 + 13122kt + 2008t^2 + 2072kt^2 + 64k^2t^2 + 13122k(k-t)z - 6561k^2z^2 + \frac{209952(1+k)(1+k-t)^2t^2}{(9-5t+k(5+4z))^2} - \frac{52488(1+k)(1+k-t)t^2}{9-5t+k(5+4z)}}{13122(1+k)^2t}$$

For the expression above, none analytically tractable formula can be obtained. Nonetheless, we can still check for the sign of $R_t - R_{tb}$ by contour-plotting the [23]-[33] difference over a grid of our critical z and k parameters.
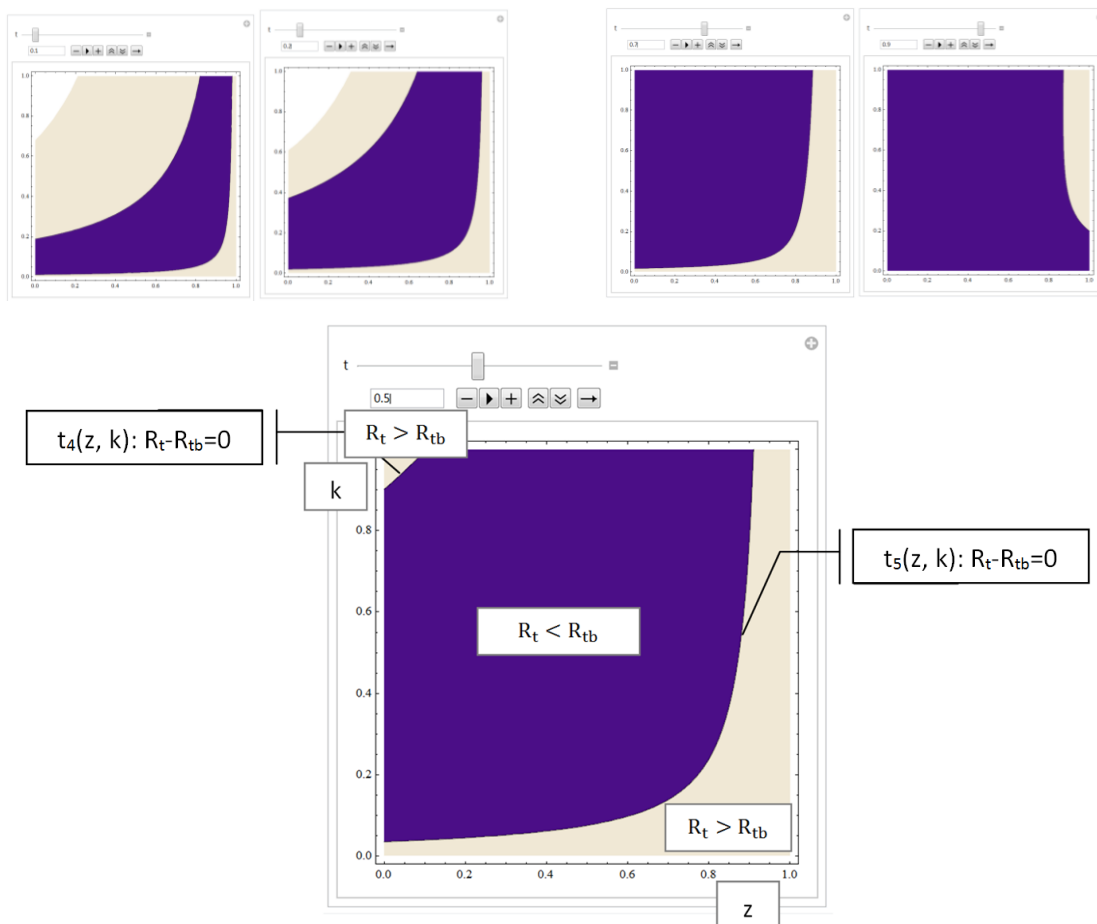
Public Revenues from Contributions:

Abstracting expression [32] from expression [22], we obtain the following results:

$$R_c - R_{cb} = \frac{k(-4 + k(-4 - \frac{27}{t}) + \frac{27kz}{t} - \frac{432(1+k)(1+k-t)^2}{(9-5t+k(5+4z))^2} + \frac{108(1+k)(1+k-t)}{9-5t+k(5+4z)})}{27(1+k)^2}$$

For the expression above, none analytically tractable formula can be obtained. Nonetheless, we can still check for the sign of $R_c - R_{cb}$ by contour-plotting the [22]-[32] difference over a grid of our critical z and k parameters.
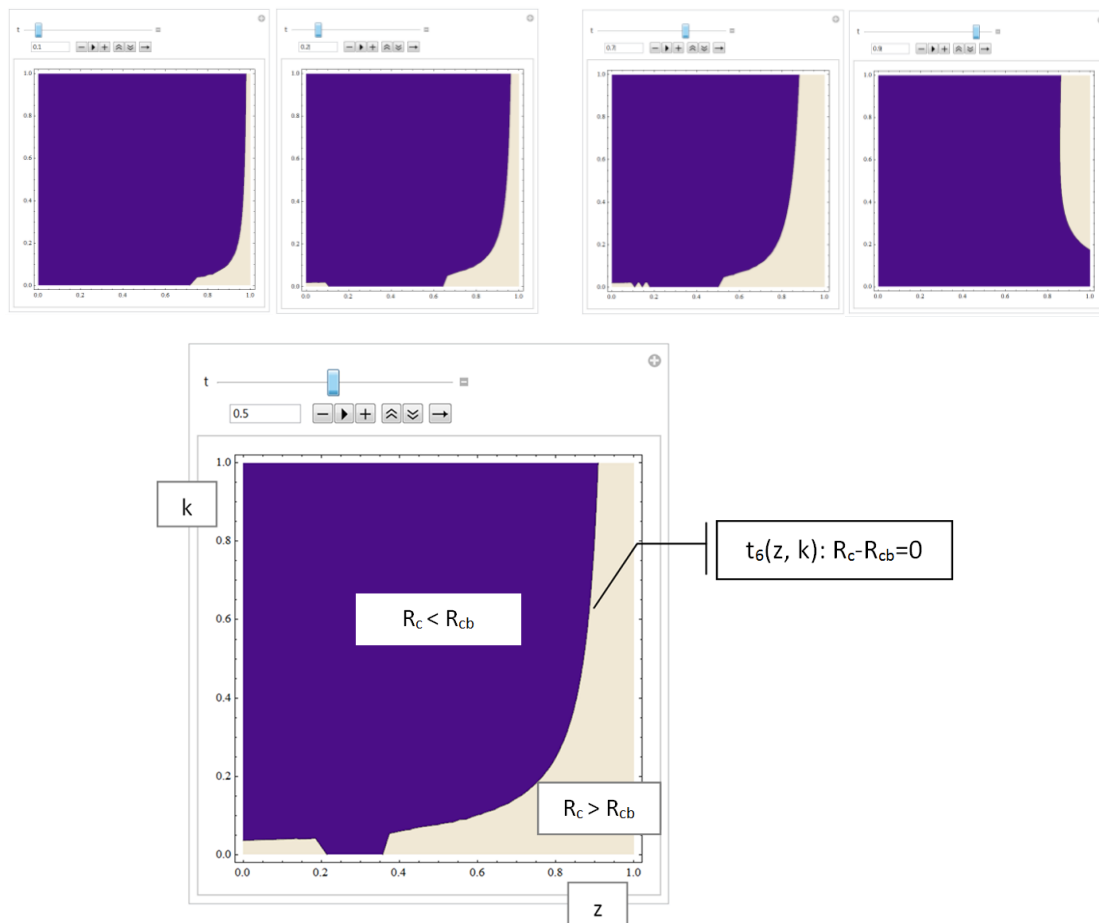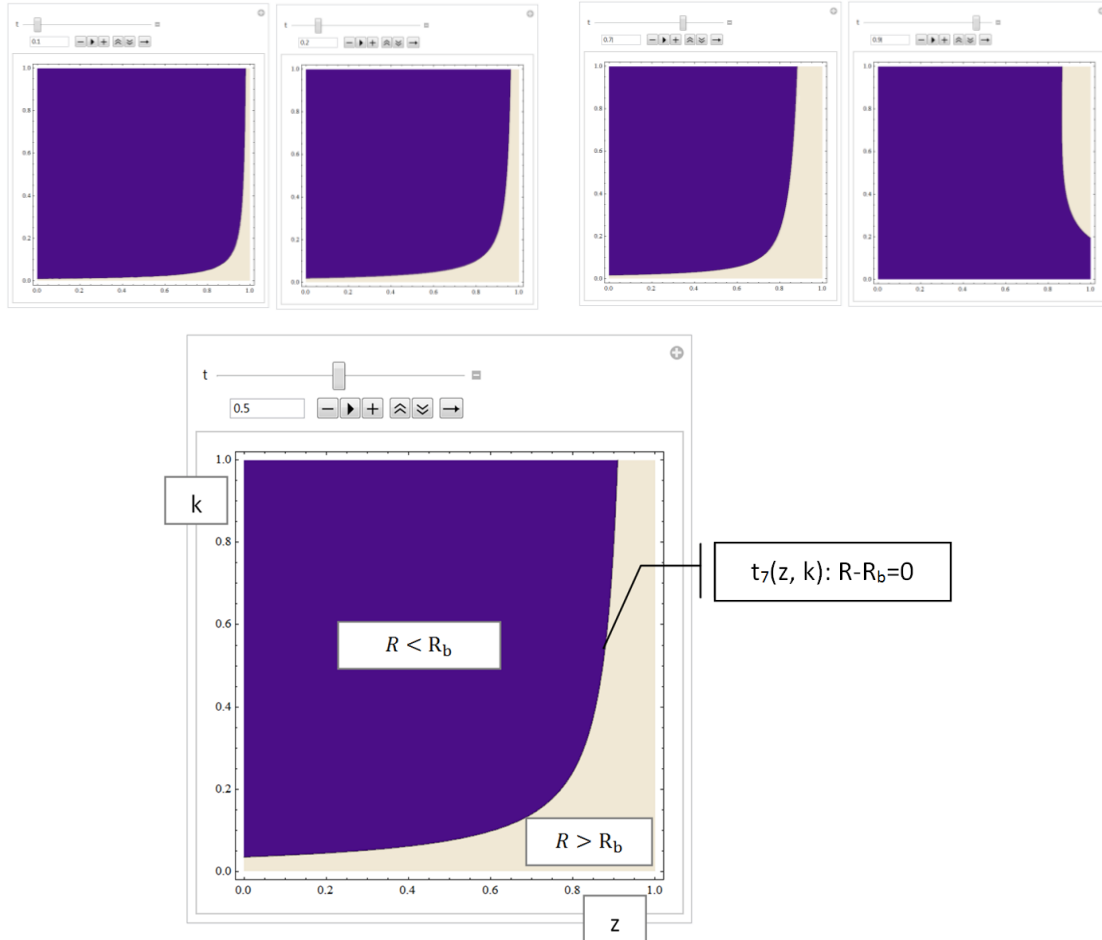

