# Welfare Improving Cartel Formation in a Union-Oligopoly Static Framework

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**Abstract**

In a union-oligopoly static framework we study the role of unions regarding the possibility and the effects of endogenous cartel formation. Given that firms independently adjust their own quantities, we show that, if union members are not sufficiently risk-averse and firms' products are sufficiently close substitutes, then collusion among firms may emerge in equilibrium, and that − in contrast to conventional wisdom − cartel formation proves to be a welfare improving market arrangement. Quite remarkably, the latter gain in social welfare materializes at the cost of union rents despite it is the union's presence which effectively sustains collusion.

**JEL Classification:** D43; J51; L13

**Keywords:** Oligopoly; Unions; Collusion

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## 1. Introduction

One of the ever interesting fields of Industrial Organization is whether and when firms may choose collusion, e.g., to informally engage to cartel formation, over competition. The conventional conclusion is that in static (one-shot) games collusion is not feasible, while on dynamic games (of infinite or finite time span) it may emerge in equilibrium if and only if a sufficiently high discount factor exists.

Yet, and despite the fact that modern industrial sectors are heavily unionized, what has not been investigated in depth is the interaction that may arise among firms and their workers' unions over firms' decisions for collusive play, and which might be the outcome and the welfare implications of such interaction. To our best knowledge, the relevant literature mainly focuses on the interpretation of market conditions which may strengthen collusion − including the presence of workers and their unions. Being the first to incorporate the latter factor into analysis, McDonald and Solow (1981), Clark (1984), Karier (1985), and Mishel (1986), suggest that product market imperfections allow the generation of economic rents over which unions and firms may bargain, while later on Dowrick (1989), focusing on the interaction between oligopolistic price setting and union-firm bargaining, suggests that the union-oligopoly wage contract is an increasing function of the degree of collusion among firms. Taking another path, Compte (1998), and Kandori and Matsushima (1998), argue that the existence of private information imposes an additional barrier to collusion and, therefore, communication-enhancing devices, such as trade unions, may promote collusion by transferring private information and setting it as public. More recently, and like us in an upstream (union) – downstream (oligopoly) static framework, Symeonidis (2008) suggests that if unions are risk neutral, the firms' products are close substitutes, and union bargaining power is sufficiently high, then social welfare may be higher under an ad-hoc scheme of joint profit maximization than under competition. Unlike us, however, the above author does not examine whether and how a joint profit maximization/collusion scheme may endogenously emerge in a static equilibrium. Yet, as it will become evident later on, his major finding is confirmed by our research thus strengthening our results.

In the present paper we consider a simple union-oligopoly model where in the product market two technologically identical firms producing differentiated goods may compete, or collude, by independently adjusting their own quantities. We further argue that either of these decisions is taken cooperatively inside each firm/union unit (yet non-cooperatively across the different firm/union units) in a firm-specific bargain where the firm decides on its output and the union decides on the firm-specific wage.[1] In this context, a (two-stage) static game arises, with the following envisaged events: At the first stage, firms independently decide to proceed to competitive or to collusive play (e.g. to cartel formation) in the continuation of the game, and unions independently choose the (firm-specific) wage to set in either instance. At the second stage, firms independently adjust their own quantities in the product market, in order either to maximize their own profits or to maximize joint profits, according to the decisions taken, inside each firm/union unit, at the first stage.

Solving this game, our findings suggest that, if union members are not sufficiently risk-averse, and the firms' products are sufficiently close substitutes, then collusion among firms may quite interestingly emerge in the (static) equilibrium and that − in contrast to conventional wisdom – cartel formation may prove to be a welfare improving market arrangement compared to competition. Remarkably, moreover,

[1] Effectively, therefore, we postulate a special version of "Efficient Bargains" [see, e.g., McDonald and Solow (1981), Petrakis and Vlassis (2000)], where for simplicity assume that the union (the firm) possesses all the power over the wage (the output/employment) bargain.

such a gain in social welfare materializes at the cost of union rents, despite it is the union's presence that effectively sustains collusion.

The rest of paper is organized as follows. Section 2 analytically presents our union-duopoly model, while in Section 3 we demonstrate the conditions under which one or more Nash equilibria may arise. Hence, the effects of critical structural parameters on wages and employment/output are consistently predicted and interpreted, in Section 4. By these means we subsequently proceed to welfare analysis, in Section 5. Our findings are conclusively evaluated in Section 6.

## 2. The Model

Assume a sectoral product market where two technologically identical firms, denoted by $i \neq j = 1,2$, producing differentiated goods, may compete or collude by independently adjusting their own quantities. Each firm faces an inverse linear demand function[2] which is given by:

$$p_i(q_i, q_j) = 1 - q_i - \gamma q_j \tag{1}$$

Where, $p_i$, $q_i$ respectively are the price and output of firm $i \neq j = 1,2$, and $\gamma \in (0,1)$ denotes the degree of substitutability among the goods $i \neq j = 1,2$: As $\gamma \rightarrow 1$ the firms' products become more close substitutes.

For simplicity, we assume that the production technology exhibits constant returns to scale, and labor productivity equals to one for both firms, namely one unit of labor is needed to produce one unit of output:[3]

$$\mathrm{L_i = q_i} \tag{2}$$

[2] Like in Dixit (1979), this function is derived by maximizing (w.r.t. $q_i, q_j$) the quadratic and strictly concave utility function $u(q_i, q_j) = aq_i + aq_j - \frac{b}{2}(q_i + q_j + 2\gamma q_i q_j) + m$, where m is the competitive numeraire sector. For simplicity, we assumed that both $a$ and $b$ are equal to one.

[3] Hence, (2) implies a specific version of a two-factor Leontief technology in which the (minimum cost) capacity - labour ratio is equal to one.

Where $L_i$ and $q_i$ respectively are the employment and the quantity of firm $i(\neq j = 1,2)$.[4]

The firm's unit transformation cost of labor into product equals the wage rate, denoted by $w_i$. Hence, the profit function of firm $i$ is defined by:

$$\Pi_\iota = (p_i - w_i)q_i \tag{3}$$

The sectoral labor market is unionized: Workers are organized into two separate firm-specific unions, and firm-union bargaining is decentralized. Hence, each firm enters into (any) negotiation(s) exclusively with the firm's union of workers The union $i$'s objective is to maximize the sum of its members' rents, given by the following equation:

$$u_i(w_i, L_i) = (w_i - w_0)^\varphi L_i \tag{4}$$

Where, $w_i$ is firm $i$'s wage rate, $0 < w_0 < 1$ is the workers' outside option[5], and $\varphi \in (0,1]$ reflects the representative unions member's relative rate of risk aversion, or alternatively the representative union member's elasticity of substitution between wages and employment, provided that union membership is fixed and all members are (or the union leadership treats them as being) identical [see, e.g. Oswald (1982), Pencavel (1991), Booth, (1995)]: As $\varphi \to 1$ union members become less risk-averse.

In the above context, our envisaged two-stage game unfolds as follows:

- At the 1st stage, both firms simultaneously and independently decide whether to collude or to compete in quantities, and each firm's union (independently from the rival firm's union) dictates the firm-specific wage in either instance.

[4] We are aware of the limitations of our analysis in assuming specific functional forms and constant returns to scale. However, the use of more general forms would jeopardize the clarity of our findings, without significantly changing their qualitative character.

[5] As it is generally accepted in the trade unions literature, $w_0$ represents a weighted average of the competitive wage and the unemployment benefits, the weights respectively being the probability of a worker to find a job or not in the competitive sector.

- At the 2nd stage, if (at the first stage) one or both firms have independently decided to play collusively, they simultaneously and independently adjust their output (hence employment) levels so that each, either on its part maximize the monopoly (cartel's) profits, or maximize its own profits. If, however, both firms have (at the first stage) independently decided to play competitively they both adjust their own capacities and quantities in order each one to maximize its own profits.

Note that, unlike in matrix-type games, in our static context of analysis the decisions taken within each firm/union pair, at stage one, are not observable by the rival pair before product market competition is in place. Hence, the above sequence of events effectively comprises a one shot-game which is conceptually arranged on two sequential sub-stages −all of which materialize without delay in- between.[6]

## 3. Equilibrium Analysis

Like in standard game-theoretic analysis, backwards inducting, we propose a candidate equilibrium and subsequently validate (or reject) it by checking for all possible unilateral deviations, on the part of any involved agent− considering such a deviation. In our model three such candidate equilibria arise: In Section 3.1 the candidate equilibrium is the one where a cartel is effectively formed (e.g., firms independently collude in quantities) and the possible deviation, on the part of any firm, is to adjust its own quantity in order to maximize its own profits given that the other firm sticks to collusive play. In Section 3.2, the candidate equilibrium is Cournot competition and the possible deviation, on the part of any firm, is to adjust its own quantity in order to maximize the cartel's profits, given that the other firm still

[6] A matrix-type approach should rather be employed in a dynamic (super-game) version of our present model.

behaves as a Cournot competitor. In Section 3.3, the candidate equilibrium is the one where one firm acts collusively, while its rival firm acts competitively, and the possible deviations arise by unilaterally switching each firm's strategy to its rival's one. Recall that, in any of the above configurations, wages are consistently adjusted within each firm/union pair so as to maximize the firm-specific union rents, independently from, and not being observable (until the product market outcome unfolds) by, the rival firm/union pair.

### 3.1 Collusive Play(*c*)

Assume that, at the last stage of the game, both firms independently adjust their quantities (thus their own employment levels) in order to maximize joint profits. Hence, according to (1) and (3), the firm's $i$'s objective is:

$$max_{q_i}\begin{bmatrix}\Pi_i+\Pi_j=q_i\left(1-q_i-\gamma q_j-w_i\right)\\ +q_j\left(1-q_j-\gamma q_i-w_j\right)\end{bmatrix} \qquad (5)$$

The first order condition (*f.o.c.*) of (5) provides the reaction function of firm $i$:

$$R_{ic}(q_{jc})=\left(1-2\gamma q_{jc}-w_{ic}\right)/2 \qquad (6)$$

Taking the reaction functions of both firms and solving the system of equations we then get the optimal output/employment rules in the candidate equilibrium:

$$q_{ic}\left(w_{ic},w_{jc}\right)=\frac{1-\gamma-w_i+\gamma\cdot w_{jc}}{2(1-\gamma)(1+\gamma)} \qquad (7)$$

Taking in to consideration (7) union $i$ chooses $w_i$ so as to maximize:

$$[u_i(w_i,q_i)\{=(w_i-w_0)^{\varphi}q_i\}] \qquad (8)$$

From the *f.o.c*$_s$ of (8) we subsequently derive the union $i$'s wage reaction function:

$$w_{ic}(w_{jc}) = \frac{(1-\gamma)\varphi + w_0 + \gamma\varphi w_{jc}}{1+\varphi} \tag{9}$$

Note that $dw_{ic}/dw_{jc} = (\gamma\varphi)/(1+\varphi) > 0 \ \ \forall\, s, \varphi \in (0,1)$, hence, wages are strategic complements on the part of unions. Solving the system of (9) we get the (candidate) equilibrium wages:

$$w_{ic}^{*} = \frac{w_0 + (1-\gamma)\varphi}{1+(1-\gamma)\varphi} \tag{10}$$

Observe that $dw_{ic}/d\varphi = (1-\gamma)(1-w_0)/\big(1+\varphi(1-\gamma)\big)^2 > 0$, i.e. the less risk-averse are union members, the higher is the wage set by the union. To grasp this, notice that if unions abominate any risk (hence $\varphi = 0$), the wage ($w_i$) will be equal to the workers' outside option ($w_0$).

