Pure Strategies and Undeclared Labour in Unionized Oligopoly

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Abstract

In a unionized Cournot duopoly under decentralized wage bargaining regime, we analyzed undeclared labour in a matrix game. We reveal the opportunity cost between taxation and contributions for social insurance that firms and unions face, while we examine all relevant possible unilateral deviations from firms and unions. Our research concludes in three different possible equilibria that all three of them – under certain circumstances – may constitute a Nash SPE. Further, we conclude that if both firms declare their labour, then the incentive for firm's deviation will arise if the bargaining power of unions is low enough ($b < b_{cr1}$), while unions will silently consent to undeclared labour if the rate for social insurance's contributions is great enough ($k > k_{cr1}$). If both firms practice undeclared labour, then there can be none critical value that will alter firms' policy to declared labour; thus, in this case, unions will consent to undeclared labour only if k is low enough ($k < k_{cr2}$). Finally, in the case that one firm declares its labour while the other one not, firm's incentive to alter its policy to declared labour occurs if the direct tax rate is great enough ($t_a > 1- t_e$), while the incentive to discontinue practicing undeclared labour occurs if b is low enough ($b < b_{cr2}$)*. However, in this latter case, there can be none incentive for unions to consent to the change of declared to undeclared labour.*

Keywords: Undeclared Labour, Cournot Duopoly, Labour Unions, Unionisation, Endogenous Objectives

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Introduction

Undeclared work is defined as "any paid activities that are lawful as regards their nature but not declared to public authorities". It is a complex phenomenon associated with tax evasion and social security fraud. Undeclared labour concerns various types of activities, ranging from informal household services to clandestine work by illegal residents, but excludes criminal activities.

It is a process that may engage both employers and employees voluntarily, because of the potential gain in avoiding taxes and social security contributions, social rights and the cost of complying with regulations.

From a macroeconomic point of view, undeclared labour reduces tax revenues (since employees declare no income and then no taxes are imputed) and undermines the financing of social security systems. To the extent that undeclared work competes with and even crowds out activities that comply with regulations, it is the main source of social dumping. In the case of undeclared work performed by individuals who are receiving benefits compensating their inactivity, there is also a dimension of social fraud.

From a microeconomic perspective, undeclared labour distorts fair competition among firms and causes productive inefficiencies, as informal businesses typically avoid access to formal services and inputs (e.g. credit) and prefer to stay small.

Undeclared labour is a decomposite phenomenon, that is influenced by a great range of economic, social, structural and cultural factors, tending to comprise a constraint to economic, fiscal, and social policies applied for the economic growth of an economy.

The fact that undeclared labour on one hand cannot be observed and on the other hand may be otherwise defined among countries, makes it even more difficult to establish credible evaluations about the growth of this phenomenon. However, a research, conducted on behalf of European Committee at 2004, while it accented important differences among countries regarding the qualitative characteristics as well as the size of undeclared labour, estimated undeclared labour's maximum values at 20% at some countries of Eastern and South Europe.

Given the complexity and the heterogeneity of the phenomenon, there is no simple solution to confront it. Nevertheless, the resolution of the European Union's Council of 29 October 2003 on transforming undeclared work into regular employment proposed the following policies:

- Reducing the financial attractiveness of undeclared work stemming from the design of tax and benefit systems, and the permissiveness of the social protection system with regard to the performing of undeclared work;
- Administrative reform and simplification, with a view to reducing the cost of compliance with regulations;
- Strengthening the surveillance and sanction mechanisms, with the involvement of labour inspectorates, tax offices and social partners;
- Trans-national cooperation between Member States, and
- Awareness raising activities.

Regarding the first policy group of meters, European Committee concluded that there is still a great deal of actions to be done in order to balance both the motives and the disincentives offered by the social security systems. In particular, proposed policies concern the reservation of adequate income levels (taking into account the relation between benefits and contributions), the enforcement of exercising control over the labour market and over the persons entitled to social benefits and the imposition of proper economic penalties for tax and contribution evasion.

To gain all the above, policies should emphasize in:

(i) Proper taxation of overtime work;

(ii) Maintaining the institutional minimum wages;

(iii) Regulating tax distortions between tax systems applied in wage earners and those applied to self-employed;

(iv) Reducing the taxation of low productivity activities.

Even though during the past decades a broad range of methods has been developed to analyze the undeclared labour phenomenon, to understand its dimensions and causes, to formulate an appropriate policy to constrain its spread, neither this phenomenon has been examined with any available method, nor the discussion about which methodology is the most appropriate has still not come to an end. In particular, there has been an extended use of econometrics and applied statistics in the relevant researches. Surveys from international organizations (such as OECD, ILO, EU etc) based mostly on evidence and results of state audits also consist a notable framework. However, undeclared labour has not yet been approached or analyzed using the framework of industrial organization and game theoretic analytical toolkit.

With this research, we aspire to deliver a different approach, using the industrial's organization framework. Moreover, one of the main goals of this work is to propose a different policy for restraining the phenomenon of undeclared labour. As it is shown, the use of proper tax rates relative to those of social insurance could – under certain circumstances – restrain the economic attractiveness of this phenomenon.

1. The Model

Consider a homogeneous good market, where two symmetric firms compete by adjusting their quantities. Production exhibits constant returns to scale and requires only labour input to produce the good. Moreover, each firm possesses a Leontief technology, so the capital stock is always sufficient to produce the good.

The production function of the first firm (second firm) can be defined as $q_1 = L_1$ ($q_2 = L_2$), where q (L) denotes output (employment), and the productivity of labour is normalized to unity. Moreover, let the inverse demand function specified of the simple normalized linear form, $P(Q) = 1 - Q$, where Q is the aggregate output: $Q = q_1 + q_2$.

Firms have the option either to declare all their workers and pay contributions for social security, or to employ their staff undeclared to the authorities. If any firm decides to declare its employees, then an additional insurance cost will arise, calculated as a percentage of $k \epsilon (0,1)$ on employees' wages. Moreover, if a firm insures its personnel, then all payroll costs will be deducted from its profits, including insurance costs, and thus fewer taxes will be paid; whereas, if the firm does not insure its personnel, then – for the tax calculation only – payroll costs will not consist a deduction element of profits and therefore more taxes will be defrayed. Considering the imposition of two different types of taxation, indirect tax rate t_e, imposed on firm's revenues, and proportional direct tax rate t_a, imposed on firm's profits $\left(t_a, t_e\ \epsilon (0,1)\right)$, the profit functions form as follows:

- Case of undeclared labour:

$$\Pi_i = p \cdot q_i - w_i \cdot q_i - t_e \cdot p \cdot q_i - t_a \cdot (p \cdot q_i - t_e \cdot p \cdot q_i) \tag{1}$$

- Case of declared labour:

$$\Pi_i = p \cdot q_i - (1+k) \cdot w_i \cdot q_i - t_e \cdot p \cdot q_i - t_a \cdot (p \cdot q_i - (1+k) \cdot w_i \cdot q_i - t_e \cdot p \cdot q_i) \tag{2}$$

Given risk-neutral fixed membership and immobile labour, according to the utilitarian hypothesis, unions are assumed to maximize rents, $U(w_i, L_i) = (w_i - w_0) \cdot L_i = (w_i - w_0) \cdot q_i$, where w_i and L_i are the wage and employment arguments, i stands for first or second firm, and w_0 stands for the reservation wage - unemployment benefit. For simplicity, we normalize w_0 to zero, as such a normalization does not qualitatively affect the final state of the equilibrium. Furthermore, if employees are declared, then social insurance will consist an additional – fringe – benefit for them; thus, it should be included to their utility. Additionally, declared employees reveal their income and, thus, they pay proportional taxes, calculated as a percentage t_a of their income. So, in the case of declared employees, the utility function forms as $U_i(w_i, L_i) = (1+k) \cdot w_i \cdot q_i - t_a \cdot (w_i \cdot q_i)$.

