On the mode of Competition as a Collusive Perspective in Unionized Oligopoly

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Abstract

In a union-oligopoly framework with differentiated products, this paper endogenizes the mode of product market competition by exploring its strategic role on firms' incentives for collusion. It is shown that in a one-shot game setup, provided that union members are endowed with risk-neutral and monopoly bargaining power during the negotiations, cartel formation is an unavoidable equilibrium in the product market, hence industry's outcomes and market participants surpluses/rents equal to that of collusive play. The cartel is proved to be welfare improving, if and only if products' substitutability is sufficiently high under Cournot competition. Moreover and given firms' competition, we conclude that among modes of competition, under Bertrand competition Social Welfare is higher than Cournot, while under a Mix of Strategies it lies in-between. Consequently, it is welfare improving to be a benevolent policy maker that deters cartel formation and gives firms' incentive for Bertrand competition.

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1. Introduction

The mode of competition and cartel formation are fundamental concerns of Industrial Organization. The majority of studies suggest that, regardless the mode of competition, in one-shot games collusive play is a weak equilibrium condition, while in infinitely repeated games it may emerge in equilibrium under a sufficiently high discount factor. Albaek and Lambertini (*1998*) compare the stability of collusion in quantities and in prices and show that if the degree of product substitutability is low (high), collusion in quantities (prices) is more stable than collusion in prices. In a dynamic framework, Lambertini and Schultz (*2001*) conclude that if the discount factor is high, the cartel can realize the monopoly profits in Bertrand and Cournot. Otherwise, it is optimal for the cartel to rely on quantities in the collusive phase if goods are substitutes and prices if goods are complements. More recently, Suetens and Potters (*2007*) on the basis of experimental data from oligopoly experiments, show that there is more tacit collusion in price-choice than in quantity-choice experiments.

The present paper is a further step in our research, on welfare improving cartel formation in oligopoly[1], which suggests that if in a Cournot duopoly, union members are not sufficiently risk-averse and firm products are sufficiently close substitutes, then collusion among firms may emerge in the static equilibrium and this may improve social welfare. Remarkably, the gain in social welfare materializes at the cost of union rents despite the union's presence being that which effectively sustains

[1] The present paper is also a further step in our research on the mode of competition as a collusive perspective with exogenous wages. It proposes that, in an infinitely repeated game setup, firms may (ex-post) collude by (ex-ante) choosing their mode of competition in the product market with exogenous wages. It is shown that, if the discount factor is not high enough whilst the degree of product substitutability is sufficiently high, firms independently choose (in case of competition) to adjust their own prices, because this minimizes the gains from deviation from collusive play and consequently enables collusion and higher profits. Otherwise, collusion is weak / unstable and each firm's dominant strategy is (then) to compete by adjusting its own quantity.

collusion. In the present paper, in the context of a unionized duopoly model with differentiated goods and decentralized *Right-to-Manage bargaining*[2], we endogenize the firms' mode of competition, as well their perspective for cartel formation. That is, firms may compete or collude by simultaneously and independently adjusting their own quantities (*Cournot Competition*) or their own prices (*Bertrand Competition*). We further argue that either of these decisions corresponds to a long-run commitment on the part of each firm, since a higher or lower capacity, hence, sunk cost, is implied in order to efficiently produce a lower or higher level of output –for any given unit cost. The latter is in turn determined in the labour market, where each firm separately engages into wage contracting with its own workers' union, each firm retaining its discretion over employment/output. Union members are endowed with a (symmetric) rate of risk aversion, whilst union power over the firm-specific wage bargain is assumed to be unity ("monopoly unions"). In this context we subsequently postulate a static game with the following sequence of events: At the first stage, firms simultaneously and independently decide to compete in quantities or prices. At the second stage, firms simultaneously and independently decide to proceed to collusive or competitive play. At the third stage, unions and firms enter into negotiations about wages (*w-bargaining*). At the fourth stage, firms simultaneously and independently adjust their own quantities or set their own prices in order to maximize their own profits or the cartel's ones, depending on their decision at previous stages.

Our findings suggest that firm cartel formation is an unavoidable result in equilibrium, regardless of the chosen mode of competition. Moreover, we show that if product substitutability is sufficiently high, then cartel formation is Welfare improving under Cournot competition. Apart from this exception, competitive play is

[2] Right-to-manage literature was initially developed by the British school during the 1980s (Nickell). It implies that the union-firm negotiations agenda includes only the wage rate, according to a typical Nash Bargaining Maximization.

shown to be superior in terms of Social Welfare. In particular, under Bertrand competition Social Welfare is higher than under Cournot competition, while a Mix of Strategies lies in-between. Consequently, our analysis suggests that in order to improve social welfare, a benevolent policy maker should deter cartel formation while at the same time give firms' incentives for Bertrand competition.

The rest of the paper is organized as follows: Section 2 introduces our unionized duopoly model in a one-shot game context of analysis. In Section 3, we develop our proposed four-stage game. Sections 4 – 6 demonstrate the conditions under which firms proceed to cartel formation, given their mode of competition. In Section 7, our model endogenizes firms' mode of competition. Sections 7 and 8 proceed to comparative and welfare analysis of our findings, respectively. Our results are summarized in Section 8.

2. The Model

We consider a duopoly with differentiated goods, in where firms, denoted by $i \neq j = 1,2$, may compete or collude by adjusting their own quantities or prices. Each one faces the following inverse linear demand function[3]:

$$p_i(q_i, q_j) = 1 - q_i - \gamma q_j \quad (1)$$

Where, p_i and q_i denotes the price and output of firm $i \neq j = 1,2$, respectively. The factor $\gamma \in (0,1)$ presents the degree of substitutability among the goods $i \neq j = 1,2$: As $\gamma \to 1$ the firms' products tends to be perfect substitutes.

[3] Like in Dixit (1979), this function is derived by maximizing (w.r.t. q_i, q_j) the quadratic and strictly concave utility function $u(q_i, q_j) = aq_i + aq_j - \frac{b}{2}(q_i + q_j + 2\gamma q_i q_j) + m$, where m is the competitive numeraire sector. For simplicity, we assumed that both a and b are equal to one.

Firms' production technology exhibits constant returns to scale and their production function is normalized by assuming that labor productivity equals to one for both firms[4]:

$$\mathrm{L_i} = \mathrm{q_i} \tag{2}$$

Where L_i and q_i are employment and output level, respectively.[5]

According to the above, we obtain that firm i's profit function is defined by the following equation:

$$\Pi_\iota = (p_i - w_i)q_i \tag{3}$$

Where w_i denotes firm i's unit transformation cost of labor into product, i.e. wage rate.

Regarding the labor market, we assume that workers are organized into two separate firm-specific unions. Under a decentralized *Right-to-Manage* bargaining model, unions enter into negotiations with their specific firm about their wages (only). Unions are endowed with monopoly bargaining power; therefore unions act as firm-specific monopoly unions. The union i's objective is to maximize the sum of its member rents, given by the following equation:

$$u_i(w_i, L_i) = (w_i - w_0)L_i \tag{4}$$

Where, w_i is firm i's wage rate, $0 < w_0 < 1$ is the workers' outside option[6].

In the above context, our envisaged four-stage game unfolds as follows:

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[4] In more general terms, a two-factor Leontief technology is assumed in which the (minimum cost) capital (capacity) over labour ratio is equal to one.

[5] We are aware of the limitations of our analysis in assuming specific functional forms and constant returns to scale. However, the use of more general forms would jeopardize the clarity of our findings, without significantly changing their qualitative character.

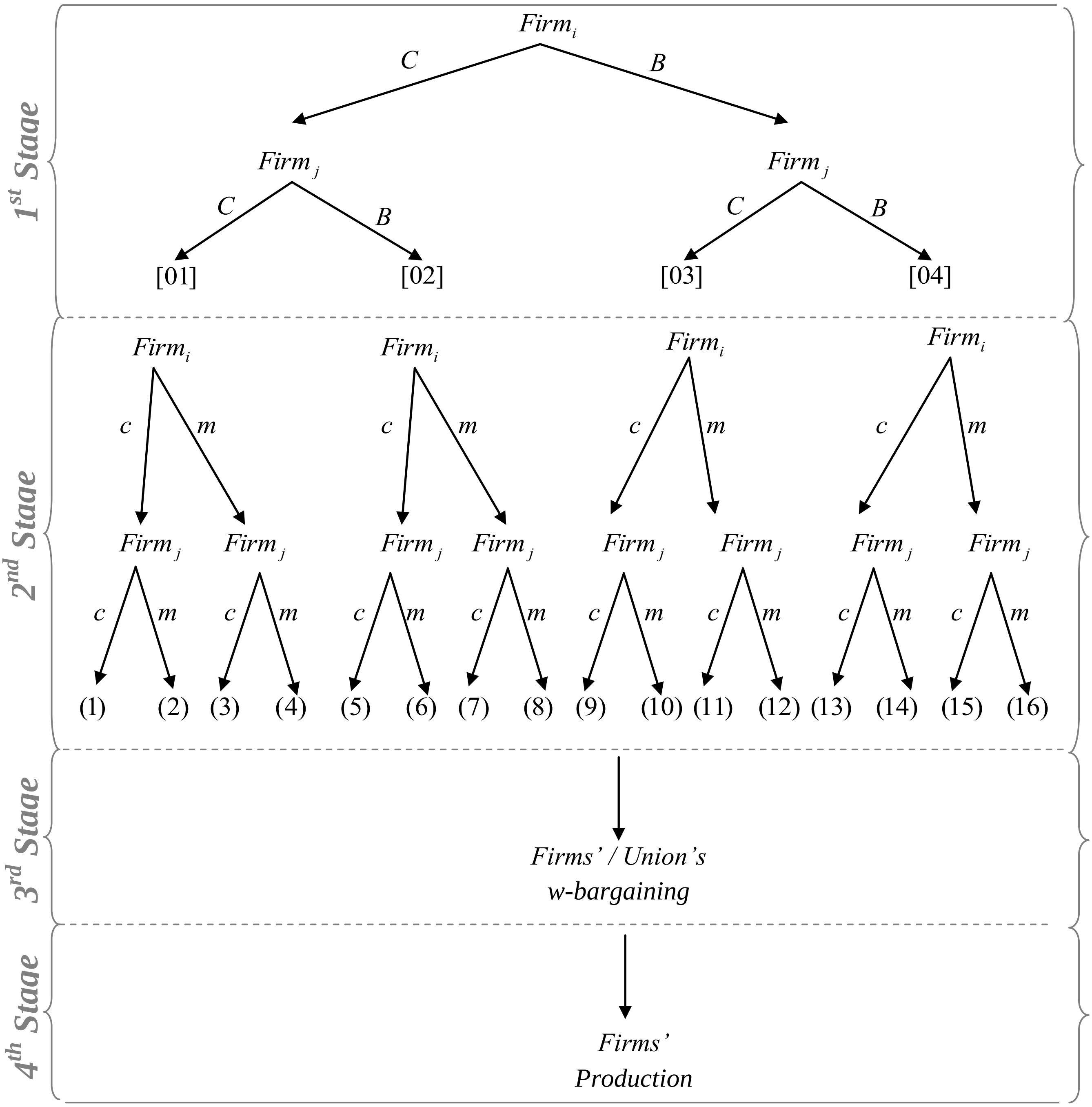
[6] As it is generally accepted in the trade unions literature, w_0 represents a weighted average of the competitive wage and the unemployment benefits, the weights respectively being the probability of a worker to find a job or not in the competitive sector.

- At the 1st stage, both firms simultaneously and independently decide their mode of competition, namely whether to adjust their own quantities (*Cournot Competition*) or their own prices (*Bertrand Competition*), in order to maximize monopoly (cartel's) profits or their own profits.
- At the 2nd stage, both firms simultaneously and independently decide whether to collude or to compete, by maximizing the monopoly (cartel's) profits or their own profits, respectively.
- At the 3rd stage, both unions simultaneously and independently set the wages for their own firms, so that each union maximizes its own member rents.
- At the 4th stage, each firm simultaneously and independently adjusts either its own output level or its product's price (depends on firms decision at the 2nd stage), in order to maximize either the monopoly (cartel's) profits or their own profits (depends on firms decision at the 1st stage).

3. Equilibrium Analysis

Following the backwards inducting method of game theory, subgame perfect equilibrium is the candidate one that no game player has incentive for deviation.

Due to the complicated form of our four-stage game, we determine all game's candidate equilibria by developing the complete game tree of our model in the following directed graph. The development of the tree is consistent with the sequence of the proposed game's stages. The graph's nodes present a specific player's position and the graph's edges present a specific player's decision. In each stage, players take their decisions simultaneously and independently.



Where *C*, *B* and *c*, *m* denote firm decisions about Cournot or Bertrand competition and Collusive or Competitive play, respectively.

At the first two stages, the numbered nodes denote the pair of firm decisions in each given stage. Each numbered node presents the following pair of decisions:

[01]: $(Cournot_i, Cournot_j)$

(1): $(Collusion_i, Collusion_j)$

(2): $(Collusion_i, Cournot_j)$

(3): $(Cournot_i, Collusion_j)$

(4): $(Cournot_i, Cournot_j)$

[03]: $(Bertrand_i, Cournot_j)$

(9): $(Collusion_i, Collusion_j)$

(10): $(Collusion_i, Cournot_j)$

(11): $(Bertrand_i, Collusion_j)$

(12): $(Bertrand_i, Cournot_j)$

[02]: $(Cournot_i, Bertrand_j)$

(5): $(Collusion_i, Collusion_j)$

(6): $(Collusion_i, Bertrand_j)$

(7): $(Cournot_i, Collusion_j)$

(8): $(Cournot_i, Bertrand_j)$

[04]: $(Bertrand_i, Bertrand_j)$

(13): $(Collusion_i, Collusion_j)$

(14): $(Collusion_i, Bertrand_j)$

(15): $(Bertrand_i, Collusion_j)$

(16): $(Bertrand_i, Bertrand_j)$

The first two stages can also be presented by the following modified matrix game:

Firm i

Firm j		***Cournot***			***Bertrand***	
		Collusion	Competition		Collusion	Competition
Cournot	Collusion	(1)	(3)	Collusion	(9)	(11)
	Competition	(2)	(4)	Competition	(10)	(12)
		Collusion	Competition		Collusion	Competition
Bertrand	Collusion	(5)	(7)	Collusion	(13)	(15)
	Competition	(6)	(8)	Competition	(1)	(16)

The four enclosed matrix games represent the firm decisions about playing collusively or competitively at the second stage of the game. The outside matrix game corresponds to firm decisions about their competition in quantities or in prices at the first stage of the game.

According to the above analysis of game tree and matrix game, four candidate equilibria arise at the first stage and sixteen arise at the second stage of the proposed game, which are presented by numbered nodes [01] to [04] and (1) to (16), respectively.

Due to symmetry, it applies that candidate equilibria [02] and [03] are identical and thus the number of candidate equilibria at the first stage are reduced to three.

For convenience of subgame perfect equilibrium determination of the proposed game, we investigate separately the candidate equilibria of the first stage of the game, in Sections 4 – 6 to follow. More specifically, in Section 4 we analyze the candidate equilibrium in where both firms decide to adjust their own quantities. Section 5 investigates the candidate equilibrium in where both firms decide to adjust their own prices. In Section 6, the candidate equilibrium in where one firm (let it be firm i) adjusts its own output is presented, while the other firm (let it be firm j) adjusts its own prices. Subsequently, in Section 7, we endogenize firms' mode of competition and demonstrate the Nash equilibrium of our proposed game.

4. Cournot Competition (*C*): [01]

Given that both firms decide to adjust their own quantities at the first stage, at the second stage three candidate equilibria arise: In Section 4.1, the candidate equilibrium is the one where a cartel is effectively formed and the possible deviation, on the part of any firm, is to adjust its own quantity in order to maximize its own. In Section 4.2, the candidate equilibrium is Cournot competition and the possible deviation, on the part of any firm, is to adjust its own quantity in order to maximize collusive profits. In Section 4.3, the candidate equilibrium is the one where one firm acts collusively, while its rival firm acts competitively, and the possible deviations arise by unilaterally switching each firm's strategy to its rival's one.