The firms' output /employment levels in the candidate equilibrium are then derived by substituting (10) into (7):

$$q_{ic}^{*} = \frac{1-w_0}{2(1+\gamma)\big(1+\varphi(1-\gamma)\big)} \tag{11}$$

Moreover, we get that:

$$\Pi_{ic}^{*} = \big(\Pi_{ic}^{*} + \Pi_{jc}^{*}\big)/2 = (1+\gamma)({Q_c}^{*})^2/2 = 2(1+\gamma)(q_{ic}^{*})^2 \tag{12}$$

Where, ${Q_c}^{*}$is the sum of the firms' outputs in the candidate equilibrium.

Collusion is the equilibrium configuration only if no firm has incentive to unilaterally deviate, by independently adjusting its own quantity so that to maximize its own profits − while its rival on its part sticks to collusive output. In the standard oligopoly literature the deviant firm would then achieve higher own profits. However, in our model, where the labor market is unionized, the deviant firm, by increasing its output level and therefore labor demand, would also create an extra unit cost− in terms of a higher wage set by its union of workers. Moreover, this increment in unit (labor) cost would be higher the less risk-averse union members are (e.g., the higher is

$\varphi$). Consequently, the gains from deviation from collusive play would be lower than those in the case of fixed wages, and the deviant firm may lose more due to the extra unit cost than gain from business-stealing against its rival firm. Hence, the firm's dominant strategy would be to stick on its share of the collusive output.

Summing up the above, a deviation from collusive play causes two opposing effects on the deviant firm's profits: A positive one, from increasing its market share, and a negative one, due to the higher wage − whose magnitude depends on the representative unions' member's relative rate of risk aversion ($\varphi$). The following proposition encapsulates both effects, suggesting that if $\varphi$ is sufficiently high, firms would be deterred to deviate from collusive play.

**Proposition 1:** *If union members are not enough risk-averse:* $\boldsymbol{\varphi} > \boldsymbol{\varphi_1}(\boldsymbol{\gamma})\{= \frac{\boldsymbol{\gamma}}{\mathbf{2}\left(\boldsymbol{\gamma}^{\mathbf{2}}+(\mathbf{1}+\mathbf{2}\boldsymbol{\gamma})\left(\mathbf{1}+\sqrt{\mathbf{1}+\boldsymbol{\gamma}}\right)\right)} < 0.06\}$, *then collusion among firms is a subgame perfect equilibrium configuration in the product market.*

[Proof: See Appendix (A.1)]

In *Figure 1* below, notice that $\vartheta\varphi_1/\vartheta\gamma > 0$. Hence, as their products become more close substitutes collusion among firms becomes more unstable. The reason is that the higher is $\gamma$, the stronger business-stealing becomes on the part of the deviant firm. It is then needed a high enough $\varphi$ so that the ensuing higher wage to deter deviation from collusive play.

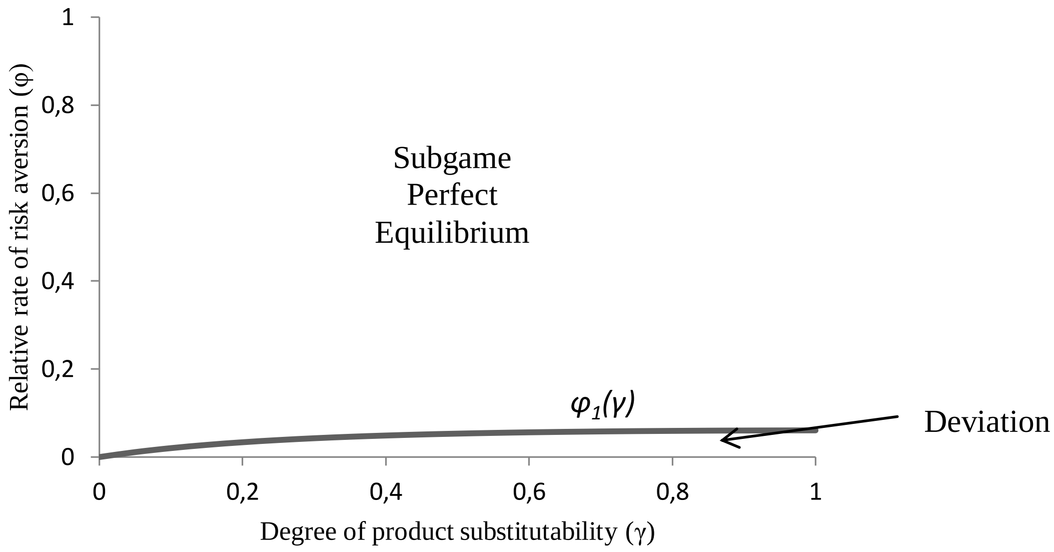


*Figure 1*: *The φ(γ) conditions under which Cartel formation/Collusion emerges in equilibrium*

### 3.1 Competitive Play (*m*)

Assume next that at the last stage of the game each firm aims to maximize its own profits by independently adjusting its own quantity (hence, its employment level). According to (1) and (3), the firm $i$'s objective is then as follows:

$$max_{q_i}[\Pi_i\{= q_i(1 - q_i - \gamma q_j - w_i)\}] \tag{13}$$

The *f.o.c*$_s$ of (13) deliver the following system of reaction functions of firms $i \neq j = 1,2$:

$$R_{im}(q_{jm}) = (1 - \gamma q_{jm} - w_{im})/2 \tag{14}$$

Solving this system we subsequently get the optimal output/employment rules in the candidate equilibrium:

$$q_{im}(w_{im}, w_{jm}) = \frac{2 - \gamma - 2w_{im} + \gamma w_{jm}}{(2 - \gamma)(2 + \gamma)} \tag{15}$$

Therefore, each union $i$ chooses the firm-specific wage $(w_i)$ in order to maximize its rents [given by (8)], taking as given the outcomes of the production game [given by

(15)]. From the *f.o.c*$_s$ of that maximization we may then derive the unions' $i \neq j = 1,2$, wage reaction functions which are as follows:

$$w_{im}(w_{jm}) = \frac{(2-\gamma)\varphi + 2w_0 + \gamma\varphi w_{jm}}{2(1+\varphi)} \quad (16)$$

Observe that, like in 3.1, wages are strategic complements for the unions, since: $dw_i/dw_j = (\gamma\varphi)/\big(2(1+\varphi)\big) > 0 \;\; \forall\, s, \varphi \in (0,1)$

Solving system (16) we get the wage outcome (s) in the candidate equilibrium:

$$w_{im}^{*} = \frac{2w_0 + (2-\gamma)\varphi}{2 + (2-\gamma)\varphi} \quad (17)$$

Consequently [by virtue of (15)], the firms' $i \neq j = 1,2$, output/employment levels in the candidate equilibrium are:

$$q_{im}^{*} = \frac{2(1-w_0)}{(2+\gamma)\big(2+\varphi(2-\gamma)\big)} \quad (18)$$

And, from Cournot's lemma, we have that:

$$\Pi_{im}^{*} = (q_{im}^{*})^2 \quad (19)$$

To check whether the above version of Cournot competition can be sustained as an equilibrium configuration in the product market, assume that firm $i$ unilaterally deviates by adjusting its quantity in order to maximize joint profits, while firm $j$ sticks to the competitive output. It turns out that if union members are sufficiently risk-averse, e.g., if $\varphi$ is sufficiently low, then firm *i* would have no incentive to deviate from competition. Proposition 2 summarizes.

**Proposition 2:** *If union members are sufficiently risk-averse:* $\boldsymbol{\varphi} < \boldsymbol{\varphi_2}(\boldsymbol{\gamma})\{= \frac{2\gamma}{4(1-\gamma)+(2-\gamma)\sqrt{4+\gamma^2}} < 0.89\}$, *then Cournot competition is a subgame perfect equilibrium configuration in the product market.*

Where, $\vartheta\varphi_2/\vartheta\gamma > 0$ (see *Figure 2*)

[Proof: See Appendix (A.2)]

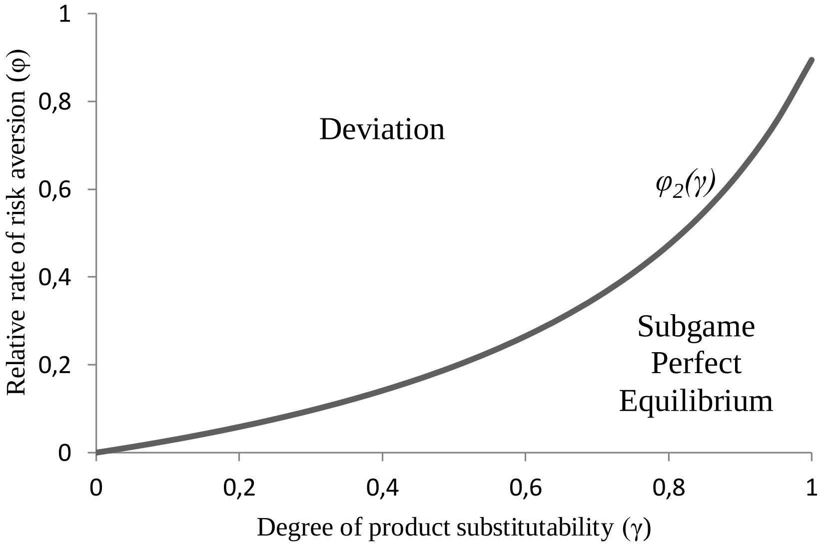


*Figure 2: The $\varphi(\gamma)$ conditions under which Cournot competition emerges in equilibrium*

The economic intuition behind the above finding is of the same pattern with that of the previous section; by simply inversing reasoning one may easily interpret why firms might unilaterally deviate from competition to collusion.

### 3.2 Mix of Strategies (*mos*)

The candidate equilibrium here is the one where one firm (let firm $j$) adjusts its own output competitively, while its rival's (let firm's $i$'s) strategy is to adjust its own output in order to maximize joint profits. The possible deviation on the part of each firm then is to unilaterally switch its own strategy to its rival's one.