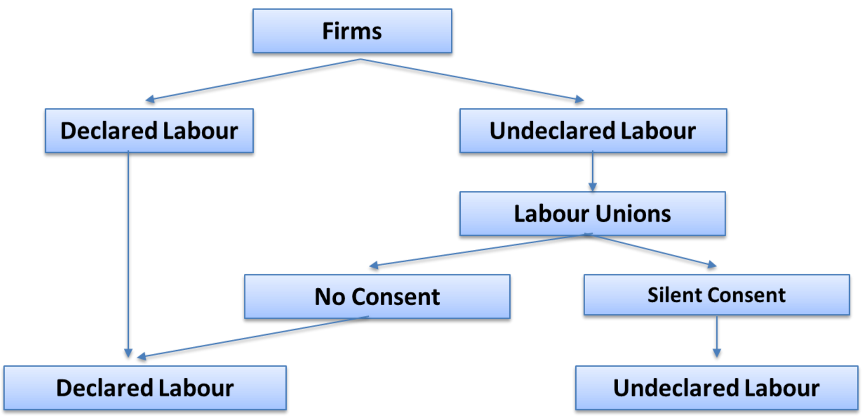
It is clear that an opportunity cost arises for unions; if unions consent to undeclared labor, it is more possible that more union's members will be employed (employment will increase), while its members will pay no taxes at all. In the case that employees are properly declared, they will benefit social security, but they will have to pay taxes, since their income will be declared to public authorities, and, thus, it will be taxable.

Regarding the wage-setting structure, we assume de facto decentralized wage bargaining regime; each union will negotiate the wage (and thus the employment level) with the relevant firm, considering the maximization of its utility. Unions (firms) are moreover assumed to possess a bargaining power of b (1-b) during labour-management negotiations.

Note that in order undeclared labour to be applied, firm and union (the latter most likely silently) must collude. On the other hand, declared labour may be practiced unilaterally; if one firm decides to properly declare its personnel to the authorities, union has to comply. Else, if unions disown undeclared labour for their members, then they will denounce any illegal practices to the authorities and restore declared labour status.

Arising from the above, a three-stage game can be formally addressed as follows:

1. Firms and unions mutually decide whether labour should be declared or not.



2. Decentralized wage bargaining takes place, where firms and unions bargain over wages – and, thus, employment.
3. Firms determine their quantities in the market (Cournot competition).

Regarding to the first stage of the game, four alternative cases are clearly distinguished, as displayed in the following matrix:

	f_2 Declared Labor	f_2 Undeclared Labor
f_1 Declared Labour	*f_1 Declared Labour, f_2 Declared Labour* ❶	*f_1 Declared Labour, f_2 Undeclared Labour* ❷
f_1 Undeclared Labour	*f_1 Undeclared Labour, f_2 Declared Labour* ❸	*f_1 Undeclared Labour, f_2 Undeclared Labour* ❹

Since cases 2 and 3 are symmetrical, the number of alternative cases is reduced to three.

We shall proceed with the further research of the model, using backward induction. Having the model solved, we will examine which case consist a possible Nash Subgame Perfect Equilibrium (SPE). Further, we will determine those circumstances, under which any agent of the game (e.g. firms or unions) are motivated to deviate from the equilibrium.

2. Pure Strategies Focusing on Undeclared Labour in Unionized Oligopoly

Let us now proceed solving the model. As mentioned before, 3 alternative possible equilibria are formed. Thus, we shall solve each case discrete.

a. 1st Case, Both Firms Declare Their Employees

Using backward induction, let us start with the final stage, Cournot competition. The profit functions of both firms have as follows[1]:

$$\Pi_{1i} = p_i q_{1i} - (1+k)w_{1i}q_{1i} - t_e p_i q_{1i} - t_a(p_i q_{1i} - (1+k)w_{1i}q_{1i} - t_e p_i q_{1i}) \quad (3)$$

$$\Pi_{2i} = p_i q_{2i} - (1+k)w_{2i}q_{2i} - t_e p_i q_{2i} - t_a(p_i q_{2i} - (1+k)w_{2i}q_{2i} - t_e p_i q_{2i}) \quad (4)$$

Taking first order conditions as to quantities and solving both equations simultaneously, we result to:

$$\mathrm{q_{1i}} = \frac{1 - \mathrm{t_e} - 2\mathrm{w_{1i}} - 2k\mathrm{w_{1i}} + \mathrm{w_{2i}} + k\mathrm{w_{2i}}}{3(1 - \mathrm{t_e})} \quad (5)$$

$$\mathrm{q_{2i}} = \frac{1 - \mathrm{t_e} - 2\mathrm{w_{2i}} - 2k\mathrm{w_{2i}} + \mathrm{w_{1i}} + k\mathrm{w_{1i}}}{3(1 - \mathrm{t_e})} \quad (6)$$

Let us now proceed to the 2nd stage, decentralized wage bargaining. Unions' utility functions have as follows:

$$\mathrm{U_{1i}} = ((1+k)\cdot \mathrm{w_{1i}} \cdot \mathrm{q_{1i}}) - \mathrm{t_a}\cdot(\mathrm{w_{1i}}\cdot \mathrm{q_{1i}}) \quad (7)$$

$$\mathrm{U_{2i}} = ((1+k)\cdot \mathrm{w_{2i}} \cdot \mathrm{q_{2i}}) - \mathrm{t_a}\cdot(\mathrm{w_{2i}}\cdot \mathrm{q_{2i}}) \quad (8)$$

The agreed wages will occur by the maximization as to w_{1i} and w_{2i} of the following expressions:

$$\text{Max}\left\{(\mathrm{U_{1i}})^b \cdot (\Pi_{1i})^{(1-b)}\right\} \quad (9)$$

$$\text{Max}\left\{(\mathrm{U_{2i}})^b \cdot (\Pi_{2i})^{(1-b)}\right\} \quad (10)$$

Maximizing as above, we obtain the following results:

$$\mathrm{w_{1i}} = \frac{b(1-\mathrm{t_e})}{(4-b)(1+k)} \quad (11)$$

$$\mathrm{w_{2i}} = \frac{b(1-\mathrm{t_e})}{(4-b)(1+k)} \quad (12)$$

[1] Index ***i*** is used to denote the case that both firms **i**nsure their employees.

$$q_{1i} = \frac{2(2-b)}{3(4-b)} \tag{13}$$

$$q_{2i} = \frac{2(2-b)}{3(4-b)} \tag{14}$$

$$\Pi_{1i} = \frac{4(-2+b)^2(1-t_a)(1-t_e)}{9(-4+b)^2} \tag{15}$$

$$\Pi_{2i} = \frac{4(-2+b)^2(1-t_a)(1-t_e)}{9(-4+b)^2} \tag{16}$$

$$U_{1i} = \frac{2(2-b)b(1+k-t_a)(1-t_e)}{3(-4+b)^2(1+k)} \tag{17}$$

$$U_{2i} = \frac{2(2-b)b(1+k-t_a)(1-t_e)}{3(-4+b)^2(1+k)} \tag{18}$$

$$p_i = \frac{4+b}{12-3b} \tag{19}$$

b. 2nd Case, Both Firms Practice Undeclared Labour

As above, let us start with the final stage, Cournot competition. The profit functions of both firms have as follows[2]:

$$\Pi_{1u} = (p_u \cdot q_{1u}) - w_{1u} \cdot q_{1u} - t_e \cdot (p_u \cdot q_{1u}) - t_a \cdot \left(p_u \cdot q_{1u} - t_e \cdot (p_u \cdot q_{1u})\right) \tag{20}$$

$$\Pi_{2u} = (p_u \cdot q_{2u}) - w_{2u} \cdot q_{2u} - t_e \cdot (p_u \cdot q_{2u}) - t_a \cdot \left(p_u \cdot q_{2u} - t_e \cdot (p_u \cdot q_{2u})\right) \tag{21}$$

Taking first order conditions as to quantities and solving both equations simultaneously, we result to:

$$q_{1u} = \frac{1 + t_a(-1+t_e) - t_e - 2w_{1u} + w_{2u}}{3(1-t_a)(1-t_e)} \tag{22}$$

$$q_{2u} = \frac{1 + t_a(-1+t_e) - t_e + w_{1u} - 2w_{2u}}{3(1-t_a)(1-t_e)} \tag{23}$$

[2] Index ***u*** is used to denote the case that both firms practice **u**ndeclared labour.