4.1. Collusive Play (*c*): (*1*)

Given that both firms decide on Collusive play (L) at the second stage, then at the last stage of the game both firms simultaneously and independently adjust their own quantities, hence their employment, in order to maximize collusive profits. Therefore, firm i's objective is presented by the following equation:

$$max_{q_i}\left[\Pi_j + \Pi_j\{= q_i\left(1 - 2\gamma q_j - w_i\right) + q_j\left(1 - 2\gamma q_i - w_j\right)\}\right] \tag{5}$$

We get firm i's reaction function by taking the first order condition (*f.o.c.*) of (5), as follows:

$$R_{ic}(q_{jc}) = \left(1 - 2\gamma q_{jc} - w_{ic}\right)/2 \tag{6}$$

Solving the system of both firms' reaction function in (6), we get the optimal output/employment rule in the candidate equilibrium:

$$q_{ic}\left(w_{ic}, w_{jc}\right) = \frac{1 - \gamma - w_i + \gamma w_{jc}}{2(1-\gamma)(1+\gamma)} \tag{7}$$

At the third stage of the game, both unions simultaneously and independently set the wages for their own firms in order to maximize their own member rents. Hence, according to (2) and (4), union i's objective is:

$$max_{w_i}[u_i(w_i, q_i)\{= (w_i - w_0)q_i\}] \tag{8}$$

The union i's wage reaction function is derived by the $f.o.c_s$ in (8):

$$w_{ic}\left(w_{jc}\right) = \left(1 - \gamma + w_0 + \gamma w_{jc}\right)/2 \tag{9}$$

Solving the system in (9), we get the (candidate) optimal wage rule:

$$w_{ic}^{*} = \frac{1-\gamma+w_0}{2-\gamma} \tag{10}$$

Substituting now (10) for (7), we get the (candidate) equilibrium output level, hence the employment level:

$$q_{ic}^{*} = \frac{1-w_0}{2(1+\gamma)(2-\gamma)} \tag{11}$$

Moreover, we get that:

$$\Pi_{ic}^{*} = \left(\Pi_{ic}^{*} + \Pi_{jc}^{*}\right)/2 = (1+\gamma)(q_{ic}^{*})^2 \tag{12}$$

Substituting now the candidate equilibrium output level (11) for firms' profit function (12), we get that:

$$\Pi_{ic}^{*} = \frac{(1-w_0)^2}{4(1+\gamma)(2-\gamma)^2} \tag{13}$$

4.2. Competitive Play (*m*): (*4*)

Assume next that both firms decide to play competitively (*M*) at the second stage of the game, while at the last stage of the game both firms simultaneously and independently adjust their own quantities in order to maximize their own profits. Therefore, firm i's objective is given by the following equation:

$$max_{q_i}\left[\Pi_i\left\{= q_i\left(1-2\gamma q_j - w_i\right)\right\}\right] \tag{14}$$

From the *f.o.c*$_s$ of (14), firm i's reaction function is derived, as follows:

$$R_{im}(q_{jm}) = \left(1-\gamma q_{jm} - w_{im}\right)/2 \tag{15}$$

Solving this system of both firms' reaction function in (15), we subsequently get the (candidate) equilibrium output rules under Cournot competition:

$$q_{im}\left(w_{im}, w_{jm}\right) = \frac{2-\gamma-2w_{im}+\gamma w_{jm}}{(2-\gamma)(2+\gamma)} \tag{16}$$

Given now the optimal output/employment level in candidate equilibrium [given in (16)], at the third stage each union aims to maximize the sum of its member rents [given in (8)] by setting a firm-specific optimal wage (w_i). The *f.o.c*$_s$ of that maximization delivers the unions' reaction function:

$$w_{im}(w_{jm}) = (2 - \gamma + 2w_0 + \gamma w_{jm})/4 \tag{17}$$

Solving system (17), we get the candidate equilibrium wage:

$$w_{im}^* = \frac{2(1 + w_0) - \gamma}{4 - \gamma} \tag{18}$$

By means of (16) and (18), we obtain that each firm's output/employment levels in the candidate equilibrium is given by the following:

$$q_{im}^* = \frac{2(1 - w_0)}{(2 + \gamma)(4 - \gamma)} \tag{19}$$

In addition, we get from Cournot's lemma that:

$$\Pi_{im}^* = (\Pi_{im}^* + \Pi_{jm}^*)/2 = (q_{im}^*)^2 \tag{20}$$

Substituting now the candidate equilibrium output level (19) for firms' profit function (20), we get that:

$$\Pi_{im}^* = \frac{4(1 - w_0)^2}{(2 + \gamma)^2(4 - \gamma)^2} \tag{21}$$

4.3. Mix of Strategies (*mos*): (*2*), (*3*)

According to the Mix of Strategies in the second stage, the one firm (let it be firm j) adjusts its own output competitively, while the other firm (let it be firm i) adjusts its own output in order to maximize joint profits. Thence, at the last stage of the game we must consider as firm i's and firm j's objective function the pair of (5) and (14), respectively. Correspondingly, if we consider now the firm-specific reaction functions (6) and (15), we get the following candidate equilibrium output level, hence employment, by solving that system of equations:

$$q_i(w_i, w_j) = \frac{1-\gamma-w_i+\gamma w_j}{2-\gamma^2} \tag{22}$$

$$q_j(w_i, w_j) = \frac{2-\gamma-2w_j+\gamma w_i}{2(2-\gamma^2)} \tag{23}$$

Substituting now firms' output level [given in (22) and (23)] for the unions' objective function [given in (8)], at the third stage we derive the unions' wage reaction function:

$$w_i(w_j) = (1-\gamma+w_0+\gamma w_j)/2 \tag{24}$$

$$w_j(w_i) = (2-\gamma+2w_0+\gamma w_i)/4 \tag{25}$$

The firm-specific wage outcomes in the candidate equilibrium are derived by solving the system of unions' wage reaction function [given in (24) and (25)], as follows:

$$w_i^* = \frac{4-\gamma(2+\gamma)+2(2+\gamma)w_0}{8-\gamma^2} \tag{26}$$

$$w_j^* = \frac{4-\gamma(1+\gamma)+(4+\gamma)w_0}{8-\gamma^2} \tag{27}$$

We get the following firm-specific output levels in the candidate equilibrium by substituting now (26) and (27) for (22) and (23), respectively:

$$q_i^* = \frac{\big(4-\gamma(2+\gamma)\big)(1-w_0)}{(2-\gamma^2)(8-\gamma^2)} \tag{28}$$

$$q_j^* = \frac{\big(4-\gamma(1+\gamma)\big)(1-w_0)}{(2-\gamma^2)(8-\gamma^2)} \tag{29}$$

Comparing the firm's candidate equilibrium wages [given in (26) and (27)] and employment [given in (28) and (29)] outcomes, we obtain that firm j, which plays competitively, not only achieves for its specific-union higher employment, but also higher wage rates. Consequently, in candidate equilibrium it is applies that union$j$'s utility is higher than union$j$'s one.

Substituting now firms' candidate equilibrium output levels in (26) and (27) for firms' profit function (28) and (29), we get that:

$$\Pi_i^* = \frac{(2+\gamma)\big(4-\gamma(2+\gamma)\big)(1-w_0)^2}{(2-\gamma^2)(8-\gamma^2)^2} \tag{30}$$

$$\Pi_j^* = \frac{\big(4-\gamma(1+\gamma)\big)^2(1-w_0)^2}{(2-\gamma^2)^2(8-\gamma^2)^2} \tag{31}$$

4.4. Endogenous Selection of Final Market Structure

Given that both firms decide on Cournot competition (*C*) at the first stage of the game, at the second stage of the game both firms simultaneously and independently decide to play collusively or competitively by adjusting their own output level in order to maximize the monopoly (cartel's) profits or their own profits, respectively.

Given our findings in Subsections 4.1.− 4.3., at the second stage of the game firms deal with the following matrix game, which presents firm payoffs when both firms simultaneously and independently decide about their objective:

		Firm i: *Collusion*	Firm i: *Cournot Competition*
Firm j	*Collusion*	$\{\Pi_{ic}^*, \Pi_{jc}^*\}$	$\{\Pi_{id_i}^*, \Pi_{jd_i}^*\}$
Firm j	*Cournot Competition*	$\{\Pi_{id_j}^*, \Pi_{jd_j}^*\}$	$\{\Pi_{im}^*, \Pi_{im}^*\}$

Table 1: The Matrix Game that firms deal with at the second stage of the game.

Due to symmetry, the number of candidate equilibria is reduced to three, as it applies that $\left(\Pi_{id_j}^*, \Pi_{jd_j}^*\right) = \left(\Pi_{jd_i}^*, \Pi_{id_i}^*\right)$

In contrast to conventional belief, in our model it is proved that in a unionized Cournot duopoly static framework, firm cartel formation is the only candidate equilibrium that no firm has an incentive to deviate from; hence collusion among firms is the only Subgame Perfect equilibrium.

In order to grasp it, consider the standard oligopolistic market in where firms' decide whether to collude or compete in quantities. It is well-known according to relevant literature, that even if collusive play is Pareto Optimal as regards profits, it does not emerge in equilibrium, as a deviant firm would achieve higher profits, and the only equilibrium configuration is derived by competition (*Prisoners' Dilemma*). Assuming now a unionized Cournot-oligopolistic market (our model), the above results are reversed, i.e. the cartel formation is not only Pareto Optimal as regards profits, but also Subgame Perfect equilibrium and competition does not sustain in equilibrium. Firm's dominant strategy is the cartel formation, as collusive play increases its own profits, because by decreasing its output level/market share (labor demand) it achieves a reduction in labor costs. Thus, the deviated gains from the cartel are lower than those under the market case of fixed wages, and eventually the deviant firm loses more due to the extra unit cost than gains from stealing business from its rival firm. The following proposition encapsulates both effects, suggesting that firms would be deterred to deviate from collusive play.

Proposition 1: *In Cournot duopoly, collusion among firms is a subgame perfect equilibrium in the product market.*

[Proof: See Appendix (A.1)]

4.5. Equilibrium in Product and Labor Market

Let us now investigate how the oligopoly market eventually reaches the equilibrium point in product and labor market. For convenience, we investigate the case of firms' collusion and competition.

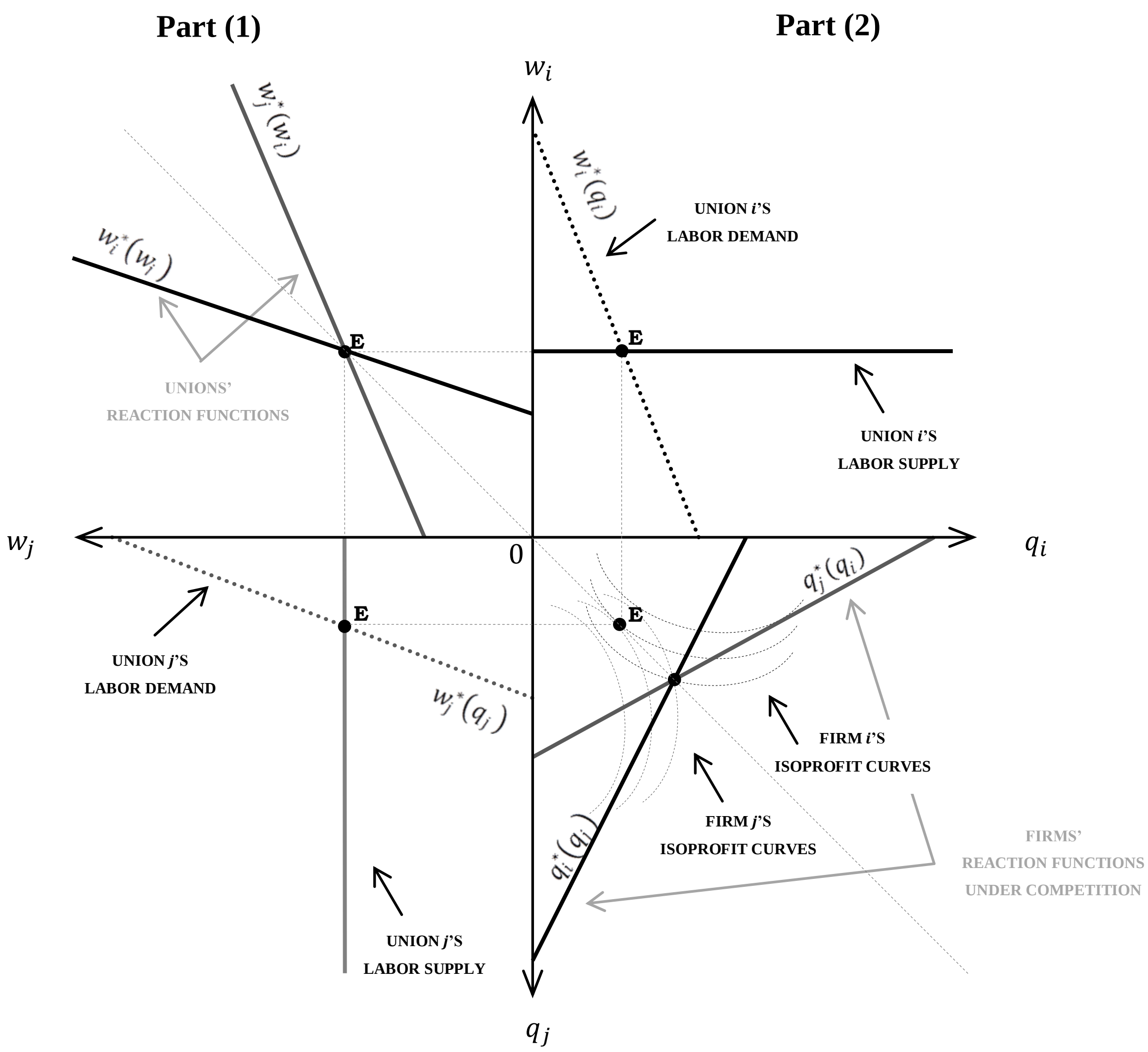


Figure 1: Labor and Product market equilibrium under firms' collusive play.

Assuming first that the firms form a cartel, at the last stage of the game firms determine their optimal output rule, hence employment, given in (7). At the previous stage, unions set their own wages in order to maximize their revenue given their bargaining power over negotiations with their own specific-firm and firms' optimal output rule, hence employment, [given in (7)]. The *f.o.c$_s$* of that maximization deliver unions' reaction function [given in (9), respectively for each union]. The intersection of unions' reaction function denotes the optimal union wage rule [given in (10)] under firms' collusive play. Due to unions' monopoly bargaining power, unions' labor supply functions are perfectly inelastic and are presented in Part (2) of Figure 1.

Labor demand functions are also presented in Part (2) of Figure 1 and are derived by substituting the rivals' union reaction function [let $w_j^*(w_i)$, given in (9)] for the specific firm optimal output rule [let $q_i^*\left(w_i, w_j\right)$, given in (7)], as follows:

$$q_i^*(w_i) = \frac{(1-\gamma)(2+\gamma) + \gamma w_0 - (2-\gamma^2)w_i}{4(1+\gamma)(1-\gamma)} \qquad (32)$$

The intersection of labor demand and supply functions denotes the equilibrium union wages, which is given in (10). Taking into consideration now the equilibrium wages, we can easily get the plots of firms' competitive reaction functions as are presented in Part (3) of Figure 1 in (10) and (15), as follows:

$$q_i = \frac{1}{2}\left(\frac{1-w_0}{2-\gamma} - \gamma q_j\right) \qquad (33)$$

Diagrammatically the equilibrium output level is the only one point in where firms' isoprofit curves osculate, whichformula is given in (11).

We easily conclude that the firms' reaction function curves are not constant over Cartesian plane but they shift, inward or outward, depending on the different values of labor cost. In fact, this implies that unions, as "first movers" in our model, force the point on the Cartesian plane $\left(q_i, q_j\right)$ at which firms reach their equilibrium point eventually.

The analysis of the case of competition follows excactly the same path, except that product market equilibrium is denoted by the intersection of the firm's reaction function and not by the tangent point of the firms' isoprofit curves.