According to this Mix of Strategies configuration, at the last stage we must consider the $f.o.c_s$ of the pair (5) and (13) separately. Thus, respectively considering the firm-specific reaction functions (6) and (14), and solving that system, we get the following optimal output/employment rules in the candidate equilibrium:

$$q_i(w_i, w_j) = \frac{1 - \gamma - w_i + \gamma w_j}{2 - \gamma^2} \tag{20}$$

$$q_j(w_i, w_j) = \frac{2 - \gamma - 2w_j + \gamma w_i}{2(2 - \gamma^2)} \tag{21}$$

Subsequently substituting (20) and (21) into (8), from the latter expressions' *f.o.c*$_s$ *w.r.t.* $w_i$,$w_j$, we derive the unions' $i's \neq j's$ wage reaction functions emerging at the first stage of the game:

$$w_i(w_j) = \frac{(1 - \gamma)\varphi + w_0 + \gamma\varphi w_j}{1 + \varphi} \tag{22}$$

$$w_j(w_i) = \frac{(2 - \gamma)\varphi + 2w_0 + \gamma\varphi w_i}{2 \cdot (1 + \varphi)} \tag{23}$$

Solving the system of (22) and (23), we then get the following firm-specific wage outcomes in the candidate equilibrium:

$$w_i^* = \frac{\varphi\big(2(1 - \gamma) + \varphi(2 - \gamma^2)\big) + 2\big(1 + \varphi(1 + \gamma)\big)w_0}{2 + \varphi\big(4 + \varphi(2 - \gamma^2)\big)} \tag{24}$$

$$w_j^* = \frac{\varphi\big(1 - \gamma + \varphi(2 - \gamma^2)\big) + \big(2 + \varphi(2 + \gamma)\big)w_0}{2 + \varphi\big(4 + \varphi(2 - \gamma^2)\big)} \tag{25}$$

Substituting in turn (24) and (25) into (20) and (21), respectively, we obtain the following firm-specific output/employment levels in the candidate equilibrium:

$$q_i^* = \frac{\big(2(1 - \gamma) + \varphi(2 - \gamma^2)\big)(1 - w_0)}{(2 - \gamma^2)\Big(2 + \varphi\big(4 + \varphi(2 - \gamma^2)\big)\Big)} \tag{26}$$

$$q_j^* = \frac{\big(2 - \gamma + \varphi(2 - \gamma^2)\big)(1 - w_0)}{(2 - \gamma^2)\Big(2 + \varphi\big(4 + \varphi(2 - \gamma^2)\big)\Big)} \tag{27}$$

While we cannot derive a specific pattern for $\Pi_i^*$ as a function of $q_i^*$, we still get that:

$$\Pi_j^* = \left(q_j^*\right)^2 \tag{28}$$

The proposed Mix of Strategies is an equilibrium configuration only if no firm has an incentive to unilaterally deviate from its own strategy. Along the same path of reasoning with the previous sub-sections, Proposition 3 concludes.

**Proposition 3:** *If* $\boldsymbol{\varphi_{3A} < \varphi < \varphi_{3B}}$, *then a Mix of Strategies is a subgame perfect equilibrium configuration in the product market.*

Where, $\vartheta\varphi_{3B(A)}/\vartheta\gamma > 0$ (see *Figure 3*); $\begin{Bmatrix} \varphi_{3A} < 0.393 \;\forall\, \gamma \in [0,1] \\ \varphi_{3B} < 1 \;\forall\, \gamma \in [0,0.608] \end{Bmatrix}$

[Proof: See Appendix (A.3)]

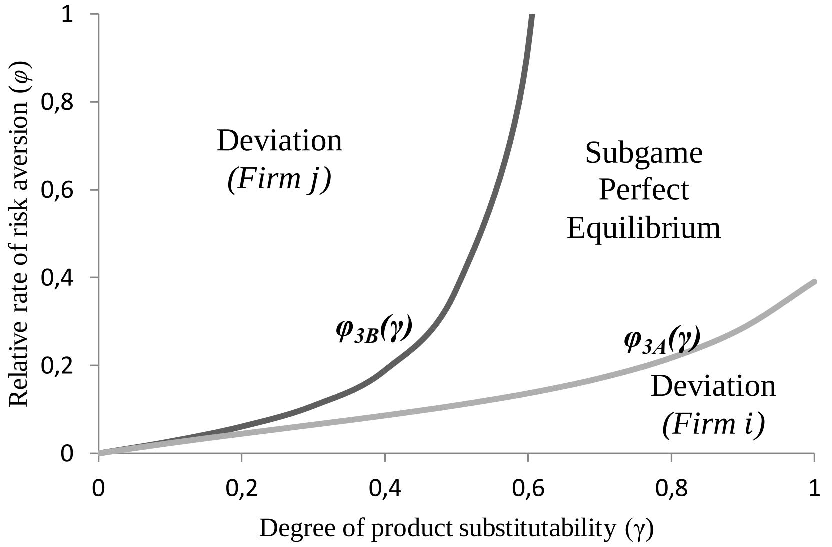


*Figure 3: The φ(γ) conditions under which Mix of Strategies emerge in equilibrium*

The intuition here is a bit more complicated. As it first regards the own profits maximizer − firm *j*, it would have no incentive to deviate to a joint profit maximizing behavior, so long as it expects that its gain, in terms of lower wage, would then be lower than its loss, due to "obedience" in a lower output: The latter effect would be strong enough to dominate over the former one, if $\varphi$ is low enough [lower than $\varphi_{3B}(\gamma)$] so that to induce an insufficient reduction in the firm-specific wage. As on the other hand regards the joint profit maximizer − firm *i*, it would have no incentive to deviate to own profit maximization when $\varphi$ is high enough [higher than $\varphi_{3A}(\gamma)$]: If it does so, it would lose more in terms of a higher firm-specific wage than gain in terms of higher market share. Note, moreover, that the latter lower bound of $\varphi$ for firm *i* to play its part in the Mix of Strategies equilibrium should be lower than the respective one for firm $j$ to sustain it by sticking to own profit maximization [ $\varphi_{3B}(\gamma)$].

### 3.3 Equilibrium Analysis

Given our findings in 3.1.− 3.3., we may now investigate the conditions under which a single or multiple Nash equilibria arise in our static framework. For convenience, our suggested equilibria are summarized in *Table 1*.

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| | | Firm $i$ | |
|---|---|---|---|
| | | *Collusion* | *Competition* |
| Firm $j$ | *Collusion* | Equilibrium if: $\varphi > \varphi_1(\gamma)$ | Equilibrium if: $\varphi \in \{\varphi_{3A}(\gamma), \varphi_{3B}(\gamma)\}$ |
| | *Competition* | Equilibrium if: $\varphi \in \{\varphi_{3A}(\gamma), \varphi_{3B}(\gamma)\}$ | Equilibrium if: $\varphi < \varphi_2(\gamma)$ |

*Table 1: Conditions under which the various firm-specific strategy combinations emerge in equilibrium.*

Where, $\varphi_{3B}(\gamma) > \varphi_2(\gamma) > \varphi_{3A}(\gamma) > \varphi_1(\gamma) \forall \gamma \in [0,1]$, as depicted in *Figure 4*.

Combining the information contained in *Table 1* and *Figure 4*, our suggested equilibria can be then arranged in the more informative *Table 2*.[7] Yet, as it can be there clearly seen (: in the *d*, *c*, *b*, *Figure 4*-regions/2$^{nd}$, 3$^{rd}$,4$^{th}$ -*Table 2* rows), for certain values of the union member's relative rate of risk aversion there arise up to three multiplicities of subgame perfect Nash equilibria.

[7] Where ✓ denotes equilibrium.

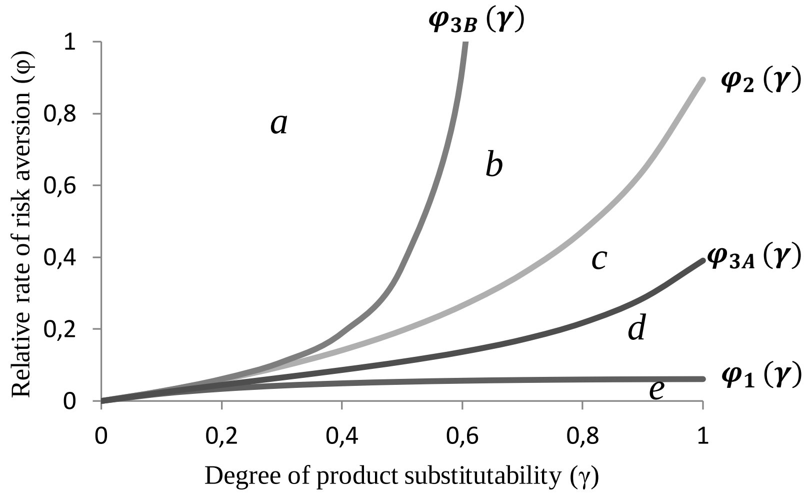


*Figure 4: The $\varphi(\gamma)$ – critical values for one or more equilibria to arise*

| | **Collusion** | **Competition** | **Mix of Strategies** | **Figure 5-regions** |
|---|---|---|---|---|
| $(\mathbf{0}, \boldsymbol{\varphi_1})$ | | ✓ | | *e* |
| $[\boldsymbol{\varphi_1}, \boldsymbol{\varphi_{3A}})$ | ✓ | ✓ | | *d* |
| $[\boldsymbol{\varphi_{3A}}, \boldsymbol{\varphi_2})$ | ✓ | ✓ | ✓ | *c* |
| $[\boldsymbol{\varphi_2}, \boldsymbol{\varphi_{3B}})$ | ✓ | | ✓ | *b* |
| $[\boldsymbol{\varphi_{3B}}, \mathbf{1}]$ | ✓ | | | *a* |

*Table 2: Subgame Perfect Nash equilibria – arising for various $\varphi$ values*

Nonetheless, in order to narrow down as much as possible this multiplicity of equilibria, we can reasonably make use of the criterion of Pareto optimality in the space of profits. According to this criterion, we may select that (those) subgame perfect Nash equilibrium (equilibria) where both firms are better off, by each achieving higher profits in comparison to the remainder ones. The following Proposition (see also *Figure 5*) summarizes our refined findings:

**Proposition 4:**

***(i)*** *Cartel formation/collusion is the unique Pareto Optimal Nash equilibrium, in the space of profits, if union members are not sufficiently risk averse, i.e., if* $\boldsymbol{\varphi} > \boldsymbol{\varphi_a}(\boldsymbol{\gamma}) > \boldsymbol{\varphi_1}(\boldsymbol{\gamma})(< 0.06)$, *while if* $\boldsymbol{\varphi_1}(\boldsymbol{\gamma}) < \boldsymbol{\varphi_{3A}}(\boldsymbol{\gamma}) < \boldsymbol{\varphi} < \boldsymbol{\varphi_a}(\boldsymbol{\gamma})$ *then Cartel formation/collusion and Mix of Strategies are both emerging in equilibrium.*

***(ii)*** *Otherwise, i.e., if* $\boldsymbol{\varphi} < \boldsymbol{\varphi_1}(\boldsymbol{\gamma})$, *Cournot competition is the unique Nash equilibrium.*

Where, $\varphi_2(\gamma) > \varphi_\alpha(\gamma) > \varphi_{3A}(\gamma)\ \forall\ \gamma \in [0,1]$

[Proof: See Appendix (A.4)]