Let us now proceed to the 2nd stage, decentralized wage bargaining. Unions' utility functions have as follows:

$$U_{1u} = (w_{1u} \cdot q_{1u}) \tag{24}$$

$$U_{2u} = (w_{2u} \cdot q_{2u}) \tag{25}$$

The agreed wages will occur by the maximization as to w_{1u} and w_{2u} of the following expressions:

$$\text{Max}\left\{(U_{1u})^{b} \cdot (\Pi_{1u})^{(1-b)}\right\} \tag{26}$$

$$\text{Max}\left\{(U_{2u})^{b} \cdot (\Pi_{2u})^{(1-b)}\right\} \tag{27}$$

Maximizing as above, we conclude to the following results:

$$w_{1u} = \frac{b(1 - t_a - t_e + t_a t_e)}{4 - b} \tag{28}$$

$$w_{2u} = \frac{b(1 - t_a - t_e + t_a t_e)}{4 - b} \tag{29}$$

$$q_{1u} = \frac{2(2 - b)}{3(4 - b)} \tag{30}$$

$$q_{2u} = \frac{2(2 - b)}{3(4 - b)} \tag{31}$$

$$\Pi_{1u} = \frac{4(-2 + b)^2(1 - t_a)(1 - t_e)}{9(-4 + b)^2} \tag{32}$$

$$\Pi_{2u} = \frac{4(-2 + b)^2(1 - t_a)(1 - t_e)}{9(-4 + b)^2} \tag{33}$$

$$U_{1u} = \frac{2(2 - b)b(1 - t_a)(1 - t_e)}{3(-4 + b)^2} \tag{34}$$

$$U_{2u} = \frac{2(2 - b)b(1 - t_a)(1 - t_e)}{3(-4 + b)^2} \tag{35}$$

$$p_u = \frac{4 + b}{12 - 3b} \tag{36}$$

c. 3rd Case, f_1 Declares, f_2 Doesn't Declare Its Employees

Once more, we begin solving from the final stage, Cournot competition. The profit functions of both firms have as follows[3]:

$$\Pi_1 = (p \cdot \mathrm{q}_1 - (1+k) \cdot \mathrm{w}_1 \cdot \mathrm{q}_1) - \mathrm{t_e} \cdot (p \cdot \mathrm{q}_1) - \mathrm{t_a} \cdot (p \cdot \mathrm{q}_1 - (1+k) \cdot \mathrm{w}_1 \cdot \mathrm{q}_1) \tag{37}$$

$$\Pi_2 = (p \cdot \mathrm{q}_2) - \mathrm{w}_2 \cdot \mathrm{q}_2 - (\mathrm{t_e} + \mathrm{t_a}) \cdot (p \cdot \mathrm{q}_2) \tag{38}$$

Taking first order conditions as to quantities and solving both equations simultaneously, we result to:

$$\mathrm{q}_1 = \frac{-1 + \mathrm{t_a} + \mathrm{t_e} + 2(1+k)\mathrm{w}_1 - 2(1+k)\mathrm{t_a}\mathrm{w}_1 - \mathrm{w}_2}{3(-1 + \mathrm{t_a} + \mathrm{t_e})} \tag{39}$$

$$\mathrm{q}_2 = \frac{-1 + \mathrm{t_e} - \mathrm{w}_1 - k\mathrm{w}_1 + \mathrm{t_a}(1 + \mathrm{w}_1 + k\mathrm{w}_1) + 2\mathrm{w}_2}{3(-1 + \mathrm{t_a} + \mathrm{t_e})} \tag{40}$$

Let us now proceed to the 2nd stage, decentralized wage bargaining. Unions' utility functions have as follows:

$$\mathrm{U}_1 = \left((1+k) \cdot \mathrm{w}_1 \cdot \mathrm{q}_1\right) - \mathrm{t_a} \cdot (\mathrm{w}_1 \cdot \mathrm{q}_1) \tag{41}$$

$$\mathrm{U}_2 = (\mathrm{w}_2 \cdot \mathrm{q}_2) \tag{42}$$

The agreed wages will occur by the maximization as to w_1 and w_2 of the following expressions:

$$\text{Max}\left\{(\mathrm{U}_1)^b \cdot (\Pi_1)^{(1-b)}\right\} \tag{43}$$

$$\text{Max}\left\{(\mathrm{U}_2)^b \cdot (\Pi_2)^{(1-b)}\right\} \tag{44}$$

Maximizing as above, we conclude to the following results:

$$\mathrm{w}_1 = \frac{b(1 - \mathrm{t_a} - \mathrm{t_e})}{(4-b)(1+k)(1-\mathrm{t_a})} \tag{45}$$

$$\mathrm{w}_2 = \frac{b(1 - \mathrm{t_a} - \mathrm{t_e})}{4-b} \tag{46}$$

$$\mathrm{q}_1 = \frac{2(2-b)}{3(4-b)} \tag{47}$$

$$\mathrm{q}_2 = \frac{2(2-b)}{3(4-b)} \tag{48}$$

[3] We shall use no index for this case.

$$\Pi_1 = \frac{4(-2+b)^2(1-\mathrm{t_a}-\mathrm{t_e})}{9(-4+b)^2} \tag{49}$$

$$\Pi_2 = \frac{4(-2+b)^2(1-\mathrm{t_a}-\mathrm{t_e})}{9(-4+b)^2} \tag{50}$$

$$\mathrm{U_1} = \frac{2\,(2-\mathrm{b})\,\mathrm{b}\,(1+\mathrm{k}-\mathrm{t_a})\,(1-\mathrm{t_a}-\mathrm{t_e})}{3\,(-4+\mathrm{b})^2\,(1+\mathrm{k})\,(1-\mathrm{t_a})} \tag{51}$$

$$\mathrm{U_2} = \frac{2(-2+b)b(-1+\mathrm{t_a}+\mathrm{t_e})}{3(-4+b)^2} \tag{52}$$

$$p = \frac{4+b}{12-3b} \tag{53}$$

3. Subgame Perfect Equilibrium

In this section, we check whether any (and which) of the candidate equilibria is a Nash equilibrium or there exists any motivation for any of the agents to deviate unilaterally from the proposed equilibrium.

Both firms and labour unions may have incentives to deviate from the proposed equilibrium. On one hand, firms make their choices opting to maximize their profits. Unions on the other hand may connive with firms at undeclared labor, and therefore effectively sustain undeclared labor, if their overall utility (taking into account wages, employment, social insurance and taxation) increases under such an arrangement. In any opposite case, unions will denounce firms to public authorities, forcing firms to comply with the regulations about social security.

All possible unilateral deviations are illustrated in the matrix below:

	Proposed SPE				**Possible Unilateral Deviation**	
Index	f_1	f_2		Index	f_1	f_2
(a)	Insures	Insures	⇨	(i)	Insures	NOT Insures
(b)	NOT Insures	NOT Insures	⇨	(ii)	Insures	NOT Insures
(c)	Insures	NOT Insures	⇨	(iii)	NOT Insures	NOT Insures
(c)	Insures	NOT Insures	⇨	(iv)	Insures	Insures

The rest cases of unilateral deviations (i.e. the reverse of the reported above) are skipped from the analysis, as being symmetrical to the above. Note that, since undeclared labor is a phenomenon generally blinded due to the consequences that may incur, we assume that any agent (firm or union) may deviate, given that the rival unit is not able to find it out. Therefore, the rival unit will act as if the deviant unit was maintaining its assumed decision.