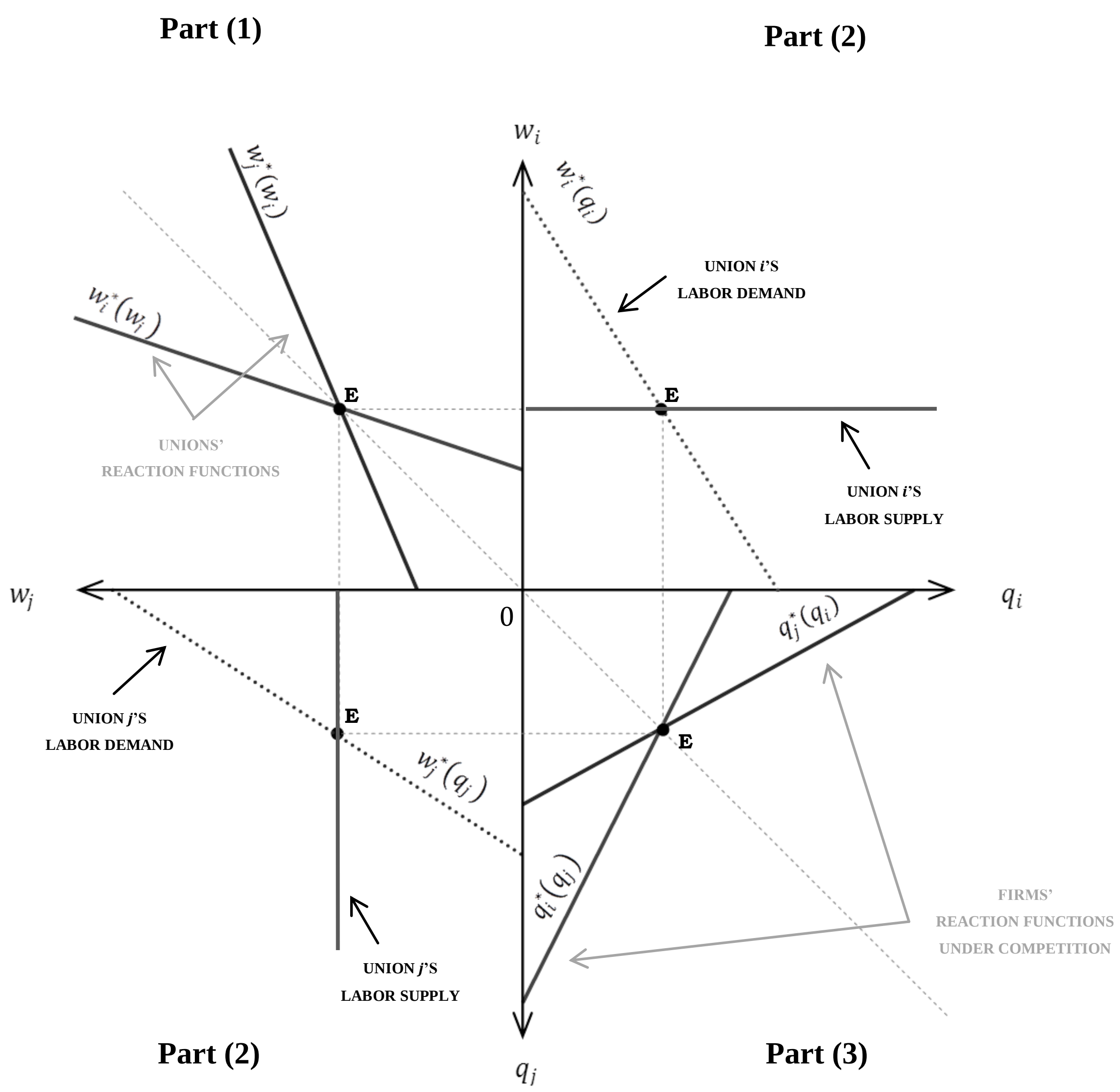


Figure 2: How the labor and product markets reach the equilibrium point under the firms' competition.

In brief, Part (1) of Figure 2 presents the unions' reaction functions [given in (17)], with the point of intersection being the optimal wage rule [given in (18)]. Part (2) depictsthe labor supply functions [given in (18)], which is perfectly anelastic due to unions' monopoly bargaining power. Part (2) presents also the labor demand functions, which is derived by substituting the rivals' union reaction function [let $it\ be\ w_j^*(w_i)$, given in (17)] for its specific firm optimal output rule [let $it\ be\ \ q_i^*(w_i, w_j)$, given in (16)], as follows:

$$q_i^*(w_i) = \frac{(2-\gamma)(4+\gamma) + 2\gamma w_0 - (8-\gamma^2)w_i}{4(2+\gamma)(2-\gamma)} \tag{34}$$

The firms reaction functions appear in Part (4) and are plotted given the wages from the labor market equilibrium. Their equations are derived from the substitution of the equilibrium wages [given in (18)] for the firms' reaction function [given in (15)]:

$$q_i = \frac{1}{2}\left(\frac{2(1-w_0)}{4-\gamma} - \gamma q_j\right) \tag{35}$$

Solving the system of equations, we get the equilibrium point of product output [given in (19)]. The equilibrium is the intersection of the firms' reaction functions in Part (4) of Figure 2.

4.6. The Effects of Alternative Final Market structure on Market Outcomes

In this section we analyze the impact of firms' strategic play in the product market, i.e. collusive, competitive and mix of strategies regimes, on equilibrium outcomes, i.e. on output level and union wages.

In Cournot duopoly, as product substitutability increases, the reduction of collusive wages is high enough to inverse the output differential among competition and collusion. The following Proposition summarizes our findings:

Proposition 2:

(i) *Regarding total output/employment level in Cournot duopoly:*

- *If $\gamma > \frac{2}{3}$, then total output under Collusion is higher than under Competition, the latter being higher than total output under Mix of Strategies Configuration, i.e. $\boldsymbol{Q_c} > \boldsymbol{Q_m} > \boldsymbol{Q_{mos}}$.*

- *If $\gamma < \frac{2}{3}$, then total output under Competition is higher than under Collusion and Mix of Strategies Configuration, i.e.* $\boldsymbol{Q_m} > \boldsymbol{Q_c}, \boldsymbol{Q_{mos}}$

(ii) *Regarding firm-union wages in Cournot duopoly, the wages are always higher under Competition than under Collusion, while under Mix of Strategies they lay in-between, i.e.* $\boldsymbol{w_m} > \boldsymbol{w_{mos}} > \boldsymbol{w_c}$.

[Proof: See Appendix (A.2)]

4.7. Welfare Analysis

In this section we proceed to a comparative analysis of the candidate equilibrium in terms of social welfare. Social Welfare is the sum of Consumer Surplus (*CS*), Producer Surplus (*PS*) and Union Rents (*UR*):

$$SW_s = CS_s + PS_s + UR_s\ ;\ \ s = c, m, mos \tag{36}$$

Where$c$, m and mos respectively denote collusive, competitive, and mix of strategies, equilibria. The ingredients of the above equation are defined by:

$$CS_s = \sum_{k=i,j}[(1 - p_{ks})q_{ks}] - \frac{1}{2}\left(q_{is}{}^2 + q_{js}{}^2 + 2\gamma q_{is}q_{js}\right) \tag{37}$$

$$PS_s = \sum_{k=i,j}[\Pi_{ks}] = \sum_{k=i,j}[(p_{ks} - w_{ks})q_{ks}] \tag{38}$$

$$UR_s = \sum_{k=i,j}[u_{ks}^N] = \sum_{k=i,j}[(w_{ks} - w_0)q_{ks}] \tag{39}$$

In accordance with Cournot Lemma, we also get that:

$$CS_{c,m} = \frac{1+\gamma}{4}{Q_{c,m}}^2 \tag{40}$$

$$PS_{c,m} = \frac{{Q_{c,m}}^2}{2} \tag{41}$$

The results of the comparative evaluation of market participant surpluses/rents in (36), (37), (38) and social welfare in (39), across the s, are summarized in the following propositions:

Proposition 3: *Regarding ConsumerSurplus in Cournot duopoly:*

- *If* $\gamma > \frac{2}{3}$, *then Consumer Surplus under Collusion is higher than under Competition, the latter being higher than total output under Mix of Strategies Configuration, i.e.* $\boldsymbol{CS_c} > \boldsymbol{CS_m} > \boldsymbol{CS_{mos}}$.
- *If* $\gamma < \frac{2}{3}$, *then Consumer Surplus under Competition is higher than under Collusion and Mix of Strategies Configuration, i.e.* $\boldsymbol{CS_m} > \boldsymbol{CS_c}, \boldsymbol{CS_{mos}}$.

[Proof: See Appendix (A.3)]

Proposition 4: *In Cournot duopoly, Producer Surplus under Collusion is always higher than under a Mix of Strategies configuration, the latter being always higher than under Competition, i.e.* $\boldsymbol{PS_c} > \boldsymbol{PS_{mos}} > \boldsymbol{PS_m}$.

Precisely the opposite order applies regarding Union Rents, i.e. $\boldsymbol{UR_m} > \boldsymbol{UR_{mos}} > \boldsymbol{UR_c}$

[Proof: See Appendix (A.4)]

In contrast to conventional belief, we proved that in a unionized Cournot duopoly static framework, collusion among firms not only emerges in equilibrium, but may also be Social Welfare improving.

Proposition 5: *Regarding Social Welfare in Cournot duopoly:*

- *If $\gamma > \frac{2}{3}$, then Social Welfare under Collusion is higher than under Competition, the latter being higher than total output under Mix of Strategies Configuration, i.e.* $\boldsymbol{SW_c} > \boldsymbol{SW_m} > \boldsymbol{SW_{mos}}$.
- *If $\gamma < \frac{2}{3}$, then Social Welfare under Competition is higher than under Collusion and Mix of Strategies Configuration, i.e.* $\boldsymbol{SW_m} > \boldsymbol{SW_c}, \boldsymbol{SW_{mos}}$.

[Proof: See Appendix (A.5)]

5. Bertrand Competition (*B*): [04]

Given that both firms decide to set their own prices at the first stage, each firm faces the following linear demand function, which is derived in (1):

$$q_i(p_i, p_j) = \frac{(1 - \gamma - p_i + \gamma p_j)}{(1 - \gamma)(1 + \gamma)} \tag{42}$$

By getting the linear demand function in (42) and firm i's profit function in (3), we get that firm i's profit is defined by:

$$\Pi_i = (p_i - w_i)\frac{(1 - \gamma - p_i + \gamma p_j)}{(1 - \gamma)(1 + \gamma)} \tag{43}$$

In our model, at the second stage three candidate equilibria arise: In Section 5.1, the candidate equilibrium is the one where both firms set their own prices in order to maximize their joint profits and the possible deviation, on the part of any firm, is to set its own price in order to maximize its own profits. In Section 5.2, the candidate equilibrium is Bertrand competition and the possible deviation, on the part of any firm, is to set its own quantity in order to maximize the cartel's profits. In Section 5.3, the candidate equilibrium is the one where one firm acts collusively, while its rival firm acts competitively, and the possible deviations arise by unilaterally switching each firm's strategy to its rival's one.

5.1. Collusive Play (*c*): (*13*)

Under firm cartel formation, at the last stage of the game both firms simultaneously and independently set their own product price in order to maximize joint profits. Hence, firm i's objective is presented by the following equation:

$$max_{p_i}[\Pi_j + \Pi_j] \tag{44}$$

From *f.o.c.*'s of (44), we get firm i's reaction function, as follows:

$$R_{ic}(p_{jc}) = (1 - \gamma + 2\gamma p_{jc} + w_{ic} - \gamma w_{jc})/2 \tag{45}$$

The candidate equilibrium pricing policy is derived by solving the system of both firms' reaction function in (45):

$$p_{ic}(w_{ic}, w_{jc}) = (1 + w_{ic})/2 \tag{46}$$

At the third stage of the game, we derive the union i's wage reaction function by substituting the optimal output level [given by substituting (46) for (42)] and taking the *f.o.c.*$_s$ to union i's objective function [given in (8)]:

$$w_{ic}(w_{jc}) = (1 - \gamma + w_0 + \gamma w_{jc})/2 \tag{47}$$

Solving the system of (47), we get the (candidate) optimal wage rule:

$$w_{ic}^* = \frac{1 - \gamma + w_0}{2 - \gamma} \tag{48}$$

From (46) and (48), we get the optimal price of product i in candidate equilibrium, as follows:

$$p_{ic}^* = \frac{3 - 2\gamma + w_0}{2(2 - \gamma)} \tag{49}$$

We derive now the candidate equilibrium output level, hence employment, by substituting the optimal price [given in (49)] for the product demand function [given in (42)]:

$$q_{ic}^{*} = \frac{1 - w_0}{2(1+\gamma)(2-\gamma)} \tag{50}$$

Taking now into consideration the candidate equilibrium product price in (49) and firms' profit function (43), we get that:

$$\Pi_{ic}^{*} = \frac{(1 - w_0)^2}{4(1+\gamma)(2-\gamma)^2} \tag{51}$$

Notice that regardless of the chosen mode of competition, under firm cartel formation the market outcomes i.e. output/employment level [given in (11) and (50)] and union wages [given in (10) and (48)], are the same. In fact, firms' collusive play generates a monopoly market, in where by either setting prices or adjusting quantities, the optimal output level and product prices that maximize profits is given.

5.2. Competitive Play (*m*): (*16*)

Suppose now that both firms decide to play competitively (*M*) at the second stage of the game and thus at the last stage of the game both firms simultaneously and independently set their own product price in order to maximize their own profits. Consequently, firm i's objective is given by the following equation:

$$max_{p_i}\left[\Pi_i\left\{= \frac{(p_i - w_i)\left(1-\gamma-p_i+\gamma p_j\right)}{(1-\gamma)(1+\gamma)}\right\}\right] \tag{52}$$

Firm i's reaction function is derived from the *f.o.c*$_s$ of (52):

$$R_{im}(p_{jm}) = \left(1-\gamma-\gamma p_{jm}+w_{im}\right)/2 \tag{53}$$

Under Bertrand competition, we get the product i's price in candidate equilibrium by solving the system of both firms' reaction function in (53):

$$p_{im}\left(w_{im}, w_{jm}\right) = \frac{(1-\gamma)(2+\gamma) + 2w_{im} + \gamma w_{jm}}{(2-\gamma)(2+\gamma)} \tag{54}$$

Taking into consideration the candidate equilibrium employment level by substituting (54) for (42), at the third stage we derive the unions' reaction functions by taking the *f.o.c*$_s$ of unions' rent maximization in (8), as follows:

$$w_{im}\left(w_{jm}\right) = \frac{(1-\gamma)(2+\gamma) + (2-\gamma^2)w_0 + \gamma w_{jm}}{2(2-\gamma^2)} \tag{55}$$

The candidate equilibrium wage arises by solving the system of unions' reaction functions (55):

$$w_{im}^* = \frac{(1-\gamma)(2+\gamma) + (2-\gamma^2)w_0}{4-\gamma(1+2\gamma)} \tag{56}$$

Therefore, we get the optimal firms' pricing policy in the candidate equilibrium by substituting (56) for (54), as is presented below:

$$p_{im}^* = \frac{2(1-\gamma)(3-\gamma^2) + (2-\gamma^2)w_0}{(2-\gamma)\left(4-\gamma(1+2\gamma)\right)} \tag{57}$$

The optimal output/employment level is derived by substituting the optimal firm pricing policy in (57) for the demand function in (42):

$$q_{im}^* = \frac{(2-\gamma^2)(1-w_0)}{(1+\gamma)(2-\gamma)\left(4-\gamma(1+2\gamma)\right)} \tag{58}$$

In addition, we get that:

$$\Pi_{im}^* = (1+\gamma)(1-\gamma)(q_{im}^*)^2 \tag{59}$$

Substituting now the candidate equilibrium output level (58) for firms' profit function (59), we get that:

$$\Pi_{im}^* = \frac{(1-\gamma)(2-\gamma^2)^2(1-w_0)^2}{(1+\gamma)(2-\gamma)^2\left(4-\gamma(1+2\gamma)\right)^2} \tag{60}$$

5.3. Mix of Strategies (*mos*): (*14*), (*15*)

In accordance with the Mix of Strategies configuration, at the second stage the one firm (let it be firm j) sets its own price competitively, while the other firm (let it be firm i) adjusts its own price in order to maximize cartel profits. Consequently, at the last stage of the game we must consider as firm i's and firm j's objective functions and reaction functions the pair of (44), (45) and (52), (53) respectively. Solving now the system of firms' reaction functions, we get the following candidate equilibrium product price:

$$p_i(w_i, w_j) = \frac{(1+\gamma)(1-\gamma) + w_i}{2-\gamma^2} \tag{61}$$

$$p_j(w_i, w_j) = \frac{(2+\gamma)(1-\gamma) + \gamma w_i + (2-\gamma^2)w_j}{2(2-\gamma^2)} \tag{62}$$

At the third stage we derive the unions' reaction functions by taking the *f.o.c*$_s$ of unions' rent maximization in (8), as follows:

$$w_i(w_j) = (1-\gamma+w_0+\gamma w_j)/2 \tag{63}$$

$$w_j(w_i) = \frac{(2+\gamma)(1-\gamma) + (2-\gamma^2)w_0 + \gamma w_i}{2(2-\gamma^2)} \tag{64}$$

Solving the system of unions' wage reaction functions [given in (63) and (64)], we get the firm-specific wage outcomes:

$$w_i^* = \frac{(1-\gamma)\big(4+\gamma(2-\gamma)\big) + (2+\gamma)(2-\gamma^2)w_0}{8-5\gamma^2} \tag{65}$$

$$w_j^* = \frac{4-\gamma(1+3\gamma) + \big(4+\gamma(1-2\gamma)\big)w_0}{8-5\gamma^2} \tag{66}$$

Given the equilibrium wages in (65) and (66), we derive the following firm-specific output levels in the candidate equilibrium from the demand function [given in (42)], respectively:

$$q_i^* = \frac{\big(4+\gamma(2-\gamma)\big)(1-w_0)}{2(1+\gamma)(8-5\gamma^2)} \tag{67}$$

$$q_j^* = \frac{(4+3\gamma)(1-w_0)}{2(1+\gamma)(8-5\gamma^2)} \tag{68}$$

Observe that like Cournot competition, under Bertrand competition the comparison of firm wages [given in (65) and (66)] and employment [given in (67) and (68)] outcomes in candidate equilibrium deliver that competitive firm, i.e. firm j, which accomplishes higher employment and wages rates for its union. Thus, we easily conclude that union j's utility is higher than union j's one.