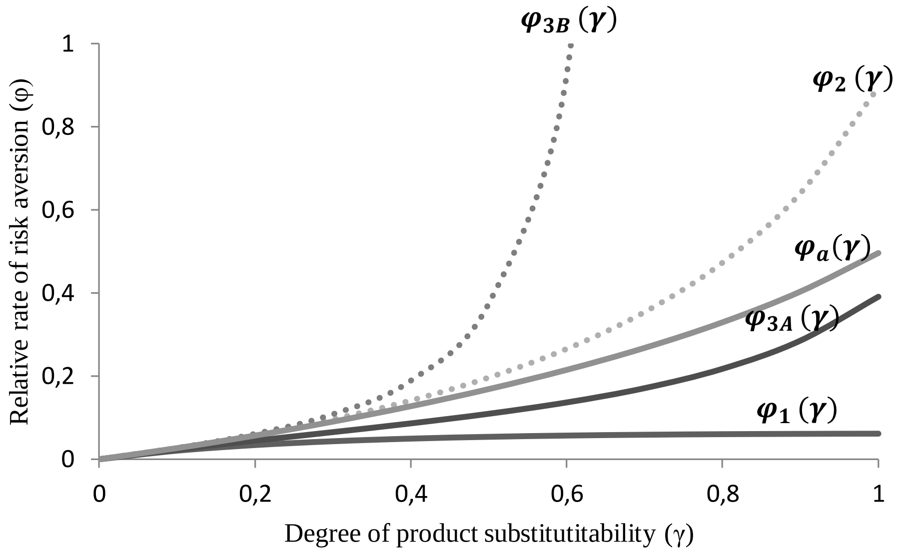


*Figure 5: The refined* $\varphi(\gamma)$ *– critical values for Pareto optimal equilibria to arise*

For the reader's convenience, the Pareto optimal Nash Equilibria are also summarized in *Table 3.*

| | *Pareto optimal Nash Equilibrium* | *Collusion (1)* | *Competition (2)* | *Mix of Strategies (3)* |
|---|---|---|---|---|
| $[\boldsymbol{0}, \boldsymbol{\varphi_1}]$ | *(2)* | | ✓ | |
| $[\boldsymbol{\varphi_1}, \boldsymbol{\varphi_{3A}}]$ | *(1)* | ✓ | | |
| $[\boldsymbol{\varphi_{3A}}, \boldsymbol{\varphi_a}]$ | *(1), (3)* | ✓ | | ✓ |
| $[\boldsymbol{\varphi_a}, \boldsymbol{1}]$ | *(1)* | ✓ | | |

*Table 3: The Pareto optimal Nash equilibria − arising for various φ values*

## 4. Risk Aversion and Product Substitutability: Wage and Output Effects

Given the findings of the previous sections let us now proceed to an informative analysis of the effects of our critical structural parameters, namely of $\varphi$ and $\gamma$, on wages and employment/output. We will thus become able to interpret the circumstances under which cartel formation/collusion may endogenously arise in our static framework, as well as, and more importantly, to configure the ingredients of such a market arrangement. By the latter means, we may subsequently proceed to our welfare analysis in Section 5.

For clarity and comprehension, consider first the standard/ad-hoc − collusive versus competitive – hypotheses in both of which wages/unit costs of production are assumed to be exogenous and firm-union bargaining is absent. As, there ,the degree of product substitutability ($\gamma$) switches from 0 to 1, the difference in sectoral output between competition and collusion respectively switches from minimum to maximum, while for $0 < \gamma < 1$ the $(Q_m - Q_c)$ positive difference lies in-between: In terms of our model, letting $w_{ic} = w_{im}$=$w_0$ and $\varphi$=0, it can be easily checked that when their products are independent ($\gamma$=0) the two firms together (no matter collusively or competitively) produce the quantity of two monopolists [e.g., $q_{ic} = q_{im} = \frac{1-w_0}{2}$; $Q_m = Q_c = (1 - w_0)$], each of them effectively operating in an isolated market[8]. On the other hand, when products are perfect substitutes ($\gamma$=1), collusive production is equal to that of a monopolist who produces a single product sold in the market (e.g., $Q_c = (1 - w_0)/2$), while (Cournot) competitive production ($Q_m = \frac{[2(1-w_0)]}{3}$) is clearly higher, though lower than that of the two monopolists together. Subsequently to check for the sign of $(Q_m - Q_c)/\partial\gamma \ \forall\gamma \in (0,1)$, it only needs to calculate $\left|\frac{dq_{im}}{dw_m}\right|$ and $\left|\frac{dq_{ic}}{dw_c}\right|$, from (15) and (7) respectively, given that, $w_{ic} = w_{jc} = w_c \equiv w_0$ ; $w_{im} = w_{jm} = w_m \equiv w_0$. The following difference thus arises:

$$\left|\frac{dq_{im}^*}{dw_0}\right| - \left|\frac{dq_{ic}^*}{dw_0}\right| = \frac{\gamma}{4+\gamma(6+2\gamma)} > 0 \ \forall \ \gamma \in (0,1) \qquad (29)$$

In conclusion, in the case of exogenously determined wages, as product substitutability rises output reduction under collusive play becomes higher than under competitive play, hence, $(Q_m - Q_c) > 0 \ \forall \ \gamma \in (0,1]$ .

Consider now what – quite significantly − changes when wages are simultaneously and independently set by the firm-specific monopoly unions, each of them (and only it) being informed about its own firm's decision to collude with (or compete against) the rival firm in the continuation of the game: Since the latter choice implies the firm's optimal output rule, hence, its labour demand for any (given) wage, the firm's workers' union will optimally choose the firm-specific wage along the latter schedule, at the second stage of the game. Restricting interest on symmetric strategies, recall that the labour demand schedules in the cases of collusive and

[8] Note that, in our context, the absence of firm-union bargaining effectively implies that $\varphi$=0. Simply because, $\varphi=0 \rightarrow u_i = q_i$, therefore, unions have nothing to choose, at the second stage.

competitive equilibria are respectively given by (7) and (15)[9] , and let there $0 < w_c = w_m \equiv w_0 < 1$. It can be then checked that,

$$Difld_s \equiv [q_m(w_0) - q_c(w_0)] = \frac{(1-\gamma)w_0}{4+2\gamma(3+\gamma)} > 0;$$

$$\frac{\partial Difld_s}{\partial \gamma} = \frac{(1-w_0)}{2[2+\gamma(3+\gamma)]^2} > 0 \;\forall\, \gamma \in (0,1) \tag{30}$$

Moreover, from (10) and (17), it can subsequently be checked that,

$$Difwg_s \equiv (w_m^* - w_c^*) \propto (1 + w_0)\gamma\varphi > 0\,;$$

$$\frac{\partial Difwg_s}{\partial \gamma} > 0\,; \frac{\partial Difwg_s}{\partial \varphi} > 0 \;\forall\, \gamma, \varphi \in (0,1) \tag{31}$$

Combining the information contained in (30) and (31), the following configuration of $\varphi; \gamma-$ effects, on$(Q_m - Q_c)$, in turn arises in equilibrium:

*(I)* As γ increases, the $(Q_m - Q_c) > 0$ differential increases, since then the labour demand differential $[q_m(w_m) - q_c(w_c)] > 0$ increases, for any symmetric (*m* and *c*) wages.

*(II)* The less risk-averse union members are, e.g., the higher is $\varphi$, the more unions respond to the higher$-q_m$ (lower$-q_c$) labour demand, by respectively setting a higher (lower) wage. Moreover, by virtue of *(I)*, the ensuing $(w_m^* - w_c^*) > 0$ differential increases with γ.

It is now clear that *(I)* and *(II)* above identify two opposite $\varphi$ and/or $\gamma$ effects on the $(Q_m^* - Q_c^*)$ differential; a positive one and a negative one, respectively. Quite interestingly, it proves that the order configuration among those effects may be such that the output differential among competition and collusion can – in contrast to conventional wisdom – be reversed: $(Q_m^* - Q_c^*) < 0$. Namely, the latter happens if the $[\varphi; \gamma]$ effects− set out in *(II)* are of a first order, while the $[\gamma]$ effect− set out in *(I)* is of

[9] Note that, in the case of exogenous wages, (7) and (15) at the same time determine the labour demand schedules and the optimal employment/output levels, under collusion and competition respectively.

a second order. Proposition 5 (with reference to *Figure 6*) summarizes our relevant findings.[10]

**Proposition 5:**

***(i)*** *Regarding total (sectoral) output in equilibrium:*

- *If* $\boldsymbol{\varphi > \varphi_{Q(c,m)}(\gamma) = \frac{2}{3\cdot\gamma}\ \forall\ \gamma \in \left(\frac{2}{3}, 1\right)}$ *then total output under Collusion is higher than under Cournot competition.*
- *If* $\boldsymbol{\varphi_{Q(c,m)}(\gamma) > \varphi > \varphi_{Q(c,d)}(\gamma) \forall\ \gamma \in (0.472, 1)}$ *then total output under Collusion is lower than under Cournot competition, but higher than under a Mix of Strategies configuration.*

***(ii)*** *Regarding firm-union wage contracts in equilibrium:*

*The firm-union wage contracts under Cournot competition are always higher than under Collusion, while under a Mix of Strategies they lay in-between, i.e.* $\boldsymbol{w_m > w_{mos} > w_c}$.

[Proof: See Appendix (A.5)]

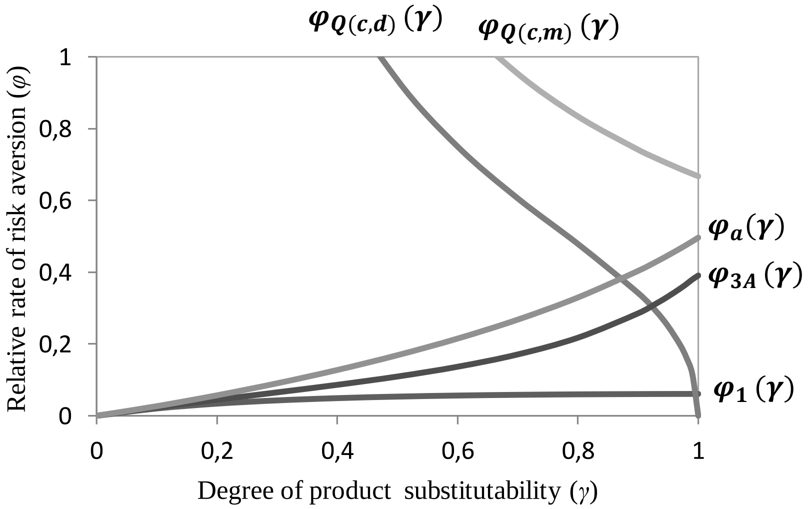


*Figure 6: The φ(γ) critical values for − c, m, mos, – total output comparisons*

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[10] Note that in our preceding analysis (in Section 4) we have not comprehensively considered *mos* in comparison to *m* and/or *c*. Yet, as dictated in Proposition 5 a Mix of Strategies configuration always possess the last place, in terms of sectoral output. On the other hand, regarding wages, *mos* always lies in-between *m* and *c*.

## 5. Welfare Analysis

Furnished by the preceding findings, the present section proceeds to a comparative analysis of the emerging equilibria (*s*, *m*, and *mos*) in terms of social welfare. The latter is typically defined to be the sum of total Consumer Surplus (*CS*), total Profits (*PS*) and total Union Rents (*UR*):

$$SW_s = CS_s + PS_s + UR_s \quad ; \; s = c, m, mos \tag{32}$$

Where, the ingredients of (32) can be easily calculated by means of the following formulae:

$$CS_s = q_{is} + q_{js} - \frac{1}{2}\left({q_{is}}^2 + {q_{js}}^2 + 2\gamma q_{is} q_{js}\right) - p_{is} q_{is} + p_{js} q_{js} \tag{33}$$

$$PS_s = \Pi_{is} + \Pi_{js} = (p_{is} - w_{is}) q_{is} + (p_{is} - w_{is}) q_{is} \tag{34}$$

$$UR_s = u_{is}^N + u_{js}^N = (w_{is} - w_0)^{\varphi} q_{is} + \left(w_{js} - w_0\right)^{\varphi} q_{js} \tag{35}$$

Our findings regarding the comparative evaluation of (33), (34), and (35) across the *s*, *m*, *mos*, configurations are respectively summarized in Propositions 7, and 8.