Total Public Revenues:

Abstracting expression [34] from expression [24], we obtain the following results:

$$R - R_b = \frac{-486k(31+4k) - \frac{6561k^2}{t} - 8(1+k)(251+8k)t + 13122kz + \frac{6561k^2z^2}{t} - \frac{209952(1+k)(1+k-t)^2(k+t)}{(9-5t+k(5+4z))^2} + \frac{52488(1+k)(1+k-t)(k+t)}{9-5t+k(5+4z)}}{13122(1+k)^2}$$

For the expression above, none analytically tractable formula can be obtained. Nonetheless, we can still check for the sign of $R - R_b$ by contour-plotting the [24]-[34] difference over a grid of our critical z and k parameters.


Proof of Proposition 6:

Abstracting expression [36] from expression [26], we obtain the following results:

$$SW - SW_b = \frac{-162(28 + k(-25+28k)) - \frac{6561k^2}{t} + 8(1+k)(251+8k)t + \frac{13122k(k-t)z}{t} - \frac{6561k^2z^2}{t} - \frac{104976(1+k)(1+k-2t)(1+k-t)^2}{(9-5t+k(5+4z))^2} + \frac{52488(1+k)(1+k-t)^2}{9-5t+k(5+4z)}}{13122(1+k)^2}$$

For the expression above, none analytically tractable formula can be obtained. Nonetheless, we can still check for the sign of $SW - SW_b$ by contour-plotting the [26]-[36] difference over a grid of our critical z and k parameters.
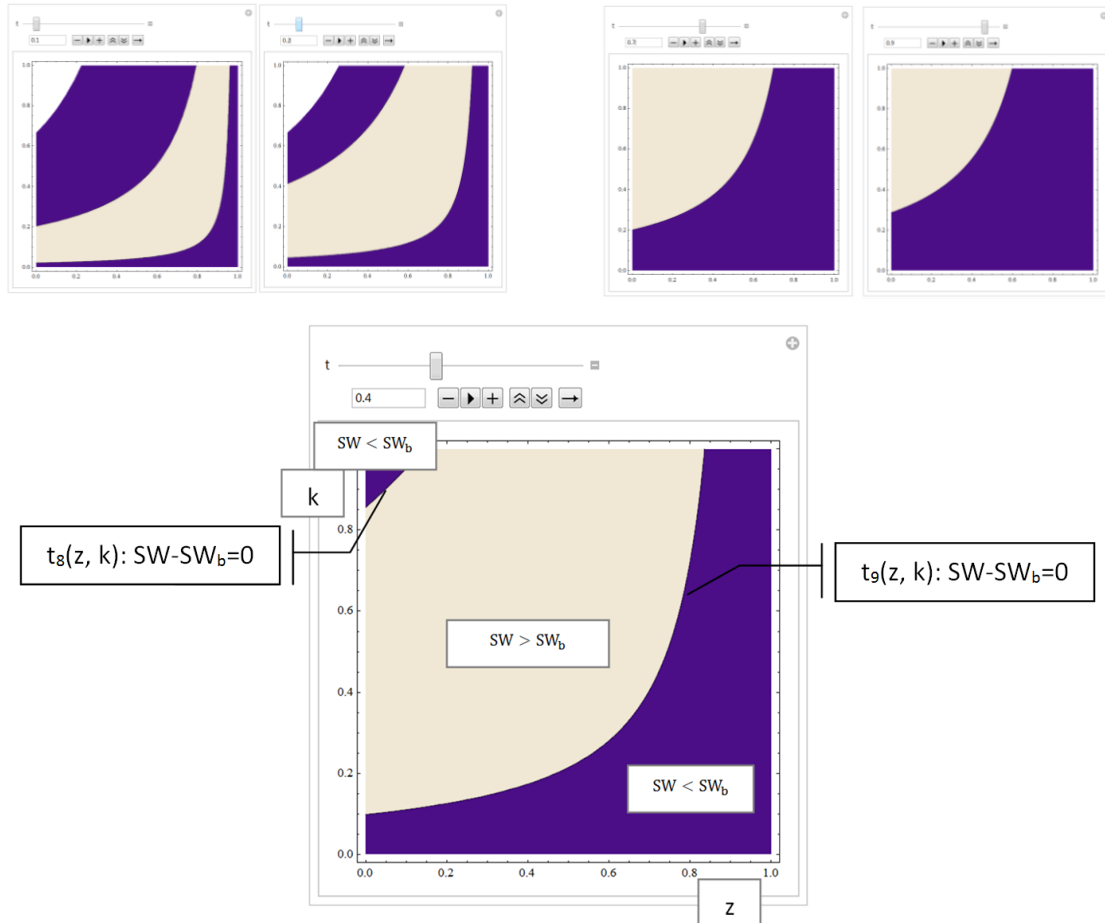
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