Taking the optimal firm output levels in (67) and (68) and substituting them for firms' profit function in (44) and (52), respectively, we obtain that:

$$\Pi_i^* = \frac{\Big(8-\gamma^2\big(8+\gamma(1-\gamma)\big)\Big)(1-w_0)^2}{2(8-5\gamma^2)^2} \tag{69}$$

$$\Pi_j^* = \frac{(1-\gamma)(4+3\gamma)^2(1-w_0)^2}{4(1+\gamma)(8-5\gamma^2)^2} \tag{70}$$

5.4. Endogenous Selection of Final Market Structure

Turn now to the second stage of the game, both firms simultaneously and independently decide whether to collude or to compete in prices, given that both firms decide on Bertrand competition (*C*) at the first stage of the game.

The firms deal with the matrix game presented in Subsection 4.4., except that competition that takes place in prices and payoffs for each union are given in Subsections 5.1-5.3.

In a unionized Bertrand-oligopolistic market, the firms' collusive play is Subgame Perfect equilibrium, as well as Pareto Optimal as regards profits. The economic intuition is of the same pattern with that of Cournot duopoly.

As it is well-known, the degree of competition decreases market prices, thus increasing output level/employment. Therefore, collusive play decreases labor demand, which causes reduction in per unit labor cost. The gains from the reduction in unit cost by collusive play are higher than the gains from stealing business from its rival firm by competitive play. Consequently, we easily conclude that unions' rent maximizing behavior deters firms to deviate from a cartel formation. Our findings are summarized in Proposition 6.

Proposition 6: *In Bertrand duopoly, collusion among firms is a subgame perfect equilibrium in the product market.*

[Proof: See Appendix (A.2)]

5.5. The Effects of Alternative Final Market Structure on Market Outcomes

Let us now compare market outcomes under firms' collusive, competitive and mix of strategies regimes.

Unlike Cournot duopoly, in Bertrand duopoly the reduction in collusive wages is not high enough to inverse the output differential among competition and collusion, but high enough to inverse the output differential among competition and mix of strategies configuration. The following Proposition summarizes our findings:

Proposition 7:

(i) *Regarding total output/employment level in Bertrand duopoly:*

- *If $\gamma < 0.77$, then total output under Competition is always higher than under Collusion, while under a Mix of Strategies it lies in-between, i.e.* $\boldsymbol{Q_m} > \boldsymbol{Q_{mos}} > \boldsymbol{Q_c}$.

- *If* $\gamma > 0.77$*, , then total output under a Mix of Strategies configuration is higher than under Competition, the latter being higher under Collusion, i.e.* $\boldsymbol{Q_{mos} > Q_m > Q_c}$

(ii) *Regarding firm-union wages in Bertrand duopoly, the wages are always higher under Competition than under Collusion, while under Mix of Strategies they lay in-between, i.e.* $\boldsymbol{w_m > w_{mos} > w_c}$.

[Proof: See Appendix (A.7)

5.6. Welfare Analysis

Social Welfare is defined by the sum of Consumer Surplus (*CS*), Producer Surplus (*PS*) and Union Rents (*UR*), which in turn are defined by (37), (38) and (39), respectively.

The results of comparative evaluation of market participant surpluses/rents and social welfare, across the s, are summarized in the following propositions:

Proposition 8: *Regarding Consumer Surplus in Bertrand duopoly:*

- *If* $\gamma < 0.77$*, then Consumer Surplus under Competition is always higher than under Collusion, while under a Mix of Strategies it lies in-between, i.e.* $\boldsymbol{CS_m > CS_{mos} > CS_c}$.
- *If* $\gamma > 0.77$*, then Consumer Surplus under a Mix of Strategies configuration is higher than under Competition, the latter being higher under Collusion, i.e.* $\boldsymbol{CS_{mos} > CS_m > CS_c}$

[Proof: See Appendix (A.8)]

Proposition 9: *In Bertrand duopoly, Producer Surplus under Collusion is always higher than under a Mix of Strategies configuration, the latter being always higher than under Competition, i.e.* $\boldsymbol{PS_c} > \boldsymbol{PS_{mos}} > \boldsymbol{PS_m}$.

Precisely the opposite order applies regarding Union Rents, i.e. $\boldsymbol{UR_m} > \boldsymbol{UR_{mos}} > \boldsymbol{UR_c}$

[Proof: See Appendix (A.9)]

Proposition 10: *Regarding Social Welfare in Bertrand duopoly:*

- *If* $\gamma < 0.79$*, then Social Welfare under Competition is always higher than under Collusion, while under a Mix of Strategies it lies in-between, i.e.* $\boldsymbol{SW_m} > \boldsymbol{SW_{mos}} > \boldsymbol{SW_c}$.
- *If* $\gamma > 0.79$*, then Social Welfare under a Mix of Strategies configuration is higher than under Competition, the latter being higher under Collusion, i.e.* $\boldsymbol{SW_{mos}} > \boldsymbol{SW_m} > \boldsymbol{SW_c}$

[Proof: See Appendix (A.10)]

6. Mix of Strategies (*MOS: C-B*): [*02*], [*03*]

According to Mix of Strategies configuration as regards the firms' mode of competition, the one firm (let it be firm i) adjusts its own output, while the other firm (let it be firm j) set its own price in the product market.

Given the above mode of competition at the first stage, firms face the following demand functions, which are derived in (1):

$$p_i(q_i, p_j) = 1 - \gamma - (1 - \gamma^2)q_i + \gamma p_j \tag{71}$$

$$q_j(q_i, p_j) = 1 - p_j - \gamma q_i \tag{72}$$

At the second stage four candidate equilibria arise: In Section 6.1, the candidate equilibrium is firms' cartel formation in where both firms aim to maximize their joint profits and the possible deviation, on the part of any firm, is to maximize its own profits. In Section 6.2, the candidate equilibrium is firms' competition and the possible deviation, on the part of any firm, is to maximize the cartel profits. In Section 5.3, the candidate equilibrium is the one where the one firm, which adjusts its own quantities, acts collusively, while its rival firm, which sets its own prices, acts competitively, and the possible deviations arise by unilaterally switching each firm's strategy to its rival's one. In Section 5.3, the candidate equilibrium is the one where the one firm, which adjust its own quantities, acts competitively now, while its rival firm, which sets its own prices, acts collusively, and the possible deviations arise by unilaterally switching each firm's strategy to its rival's one.

6.1. Collusive Play (*c*): (*5*), (*9*)

At the last stage of the game each firm simultaneously and independently adjusts its own quantity or sets its price, according to its selection of mode of competition at the first stage, in order to maximize cartel profits.

At this stage we get firm i's output level and firm j's pricing policy in candidate equilibrium, by solving the system of *f.o.c.*$_s$ of firm i's and j's objective [given in (5) and (44), respectively], as follows:

$$q_{ic}(w_{ic}, w_{jc}) = \frac{1-\gamma-w_i+\gamma w_{jc}}{2(1-\gamma)(1+\gamma)} \quad (73)$$

$$p_{ic}(w_{ic}, w_{jc}) = (1+w_{ic})/2 \quad (74)$$

At the third stage of the game, we derive the (candidate) optimal wage rule by solving the system of *f.o.c.*$_s$ of union i's objective function [given in (8)], given the optimal firm i's output level and firm j's pricing policy rules in (73) and (74):

$$w_{ic}^{*} = w_{jc}^{*} = \frac{1-\gamma+w_0}{2-\gamma} \tag{75}$$

Taking now into consideration the market demand function and (75), (74), (73), we get the optimal firm' quantities and prices in candidate equilibrium, as follows:

$$q_{ic}^{*} = q_{jc}^{*} = \frac{1-w_0}{2(1+\gamma)(2-\gamma)} \tag{76}$$

$$p_{ic}^{*} = p_{jc}^{*} = \frac{3-2\gamma+w_0}{2(2-\gamma)} \tag{77}$$

We can derive now firms' profit function in (3), as follows:

$$\Pi_{ic}^{*} = \Pi_{jc}^{*} = \frac{(1-w_0)^2}{4(1+\gamma)(2-\gamma)^2} \tag{78}$$

Notice that like in the previous section, regardless of the chosen mode of competition, collusive outcomes are the same and equal to those of monopoly markets. It is easy to check the above conclusion by comparing equations (11), (50) and (76) of output/employment level and equations (10), (48) and (75) of union wages.

6.2. Competitive Play (*m*): (*8*), (*12*)

Under the assumption of firms' competitive play (*M*) at the second stage, at the last stage of the game we get the competitive firm i's output level and firm j's pricing policy rules by solving the system of firms' reaction function which is derived from the objective [given in (14) and (52)]:

$$q_{im}(w_{im}, w_{jm}) = \frac{2-\gamma-2w_{im}+\gamma w_{jm}}{4-3\gamma^2} \tag{79}$$

$$p_{jm}(w_{im}, w_{jm}) = \frac{(2+\gamma)(1-\gamma)+2(1+\gamma)(1-\gamma)w_{im}+\gamma w_{jm}}{4-3\gamma^2} \tag{80}$$

Turning now to the third stage, the candidate equilibrium wage arises by solving the system of unions' reaction functions, which are the $f.o.c_s$ of unions' rent maximization in (8), as follows:

$$w_{im}^{*}=\frac{8-\gamma\big(2+(5-\gamma)\big)+(4+\gamma)(2-\gamma^{2})w_{0}}{(4+3\gamma)(4-3\gamma)} \tag{81}$$

$$w_{jm}^{*}=\frac{8-\gamma(2+5\gamma)+2\big(4+\gamma(1-2\gamma)\big)w_{0}}{(4+3\gamma)(4-3\gamma)} \tag{82}$$

According to candidate equilibrium wages in (81) and (82), firm i's output level rule in (79), firm j's pricing policy rules in (80) and demand function in (42), we obtain the optimal output/employment level:

$$q_{im}^{*}=\frac{2\Big(8-\gamma\big(2+(5-\gamma)\big)\Big)(1-w_{0})}{64-3\gamma^{2}(28-9\gamma^{2})} \tag{83}$$

$$q_{jm}^{*}=\frac{(2-\gamma^{2})\big(8-\gamma(2+5\gamma)\big)(1-w_{0})}{64-3\gamma^{2}(28-9\gamma^{2})} \tag{84}$$

In addition, we get that:

$$\Pi_{im}^{*}=(1+\gamma)(1-\gamma)(q_{im}^{*})^{2} \tag{85}$$

$$\Pi_{jm}^{*}=\big(q_{jm}^{*}\big)^{2} \tag{86}$$

Substituting now firms' output level in candidate equilibrium in (83) and (84) for their profit function in (85) and (86), respectively, we get that:

$$\Pi_{im}^{*}=\frac{4(1-\gamma^{2})\Big(8-\gamma\big(2+(5-\gamma)\big)\Big)^{2}(1-w_{0})^{2}}{\big(64-3\gamma^{2}(28-9\gamma^{2})\big)^{2}} \tag{87}$$

$$\Pi_{jm}^{*}=\frac{(2-\gamma^{2})^{2}\big(8-\gamma(2+5\gamma)\big)^{2}(1-w_{0})^{2}}{\big(64-3\gamma^{2}(28-9\gamma^{2})\big)^{2}} \tag{88}$$

6.3. Mix of Strategies within Bertrand Competition (*mos*$_C$): (*6*), (*11*)

Under Mix of Strategies configuration within Bertrand Competition, at the second stage the one firm (let it be firm j) sets its own price competitively, while the other firm (let it be firm i) adjusts its own quantity collusively. Hence, at the last stage of the game we must consider as firm i's and firm j's objective function (5) and (52), respectively. Getting the *f.o.c.*$_s$ of firms' objective and solving the system of equations, we obtain firm i's output level and firm j's pricing policy rules in candidate equilibrium:

$$q_i\left(w_i, w_j\right) = \frac{1 - \gamma - w_i + \gamma w_j}{2(1+\gamma)(1-\gamma)} \tag{89}$$

$$p_j\left(w_i, w_j\right) = \frac{(2+\gamma)(1-\gamma) + \gamma w_i + (2 - 3\gamma^2) w_j}{4(1+\gamma)(1-\gamma)} \tag{90}$$

At the third stage we derive the unions' reaction functions by taking the *f.o.c*$_s$ of unions' rent maximization in (8). Solving the system of unions' wage reaction functions, we get firm-specific wage outcomes, as follows:

$$w_i^* = \frac{(1-\gamma)\big(4 + \gamma(2-\gamma)\big) + (2+\gamma)(2-\gamma^2) w_0}{8 - 5\gamma^2} \tag{91}$$

$$w_j^* = \frac{4 - \gamma(1+3\gamma) + \big(4 + \gamma(1-2\gamma)\big) w_0}{8 - 5\gamma^2} \tag{92}$$

Taking now into consideration the candidate equilibrium wages in (91) and (92), firm i's output level rule in (89), firm j's pricing policy rules in (90) and demand function in (42), we obtain the optimal output/employment level:

$$q_i^* = \frac{\big(4 + \gamma(2-\gamma)\big)(1 - w_0)}{2(1+\gamma)(8 - 5\gamma^2)} \tag{93}$$

$$q_j^* = \frac{(4 + 3\gamma)(1 - w_0)}{2(1+\gamma)(8 - 5\gamma^2)} \tag{94}$$

Moreover, we can evaluate now firms' functions, as presented below:

$$\Pi_i^* = \frac{\left(32 + \gamma\left(64 - \gamma^2\left(52 - \gamma(2-\gamma)(8+\gamma)\right)\right)\right)(1 - w_0)^2}{8(1+\gamma)^2(8 - 5\gamma^2)^2} \tag{95}$$

$$\Pi_j^* = \frac{(2-\gamma^2)^2(4+3\gamma)^2(1-w_0)^2}{16(1+\gamma)^2(8-5\gamma^2)^2} \tag{96}$$

6.4. Mix of Strategies within Cournot Competition (*mos*$_B$): (*7*), (*10*)

Under Mix of Strategies configuration within Cournot Competition, at the second stage the one firm (let it be firm j) sets its own price in order to maximize cartel profits, while the other firm (let it be firm i) adjusts its own quantity in order to maximize its own profits. Therefore, at the last stage of the game we must consider as firm i's and firm j's objective functions (14) and (44), respectively. Firm i's output level and firm j's pricing policy rules in candidate equilibrium is derived by solving the system of *f.o.c.*$_s$ of firms' objective, i.e. firms' reaction functions:

$$q_i(w_i, w_j) = \frac{2 - \gamma - 2w_i + \gamma w_j}{2(2-\gamma^2)} \tag{97}$$

$$p_j(w_i, w_j) = \frac{1 - w_j}{2} \tag{98}$$

At the third stage, we derive firm-specific wage outcomes by solving the system of unions' wage reaction functions, which arise from the *f.o.c*$_s$ of unions' rent maximization in (8), as follows:

$$w_i^* = \frac{8 - \gamma\left(2 + \gamma(6-\gamma)\right) + (4+\gamma)(2-\gamma^2)w_0}{2(8-5\gamma^2)} \tag{99}$$

$$w_j^* = \frac{4 - \gamma(2+3\gamma) + 2(1+\gamma)(2-\gamma)w_0}{8-5\gamma^2} \tag{100}$$

The candidate equilibrium wages in (99) and (100), firm i's output level rule in (94), firm j's pricing policy rules in (95) and demand function in (42) give us the output/employment level in candidate equilibrium:

$$q_i^* = \frac{\left(8-\gamma\big(2+\gamma(6-\gamma)\big)\right)(1-w_0)}{4(1-\gamma^2)(8-5\gamma^2)} \tag{101}$$

$$q_j^* = \frac{(2-\gamma^2)\big(4-\gamma(2+3\gamma)\big)(1-w_0)}{4(1-\gamma^2)(8-5\gamma^2)} \tag{102}$$

Additionally, we get that:

$$\Pi_i^* = \frac{(1-\gamma^2)\left(8-\gamma\big(2+\gamma(6-\gamma)\big)\right)^2(1-w_0)^2}{16(1-\gamma^2)^2(8-5\gamma^2)^2} \tag{103}$$

$$\Pi_j^* = \frac{(2-\gamma)(2-\gamma^2)\big(4-\gamma(2+3\gamma)\big)(1-w_0)^2}{4(1-\gamma)(8-5\gamma^2)^2} \tag{104}$$

Where $\Pi_j^* > 0 \ \forall \ \gamma \in (0, \gamma_{C,L})$ and $\gamma_{C,L} = \frac{1}{3}(\sqrt{13}-1) \cong 0.87$.