Let us next examine each of the above cases separately.

(a) Deviation from {f_1: Insure, f_2: Insure} to { f_1: Insure, f_2: Not Insure}

First we examine if there is any motivation for any firm to unilaterally deviate from the state (the proposed equilibrium) where both firms declare their employees. Suppose that f_2 deviates. Then, its profit function becomes:

$$\Pi_{2id} = (p_{id} \cdot q_{2id}) - w_{2id} \cdot q_{2id} - t_e \cdot (p_{id} \cdot q_{2id}) - t_a \cdot \left(p_{id} \cdot q_{2id} - t_e \cdot (p_{id} \cdot q_{2id})\right) \tag{54}$$

Taking first order conditions for Π_{2id} as to q_{2id} and setting $q_{1id} = \frac{2(-2+b)}{3(-4+b)}$ [4], the output of f_2 is

$$q_{2id} = \frac{(-8+b)(-1+t_a+t_e)+3(-4+b)w_{2id}}{3(-4+b)(-3+2t_a+2t_e)} \tag{55}$$

The utility of f_2 firm's union is given by the following expression:

$$U_{2id} = (w_{2id} \cdot q_{2id}) \tag{56}$$

Taking first order conditions for the expression $\left\{U_{2id}^{b} \cdot pr_{2id}^{(1-b)}\right\}$ as to w_{2id} we obtain the following results:

$$w_{2id} = \frac{(8-b)b(1-t_a-t_e)}{6(4-b)} \tag{57}$$

$$q_{2id} = \frac{(8-b)(2-b)(1-t_a-t_e)}{6(4-b)(3-2t_a-2t_e)} \tag{58}$$

[4] Even though f_2 may deviate, f_1 shall not be able to observe the deviation, so f_1 and its labor union will act and play like f_2 would insure its employees. That means we will have the same quantity and wage set up as for f_1 concerns, like there was no deviation.

$$\Pi_{2id} = \frac{(-8+b)^2(-2+b)^2(2-\mathrm{t_a}-\mathrm{t_e})(-1+\mathrm{t_a}+\mathrm{t_e})^2}{36(-4+b)^2(-3+2\mathrm{t_a}+2\mathrm{t_e})^2} \tag{59}$$

To examine if f_2 has any motive to deviate, we abstract f_2's profit before and after deviation and we obtain the following results:

$$\Pi_{2id} - \Pi_{2i} = -\frac{1}{36(-4+b)^2(-3+2\mathrm{t_a}+2\mathrm{t_e})^2}(-2+b)^2 \cdot (-16(-1+\mathrm{t_a}+\mathrm{t_e}) - 16b(-2+\mathrm{t_a}+\mathrm{t_e})(-1+\mathrm{t_a}+\mathrm{t_e})^2 + b^2(-2+\mathrm{t_a}+\mathrm{t_e})(-1+\mathrm{t_a}+\mathrm{t_e})^2 + 16\mathrm{t_a}\mathrm{t_e}(-3+2\mathrm{t_a}+2\mathrm{t_e})^2)$$

The expression $-\frac{1}{36(-4+b)^2(-3+2\mathrm{ta}+2\mathrm{te})^2}(-2+b)^2$ is negative, thus we continue with the rest of the expression $R = (-16(-1+\mathrm{ta}+\mathrm{t_e}) - 16b(-2+\mathrm{ta}+\mathrm{te})(-1+\mathrm{ta}+\mathrm{te})^2 + b^2(-2+\mathrm{ta}+\mathrm{te})(-1+\mathrm{ta}+\mathrm{te})^2 + 16\mathrm{tate}(-3+2\mathrm{ta}+2\mathrm{te})^2)$.

R is trinomial expression of b, and its roots are:

$$b_1 = \frac{4(2(-2+\mathrm{ta}+\mathrm{t_e})(-1+\mathrm{ta}+\mathrm{t_e})^2 - \sqrt{-(-1+\mathrm{ta})(-1+\mathrm{t_e})(-2+\mathrm{ta}+\mathrm{t_e})(-1+\mathrm{ta}+\mathrm{t_e})^2(-3+2\mathrm{ta}+2\mathrm{t_e})^2})}{(-2+\mathrm{ta}+\mathrm{t_e})(-1+\mathrm{ta}+\mathrm{t_e})^2}$$

$$b_2 = \frac{4(2(-2+\mathrm{ta}+\mathrm{t_e})(-1+\mathrm{ta}+\mathrm{t_e})^2 + \sqrt{-(-1+\mathrm{ta})(-1+\mathrm{t_e})(-2+\mathrm{ta}+\mathrm{t_e})(-1+\mathrm{ta}+\mathrm{t_e})^2(-3+2\mathrm{ta}+2\mathrm{t_e})^2})}{(-2+\mathrm{ta}+\mathrm{t_e})(-1+\mathrm{ta}+\mathrm{t_e})^2}$$

Since $\boldsymbol{b_1}$ is always greater than 1[5], we reject it, and we accept $\boldsymbol{b_2}$ as root. Therefore,

- If $b < b_2 = b_{cr1}$, then $R < 0$ and then $\Pi_{2id} > \Pi_{2i}$, meaning that, under the condition $b < b_2 = b_{cr1}$, f_2 is motivated to deviate from the proposed equilibrium. In this case, the equilibrium that both firms declare their labour is not time-consistent.

- If $b > b_2 = b_{cr1}$, then $R > 0$ and then $\Pi_{2id} < \Pi_{2i}$, meaning that, under the condition $b < b_2 = b_{cr1}$, there is no motivation for f_2 to deviate from the equilibrium and thus its choice reveals as time-consistent.

Let us now check if there is any motivation for f_2 firm's union to deviate. Union's utilities before and after the deviation have as follows.

[5] Under 0 < ta, te <1 and 0 < b < 1.

$$\mathrm{U_{2i}} = \frac{2(2-b)b(1+k-\mathrm{t_a})(1-\mathrm{t_e})}{3(-4+b)^2(1+k)} \tag{60}$$

$$\mathrm{U_{2id}} = \frac{(-8+b)^2(2-b)b(-1+\mathrm{t_a}+\mathrm{t_e})^2}{36(-4+b)^2(3-2\mathrm{t_a}-2\mathrm{t_e})} \tag{61}$$

Abstracting the expressions above, we have:

$$\mathrm{U_{2id}} - \mathrm{U_{2i}} = \frac{((-2+b)b(-16b(1+k)(-1+\mathrm{t_a}+\mathrm{t_e})^2 + b^2(1+k)(-1+\mathrm{t_a}+\mathrm{t_e})^2 - 8(1+k+\mathrm{t_a}+\mathrm{t_e}) + 8(2\mathrm{t_a}(\mathrm{t_a}+k(-5+4\mathrm{t_a})) + (\mathrm{t_a}(-5+6\mathrm{t_a}) + k(-1+10\mathrm{t_a}))\mathrm{t_e} + 2(1+k+3\mathrm{t_a})\mathrm{t_e}^2)))}{(36(-4+b)^2(1+k)(-3+2\mathrm{t_a}+2\mathrm{t_e}))}$$

The root of the expression above ($\mathrm{U_{2id}} - \mathrm{U_{2i}} = 0$) is

$$k_{cr1} = \frac{16b(-1+\mathrm{t_a}+\mathrm{t_e})^2 - b^2(-1+\mathrm{t_a}+\mathrm{t_e})^2 + 8(1+\mathrm{t_a}+\mathrm{t_e}) - 8(2\mathrm{t_e}^2 + \mathrm{t_a}\mathrm{t_e}(-5+6\mathrm{t_e}) + \mathrm{t_a}^2(2+6\mathrm{t_e}))}{-16b(-1+\mathrm{t_a}+\mathrm{t_e})^2 + b^2(-1+\mathrm{t_a}+\mathrm{t_e})^2 + 16(\mathrm{t_a}+\mathrm{t_e})(4\mathrm{t_a}+\mathrm{t_e}) - 8(1+10\mathrm{t_a}+\mathrm{t_e})}$$

Summarizing the above,

- If $0 < k < k_{cr1}$ then $\mathrm{U_{2id}} > \mathrm{U_{2i}}$; therefore, the union is motivated to deviate from the proposed equilibrium and amplify the undeclared labour phenomenon.
- If $k_{cr1} < k < 1$ then $\mathrm{U_{2id}} < \mathrm{U_{2i}}$; thus, under this condition, union's choice will be time-consistent.