**Proposition 6:**

- *If* $\boldsymbol{\varphi} > \boldsymbol{\varphi_{Q(c,m)}(\gamma)} = \frac{\mathbf{2}}{\mathbf{3}\cdot\boldsymbol{\gamma}}\ \forall\, \boldsymbol{\gamma} \in \left(\frac{\mathbf{2}}{\mathbf{3}}, \mathbf{1}\right)$ *then total Consumer Surplus under Collusion is higher than under Cournot competition.*
- *If* $\boldsymbol{\varphi_{Q(c,m)}(\gamma)} > \boldsymbol{\varphi} > \boldsymbol{\varphi_{Q(c,d)}(\gamma)} \forall\, \boldsymbol{\gamma} \in (\mathbf{0.472}, \mathbf{1})$ *then total Consumer Surplus under Collusion is lower than under Cournot competition, but higher than under a Mix of Strategies configuration.*

[Proof: See Appendix (A.6)]

**Proposition 7**: *Total Profits under Collusion are always higher than under a Mix of Strategies configuration, the latter being always higher than total profits under Cournot competition, i.e.* $\boldsymbol{PS_c} > \boldsymbol{PS_{mos}} > \boldsymbol{PS_m}$.

[Proof: See Appendix (A.7)]

**Proposition 8**: *Total Union Rents under Cournot competition are always higher than those under a Mix of Strategies configuration, the latter being always higher than total union rents under Collusion, i.e.* $\boldsymbol{UR_m} > \boldsymbol{UR_{mos}} > \boldsymbol{UR_c}$.

[Proof: See Appendix (A.8)]

Quite remarkably, and in sharp contrast to conventional wisdom, Propositions 6, 7, and 8, suggest that, apart from profits, employment/output and consumer surplus may be higher under collusion than under Cournot competition. One the other hand, total union rents are at the same time lower under collusion than under Cournot competition, yet, it is the union members' risk attitude what effectively sustains collusion as an equilibrium strategy on the part of firms. Nonetheless, as our following Propositions suggest, social welfare may − by the latter token (e.g., a high enough $\varphi$) – improve, relative to Cournot competition, in a unique collusive equilibrium.

**Proposition 9**:

- *If* $\boldsymbol{\varphi} > \boldsymbol{\varphi_{SW(c,m)}}(\boldsymbol{\gamma}, \boldsymbol{w_0}) > 0.86$ *then Social Welfare under Collusion is higher than under Cournot competition.*
- *If* $\boldsymbol{\varphi_{SW(c,m)}}(\boldsymbol{\gamma}, \boldsymbol{w_0}) > \boldsymbol{\varphi} > \boldsymbol{\varphi_{SW(c,d)}}(\boldsymbol{\gamma}, \boldsymbol{w_0}) > 0.66$ *then Social Welfare under Collusion is lower than under Cournot competition, but higher than a Mix of Strategies configuration.*

Where, $\varphi_{SW(c,m)}(\gamma, w_0) > \varphi_{SW(c,d)}(\gamma, w_0)$ $\forall\, \gamma \in [0,1]$ and $\frac{\vartheta \varphi_{SW(c,m)}(\gamma, w_0)}{\vartheta w_0} < 0$ ; $\frac{\vartheta \varphi_{SW(c,d)}(\gamma, w_0)}{\vartheta w_0} < 0$. The upper and lower bounds, for each of the above $\varphi(\gamma)$ critical values, are depicted in *Figure*.[11]

[Proof: See Appendix (A.9)]

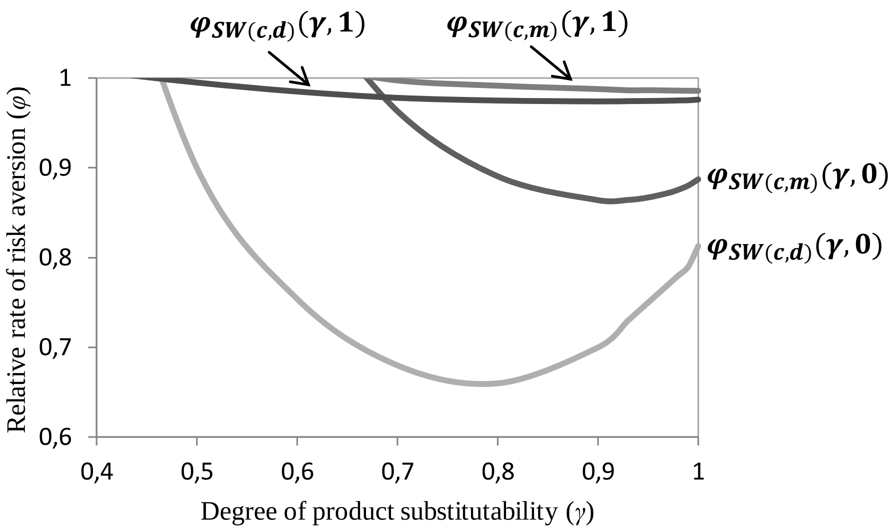



*Figure 7: The upper* $(w_0 = 1)$ *and lower* $(w_0 = 0)$ *bounds of the* $\varphi_{SW}(\gamma, w_0)$*−critical values.*

**Proposition 10**: *If* $\boldsymbol{\varphi > \varphi_{SW(c,m)}(\gamma, 0)}$ *then Cartel formation/Collusion, apart from being the unique Pareto Optimal Nash Equilibrium, in the space of profits, is a welfare improving configuration relative to both Cournot competition and Mixed of Strategies.*

[Proof: By inspecting *Figure 8* below, and recalling Proposition 4, it can be readily checked that, since $\varphi_{SW(c,m)}(\gamma, 0) > \varphi_a(\gamma)$ $\forall\, \gamma \in (0,1]$, the welfare improving

[11] The mathematical expressions of $\varphi_{SW(c,m)}(\gamma, w_0)$ and $\varphi_{SW(c,d)}(\gamma, w_0)$ were too complicated to be shaped and presented as closed forms.

condition, $\varphi > \varphi_{SW(c,m)}(\gamma, 0)$ , is also sufficient to sustain Cartel formation/collusion as the unique (Pareto optimal) equilibrium.]

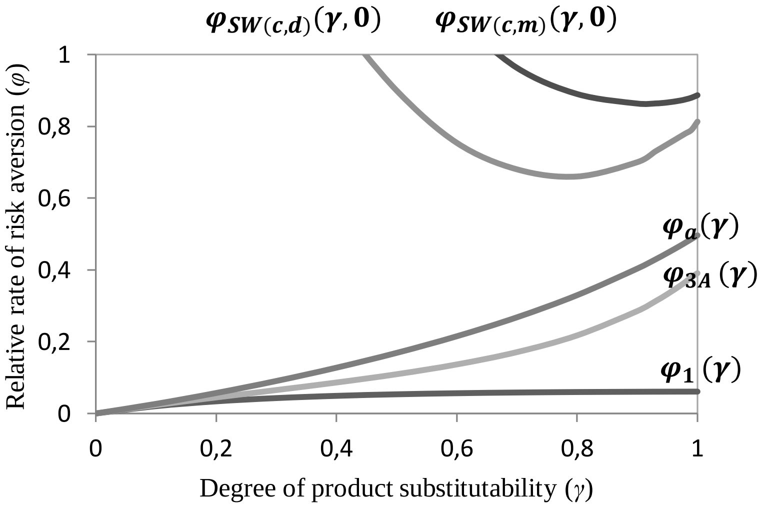



*Figure 8: The upper and lower bounds of the $\varphi_{SW}(\gamma, w_0)$−critical values Vs. the $\varphi_a(\gamma), \varphi_{3A}(\gamma), \varphi_1(\gamma)$ −critical schedules*

## 6. Concluding Remarks

In a union-oligopoly static framework this paper investigates firm-union interaction regarding the firms' incentives for collusive play. This is quite reasonable to pursue, since trade unions through their effects on unit costs, hence, on their firms output choices, may on principle play a crucial role on rivalry attitudes in the product market.

Our findings contradict conventional wisdom in a number of aspects. In one-shot games, it is a widespread belief that collusion among firms is not only weak / unstable but is also harmful for social welfare. In contrast, our analysis suggests that under certain conditions cartel formation/collusion can be endogenously sustained as an equilibrium outcome, and also improves social welfare. The key reasoning behind that is in fact quite simple: Unions would always react to their own firms' unilateral deviations from collusive play, hence, to higher firm-specific labour demand, by increasing the firm-specific wage, the latter adjustment being higher the more their members valuate wages relative to employment (i.e., the higher is $\varphi$). Firms would therefore be deterred to play competitively, thus, to commit to a high capacity in the (static) equilibrium, while at the same time the gain in terms of lower unit costs can be so high, as to drive output and consumer surplus under collusion to be higher than those under competition. Ironically for the unions, nonetheless, the ensuing loss in their members' rents can be of a second order relative to consumers' and producers' gain, so that social welfare to improve in the collusive equilibrium.

We are aware of, at least two, interesting puzzles, which our findings moreover imply. First that, unions endowed with low enough risk aversion (e.g., their members possessing a high enough $\varphi$ factor) would lose instead of gain by exercising their bargaining power over the wage, at the benefit of both firms and consumers. Second, in contrast to wide belief, benevolent policy makers should rather be in favor of militant unions, as an equivalent to antitrust policy in the absence of unions. These considerations, which raise the possibility of bargaining tricks on the part of both firms and unions, are left for future research.

## Appendix

### A.1. Proof of Proposition 1

W.l.o.g, assume that firm $i$ decides unilaterally to deviate from collusion. Then firm$i$, and also its union, taking as given the firm $j$'s output level $q_j^*$ and wages $w_j^*$ [given by (10) and (11) respectively] adjust its output level in order to maximize:

$$max_{q_i}[\Pi_\iota] = max_{q_i}\left[q_i\left(1 - q_i - \gamma q_j^* - w_i\right)\right] \tag{A1}$$

In some way, the firm $i$ is setting itself to a Stackelberg follower position by deviating from collusion. From the *f.o.c.* of (A1), we thus get the deviant (d) firm's optimal employment/output rule:

$$q_i^d(w_i) = \frac{1}{2}(1 - B_1 + B_1 w_0 - w_i) \tag{A2}$$

Where, $B_1 = \gamma / \Big(2(1+\gamma)\big(1+\varphi(1-\gamma)\big)\Big) \in [0, 1/4]\ \forall\, \gamma, \varphi \in (0,1)$, hence, it arises that $(1 - B_1) > 0$.