If products' substitutability is higher than 0.87, then firm *j* retires from the industry because its profit is negative. Consequently, firm i acts as a monopolist in the product market and then output and wage outcomes in candidate equilibrium are defined by the following equations:

$$w_i^* = (1-w_0)/2 \tag{105}$$

$$q_i^* = (1-w_0)/4 \tag{106}$$

Thus, firm *j*'s profits are presented below:

$$\Pi_i^* = (1-w_0)^2/16 \tag{107}$$

Summarizing our findings, we get that market output, unions' wages and firms' profits in candidate equilibrium:

$$w_i^* = \begin{cases} \dfrac{8-\gamma\big(2+\gamma(6-\gamma)\big)+(4+\gamma)(2-\gamma^2)w_0}{2(8-5\gamma^2)}, & \forall \ \gamma < 0.87 \\ (1-w_0)/2, & \forall \ \gamma > 0.87 \end{cases} \tag{108}$$

$$w_j^* = \begin{cases} \dfrac{4-\gamma(2+3\gamma)+2(1+\gamma)(2-\gamma)w_0}{8-5\gamma^2}, & \forall \ \gamma < 0.87 \\ 0, & \forall \ \gamma > 0.87 \end{cases} \tag{109}$$

$$q_i^* = \begin{cases} \dfrac{\left(8-\gamma\left(2+\gamma(6-\gamma)\right)\right)(1-w_0)}{4(1-\gamma^2)(8-5\gamma^2)}, & \forall \;\; \gamma < 0.87 \\ (1-w_0)/4, & \forall \;\; \gamma > 0.87 \end{cases} \tag{110}$$

$$q_j^* = \begin{cases} \dfrac{(2-\gamma^2)\left(4-\gamma(2+3\gamma)\right)(1-w_0)}{4(1-\gamma^2)(8-5\gamma^2)}, & \forall \;\; \gamma < 0.87 \\ 0, & \forall \;\; \gamma > 0.87 \end{cases} \tag{111}$$

$$\Pi_i^* = \begin{cases} \dfrac{(1-\gamma^2)\left(8-\gamma\left(2+\gamma(6-\gamma)\right)\right)^2(1-w_0)^2}{16(1-\gamma^2)^2(8-5\gamma^2)^2}, & \forall \;\; \gamma < 0.87 \\ (1-w_0)^2/16, & \forall \;\; \gamma > 0.87 \end{cases} \tag{112}$$

$$\Pi_j^* = \begin{cases} \dfrac{(2-\gamma)(2-\gamma^2)\left(4-\gamma(2+3\gamma)\right)(1-w_0)^2}{4(1-\gamma)(8-5\gamma^2)^2}, & \forall \;\; \gamma < 0.87 \\ 0, & \forall \;\; \gamma > 0.87 \end{cases} \tag{113}$$

6.5. Endogenous Selection of Final Market Structure

At the second stage of the game, we endogenize the firms' decision on playing competitively or collusively, given that their decision about their mode of competition at the first stage of the game.

The firms deal with the matrix game presented in Subsection 4.4., except that the one firm (firm *i*) adjusts its own output, while the other firm (firm *j*) set its own prices and payoffs of each union are given in Subsections 6.1-6.4.:

		Firm *i*: *Collusion*	Firm *i*: *Cournot Competition*
Firm *j*	*Collusion*	$\{\Pi_{ic}^*, \Pi_{jc}^*\}$	$\{\Pi_{id_i}^*, \Pi_{jd_i}^*\}$
	Bertrand Competition	$\{\Pi_{id_j}^*, \Pi_{jd_j}^*\}$	$\{\Pi_{im}^*, \Pi_{im}^*\}$

Table 2: The Matrix Game that firms deal with at the second stage of the game.

In a unionized Mix of Strategies configuration oligopolistic market, the firms' collusive play is Subgame Perfect equilibrium, as well as Pareto Optimal in the part of profits. The economic intuition is of the same pattern with that of Cournot and Bertrand duopoly.

Proposition 11: *Under Mix of Strategies configurations in the frames of mode of competition, collusion among firms is a subgame perfect equilibrium in the product market.*

[Proof: See Appendix (A.11)]

6.6. Welfare Analysis

Social Welfare is defined by the sum of Consumer Surplus (*CS*), Producer Surplus (*PS*) and Union Rents (*UR*), which in turn are defined by (37), (38) and (39), respectively.

The results of comparative evaluation of market participant surpluses/rents and social welfare, across the s, are summarized in the following propositions.

Proposition 12: *Regarding Consumer Surplus in Mix of Strategies, as regards the mode of competition:*

- *If* $\gamma \notin (0.83, 0.87)$, *then Consumer Surplus under Competition is always higher than under Collusion and a Mix of Strategies configuration, i.e.* $\boldsymbol{CS_m > CS_c, CS_{mos}}$.
- *If* $\gamma > 0.77$, *then Consumer Surplus under a Mix of Strategies configuration within Cournot competition is higher than under Competition, the latter being*

higher under Collusion. The Mix of Strategies configuration within Bertrand competition lies in- between Competition and Collusion, i.e. $\boldsymbol{CS_{mos_C}} > \boldsymbol{CS_m} > \boldsymbol{CS_{mos_B}} > CS_{\boldsymbol{c}}$

[Proof: See Appendix (A.12)]

Proposition 13: *In Mix of Strategies, as regards the mode of competition:*

- *Producer Surplus under Collusion is always higher than under a Mix of Strategies configuration and Competition, i.e.* $\boldsymbol{PS_c} > \boldsymbol{PS_m}, \boldsymbol{PS_{mos}}$.
- *Union Rents under Competition is always higher than under Collusion, while under a Mix of Strategies it lays in-between, i.e.* $\boldsymbol{UR_m} > \boldsymbol{UR_{mos}} > \boldsymbol{UR_c}$.

[Proof: See Appendix (A.13)]

Proposition 14: *In Mix of Strategies, as regards the mode of competition, Social Welfare under Competition is always higher than under Collusion and Mix of Strategies, i.e.* $\boldsymbol{SW_m} > \boldsymbol{SW_c}, \boldsymbol{SW_{mos}}$

[Proof: See Appendix (A.14)]

7. Endogenous Selection of Mode of Competition

At the first stage of the game, firms are asked to decide simultaneously and independently their mode of competition, namely whether to adjust their own quantities (*Cournot Competition*) or their own prices (*Bertrand Competition*), so as to maximize monopoly (cartel's) profits or their own profits.

Given that at the second stage of the game firms decide simultaneously and independently to play collusively or competitively, they deal with the following matrix game:

Firm i

Firm j	Cournot			Bertrand		
		Collusion	Competition		Collusion	Competition
Cournot	Collusion	(1)	(3)	Collusion	(9)	(11)
	Competition	(2)	(4)	Competition	(10)	(12)
		Collusion	Competition		Collusion	Competition
Bertrand	Collusion	(5)	(7)	Collusion	(13)	(15)
	Competition	(6)	(8)	Competition	(1)	(16)

Table 2: The Matrix Game that firms deal with at the first stage of the game.

Given our finding in Sections 3 – 6 [given in Propositions 1 – 3] about the four enclosed matrix games, we conclude that collusion among firms always emerges in equilibrium, regardless of the chosen mode of competition.

Proposition 15: *Collusion among firms is a subgame perfect equilibrium in the product market, regardless firms mode of competition between them.*

[Proof: Recalling Propositions 1, 6 and 11, it can be readily checked that Cartel formation/collusion is the unique (Pareto optimal) equilibrium under each firm's decision on its mode of competition.]

8. Welfare Analysis: Cournot vs. Bertrand Duopoly

According to our findings in the previous sections, we conclude that collusion among firms is an unavoidable equilibrium in the industry, regardless of the chosen mode of competition. Consequently, in our model, regardless of the chosen mode of competition, industry outcomes and market participants surpluses/rents are the same and equal to that of a cartel.

We also get that cartel formation is not Pareto Optimal solution in terms of Social Welfare, with the exception of sufficiently high product substitutability under Cournot competition.

Proposition 16: *The social welfare is promoted by developing a market policy which prevents the formation of cartels.*

An exception to our proposal is the case of sufficiently high product substitutability under Cournot competition, as then collusive play is Pareto Optimal solution in terms of Social Welfare.

[Proof: Recalling Propositions 5, 10 and 14, it can be readily checked that, regardless of the chosen mode of competition, Social Welfare is higher under competition than under collusion with the exception of sufficiently high product substitutability under Cournot competition.]

Given that the comparative analysis of competitive and collusive play denotes the superiority of competitive play for most of the cases, let's now determine the mode of competition which comparatively promotes Social Welfare. The comparative analysis of firms' mode of competition demonstrates that Bertrand provides the highest Social Welfare, due to its high degree of competition.

Proposition 17: *Given firms' competitive play, Social Welfare is higher under Bertrand than Cournot competition, while under a Mix of Strategies it lays in-between, i.e.* $\boldsymbol{SW_B > SW_{MOS} > SW_C}$.

[Proof: See Appendix (A.15)]

9. Concluding Remarks

In a static union-oligopoly framework with differentiated goods and decentralized Right-to-Manage bargaining, the present paper endogenizes the firms' mode of competition, as well their perspective for cartel formation

In our model, we show that firms' cartel formation is an unavoidable result in equilibrium, regardless of the chosen mode of competition. Therefore, regardless of the chosen mode of competition, industry outcomes and market participants surpluses/rents equal to that of a cartel. We also show that if product substitutability is sufficiently high $(\gamma > 2/3)$, then cartel formation is Welfare improving under Cournot competition. In Cournot duopoly, as product substitutability increases, the reduction of collusive wages is high enough to inverse the output differential among competition and collusion, as well as Social Welfare. Unlike Cournot, in Bertrand duopoly the reduction of collusive wages is not high enough to inverse the output differential among competition and collusion, but high enough to inverse the output differential among competition and mix of strategies configuration.

Apart from this exception, competitive play is shown to be superior in terms of Social Welfare. In particular, under Bertrand competition Social Welfare is higher than under Cournot competition, while a Mix of Strategies lies in-between. Consequently, our analysis suggests that in order to improve social welfare, a benevolent policy maker should deter cartel formation while at the same time give firms incentives for Bertrand competition.

Additionally, our previous work shows that firms may (ex-post) collude by (ex-ante) choosing their mode of competition in the product market with exogenous wages. It is shown that, if the discount factor is not high enough, whilst the degree of product substitutability is sufficiently high, firms independently choose (in case of competition) to adjust their own prices, because this minimizes the gains from deviation from collusive play and consequently enables collusion and higher profits. Otherwise, collusion is weak / unstable and each firm’s dominant strategy is (then) to compete by adjusting its own quantity. Comparing with the present paper’s findings, we easily conclude that unions, as second movers, strengthen firms’ incentives for collusive play, making the cartel an unavoidable market outcome.

Appendix

A.1 Proof of Proposition 1

Firms deal with the following matrix game, which presents the payoffs of each firm when both firms simultaneously and independently decide to collude or to compete at the second stage of the game:

		Firm i: *Collusion*	Firm i: *Cournot Competition*
Firm j	*Collusion*	(E1) $\{\Pi^*_{ic}, \Pi^*_{jc}\}$	(E3) $\{\Pi^*_{id_i}, \Pi^*_{jd_i}\}$
	Cournot Competition	(E2) $\{\Pi^*_{id_j}, \Pi^*_{jd_j}\}$	(E4) $\{\Pi^*_{im}, \Pi^*_{im}\}$

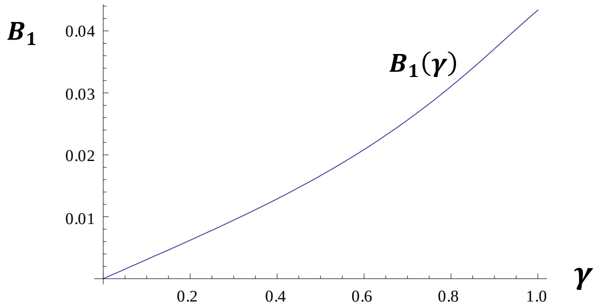
Due to symmetry, $\left(\Pi^*_{id_j}, \Pi^*_{jd_j}\right) = \left(\Pi^*_{jd_i}, \Pi^*_{id_i}\right)$ applies, hence the number of candidate equilibria is reduced to three, i.e. E1, E2 and E4.

Subgame perfect equilibrium of the game is the candidate one in where no game player has incentives to deviate from. The possible deviation on the part of each firm (player) is to unilaterally switch its own strategy, given that its rival does not.

The candidate equilibrium (E1) is the one where firms proceed to cartel formation and the possible deviation, on the part of any firm, is to adjust its own quantities in order to maximize its own profits. Taking in consideration the above subgame perfect equilibrium definition, firms' collusive play emerges in equilibrium, as no firm has incentive to deviate by playing competitively. From equations (13) and (31) it applies that:

$$\Pi^*_{ic}(= \Pi^*_{jc}) - \Pi^*_{id_i}\left(= \Pi^*_{jd_j}\right) = B_1(1 - w_0)^2 > 0 \qquad \text{(A1)}$$

$B_1 > 0 \ \forall \ \gamma \in (0,1)$[7] and its plot is presented below.

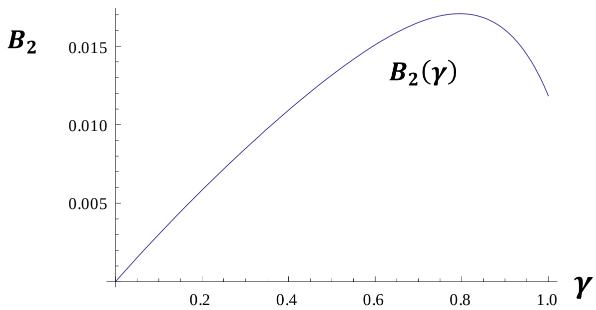


The candidate equilibrium (E2) is the one where one firm acts collusively, while the other acts competitively. The possible deviation, on the part of any firm, is to switch its strategy to its rival's one. From (A1), we conclude that the candidate equilibrium (E2) is not subgame perfect one, as a firm which acts competitively has incentive to switch its strategy by playing collusively, as its rival does, in order to increase its own profits.

The last candidate equilibrium (E4) is the one where both firms decide to play competitively and the possible deviation, on the part of any firm, is to adjust its own quantities in order to maximize cartel profits. Firms' competitive play is not subgame perfect equilibrium, as both firms have incentives to deviate by playing collusively. From equations (21) and (30) it applies that:

$$\Pi_{id_j}^*(=\Pi_{jd_i}^*) - \Pi_{im}^*(=\Pi_{jm}^*) = B_2(1-w_0)^2 > 0 \qquad \text{(A2)}$$

$B_2 > 0 \ \forall \ \gamma \in (0,1)$[8] and its plot is presented below.