Proposition 1 summarizes all the above conclusions;

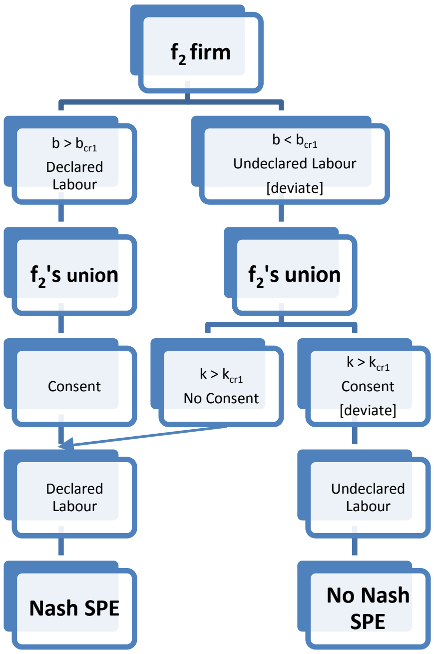
Proposition 1:

Assuming $b > b_2 = b_{cr1}$ [6], *f*$_2$ will practice anyhow declared labour and thus its union will be committed to $\mathrm{f_2}$'s choice. However, if $b < b_2 = b_{cr1}$, then *f*$_2$ will acquire an incentive to decline from the proposed equilibrium and practice undeclared labour. In this case, if k is low enough ($k < k_{cr1}$), $\mathrm{f_2}$'s union will consent (silently) to undeclared labour, and, therefore, undeclared labour will be practiced. On the other hand, assuming $k > k_{cr1}$, $\mathrm{f_2}$'s union will enjoy greater utility under declared labour

[6] $b_2 = \frac{4(2(-2+\mathrm{t}_a+\mathrm{t_e})(-1+\mathrm{t}_a+\mathrm{t_e})^2 + \sqrt{-(-1+\mathrm{t}_a)(-1+\mathrm{t_e})(-2+\mathrm{t}_a+\mathrm{t_e})(-1+\mathrm{t}_a+\mathrm{t_e})^2(-3+2\mathrm{t}_a+2\mathrm{t_e})^2})}{(-2+\mathrm{t}_a+\mathrm{t_e})(-1+\mathrm{t}_a+\mathrm{t_e})^2}$

and, thus, it will denounce any firm's illegal practice, constraining, by this way, f_2 to practice declared labour.

Interpreting the above proposition, the conclusions may be illustrated in the diagram below:



(b) Deviation from { f_1: Not Insure, f_2: Not Insure} to { f_1: Insure, f_2: Not Insure}

Let us now examine if there is any motivation for a firm to deviate from the state that both firms use undeclared labor for all employees and declare them. Suppose that ***f_1*** deviates from the proposed SPE, its profit function forms as follows:

$$\Pi_{1ud} = (\mathrm{p_{ud}} \cdot \mathrm{q_{1ud}} - (1+k) \cdot \mathrm{w_{1ud}} \cdot \mathrm{q_{1ud}}) - \mathrm{t_e} \cdot (\mathrm{p_{ud}} \cdot \mathrm{q_{1ud}}) - \mathrm{t_a} \cdot (\mathrm{p_{ud}} \cdot \mathrm{q_{1ud}} - (1+k) \cdot \mathrm{w_{1ud}} \cdot \mathrm{q_{1ud}} - \mathrm{t_e} \cdot (\mathrm{p_{ud}} \cdot \mathrm{q_{1ud}})) \tag{62}$$

Taking first order conditions for Π_{1ud} as to $\mathrm{q_{1ud}}$ and setting $q_{2ud} = \frac{2(-2+b)}{3(-4+b)}$[7], the output of f_1 is

$$\mathrm{q_{1ud}} = \frac{(\mathrm{t_a} - 1)((b-8)(\mathrm{t_e} - 1) + 3(b-4)(1+k)\mathrm{w_{1ud}})}{3(b-4)(3 + 2\mathrm{t_a}(\mathrm{t_e} - 1) - 2\mathrm{t_e})} \tag{63}$$

[7] Even though f_2 deviates, f_1 cannot observe the deviation, so f_1 and its labor union will act as f_2 would insure its employees. That means we will have the same quantity and wage set up as for f_1 concerns, like there was no deviation.

The utility of f_1 firm's union is given by the following expression:

$$\mathrm{U}_{1\mathrm{ud}} = ((1+k)\cdot \mathrm{w}_{1\mathrm{ud}}\cdot \mathrm{q}_{1\mathrm{ud}}) - \mathrm{t_a}\cdot(\mathrm{w}_{1\mathrm{ud}}\cdot \mathrm{q}_{1\mathrm{ud}}) \quad (64)$$

Taking first order conditions for the expression $\left\{\mathrm{U}_{1\mathrm{ud}}^{b} \cdot \mathrm{pr}_{1\mathrm{ud}}^{(1-b)}\right\}$ as to $\mathrm{w}_{1\mathrm{ud}}$ we conclude to the following results:

$$\mathrm{w}_{1\mathrm{ud}} = -\frac{(b-8)(\mathrm{t_e}-1)b}{6(b-4)(1+k)} \quad (64)$$

$$\Pi_{1\mathrm{ud}} = \frac{(-8+b)^2(-2+b)^2(-1+\mathrm{t_a})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2}{36(-4+b)^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2} \quad (65)$$

$$\mathrm{U}_{1\mathrm{ud}} = \frac{(-8+b)^2(-2+b)b(1+k-\mathrm{t_a})(-1+\mathrm{t_a})(-1+\mathrm{t_e})^2}{36(-4+b)^2(1+k)(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})} \quad (66)$$

To examine if f_1 has any motive to deviate, we abstract f_1's profit before and after deviation and we obtain the following results:

$$\Pi_{1\mathrm{u}} - \Pi_{1\mathrm{ud}} = -\frac{((-2+b)^2(-1+\mathrm{t_a})(-16+(-16+b)b(-1+\mathrm{t_a})(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e}))(-1+\mathrm{t_e}))}{(36(-4+b)^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2)}$$

The expression above is always positive, resulting to $\Pi_{1\mathrm{u}} > \Pi_{1\mathrm{ud}}$. Interpreting the above, if both firms do not declare their staff, then none of them will be motivated to deviate (and thus to declare its employees).