Therefore, taking as given $\left(q_j^*, q_i^d\right)$ and$\left(w_j^*\right)$, union $i$ chooses the firm-specific wage $\left(w_i^d\right)$ so as to maximize its members' rents [given by (8)]. From the *f.o.c.*, we then get wage set by the deviant firm's union $i$:

$$w_i^d = \frac{1}{1+\varphi}\big(\varphi(1 - B_1) - w_0(1 + \varphi B_1)\big) \tag{A3}$$

Consequently, by substituting for the firm-specific wage $(w_i^d)$ from (A2) into (A3), the deviant firm $i$'s output is:

$$q_i^d = \frac{1}{2(1+\varphi)}\left[(1 - B_1)(1 - w_0)\right] \tag{A4}$$

Since, moreover, $\Pi_i^d = \left(q_i^d\right)^2$, firm $i$ would have no incentive to deviate from collusion as long as $\Pi_\iota^* > \Pi_\iota^d$, that is if:

$$(\Pi_i^* - \Pi_i^d) = \left(\frac{B_1}{2\gamma \cdot (1+\varphi)}\right)^2 (1-w_0)^2 \cdot \left(\gamma \cdot \left(4 \cdot \varphi \cdot \left(1 + \varphi + \gamma \cdot (2 + \gamma + \varphi(2-\gamma^2))\right) - \gamma\right)\right) > 0 \tag{A5}$$

It can in turn be checked that (A5) is satisfied if $\varphi > \varphi_1(\gamma)$:

$$\varphi_{\mathbf{1}}(\gamma) = \frac{\gamma}{2 \cdot \left(\gamma^2 + (1 + 2 \cdot \gamma)\left(1 + \sqrt{1+\gamma}\right)\right)} \tag{A6}$$

Where, the value field of $\varphi_{\mathbf{1}}(\gamma)$is $\left(0, 1/\left(2 \cdot (4 + 3 \cdot \sqrt{2})\right)\right) \cong (0, 0.06)$ and its plot is depicted in *Figure 1*(in the main text).

**A.2. Proof of Proposition 2**

W.l.o.g, suppose that firm $i$ switches its strategy and behave collusively. Given the firm *j*'s output level and wages, $q_j^*$ and $w_j^*$ [see, e.g., (17) and (18) respectively], as well as the firm $i$'s reaction function [(6)], the firm $i$'s optimal employment/output rule subsequently becomes:

$$q_i^d(w_i) = \frac{1}{2}(1 - B_2 + B_2 w_0 - w_i) \tag{A7}$$

Where, $B_2 = 4\gamma/\left((2+\gamma)\left(2 + \varphi(2-\gamma)\right)\right) \in [0, 2/3] \quad \forall\, \gamma, \varphi \in (0,1)$, hence, it arises that $(1 - B_2) > 0$.

The deviant firm's union $i$ consequently chooses the firm-specific wage $w_i^d$ in order to maximize its members' rents [given by (8)]. Substituting in there $q_i^d(w_i)$ from (A7) and taking the *f.o.c.*, we then obtain:

$$w_i^d = \frac{1}{1+\varphi}\left(\varphi(1 - B_2) - w_0(1 + \varphi B_2)\right) \tag{A8}$$

Thus, we get the deviant firm $i$'s output, by substituting the union $i$'s wage from (A8) into (A7):

$$q_i^d = \frac{1}{2(1+\varphi)}[(1-B_2)(1-w_0)] \quad \text{(A9)}$$

Firm $i$ would have no incentive to deviate from competitive play as long as $\Pi_i^* > \Pi_i^d$, that is, if:

$$\Pi_i^* - \Pi_i^d = \left(\frac{B_2}{8\cdot\gamma\cdot(1+\varphi)}\right)^2 \cdot (1-w_0)^2 \cdot \left(4\cdot\gamma + \varphi\cdot\left(\gamma\cdot\left(8\cdot(2+\varphi)+\gamma\cdot\varphi\cdot(4-\gamma)\right) - 16\cdot(1+\varphi)\right)\right) > 0 \quad \text{(A10)}$$

The above inequality is satisfied if $\varphi < \varphi_2(\gamma)$:

$$\varphi_2(\gamma) = \frac{2\gamma}{4(1-\gamma)+(2-\gamma)(\sqrt{4+\gamma^2})} \quad \text{(A11)}$$

Where, $\varphi_2(\gamma) \in \left(0, 2/\sqrt{5}\right) \cong (0, 0.89)$ and its plot is presented in *Figure 2* (in the main text).

**A.3. Proof of Proposition 3**

Consider first, that, given firm $j$'s choice of $\left(q_j^*, w_j^*\right)$, like in [(23) and (27)], respectively − firm $i$ chooses $q_i^d$ so that to maximize its profits [given by (13)]. From the *f.o.c.* of **(13)**, the deviant firm's $i$'s optimal employment/output rule is then the following.

$$q_i^d(w_i) = \frac{1}{2}(1 - B_3 + B_3 w_0 - w_i) \quad \text{(A12)}$$

Where, $B_3 = \frac{\gamma\left((2+\gamma)(2+\varphi(2-\gamma)) - \gamma(4+\gamma\cdot\varphi)\right)}{2\cdot(2-\gamma^2)\left(2+\gamma\cdot(4+\varphi(2-\gamma^2))\right)} \in [0, 1/2] \forall\, \gamma, \varphi \in (0,1)$, hence, $(1-B_3) > 0$.

Substituting $q_i^d(w_i)$ from (A12) into (8) and taking the *f.o.c.*, *w.r.t.* $w_i$, we subsequently obtain:

$$w_i^d = \frac{1}{1+\varphi}\left(\varphi(1-B_3) - w_0(1+\varphi\cdot B_3)\right) \quad \text{(A13)}$$

By (A12) and (A13) the deviant's firm's output is then found to be:

$$q_i^d = \frac{1}{2(1+\varphi)}[(1-B_3)(1-w_0)] \tag{A14}$$

Substituting in turn (A13) and (A14) into (3), to obtain $\Pi_i^d$, we find that firm $i$ has no incentive to deviate from collusive play, e.g., $\Pi_i^* > \Pi_i^d$, if (see, e.g., *Figure 3* in the main text):

$$\varphi > \varphi_{\boldsymbol{3A}}(\gamma) \in [0,0.393] \tag{A15}$$

Consider next, whether firm $j$has an incentive to switch its own strategy, from competitive to collusive play. In the latter event, given that firm $i$ chooses $(q_i^*, w_i^*)$ like in [(22) and (26)], respectively, firm $j$ chooses $q_j^d$ to maximize the firms' joint profits. From the *f.o.c.* of (5), *w.r.t.* $q_j$ , we then get the deviant's firm $j$'s optimal employment/output rule:

$$q_j^d(w_j) = \frac{1}{2}\left(1 - B_4 + B_4 w_0 - w_j\right) \tag{A16}$$

Where, $B_4 = \frac{2\gamma\left(2(1-\gamma)+\varphi(2-\gamma^2)\right)}{(2-\gamma^2)\left(2+\varphi\left(4+\varphi(2-\gamma^2)\right)\right)} \in [0,0.293] \quad \forall\, \gamma, \varphi \in (0,1)$, hence, $(1-B_4) >$

0.

Substituting (A16) into (8) and taking the *f.o.c*, *w.r.t.* $w_j$ we subsequently obtain:

$$w_j^d = \frac{1}{1+\varphi}\left(\varphi(1-B_4) - w_0(1+\varphi \cdot B_4)\right) \tag{A17}$$

By virtue of (A16) and (A17) , the deviant firm's output then is:

$$q_j^d = \frac{1}{2(1+\varphi)}[(1-B_4)(1-w_0)] \tag{A18}$$

Therefore, firm $j$ has no incentive to deviate, from competitive to collusive play, e.g., $\Pi_j^* > \Pi_j^d$, if:

$$\varphi < \varphi_{3B}(\gamma) \tag{A19}$$

Where, the critical $\varphi_{3B}(\gamma) \in [0,1]$ $\forall \gamma \in [0,0.608]$, and its plot is presented in *Figure 3* (in the main text).[12]

**A.4. Proof of Proposition 4**

According to the Pareto criterion, an optimal allocation is this in which it is impossible to make any one better off without making at least someone else worse off. In our case, we apply the Pareto criterion in the space of profits, in order to select the optimal among the Nash equilibria.

Thus, arranging the values of the profit differentials among the various equilibria in descending order, we get that:

$$\Pi_{i,j}^{c} - \Pi_{i,j}^{m} = B_5(1-w_0)^2 > 0 \qquad \forall\ \varphi,\gamma \in (0,1) \qquad \text{(A20)}$$

$$\Pi_{i,j}^{c} - \Pi_{i}^{mos} = B_6(1-w_0)^2 > 0 \qquad \forall\ \varphi,\gamma \in (0,1) \qquad \text{(A21)}$$

$$\Pi_{j}^{d} - \Pi_{i,j}^{m} = B_7(1-w_0)^2 > 0 \qquad \forall\ \varphi,\gamma \in (0,1) \qquad \text{(A22)}$$

$$\Pi_{i,j}^{c} - \Pi_{j}^{mos} > 0 \qquad \forall\ \varphi > \varphi_{\boldsymbol{a}}(\gamma) \qquad \text{(A23)}$$

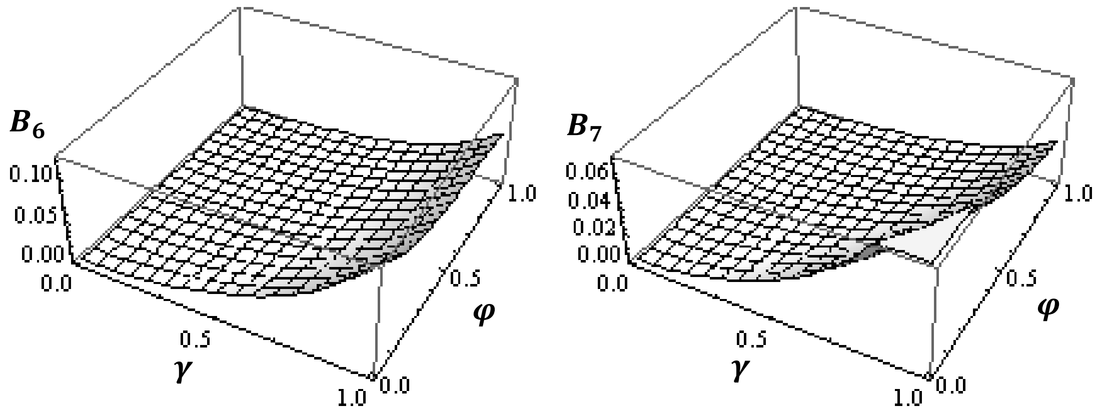
$$\Pi_{i}^{mos} - \Pi_{i,j}^{m} > 0 \qquad \forall\ \varphi > \varphi_{\boldsymbol{b}}(\gamma) \qquad \text{(A24)}$$

$$\Pi_{i}^{mos} - \Pi_{j}^{mos} > 0 \qquad \forall\ \varphi > \varphi_{\boldsymbol{c}}(\gamma) \qquad \text{(A25)}$$

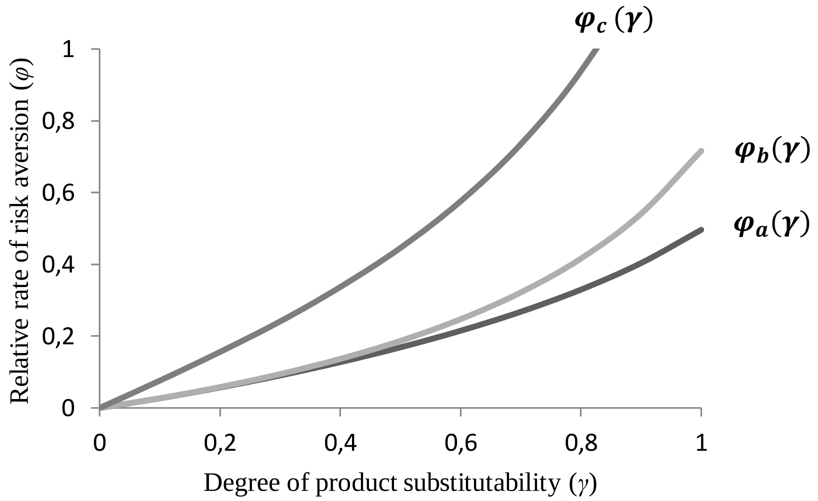
Where, $B_5 = \frac{1}{4\gamma^2}\left(4(1+\gamma){B_1}^2 - {B_2}^2\right)$, $B_6$, $B_7 > 0$[13] $\forall \gamma,\varphi \in (0,1)$, and the 3D plots of $B_6$ and $B_7$ are the following.