[7] The mathematical expression of B_1 is left out because of its wide extent. It is available by the authors upon request.
[8] The mathematical expression of B_2 is left out because of its wide extent. It is available by the authors upon request.

Summarizing the above results, we conclude that firm cartel formation (E1) emerges in equilibrium in our Cournot duopoly.

A.2 Proof of Proposition 2

(i) Total market output is the sum of firms' equilibrium output:

$$Q_s = \sum_{k=i,j} q_{ks} \qquad ; \quad s=c, m, mos \tag{A3}$$

Taking into consideration the equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (11), (19), (28) and (29), we obtain the total output levels:

$$Q_c^* = \frac{1-w_0}{(1+\gamma)(2-\gamma)} \tag{A4}$$

$$Q_m^* = \frac{4(1-w_0)}{(2+\gamma)(4-\gamma)} \tag{A5}$$

$$Q_{mos}^* = \frac{\big(8-\gamma(3+2\gamma)\big)(1-w_0)}{(2-\gamma^2)(8-\gamma^2)} \tag{A6}$$

From comparative analysis of (A4), (A5) and (A6), we obtain that:

$$Q_c^* - Q_m^* = \frac{\gamma(2-3\gamma)(1-w_0)}{(1+\gamma)(2-\gamma)(2+\gamma)(4-\gamma)} > 0 \qquad \forall \ \gamma > \frac{2}{3} \tag{A7}$$

$$Q_c^* - Q_{mos}^* = \frac{\gamma(\gamma(5-\gamma)-2)(1-w_0)}{(1+\gamma)(2-\gamma)(2-\gamma^2)(8-\gamma^2)} > 0 \qquad \forall \ \gamma > 0.46 \tag{A8}$$

$$Q_m^* - Q_{mos}^* = \frac{\gamma\big(8-\gamma(2-\gamma)(5+2\gamma)\big)(1-w_0)}{(2+\gamma)(4-\gamma)(2-\gamma^2)(8-\gamma^2)} > 0 \quad \forall \ \gamma \in (0,1) \tag{A9}$$

Summarizing our results, we conclude that:

$$Q_m^* > Q_{mos}^* > Q_c^* \qquad \forall \ \gamma \in (0, 0.46) \tag{A10}$$

$$Q_m^* > Q_c^* > Q_{mos}^* \qquad \forall \ \gamma \in \left(0.46, \frac{2}{3}\right) \tag{A11}$$

$$Q_c^* > Q_m^* > Q_{mos}^* \qquad \forall \ \gamma \in \left(\frac{2}{3}, 1\right) \tag{A12}$$

(ii) Comparing (10), (18), (26) and (27), we get that:

$$w_{im}^* - w_{ic}^* = \frac{\gamma(1-w_0)}{(2-\gamma)(4-\gamma)} > 0 \qquad \forall\ \gamma \in (0,1) \tag{A13}$$

$$w_{imos}^* - w_{ic}^* = \frac{\gamma^2(1-w_0)}{(2-\gamma)(8-\gamma^2)} > 0 \qquad \forall\ \gamma \in (0,1) \tag{A14}$$

$$w_{jmos}^* - w_{jc}^* = \frac{2\gamma(1-w_0)}{(2-\gamma)(8-\gamma^2)} > 0 \qquad \forall\ \gamma \in (0,1) \tag{A15}$$

$$w_{im}^* - w_{imos}^* = \frac{4\gamma(1-w_0)}{(4-\gamma)(8-\gamma^2)} > 0 \qquad \forall\ \gamma \in (0,1) \tag{A16}$$

$$w_{jm}^* - w_{jmos}^* = \frac{\gamma^2(1-w_0)}{(4-\gamma)(8-\gamma^2)} > 0 \qquad \forall\ \gamma \in (0,1) \tag{A17}$$

Summarizing our results, we conclude that:

$$w_m^* > w_{imos}^*, w_{jmos}^* > w_{ic}^* \qquad \forall\ \gamma \in (0,1) \tag{A18}$$

A.3 Proof of Proposition 3

By means of Consumer Surplus equation in (37) and equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (11), (19), (28) and (29), we get that:

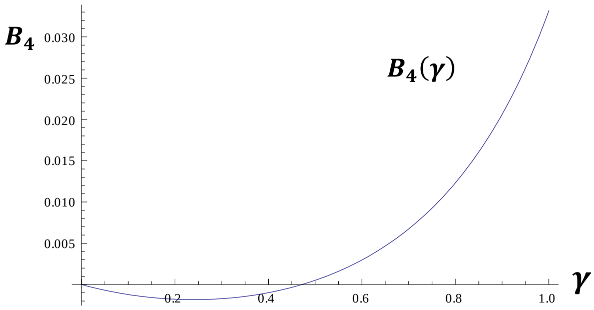
$$CS_c^* - CS_m^* = B_3(1-w_0)^2 > 0 \qquad \forall\ \gamma > \frac{2}{3} \tag{A19}$$

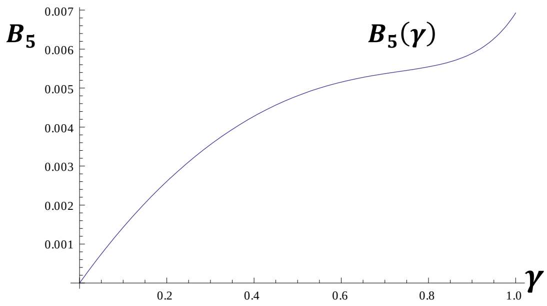
$$CS_c^* - CS_{mos}^* = B_4(1-w_0)^2 > 0 \qquad \forall\ \gamma > 0.47 \tag{A20}$$

$$CS_m^* - CS_{mos}^* = B_5(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A21}$$

Where $B_3 = \frac{\gamma(\gamma(36+\gamma(28-15\gamma))-32)}{4(1+\gamma)(2-\gamma)^2(2+\gamma)^2(4-\gamma)^2} > 0\ \forall\ \gamma \in \left(\frac{2}{3},1\right)$, $B_4 > \forall\ \gamma \in (047,1)$ and $B_5 > 0\ \gamma \in (0,1)$[9]. The plots of B_4 and B_5 are presented below.

[9] The mathematical expressions of B_4 and B_5 are left out because of their wide extent. They are available by the authors upon request.





Summarizing our results, we conclude that:

$$CS_m^* > CS_{mos}^* > CS_c^* \qquad \forall\ \gamma \in (0{,}0.47) \tag{A22}$$

$$CS_m^* > CS_c^* > CS_{mos}^* \qquad \forall\ \gamma \in \left(0.47, \frac{2}{3}\right) \tag{A23}$$

$$CS_c^* > CS_m^* > CS_{mos}^* \qquad \forall\ \gamma \in \left(\frac{2}{3}, 1\right) \tag{A24}$$

A.4 Proof of Proposition 4

Regarding Producer Surplus [given in (38)] and equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (11), (19), (28) and (29), we get that:

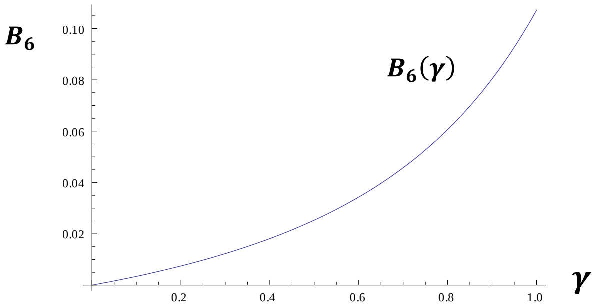
$$PS_c^* - PS_m^* = \frac{\gamma\big(32 + \gamma(2-\gamma)(18-\gamma)\big)(1-w_0)^2}{2(1+\gamma)(2-\gamma)^2(2+\gamma)^2(4-\gamma)^2} > 0 \quad \forall\ \gamma \in (0{,}1) \tag{A25}$$

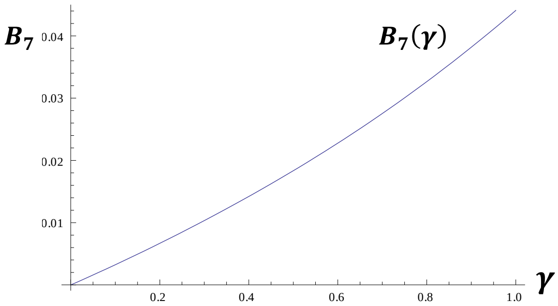
$$PS_c^* - PS_{mos}^* = B_6(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0{,}1) \tag{A26}$$

$$PS_{mos}^* - PS_m^* = B_7(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0{,}1) \tag{A27}$$

Where $B_6,\ B_7 > \forall\ \ \gamma \in (0{,}1)$[10] and their plots are presented below.

[10] The mathematical expressions of B_6 and B_7 are left out because of their wide extent. They are available by the authors upon request.





Summarizing our results, we conclude that:

$$PS_c^* > PS_{mos}^* > PS_c^* \qquad \forall \ \gamma \in (0,1) \tag{A28}$$

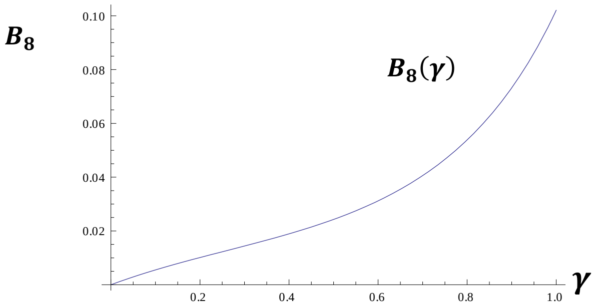
Regarding now Union Rents [given in (39)], equilibrium output levels [given in (11), (19), (28) and (29)] and wages [given in (10), (18), (26) and (27)] under firms' collusion, competition and mix of strategies configuration, we get that:

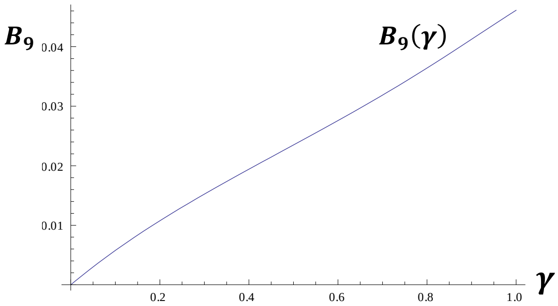
$$UR_m^* - UR_c^* = \frac{\gamma\big(\gamma\big(18-\gamma(13-3\gamma)\big)-16\big)(1-w_0)^2}{2(1+\gamma)(2+\gamma)(2-\gamma)^2(4-\gamma)^2} > 0 \qquad \forall \ \gamma \in (0,1) \tag{A29}$$

$$UR_{mos}^* - UR_c^* = B_8(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \tag{A30}$$

$$UR_m^* - UR_{mos}^* = B_9(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \tag{A31}$$

Where $B_8, \ B_9 > \forall \ \ \gamma \in (0,1)$[11] and their plots are presented below.





Summarizing our results, we conclude that:

$$UR_c^* > UR_{mos}^* > UR_c^* \qquad \forall \ \gamma \in (0,1) \tag{A32}$$

[11] The mathematical expressions of B_8 and B_9 are left out because of their wide extent. They are available by the authors upon request.

A.5 Proof of Proposition 5

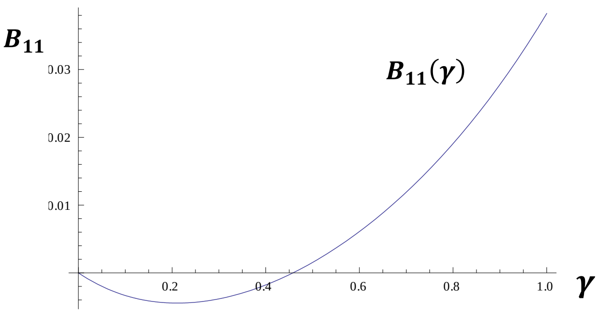
By means of market participants surpluses/rents in (37), (38) and (39) and equilibrium output levels [given in (11), (19), (28) and (29)] and wages [given in (10), (18), (26) and (27)] under firms' collusion, competition and mix of strategies configuration, we obtain about Social Welfare that:

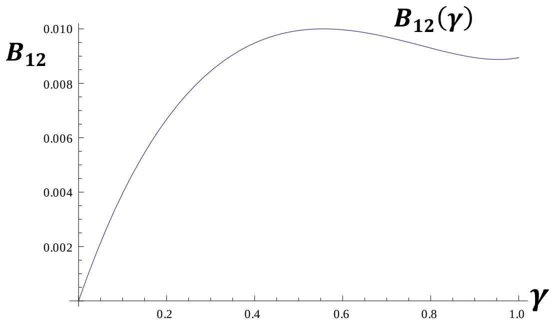
$$SW_c^* - SW_m^* = B_{10}(1-w_0)^2 > 0 \qquad \forall \;\; \gamma > \frac{2}{3} \tag{A33}$$

$$SW_c^* - SW_{mos}^* = B_{11}(1-w_0)^2 > 0 \qquad \forall \;\; \gamma > 0.46 \tag{A34}$$

$$SW_m^* - SW_{mos}^* = B_{12}(1-w_0)^2 > 0 \qquad \forall \;\; \gamma \in (0,1) \tag{A35}$$

Where $B_{10} = \frac{\gamma(2-3\gamma)(\gamma(22+\gamma(11-4\gamma))-48)}{4(1+\gamma)(2-\gamma)^2(2+\gamma)^2(4-\gamma)^2} > 0 \;\; \forall \;\; \gamma \in \left(\frac{2}{3},1\right)$, $\quad B_{11} > \forall \;\; \gamma \in (046,1)$ and $B_{12} > 0 \;\; \gamma \in (0,1)$[12]. The plots of B_4 and B_5 are presented below.





Summarizing our results, we conclude that:

$$SW_m^* > SW_{mos}^* > SW_c^* \qquad \forall \;\; \gamma \in (0, 0.46) \tag{A36}$$

$$SW_m^* > SW_c^* > SW_{mos}^* \qquad \forall \;\; \gamma \in \left(0.46, \frac{2}{3}\right) \tag{A37}$$

$$SW_c^* > SW_m^* > SW_{mos}^* \qquad \forall \;\; \gamma \in \left(\frac{2}{3}, 1\right) \tag{A38}$$

[12] The mathematical expressions of B_{11} and B_{12} are left out because of their wide extent. They are available by the authors upon request.

A.6 Proof of Proposition 6

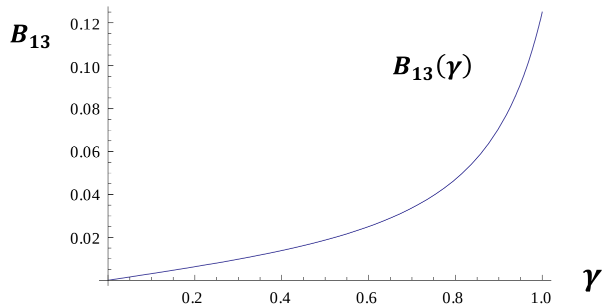
At the second stage of the game, firms deal with the matrix game presented in Subsection A.1., except that competition takes place in prices and payoffs of each firm which are given in Subsections 5.1-5.3.

Like Cournot competition, it also applies that the number of candidate equilibria is reduced to three, because $\left(\Pi^*_{id_j}, \Pi^*_{jd_j}\right) = \left(\Pi^*_{jd_i}, \Pi^*_{id_i}\right)$ due to symmetry, i.e. E1, E2 and E4.

The candidate equilibrium (E1) is the one where firms proceed to cartel formation and the possible deviation, on the part of any firm, is to play competitively. We conclude that firms' collusive play emerges in equilibrium, as no firm has incentive to deviate from it. By means of (51) and (70), we get that:

$$\Pi^*_{ic}(=\Pi^*_{jc}) - \Pi^*_{id_i}\left(=\Pi^*_{jd_j}\right) = B_{13}(1-w_0)^2 > 0 \qquad \text{(A39)}$$

$B_{13} > 0 \;\; \forall \;\, \gamma \in (0,1)$[13] and its plot is presented below.



The candidate equilibrium (E2) is the one where one firm acts collusively, while the other acts competitively, with possible deviations to switch its strategy to its rival's one. From (A1), we obtain that the candidate equilibrium (E2) is not subgame perfect one, because a firm which does not play collusively has incentive to do it.