Examining f_1 union's behavior, the utility functions, before and after the deviation, have as follows:

$$\mathrm{U}_{1\mathrm{u}} = -\frac{2(-2+b)b(-1+\mathrm{t_a})(-1+\mathrm{t_e})}{3(-4+b)^2} \quad (67)$$

$$\mathrm{U}_{1\mathrm{ud}} = \frac{(-8+b)^2(-2+b)b(1+k-\mathrm{t_a})(-1+\mathrm{t_a})(-1+\mathrm{t_e})^2}{36(-4+b)^2(1+k)(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})} \quad (68)$$

Abstracting the expressions above, we have:

$$\mathrm{U}_{1\mathrm{u}} - \mathrm{U}_{1\mathrm{ud}} = -\frac{((-2+b)b(-1+\mathrm{t_a})(-1+\mathrm{t_e})(-16b(1+k-\mathrm{t_a})(-1+\mathrm{t_e})+b^2(1+k-\mathrm{t_a})(-1+\mathrm{t_e})+8(1+k+2\mathrm{t_a}-6k\mathrm{t_a}+2(1+k-\mathrm{t_a}+3k\mathrm{t_a})\mathrm{t_e})))}{(36(-4+b)^2(1+k)(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e}))}$$

The expression above turns positive for $0 < k < k_{cr2}$, where

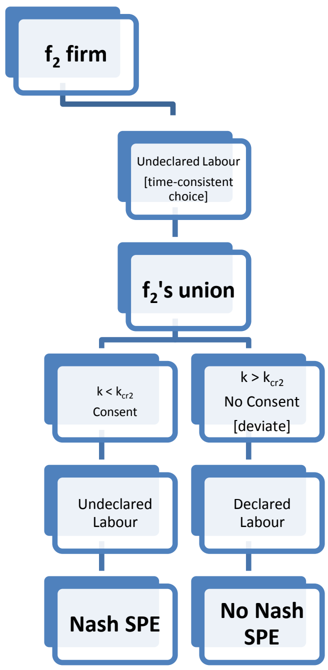
$$k_{cr2} = \frac{8(-1 + 2t_a(-1 + t_e) - 2t_e) - 16b(-1 + t_a)(-1 + t_e) + b^2(-1 + t_a)(-1 + t_e)}{-16b(-1 + t_e) + b^2(-1 + t_e) + 8(1 + 6t_a(-1 + t_e) + 2t_e)}$$

It can be shown that if k is low enough, $k < k_{cr2} \rightarrow U_{1u} > U_{1ud}$ and therefore union will prefer undeclared labor for its members. On the other hand, if k is high enough ($k > k_{cr2}$) union will then be motivated to denounce undeclared labor and deviate from the equilibrium. Proposition 2 summarizes:

Proposition 2:

If k is low enough ($0 < k < k_{cr2}$)*, then the proposed SPE, where both firms practice undeclared labour, will consist a Nash equilibrium. If, on the other hand, k is great enough* ($k_{cr2} < k < 1$)*, this proposed equilibrium is time-inconsistent.*

Interpreting the proposition above, the conclusions may be illustrated in the diagram below:



(c) Deviation from { f_1: Insure, f_2: Not Insure} to { f_1: Not Insure, f_2: Not Insure}

At this stage, we shall check the possibility of deviation from the proposed equilibrium (one firm practices declared labour while the other doesn't) to an alternative state, where both firms apply undeclared labor. Suppose ***f_1*** deviates from the proposed SPE, its profit function form as follows:

$$\Pi_{1d1} = (p \cdot q_{1d1}) - w_{1d1} \cdot q_{1d1} - (t_e + t_a) \cdot (p \cdot q_{1d1}) \tag{69}$$

Taking first order conditions for pr_{1d1} as to q_{1d1} and setting $q_{2d1} = \frac{2(-2+b)}{3(-4+b)}$[8], the output of f_1 is

$$q_{1d1} = \frac{(8-b)(1-t_a-t_e) - 3(4-b)w_{1d1}}{3(4-b)(3-2t_a-2t_e)} \tag{70}$$

The utility of f_1 firm's union is given by the following expression:

$$\mathrm{U}_{1d1} = (w_{1d1} \cdot q_{1d1}) \tag{71}$$

Taking first order conditions for the expression $\left\{\mathrm{U}_{1d1}^{b} \cdot \mathrm{pr}_{1d1}^{(1-b)}\right\}$ as to w_{1d1} we have the following results:

$$\mathrm{w}_{1d1} = \frac{-8b + b^2 + 8bt_a - b^2 t_a + 8bt_e - b^2 t_e}{6(-4+b)} \tag{72}$$

$$\mathrm{U}_{1d1} = \frac{(-8+b)^2(-2+b)b(-1+t_a+t_e)^2}{36(-4+b)^2(-3+2t_a+2t_e)} \tag{73}$$

$$\Pi_{1d1} = \frac{(-8+b)^2(-2+b)^2(2-t_a-t_e)(-1+t_a+t_e)^2}{36(-4+b)^2(-3+2t_a+2t_e)^2} \tag{74}$$

To examine if f_1 has any motive to deviate, we abstract f_1's profit before and after deviation and we obtain the following results:

$$\Pi_{1d1} - \Pi_1 = \frac{(-2+b)^2(-1+t_a+t_e)\left(16 - \frac{(-8+b)^2(-2+t_a+t_e)(-1+t_a+t_e)}{(-3+2t_a+2t_e)^2}\right)}{36(-4+b)^2}$$

The expression above

[8] Even though f_2 deviates, f_1 cannot observe the deviation, so f_1 and its labor union will act as f_2 would insure its employees. That means we will have the same quantity and wage set up as for f_1 concerns, like there was no deviation.

- If $-1+t_a+t_e>0 \Rightarrow t_a>1-t_e \Rightarrow \Pi_{1d1}-\Pi_1>0$, and thus $\Pi_{1d1}>\Pi_1$
- If $-1+t_a+t_e<0 \Rightarrow t_a<1-t_e \Rightarrow \Pi_{1d1}-\Pi_1<0$, and thus $\Pi_{1d1}<\Pi_1$

Therefore, if $t_a>1-t_e$, then f_1 has incentives to deviate from the proposed equilibrium (f_1 insures, f_2 not) and practice undeclared labour.

Examining f_1 union's behavior, the utility functions, before and after the deviation, have as follows:

$$\mathrm{U}_1=\frac{2\,(2-\mathrm{b})\,\mathrm{b}\,(1+\mathrm{k}-\mathrm{t_a})\,(1-\mathrm{t_a}-\mathrm{t_e})}{3\,(-4+\mathrm{b})^2\,(1+\mathrm{k})\,(1-\mathrm{t_a})} \tag{75}$$

$$\mathrm{U}_{1d1}=\frac{(-8+b)^2(-2+b)b(-1+t_a+t_e)^2}{36(-4+b)^2(-3+2t_a+2t_e)} \tag{76}$$

Abstracting the expressions above, we get:

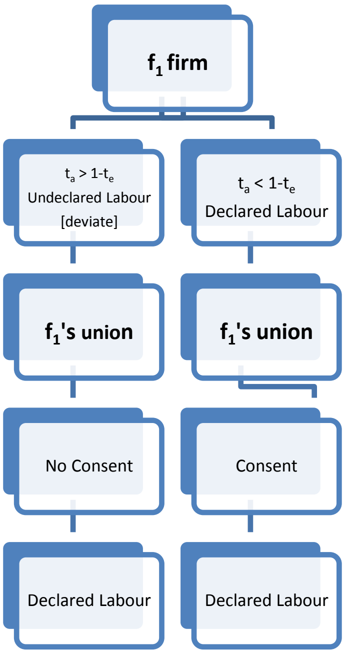
$$\mathrm{U}_{1d1}-\mathrm{U}_1=\frac{(-2+b)b(-1+t_a+t_e)\begin{pmatrix}-16b(1+k)(-1+t_a)(-1+t_a+t_e)+b^2(1+k)(-1+t_a)(-1+t_a+t_e)+\\ 8\big((-1+t_a)(1+2t_a+2t_e)+k\big(-1-2t_e+2t_a(-5+4t_a+4t_e)\big)\big)\end{pmatrix}}{36(-4+b)^2(1+k)(-1+t_a)(-3+2t_a+2t_e)}$$

The expression above is always negative, implying that $\mathrm{U}_{1d1}<\mathrm{U}_1$, thus f_1's union will prefer declared labor and therefore it will not conclude with f_1 firm's decision for undeclared labor. Eventually, union will denounce possible undeclared labor policy to public authorities and reinstate f_1 to its initial state. Proposition 3 summarizes:

Remark 1:

Within the proposed SPE (f_1 insures its workers, f_2 does not), if $t_a<1-t_e$, then f_1's choice will be considered as time-consistent and, thus, declared labour will be applied. If, on the other hand, $t_a>1-t_e$, then f_1 will acquire an incentive to deviate and practice undeclared labour. Nevertheless, its union will not consent to undeclared labour, forcing f_1 to alter its choice.