[12] As noted already, the $\varphi_{\boldsymbol{3A}}(\gamma)$, $\varphi_{\boldsymbol{3B}}(\gamma)$, and $\Pi_{\iota(j)}^{*} - \Pi_{\iota(j)}^{d}$ formulae are left out from presentation because of their wide extent. They are however available by the authors upon request.

[13] The mathematical expressions of $B_6$ and $B_7$ are left out because of their wide extent. They are available by the authors upon the request.




Moreover, $\varphi_c(\gamma) = \frac{\sqrt{4+\gamma^2}-2+3\gamma}{2(2-\gamma^2)}$; $\varphi_c(\gamma) > \varphi_b(\gamma) > \varphi_a(\gamma)$[14], as shown by the following plots.

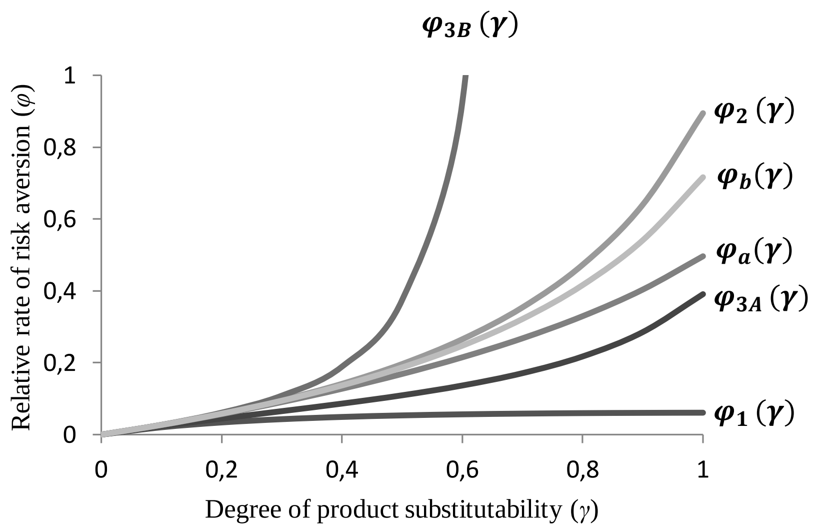



Summarizing the above, the following order configurations among profits arise:

- If $\varphi \in \left(0, \varphi_a(\gamma)\right)$ then $\Pi_j^{mos} > \Pi_{i,j}^{c} > \Pi_{i,j}^{m} > \Pi_i^{mos}$
- If $\varphi \in [\varphi_a(\gamma), \varphi_b(\gamma))$ then $\Pi_{i,j}^{c} > \Pi_j^{mos} > \Pi_{i,j}^{m} > \Pi_i^{mos}$
- If $\varphi \in [\varphi_b(\gamma), 1)$ then $\Pi_{i,j}^{c} > \left(\Pi_i^{mos}, \Pi_j^{mos}\right) > \Pi_{i,j}^{m}$

[14] The mathematical expressions of $\varphi_a(\gamma)$ and $\varphi_b(\gamma)$ are left out because of their wide extent. They are available by the authors upon the request.

Arranging all $\varphi(\gamma)$ critical schedules in descending order, we moreover get that $\varphi_{3B}(\gamma) > \varphi_2(\gamma) > \varphi_b(\gamma) > \varphi_a(\gamma) > \varphi_{3A}(\gamma) > \varphi_1(\gamma) \quad \forall\, \gamma \in [0,1]$, as shown below.




Combining the above information with the one arranged in Table (see main text), we conclude that:

- If $\varphi < \boldsymbol{\varphi_1}(\gamma)$ then Cournot competition is the single Nash Equilibrium and, consequently, the Pareto optimal one.
- If $\varphi \in [\,\varphi_1(\gamma), \varphi_{3A}(\gamma))$ and/or $(\varphi_a(\gamma),1]$ then Cartel formation/collusion is the Pareto optimal Nash equilibrium.
- If $\varphi \in [\varphi_{3A}(\gamma), \varphi_a(\gamma)]$ then both Cartel formation/collusion and Mix of Strategies are Pareto optimal Nash equilibria.

**A.5. Proof of Proposition 5**

***(i)*** Total (sectoral) output is defined as the sum of the firms' equilibrium outputs, as follows:

$$Q_s^* = q_{is}^* + q_{js}^*, \quad ; \qquad s = c, m, mos \tag{A26}$$

Where, $c$, $m$ and $mos$ respectively denote collusive, competitive, and mix of strategies, equilibria.

By means of (11), (18), and (26), (27) total equilibrium output under collusion, competition, and mix of strategies, is respectively given by:

$$Q_c^* = 2q_{ic}^* = \frac{1-w_0}{(1+\gamma)\big(1+\varphi(1-\gamma)\big)} \tag{A27}$$

$$Q_m^* = 2q_{im}^* = \frac{4(1-w_0)}{(2+\gamma)\big(2+\varphi(2-\gamma)\big)} \tag{A28}$$

$$Q_{mos}^* = \sum_{i=1}^{2} q_{imos}^* = \frac{\big(1+3(1-\gamma)+2\varphi(2-\gamma^2)\big)(1-w_0)}{(2-\gamma^2)\Big(2+\varphi\big(4+\varphi(2-\gamma^2)\big)\Big)} \tag{A29}$$

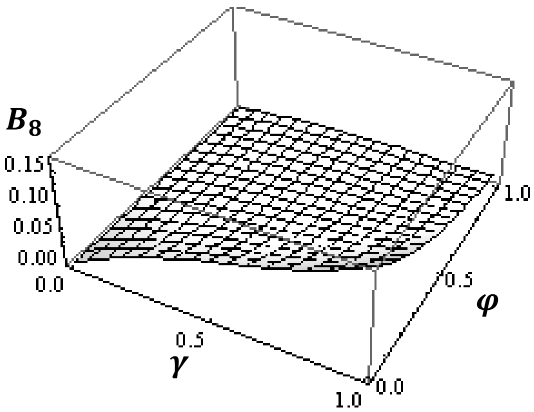
It then proves that:

$$Q_c^* > Q_m^* \qquad \forall\ \varphi > \varphi_{\boldsymbol{Q}(c,m)}(\gamma) \tag{A30}$$

$$Q_c^* > Q_{mos}^* \qquad \forall\ \varphi > \varphi_{\boldsymbol{Q}(c,d)}(\gamma) \tag{A31}$$

$$Q_m^* - Q_{mos}^* = B_8(1-w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \tag{A32}$$

Where, $B_8 > 0\ \forall\ \gamma, \varphi \in (0,1)$[15] and its 3D plot is the following.




[15] The mathematical expression of $B_8$ is left out because of their wide extent. It is available by the authors upon the request.

Regarding the validity of (A30) and (A31), it holds that $\varphi_{\boldsymbol{Q}(c,m)}(\gamma) = \frac{2}{3\gamma} \in (0,1)\ \forall\ \gamma \in (2/3\,,1)$ ; $\varphi_{\boldsymbol{Q}(c,d)}(\gamma) \forall\ \gamma \in (0.472,1)$ –with value field (0,1).[16]

Hence, as shown in *Figure 6* (see main text) depicting the $\varphi_{\boldsymbol{Q}(c,m)}(\gamma)$ , $\varphi_{\boldsymbol{Q}(c,d)}(\gamma)$ critical schedules, along with their $\varphi_1(\gamma), \varphi_{3A}(\gamma), \varphi_a(\gamma)$ counterparts, $\varphi_{Q(c,m)} > \varphi_{Q(c,d)}\ \forall\ \gamma \in [0,1]$.

***(ii)*** By means of (10), (17), (24) and (25), we obtain:

$$w_{im}^* - w_{ic}^* = B_1 B_2 \left(\frac{\varphi(1+\gamma)(2+\gamma)}{2\gamma}\right)(1-w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A33)}$$

$$w_{imos}^* - w_{ic}^* = B_1 B_5 \big(2\gamma\varphi^2(1+\gamma)\big)(1-w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A34)}$$

$$w_{jmos}^* - w_{jc}^* = B_1 B_5 \left(\frac{2\varphi(1+\varphi)(1+\gamma)}{\gamma}\right)(1-w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A35)}$$

$$w_{im}^* - w_{imos}^* = B_2 B_5 \left(\frac{\varphi(1+\varphi)(2+\gamma)}{2\gamma}\right)(1-w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A36)}$$

$$w_{jm}^* - w_{jmos}^* = B_2 B_5 \left(\frac{\varphi^2(2+\gamma)}{4}\right)(1-w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A37)}$$

Where, $B_5 = \frac{\gamma}{2+\varphi\left(4+\varphi(2-\gamma^2)\right)} \in [0,1/2]\ \ \forall\ \gamma, \varphi \in (0,1)$.

Summarizing the above results, we conclude that:

$$w_m^* > w_{mos}^* > w_c^* \qquad \text{(A38)}$$

## A.6. Proof of Proposition 6

Recall that total Consumer Surplus under collusion and competition proves to be:

$$CS_{c,m} = \frac{1+\gamma}{4} {Q_{c,m}}^2 \qquad \text{(A39)}$$

Furthermore, by means of (33), in the case of a Mix of Strategies configuration total Consumer Surplus proves to be:

[16] The mathematical expression of $\varphi_{Q(c,d)}(\gamma)$ is not reported, being too complicated to be shaped as a closed form.

$$CS_{mos} = \left( \frac{(1+\gamma)\gamma^2 + (1+\gamma)\left(4 - 3\gamma + 2\varphi(2-\gamma^2)\right)^2}{4\left(4 - 3\gamma + 2\varphi(2-\gamma^2)\right)^2} \right) Q_{mos}{}^2 \qquad \text{(A40)}$$

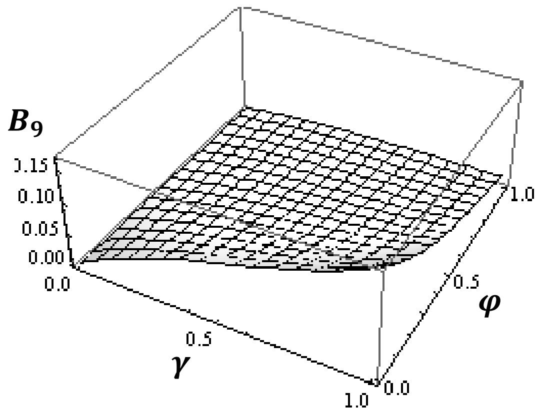
Using (A45) and (A46), it is then straightforward that, by virtue of Proposition 5:

$$CS_c^* - CS_m^* > 0 \qquad \forall\ \varphi(\gamma) > \varphi_{\boldsymbol{Q}(c,m)}(\gamma) \qquad \text{(A41)}$$

$$CS_c^* - CS_{mos}^* > 0 \qquad \forall\ \varphi(\gamma) > \varphi_{\boldsymbol{Q}(c,d)}(\gamma) \qquad \text{(A42)}$$

$$CS_m^* - CS_{mos}^* = B_9(1 - w_0) > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \qquad \text{(A43)}$$

Where, $B_9 > 0\ \forall\, \gamma, \varphi \in (0,1)$[17] and its 3D plot is presented below.