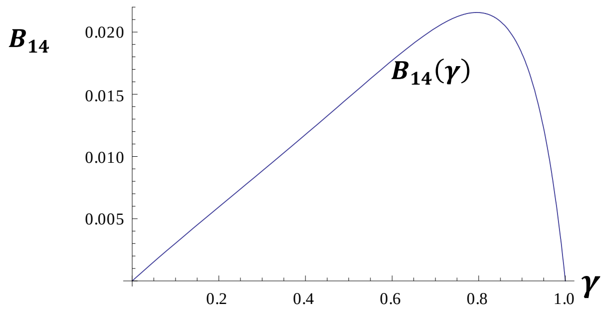
The last candidate equilibrium (E4) is the one where both firms decide to set their prices in order to maximize their own profits and the possible deviation, on the part of

[13] The mathematical expression of B_{13} is left out because of its wide extent. It is available by the authors upon request.

any firm, is to set its own prices in order to maximize cartel profits. Firms' competitive play is not subgame perfect equilibrium, as both firms have incentives to deviate from it. In equations (60) and (69), it applies that:

$$\Pi^*_{id_j}(=\Pi^*_{jd_i}) - \Pi^*_{im}(=\Pi^*_{jm}) = B_{14}(1-w_0)^2 > 0 \qquad \text{(A40)}$$

$B_{14} > 0\ \ \forall\ \gamma \in (0,1)$[14] and its plot is presented below.



Summarizing the above results, we conclude that firm cartel formation (E1) emerges in equilibrium in our Bertrand duopoly.

A.7 Proof of Proposition 7

(i) Total market output is the sum of firms' equilibrium output, as given in (A3). According to equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (50), (58), (67) and (68), we obtain the total output levels:

$$Q^*_c = \frac{1-w_0}{(1+\gamma)(2-\gamma)} \qquad \text{(A41)}$$

$$Q^*_m = \frac{2(2-\gamma^2)(1-w_0)}{(1+\gamma)(2-\gamma)\big(4-\gamma(1+2\gamma)\big)} \qquad \text{(A42)}$$

$$Q^*_{mos} = \frac{\big(8-\gamma(5-\gamma)\big)(1-w_0)}{2(1+\gamma)(8-5\gamma^2)} \qquad \text{(A43)}$$

From comparative analysis of (41), (42) and (43), we obtain that:

$$Q^*_m - Q^*_c = \frac{\gamma(1-w_0)}{(1+\gamma)(2-\gamma)\big(4-\gamma(1+2\gamma)\big)} > 0 \quad \forall\ \ \gamma \in (0,1) \qquad \text{(A44)}$$

[14] The mathematical expression of B_{14} is left out because of its wide extent. It is available by the authors upon request.

$$Q_{mos}^{*}-Q_{c}^{*}=\frac{\gamma(2+\gamma)(1-w_{0})}{2(2-\gamma)(8-5\gamma^{2})}>0 \qquad \forall\ \gamma\in(0,1) \tag{A45}$$

$$Q_{mos}^{*}-Q_{m}^{*}=B_{15}(1-w_{0})>0 \qquad \forall\ \gamma>0.78 \tag{A46}$$

Where $B_{15}=\frac{\gamma(1-\gamma)(4+\gamma)\left(2-\gamma(1+2\gamma)\right)}{2(1+\gamma)(2-\gamma)(8-5\gamma^{2})\left(4-\gamma(1+2\gamma)\right)}>0\ \forall\ \ \gamma>0.78$.

Summarizing our results, we conclude that:

$$Q_{m}^{*}>Q_{mos}^{*}>Q_{c}^{*} \qquad \forall\ \gamma\in(0,0.78) \tag{A47}$$

$$Q_{mos}^{*}>Q_{m}^{*}>Q_{c}^{*} \qquad \forall\ \gamma\in(0.78,1) \tag{A48}$$

(ii) Comparing (48), (56), (65) and (66) we get that:

$$w_{im}^{*}-w_{ic}^{*}=\frac{\gamma(1+\gamma)(1-\gamma)(1-w_{0})}{(2-\gamma)\left(4-\gamma(1+2\gamma)\right)}>0 \qquad \forall\ \gamma\in(0,1) \tag{A49}$$

$$w_{imos}^{*}-w_{ic}^{*}=\frac{\gamma^{2}(1+\gamma)(1-\gamma)(1-w_{0})}{(2-\gamma)(8-5\gamma^{2})}>0 \qquad \forall\ \gamma\in(0,1) \tag{A50}$$

$$w_{jmos}^{*}-w_{jc}^{*}=\frac{2\gamma(1+\gamma)(1-\gamma)(1-w_{0})}{(2-\gamma)(8-5\gamma^{2})}>0 \qquad \forall\ \gamma\in(0,1) \tag{A51}$$

$$w_{im}^{*}-w_{imos}^{*}=\frac{2\gamma\left(2-\gamma^{2}(3-\gamma^{2})\right)(1-w_{0})}{(8-5\gamma^{2})\left(4-\gamma(1+2\gamma)\right)}>0 \qquad \forall\ \gamma\in(0,1) \tag{A52}$$

$$w_{jm}^{*}-w_{jmos}^{*}=\frac{\gamma^{2}(1-\gamma^{2})(1-w_{0})}{(8-5\gamma^{2})\left(4-\gamma(1+2\gamma)\right)}>0 \qquad \forall\ \gamma\in(0,1) \tag{A53}$$

Summarizing our results, we conclude that:

$$w_{m}^{*}>w_{imos}^{*},w_{jmos}^{*}>w_{ic}^{*} \qquad \forall\ \gamma\in(0,1) \tag{A54}$$

A.8 Proof of Proposition 8

By means of the Consumer Surplus equation in (37) and equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (50), (58), (67) and (68), we get that:

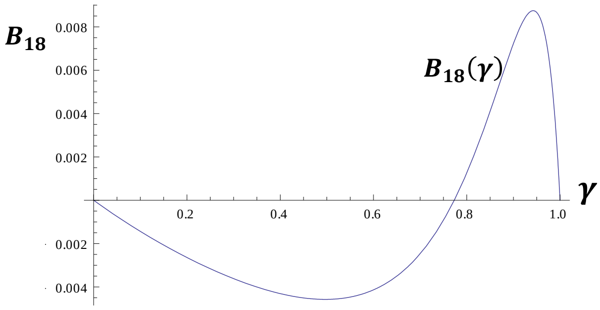
$$CS_m^* - CS_c^* = B_{16}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A55}$$

$$CS_{mos}^* - CS_c^* = B_{17}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A56}$$

$$CS_{mos}^* - CS_m^* = B_{18}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0.77,1) \tag{A57}$$

Where $B_{16} = \frac{\gamma(8-\gamma(1+4\gamma))}{4(1+\gamma)(2-\gamma)^2(4-\gamma(1+2\gamma))^2} > 0$, $B_{17} = \frac{\gamma(32+5\gamma(4-\gamma(4+\gamma)))}{8(2-\gamma)^2(8-5\gamma^2)^2} > 0\ \forall\ \gamma \in (0,1)$

and $B_{18} > 0\ \ \gamma \in (0.77,1)$[15]. The plot of B_{18} is presented below.



Summarizing our results, we conclude that:

$$CS_m^* > CS_{mos}^* > CS_c^* \qquad \forall\ \gamma \in (0,0.77) \tag{A58}$$

$$CS_{mos}^* > CS_m^* > CS_c^* \qquad \forall\ \gamma \in (0.77,1) \tag{A59}$$

A.9 Proof of Proposition 9

Regarding Producer Surplus [given in (38)] and equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (50), (58), (67) and (68), we get that:

$$PS_c^* - PS_m^* = B_{19}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A60}$$

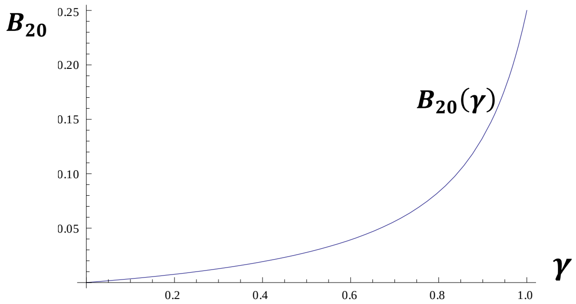
$$PS_c^* - PS_{mos}^* = B_{20}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A61}$$

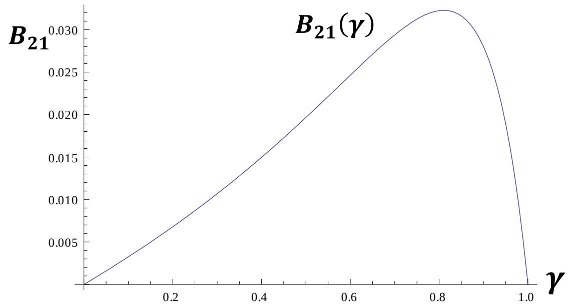
$$PS_{mos}^* - PS_m^* = B_{21}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A62}$$

[15] The mathematical expressions of B_{18} is left out because of its wide extent. It is available by the authors upon request.

Where $B_{19} = \frac{\gamma\left(8-\gamma(4\gamma(3-\gamma)-1)\right)}{2(1+\gamma)(2-\gamma)^2\left(4-\gamma(1+2\gamma)\right)^2} > 0 \;\; \forall\, \gamma \in (0,1)$ and $B_{20}, B_{21} > \;\; \forall\, \gamma \in (0,1)$[16].

The plots of B_{20} and B_{21} are presented below.





Summarizing our results, we conclude that:

$$PS_c^* > PS_{mos}^* > PS_c^* \qquad \forall\;\gamma \in (0,1) \tag{A63}$$

Regarding now Union Rents [given in (39)], equilibrium output levels [given in (50), (58), (67) and (68)] and wages [given in (48), (56), (65) and (66)] under firms' collusion, competition and mix of strategies configuration, we get that:

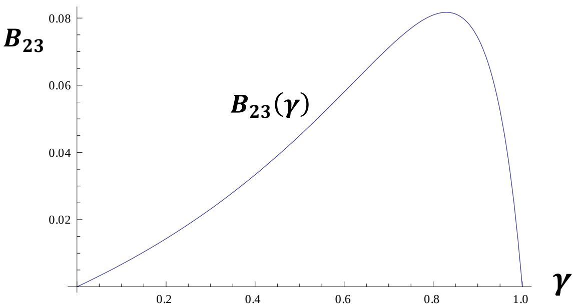
$$UR_m^* - UR_c^* = B_{22}(1-w_0)^2 > 0 \qquad \forall\;\gamma \in (0,1) \tag{A64}$$

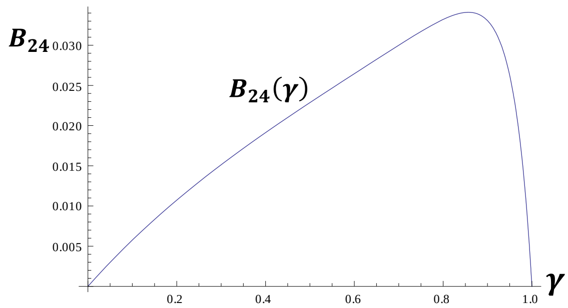
$$UR_{mos}^* - UR_c^* = B_{23}(1-w_0)^2 > 0 \qquad \forall\;\gamma \in (0,1) \tag{A65}$$

$$UR_m^* - UR_{mos}^* = B_{24}(1-w_0)^2 > 0 \qquad \forall\;\gamma \in (0,1) \tag{A66}$$

Where $B_{22} = \frac{\gamma(1-\gamma)\left(8-\gamma(2\gamma(2+\gamma)-3)\right)}{2(1+\gamma)(2-\gamma)^2\left(4-\gamma(1+2\gamma)\right)^2} > 0 \;\; \forall\, \gamma \in (0,1)$ and $B_{23}, B_{24} > \;\; \forall\, \gamma \in (0,1)$[17].

The plots of B_{23} and B_{24} are presented below.





[16] The mathematical expressions of B_{20} and B_{21} are left out because of their wide extent. They are available by the authors upon request.

[17] The mathematical expressions of B_{23} and B_{24} are left out because of their wide extent. They are available by the authors upon request.

Summarizing our results, we conclude that:

$$UR_m^* > UR_{mos}^* > UR_c^* \qquad \forall\ \gamma \in (0,1) \tag{A67}$$

A.10 Proof of Proposition 10

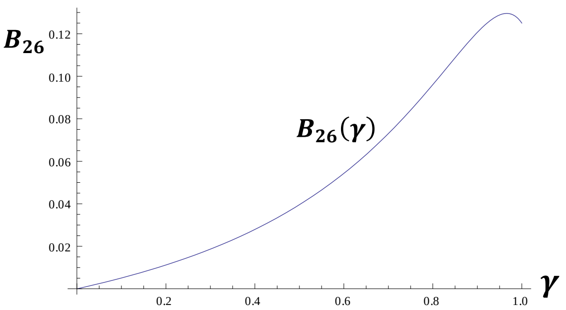
By means of market participants surpluses/rents in (37), (38) and (39) and equilibrium output levels [given in (50), (58), (67) and (68)] and wages [given in (48), (56), (65) and (66)] under firms' collusion, competition and mix of strategies configuration, we obtain about Social Welfare that:

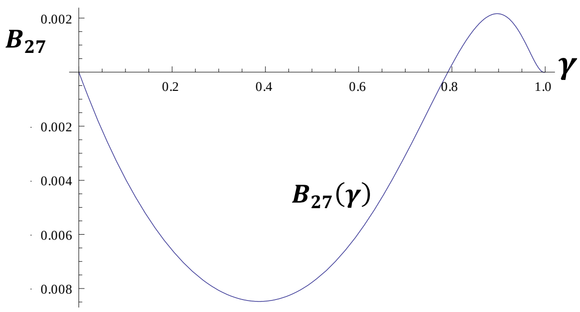
$$SW_m^* - SW_c^* = B_{25}(1-w_0)^2 > 0 \qquad \forall\ \ \gamma \in (0,1) \tag{A68}$$

$$SW_{mos}^* - SW_c^* = B_{26}(1-w_0)^2 > 0 \qquad \forall\ \ \gamma \in (0,1) \tag{A69}$$

$$SW_{mos}^* - SW_m^* = B_{27}(1-w_0)^2 > 0 \qquad \forall\ \ \gamma > 0.79 \tag{A70}$$

Where $B_{25} = \frac{\gamma(1-\gamma)\left(24-\gamma(23+8\gamma(1-\gamma))\right)}{4(1+\gamma)(2-\gamma)^2\left(4-\gamma(1+2\gamma)\right)^2} > 0\ \forall\ \ \gamma \in (0,1)$, $B_{26} > \forall\ \ \gamma \in (0,1)$ and $B_{27} > 0\ \ \gamma \in (0,0.79)$[18]. The plots of B_{26} and B_{27} are presented below.





Summarizing our results, we conclude that:

$$SW_m^* > SW_{mos}^* > SW_c^* \qquad \forall\ \gamma \in (0,0.79) \tag{A71}$$

$$SW_{mos}^* > SW_m^* > SW_c^* \qquad \forall\ \ \gamma \in (0.79,1) \tag{A72}$$

[18] The mathematical expressions of B_{26} and B_{27} are left out because of their wide extent. They are available by the authors upon request.

A.11 Proof of Proposition 11

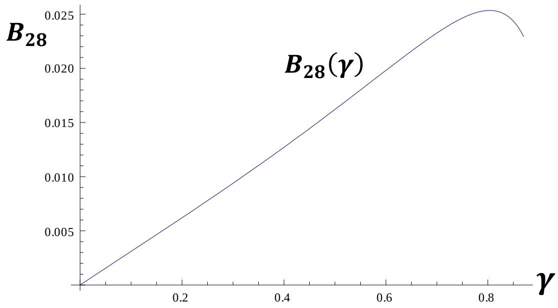
At the second stage of the game, firms deal with the matrix game which is presented in Subsection A.1., except for firms' mode of competition and payoffs which are given in Subsections 6.1-6.4. Given firms' decisions about their mode of competition, i.e. firm i adjusts its own output, while firm j set its own prices, we get four candidate equilibriums, i.e. E1, E2, E3 and E4.