Interpreting the above proposition, the conclusions may be illustrated in the diagram below:



(d) Deviation from {Insure, Not Insure} to {Insure, Insure}

The proposed SPE that one firm insures its personnel and the other doesn't has another possible deviation. Suppose f_2 deviates from the proposed SPE and decides to insure its personnel, the setup forms as follows:

$$\Pi_{1d2} = p_{d2}q_{1d2} - (1+k)w_{1d2}q_{1d2} - t_e p_{d2}q_{1d2} \\ - t_a(p_{d2}q_{1d2} - (1+k)w_{1d2}q_{1d2} - t_e p_{d2}q_{1d2}) \tag{77}$$

$$\Pi_{2d2} = p_{d2}q_{2d2} - (1+k)w_{2d2}q_{2d2} - t_e p_{d2}q_{2d2} \\ - t_a(p_{d2}q_{2d2} - (1+k)w_{2d2}q_{2d2} - t_e p_{d2}q_{2d2}) \tag{78}$$

$$p_{d2} = 1 - q_{1d2} - q_{2d2} \tag{79}$$

Taking first order conditions for Π_{2d2} as to q_{2d2} and setting $q_{1d2} = \frac{2(2-b)}{3(4-b)}$, the output of f_1 is

$$q_{2d2} = \frac{(-1+t_a)((-8+b)(-1+t_e)+3(-4+b)(1+k)w_{2d2})}{3(-4+b)(3+2t_a(-1+t_e)-2t_e)} \tag{80}$$

The utility of f_2 firm's union is given by the following expression:

$$\mathrm{U}_{2d2} = ((1+k)\cdot \mathrm{w}_{2\mathrm{d}2}\cdot \mathrm{q}_{2\mathrm{d}2}) - \mathrm{t_a}\cdot(\mathrm{w}_{2\mathrm{d}2}\cdot \mathrm{q}_{2\mathrm{d}2}) \tag{81}$$

Taking first order conditions for the expression $\left\{\mathrm{U}_{2\mathrm{d}2}^{b} \cdot \mathrm{pr}_{2\mathrm{d}2}^{(1-b)}\right\}$ as to $\mathrm{w}_{2\mathrm{d}2}$ we have the following results:

$$\mathrm{w}_{2\mathrm{d}2} = -\frac{(-8+b)b(-1+\mathrm{t_e})}{6(-4+b)(1+k)} \tag{82}$$

$$\mathrm{U}_{2d2} = \frac{(-8+b)^2(-2+b)b(1+k-\mathrm{t_a})(-1+\mathrm{t_a})(-1+\mathrm{t_e})^2}{36(-4+b)^2(1+k)(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})} \tag{83}$$

$$\Pi_{2d2} = \frac{(-8+b)^2(-2+b)^2(-1+\mathrm{t_a})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2}{36(-4+b)^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2} \tag{84}$$

To examine if f_2 has any motive to deviate, we abstract f_2's profit before and after deviation and we obtain the following results:

$$\Pi_{2\mathrm{d}2} - \Pi_2 = \frac{\begin{gathered}((-2+b)^2(16(-1+\mathrm{t_a}+\mathrm{t_e})+\\ (-1+\mathrm{t_a})(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})(-16b(-1+\mathrm{t_a})(-1+\mathrm{t_e})\\ +b^2(-1+\mathrm{t_a})(-1+\mathrm{t_e})+64\mathrm{t_a}\mathrm{t_e})))\end{gathered}}{(36(-4+b)^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2)}$$

The expression above has 3 roots,

- $b_1 = 2$, rejected as $0 < b < 1$
- $$b_2 = \frac{\begin{gathered}(8\mathrm{t_a}^3(-1+\mathrm{t_e})^3+8\mathrm{t_a}(-1+\mathrm{t_e})^2(-5+3\mathrm{t_e})\\ -8\mathrm{t_a}^2(-1+\mathrm{t_e})^2(-4+3\mathrm{t_e})-4(2(-2+te)(-1+\mathrm{t_e})^2+\\ \sqrt{-(-1+\mathrm{t_a})^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2(-1+\mathrm{t_a}+\mathrm{t_e})}))\end{gathered}}{((-1+\mathrm{t_a})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2)}$$
- $$b_3 = \frac{\begin{gathered}(-8(-2+\mathrm{t_a})(-1+\mathrm{t_a})^2+8(-1+\mathrm{t_a})^2(-5+3\mathrm{t_a})\mathrm{t_e}-\\ 8(-1+\mathrm{t_a})^2(-4+3\mathrm{t_a})\mathrm{t_e}^2+8(-1+\mathrm{t_a})^3\mathrm{t_e}^3+\\ 4\sqrt{-(-1+\mathrm{t_a})^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2(-1+\mathrm{t_a}+\mathrm{t_e})})\end{gathered}}{((-1+\mathrm{t_a})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2)}$$

  rejected as $b_3 > 12$ (while $0 < b < 1$).

We observe that:

- If $b < b_2 = b_{cr2}$, then $\Pi_{2\mathrm{d}2} - \Pi_2 > 0$, or equivalently $\Pi_{2\mathrm{d}2} > \Pi_2$, and therefore there is motivation for f_2 firm to deviate from the proposed equilibrium and declare its employees.

- If $b_2 = b_{cr2} < b$, then $\Pi_{2d2} - \Pi_2 < 0$, or equivalently $\Pi_{2d2} < \Pi_2$, and, thus, there is no motivation for f_2 firm to deviate from the proposed equilibrium and will continue to practice undeclared labour.

Let us now check f_2 union's behavior. Union's utility before and after deviation has as follows:

$$\mathrm{U}_2 = \frac{2(-2+b)b(-1+\mathrm{t_a}+\mathrm{t_e})}{3(-4+b)^2} \tag{85}$$

$$\mathrm{U}_{2d2} = \frac{(-8+b)^2(-2+b)b(1+k-\mathrm{t_a})(-1+\mathrm{t_a})(-1+\mathrm{t_e})^2}{36(-4+b)^2(1+k)(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})} \tag{86}$$

Abstracting expression [86] from [85], we obtain:

$$\mathrm{U}_{2\mathrm{d}2} - \mathrm{U}_2 = \frac{\begin{array}{c}(-2+b)b(-16b(1+k-\mathrm{t_a})(-1+\mathrm{t_a})(-1+\mathrm{t_e})^2+\\ b^2(1+k-\mathrm{t_a})(-1+\mathrm{t_a})(-1+\mathrm{t_e})^2+8\big((-1+\mathrm{t_a})(-1-2\mathrm{t_a}+k(-1+6\mathrm{t_a})\big)+\\ \big(1+k+2(-7+k)\mathrm{t_a}+2(5-3k)\mathrm{t_a}{}^2\big)\mathrm{te}+2(1+k-4\mathrm{t_a})(-1+\mathrm{t_a})\mathrm{t_e}{}^2))\end{array}}{36(-4+b)^2(1+k)(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})}$$

The expression above has one root at:

$$k_{cr3} = \frac{(\mathrm{t_a}-1)((-16+b)b(\mathrm{t_a}+\mathrm{t_e}-1)+8(1+2\mathrm{t_a}+2\mathrm{t_e}))}{(\mathrm{t_a}-1)(-8+(b-16)b+48\mathrm{t_a})+(16+(b-16)b+48\mathrm{t_a})\mathrm{t_e}}$$