### A.7. Proof of Proposition 7

Total (sectoral) Profits in the *c*, *m*, and *mos* equilibria are defined as:

$$PS_s^* = \Pi_{is}^* + \Pi_{js}^*, \qquad \text{where } s = c, m, mos \qquad \text{(A44)}$$

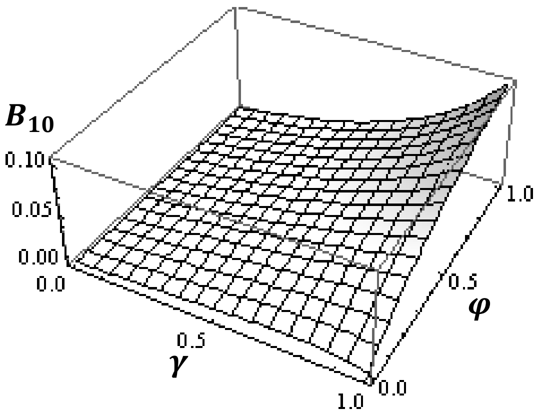
Thus, by respectively substituting (12), (19), (28a) and (28) into (A44), we get that:

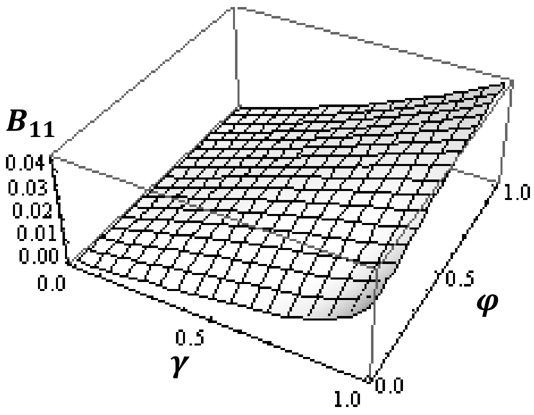
$$PS_c^* - PS_{mos}^* = B_{10}(1 - w_0)^2 > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \qquad \text{(A45)}$$

$$PS_{mos}^* - PS_m^* = B_{11}(1 - w_0)^2 > 0 \qquad \forall\ \varphi, \gamma \in (0,1) \qquad \text{(A46)}$$

Where $B_{10}, B_{11} > 0\ \ \forall\, \gamma, \varphi \in (0,1)$[18] and theirs 3D plots are the following:

[17] The mathematical expression of $B_9$ is left out because of its wide extent. It is available by the authors upon the request.
[18] The mathematical expressions of $B_{10}$ and $B_{11}$ are left out because of their wide extent. They are available by the authors upon the request.






Summarizing the above results, we conclude that:

$$PS_c^* > PS_{mos}^* > PS_m^* \quad \text{(A47)}$$

**A.8. Proof of Proposition 8**

Total (sectoral) Union Rents in the *c*, *m*, and *mos* equilibria are defined as:

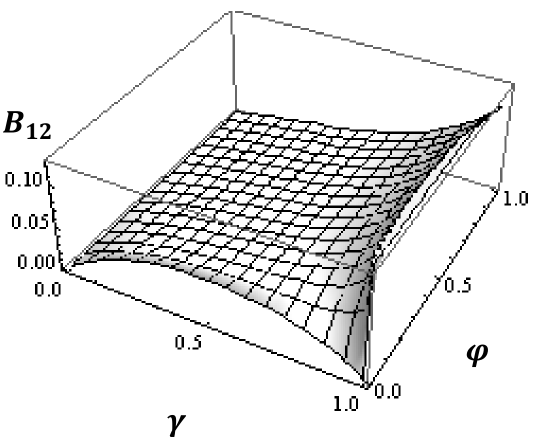
$$UR_s^* = u_{is}^* + u_{js}^* = (w_{is} - w_0)^{\varphi} q_{is} + \left(w_{js} - w_0\right)^{\varphi} q_{js}, \quad \text{where } s = c, m, d \quad \text{(A48)}$$

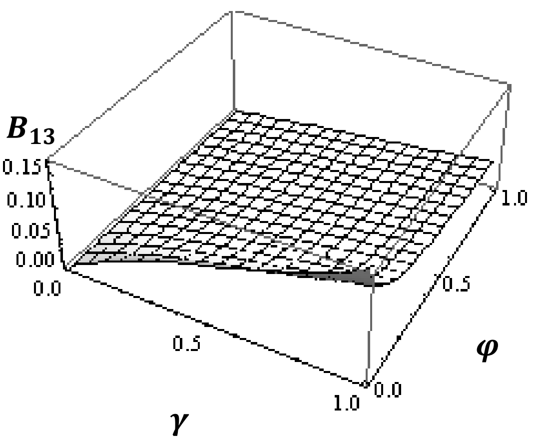
Thus, by respectively substituting (10), (17), (24) and (25) −for wages, and (11), (18), (26) and (27) −for output/employment, into (A48), we get that:

$$UR_{mos}^* - UR_c^* = B_{12}(1 - w_0)^{(1+\varphi)} > 0 \quad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A49)}$$

$$UR_m^* - UR_{mos}^* = B_{13}(1 - w_0)^{(1+\varphi)} > 0 \quad \forall\ \varphi, \gamma \in (0,1) \quad \text{(A50)}$$

Where, $B_{12}, B_{13} > 0 \ \forall\, \gamma, \varphi \in (0,1)$[19] and theirs 3D plots are the following:






[19] The mathematical expressions of $B_{12}$ and $B_{13}$ are left out because of their wide extent. They are available by the authors upon request.

Summarizing the above results, we conclude that:

$$UR_m^* > UR_{mos}^* > US_c^* \tag{A51}$$

**A.9. Proof of Proposition 9**

Social Welfare, in the *c*, *m*, and *mos* equilibria, is defined as follows:

$$SW_s = CS_s + PS_s + UR_s, \qquad \text{where } s = c, m, d \tag{A52}$$

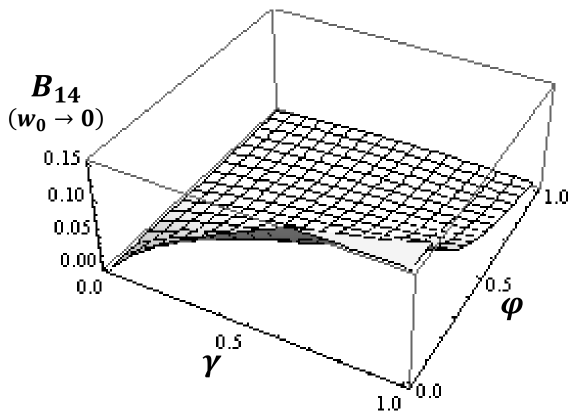
Thus, respectively substituting the $CS_s, PS_s, UR_s$ ingredients into (A52), by virtue of (A.6.), (A.7.), and (A.8.) we get that:

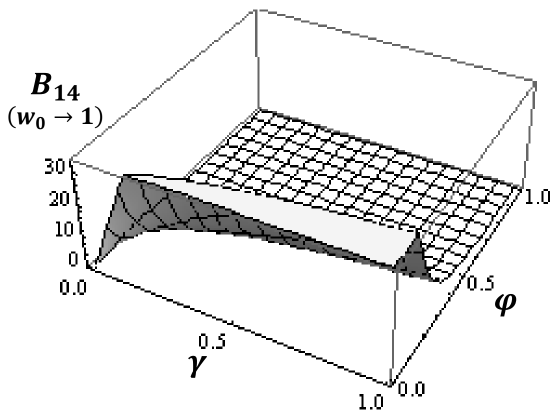
$$SW_c^* > SW_m^* \qquad \forall\ \varphi > \varphi_{SW(c,m)}(\gamma, w_0) \tag{A53}$$

$$SW_c^* > SW_{mos}^* \qquad \forall\ \varphi(\gamma) > \varphi_{SW(c,d)}(\gamma, w_0) \tag{A54}$$

$$SW_m^* - SW_{mos}^* = B_{14}(1 - w_0)^2 > 0 \quad \forall\ \varphi, \gamma \in (0,1) \tag{A55}$$

Where, $B_{14} > 0 \ \ \forall\, \gamma, \varphi, w_0 \in (0,1)$[20] and its 3D plots, for $w_0 \to 0$ and $w_0 \to 1$, are respectively the following:




[20] The mathematical expression of $B_{14}$ is left out because of its wide extent. It is available by the authors upon the request.

Regarding the validity of (A53) and (A54), it holds that $\varphi_{SW(c,m)}(\gamma, w_0) \in (0,1)\ \forall\, \gamma \in (0.67, 1), w_0 \in (0, 1)$–with value field(0.86,1) ; $\varphi_{SW(c,d)}(\gamma, w_0) \in (0,1)\ \forall\, \gamma \in (0.46, 1), w_0 \in (0, 1)$ –with value field (0.66,1)[21].

Hence, as shown in *Figure7* (see main text) depicting the $\varphi_{SW(c,m)}(\gamma, w_0)$, $\varphi_{SW(c,d)}(\gamma, w_0)$ critical schedules, $\varphi_{SW(c,m)}(\gamma, w_0) > \varphi_{SW(c,d)}(\gamma, w_0)\ \forall\, \gamma \in [0,1]$, with $\frac{\vartheta\varphi_{SW(c,m)}}{\vartheta w_0}, \frac{\vartheta\varphi_{SW(c,d)}}{\vartheta w_0} < 0$.

Summarizing the above results, we conclude that:

$$SW_m^* > SW_{mos}^* > SW_c^* \quad \forall\ \gamma \in (0,0.46) \text{ or } \gamma \in (0.46,1) \text{ and } \varphi \in \left(0, \varphi_{SW(c,d)}\right) \tag{A56}$$

$$SW_m^* > SW_c^* > SW_{mos}^* \quad \forall\ \gamma \in (0.46,0.67) \text{ and } \varphi \in \left(\varphi_{SW(c,d)}, 1\right) \text{ or } \gamma \in (0.67,1) \text{ and } \varphi \in \left(\varphi_{SW(c,d)}, \varphi_{SW(c,m)}\right) \tag{A57}$$

$$SW_c^* > SW_m^* > SW_{mos}^* \quad \forall\ \gamma \in (0.67,1) \text{ and } \varphi \in \left(\varphi_{SW(c,m)}, 1\right) \tag{A58}$$

[21] The mathematical expressions of $\varphi_{SW(c,m)}(\gamma, w_0)$ and $\varphi_{SW(c,d)}(\gamma, w_0)$ are left out, because it was complicated to be shaped as closed forms.

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