It therefore can be shown that

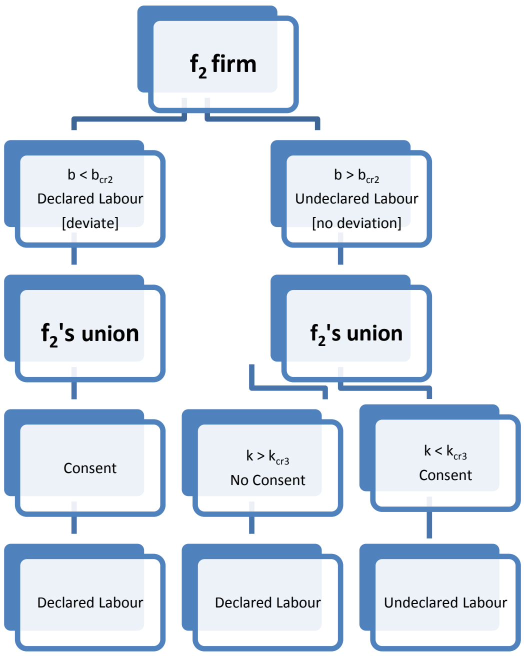
- if $k > k_{cr3} \rightarrow \mathrm{U}_{2\mathrm{d}2} - \mathrm{U}_2 > 0 \rightarrow \mathrm{U}_{2\mathrm{d}2} > \mathrm{U}_2$, else
- if $k < k_{cr3} \rightarrow \mathrm{U}_{2\mathrm{d}2} - \mathrm{U}_2 < 0 \rightarrow \mathrm{U}_{2\mathrm{d}2} < \mathrm{U}_2$

Interpreting the above, if k is great enough ($k > k_{cr3}$), then f_2's union will enjoy greater utility in the case that f_2 deviates, e.g. applies declared labour. If, on the other hand, k is low enough ($k < k_{cr3}$), then it will enjoy greater utility on the case that f_2 applies undeclared labour and remains time-consistent with its choice. Remark 2 summarizes:

Remark 2:

Within the proposed SPE (f_1 insures its workers, f_2 does not), if $b > b_{cr2}$[9]*, then f_2's choice will be considered as time-consistent and thus f_2 will practice undeclared labour. If k is low enough too ($k < k_{cr3}$*[10]*), then its union will consent to undeclared labour. On the other hand, if k is high enough (greater than k_{cr3}), then its union will not consent to undeclared labour and, thus, it will denounce any illegal practices, forcing f_2 to alter its choice. Finally, if b is low enough ($b < b_{cr3}$) f_2 has an incentive to deviate and declare its labour, and thus its union will be obliged to act along.*

The conclusions of the Remark above may be illustrated in the diagram below:



Combining Remark 1 and Remark 2, we conclude to Proposition 3.

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9 $$\mathrm{b}_2 = \frac{\begin{gathered}(8\mathrm{t_a}^3(-1+\mathrm{t_e})^3+8\mathrm{t_a}(-1+\mathrm{t_e})^2(-5+3\mathrm{t_e})\\ -8\mathrm{t_a}^2(-1+\mathrm{t_e})^2(-4+3\mathrm{t_e})-4(2(-2+\mathrm{te})(-1+\mathrm{t_e})^2+\\ \sqrt{-(-1+\mathrm{t_a})^2(3+2\mathrm{t_a}(-1+\mathrm{t_e})-2\mathrm{t_e})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2(-1+\mathrm{t_a}+\mathrm{t_e})))}\end{gathered}}{((-1+\mathrm{t_a})^2(2+\mathrm{t_a}(-1+\mathrm{t_e})-\mathrm{t_e})(-1+\mathrm{t_e})^2)}$$

10 $$k_{cr} = \frac{(\mathrm{t_a}-1)((-16+b)b(\mathrm{t_a}+\mathrm{t_e}-1)+8(1+2\mathrm{t_a}+2\mathrm{t_e}))}{(\mathrm{t_a}-1)(-8+(b-16)b+48\mathrm{t_a})+(16+(b-16)b+48\mathrm{t_a})\mathrm{t_e}}$$

Proposition 3:

If $t_a < 1 - t_e$, $b > b_{cr2}$[11] and $k < k_{cr3}$[12], then the proposed equilibrium (f_1 insures its workers, while f_2 does not) will remain time-consistent and therefore will constitute a Nash SPE.

4. Conclusions

The analysis above represents an alternative approach of the undeclared labour phenomenon with analytical tools from Industrial Organization and Game Theory framework. In a unionized duopoly, we focused on the opportunity cost that arises by the implementation of undeclared labour; if a firm properly declares its personnel to the authorities, then the firm will have to pay contributions for social insurance, while less taxes will be defrayed. Exactly the opposite occurs in the other case, highlighting the alternative cost, thereby. labour unions face the same dilemma as well; if unions – silently – consent to undeclared labour, their members will enjoy greater payments (no contributions for social insurance will be withheld) and pay fewer taxes.

In this early analysis, we considered the firms' choice for applying undeclared labour or not exogenously. Therefore, a matrix game occurred, where we examined - under pure strategies – whether any of the proposed equilibria consists a Nash Subgame Perfect Equilibrium. Regarding the formation of agents' policies, we assumed that in order for undeclared labour to be applied, the collusion between firm and its union is a prerequisite, while declared labour may occur unilaterally (either from firm's or from union's choice). Furthermore, we endogenized any possible deviations in a more realistic frame, assuming that an agent deviates, given

[11] $$b_2 = \frac{\begin{array}{c}(8t_a^3(-1+t_e)^3+8t_a(-1+t_e)^2(-5+3t_e)\\ -8t_a^2(-1+t_e)^2(-4+3t_e)-4(2(-2+te)(-1+t_e)^2+\\ \sqrt{-(-1+t_a)^2(3+2t_a(-1+t_e)-2t_e)^2(2+t_a(-1+t_e)-t_e)(-1+t_e)^2(-1+t_a+t_e)}))\end{array}}{((-1+t_a)^2(2+t_a(-1+t_e)-t_e)(-1+t_e)^2)}$$

[12] $$k_{cr} = \frac{(t_a-1)((-16+b)b(t_a+t_e-1)+8(1+2t_a+2t_e))}{(t_a-1)(-8+(b-16)b+48t_a)+(16+(b-16)b+48t_a)t_e}$$

that the rival unit is not able to find it out, and, thus, the rival unit will act as the deviant was maintaining its assumed – initial – decision.

The findings of our analysis suggest that all proposed equilibria (e.g. both firms insure, both firms do not insure, one firm insures while the other doesn't) may comprise a Nash SPE under certain circumstances. We furthermore investigated those critical values for an agent to obtain an incentive to deviate from the proposed equilibrium, and alter his policy (for example, a firm discontinue to declare its labour and practices undeclared labour). Our findings indicate that those critical values depend on the status quo of the market;

- If both firms declare their labour, then the incentive for firm's deviation will arise if b is low enough ($b < b_{cr1}$).
- If both firms practice undeclared labour, then none incentive to declare their labour may exist.
- In the case that one firm declares its personnel while the other doesn't,
    - The incentive to discontinue declaring its labour and practice undeclared labour occurs if t_a is great enough ($t_a > 1 - t_e$).
    - On the other hand, the incentive to discontinue practicing undeclared labour and insures its personnel occurs if b is low enough ($b < b_{cr2}$).

Similar conclusions are revealed for labour unions too.

- If both firms practice declared labour, union will consent to a deviation to undeclared labour only if k is great enough ($k > k_{cr1}$).
- If both firms practice undeclared labour, union will consent to undeclared labour only if k is low enough ($k < k_{cr2}$).
- In the case that one firm practices undeclared labour while the other one doesn't, unions will not accept for their members to alter from declared to undeclared labour.

Several inquiries are still left open for further research. For instance, note that we have not examined the equilibrium prospects of the centralized wage-bargaining structure. The same applies for the cost of governmental surveillance or compliance penalties (or any else relevant policy meters) imposed for undeclared labour. Consequences to social welfare are another aspect that should furthermore be examined.

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