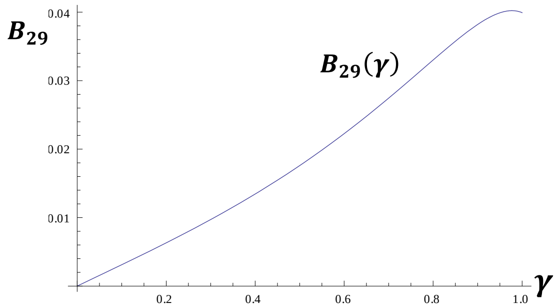
In candidate equilibrium (E1), firms proceed to cartel formation and the possible deviation, on the part of any firm, is to play competitively. We conclude that firms' cartel formation emerges in equilibrium, as no firm has incentive to deviate from it. By means of (78), (96) and (112), we get that:

$$\Pi_{ic}^{*} - \Pi_{id_i}^{*} = \begin{cases} B_{28}(1-w_0)^2 > 0, & \forall \ \gamma < 0.87 \\ \dfrac{\gamma^2(3+\gamma)(1-w_0)^2}{16(1+\gamma)(2-\gamma)^2} > 0, & \forall \ \gamma > 0.87 \end{cases} \tag{A73}$$

$$\Pi_{jc}^{*} - \Pi_{jd_j}^{*} = B_{29}(1-w_0)^2 > 0 \tag{A74}$$

$B_{28}, \ B_{29} > 0 \ \forall \ \gamma \in (0,1)$[19] and their plots are presented below.





In candidate equilibrium (E2), firm i adjusts its own output in order to maximize cartel profits, while firm j competes in prices, while in candidate equilibrium (E3), firm i competes in quantities, while firm j plays collusively. From (A73) and (A74), we obtain that none of the two candidate equilibria emerge in subgame perfect one, because both firm have incentives to deviate by forming a cartel.

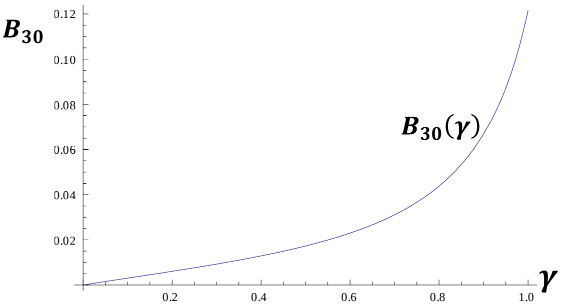
[19] The mathematical expression of B_{28} and B_{29} are left out because of their wide extent. They are available by the authors upon request.

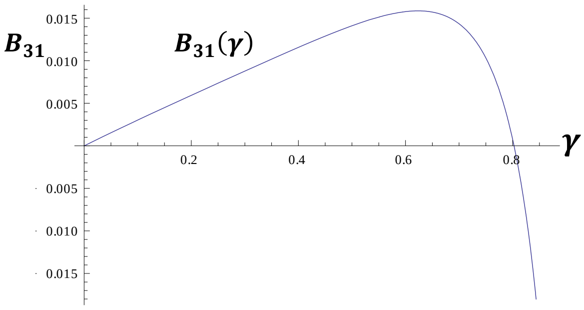
In the last candidate equilibrium (E4), both firms decide to play competitively in order to maximize their own profits and the possible deviation, on the part of any firm, is to play collusively. Firms' competitive play is not subgame perfect equilibrium, at least firm i has incentives to deviate from it. By means of (87), (88), (95) and (113), we get that:

$$\Pi_{id_j}^* - \Pi_{im}^* = B_{30}(1-w_0)^2 > 0 \tag{A75}$$

$$\Pi_{jd_i}^* - \Pi_{jm}^* = \begin{cases} B_{31}(1-w_0)^2 > 0, & \forall\ \gamma \in (0, 0.8) \\ B_{31}(1-w_0)^2 < 0, & \forall\ \gamma \in (0.8, 0.87) \\ -\Pi_{jm}^* \quad < 0, & \forall\ \gamma \in (0.87, 1) \end{cases} \tag{A76}$$

Where $B_{30} > 0\ \forall\ \gamma \in (0,1)$ and $B_{31} > 0\ \forall\ \gamma \in (0,0.8)$[20]. Their plots are presented below.





Summarizing the above results, we conclude that firm cartel formation (E1) emerges in equilibrium in our Mix of Strategy configuration.

A.12 Proof of Proposition 12

By means of the Consumer Surplus equation in (37) and equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (76), (83), (84), (93), (94), (110) and (111), we get that:

$$CS_m^* - CS_c^* = B_{31}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A77}$$

[20] The mathematical expression of B_{30} and B_{31} are left out because of their wide extent. They are available by the authors upon request.

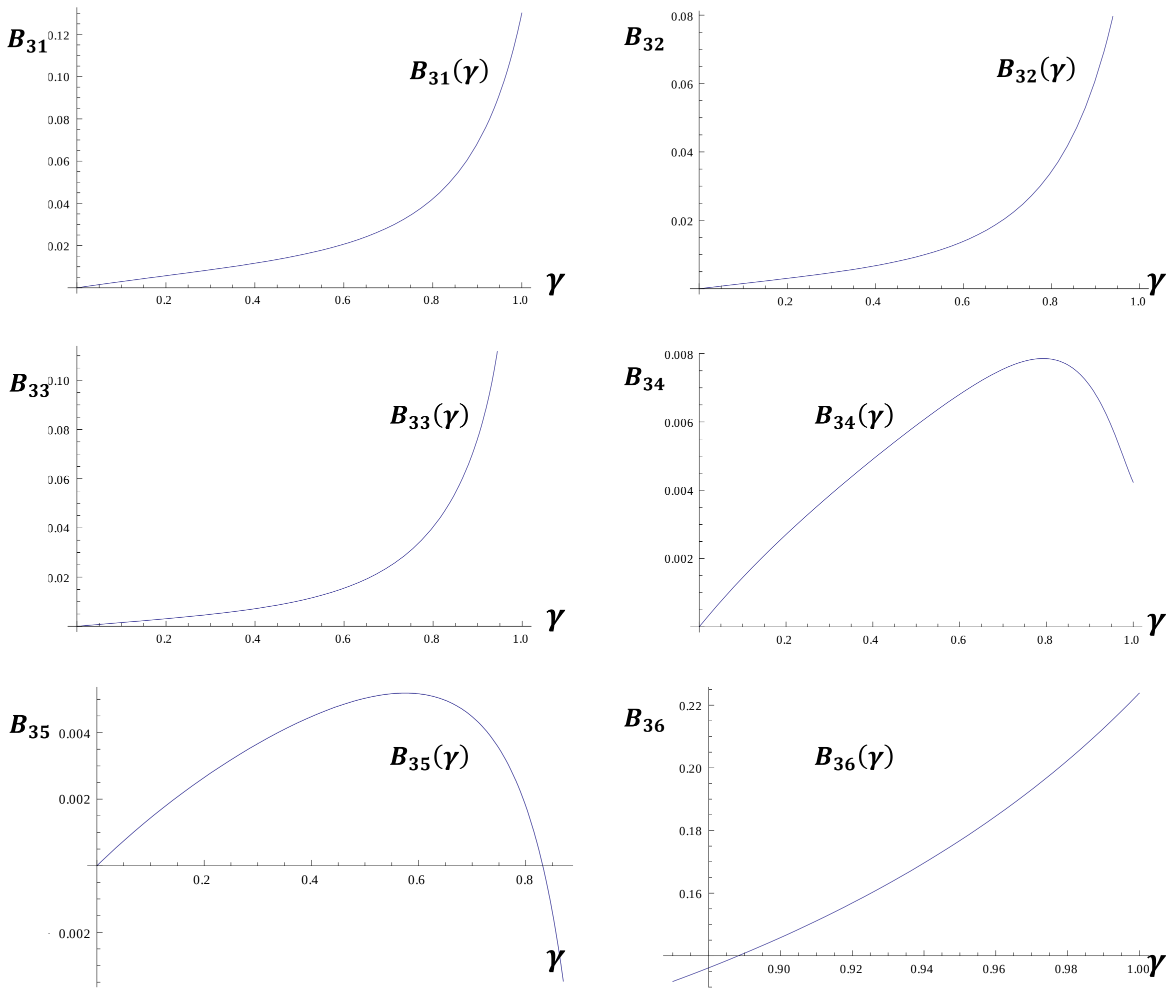
$$CS^*_{mos_B} - CS^*_c = B_{32}(1-w_0)^2 > 0 \qquad \forall \ \ \gamma \in (0,1) \tag{A78}$$

$$CS^*_{mos_C} - CS^*_c = \begin{cases} B_{33}(1-w_0)^2 > 0 & \forall \ \ \gamma \in (0,0.87) \\ -\dfrac{4+\gamma^2(1-\gamma)}{32(1+\gamma)(2-\gamma)^2} < 0 & \forall \ \ \gamma \in (0.87,1) \end{cases} \tag{A79}$$

$$CS^*_m - CS^*_{mos_B} = B_{34}(1-w_0)^2 > 0 \qquad \forall \ \ \gamma \in (0,1) \tag{A80}$$

$$CS^*_m - CS^*_{mos_C} = \begin{cases} B_{35}(1-w_0)^2 > 0 & \forall \ \ \gamma \in (0,0.83) \\ B_{35}(1-w_0)^2 < 0 & \forall \ \ \gamma \in (0.83,0.87) \\ B_{36}(1-w_0)^2 > 0 & \forall \ \ \gamma \in (0.87,1) \end{cases} \tag{A81}$$

Where $B_{31}, B_{32}, B_{33}, B_{34}, B_{36} > 0 \ \ \gamma \in (0,1)$ and $B_{35} > 0 \ \ \gamma \in (0,0.83)$[21]. Their plots are presented below.



Summarizing our results, we conclude that:

[21] The mathematical expressions of $B_{31}, B_{32}, B_{33}, B_{34}, B_{35}, B_{36}$ are left out because of their wide extent. They are available by the authors upon request.

$$CS_m^* > CS_{mos_C}^*, CS_{mos_B}^* > CS_c^* \qquad \forall\ \gamma \in (0,0.83) \tag{A82}$$

$$CS_{mos_C}^* > CS_m^* > CS_{mos_B}^* > CS_c^* \qquad \forall\ \gamma \in (0.83,0.87) \tag{A83}$$

$$CS_m^* > CS_{mos_B}^* > CS_c^* > CS_{mos_C}^* \qquad \forall\ \gamma \in (0.87,1) \tag{A84}$$

A.13 Proof of Proposition 13

Regarding Producer Surplus [given in (38)] and equilibrium output levels under firms' collusion, competition and mix of strategies configuration in (76), (83), (84), (93), (94), (110) and (111), we get that:

$$PS_c^* - PS_m^* = B_{37}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A85}$$

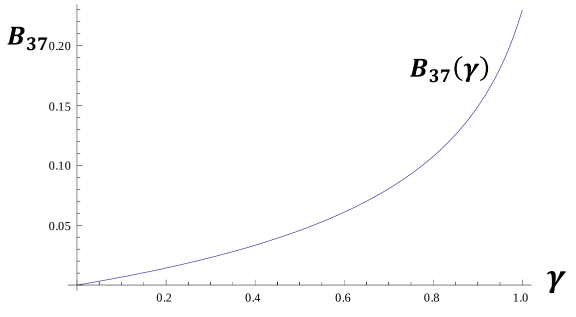
$$PS_c^* - PS_{mos_B}^* = B_{38}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A86}$$

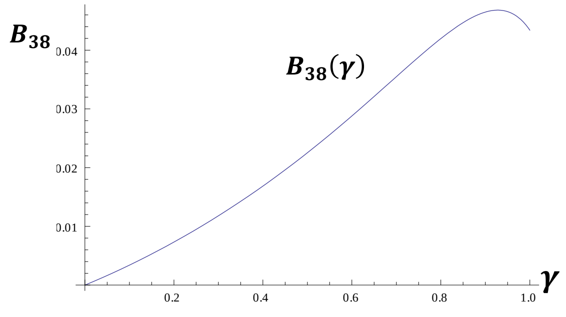
$$PS_c^* - PS_{mos_C}^* = \begin{cases} B_{39}(1-w_0)^2 > 0 & \forall\ \gamma \in (0,0.87) \\ \dfrac{4+\gamma^2(3-\gamma)}{16(1+\gamma)(2-\gamma)^2} > 0 & \forall\ \gamma \in (0.87,1) \end{cases} \tag{A87}$$

$$PS_{mos_B}^* - PS_m^* = B_{40}(1-w_0)^2 > 0 \qquad \forall\ \gamma \in (0,1) \tag{A88}$$

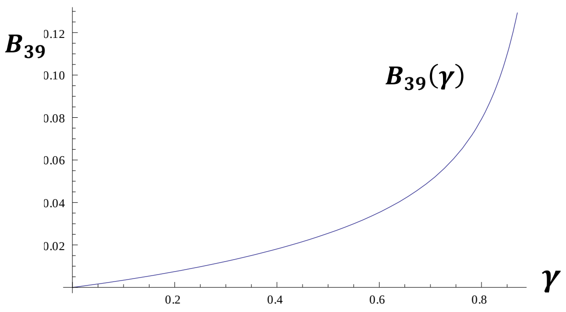
$$PS_{mos_C}^* - PS_m^* = \begin{cases} B_{41}(1-w_0)^2 > 0 & \forall\ \gamma \in (0,0.87) \\ B_{42}(1-w_0)^2 < 0 & \forall\ \gamma \in (0.87,0.92) \\ B_{42}(1-w_0)^2 > 0 & \forall\ \gamma \in (0.92,1) \end{cases} \tag{A89}$$

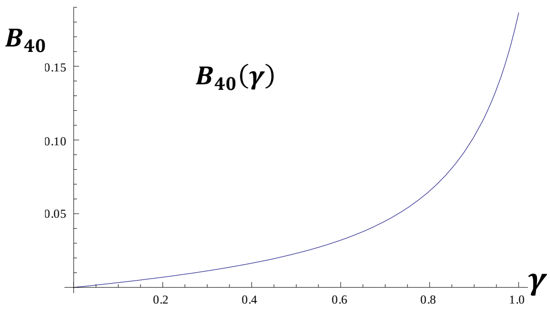
Where $B_{37}, B_{38}, B_{39}, B_{40} > 0\ \gamma \in (0,1)$, $B_{41} > 0\ \gamma \in (0,0.87)$ and $B_{42} > 0\ \gamma \in (0.92,1)$[22]. Their plots are presented below.

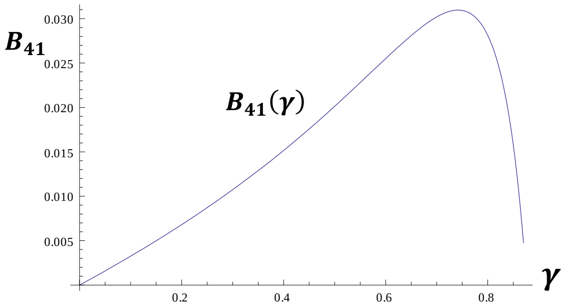


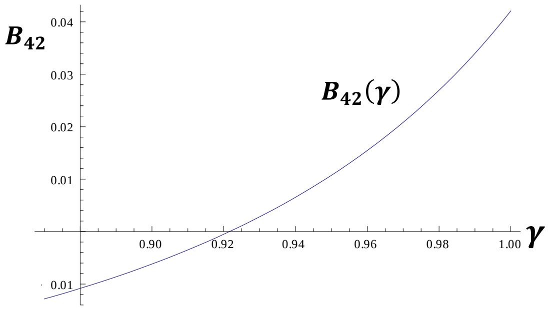


[22] The mathematical expressions of $B_{37}, B_{38}, B_{39}, B_{40}, B_{41}, B_{42}$ are left out because of their wide extent. They are available by the authors upon request.









Summarizing our results, we conclude that:

$$PS_c^* > PS_{mos_C}^*, PS_{mos_B}^* > PS_c^* \qquad \forall \ \gamma \notin (0.87,0.92) \tag{A90}$$

$$PS_c^* > PS_{mos_B}^* > PS_m^* > PS_{mos_C}^* \qquad \forall \ \gamma \in (0.92,1) \tag{A91}$$

Regarding now Union Rents [given in (39)], equilibrium output levels [given in (76), (83), (84), (93), (94), (110) and (111)] and wages [given in (75), (81), (82), (91), (92), (108) and (109)] under firms' collusion, competition and mix of strategies configuration, we get that:

$$UR_m^* - UR_c^* = B_{43}(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \tag{A92}$$

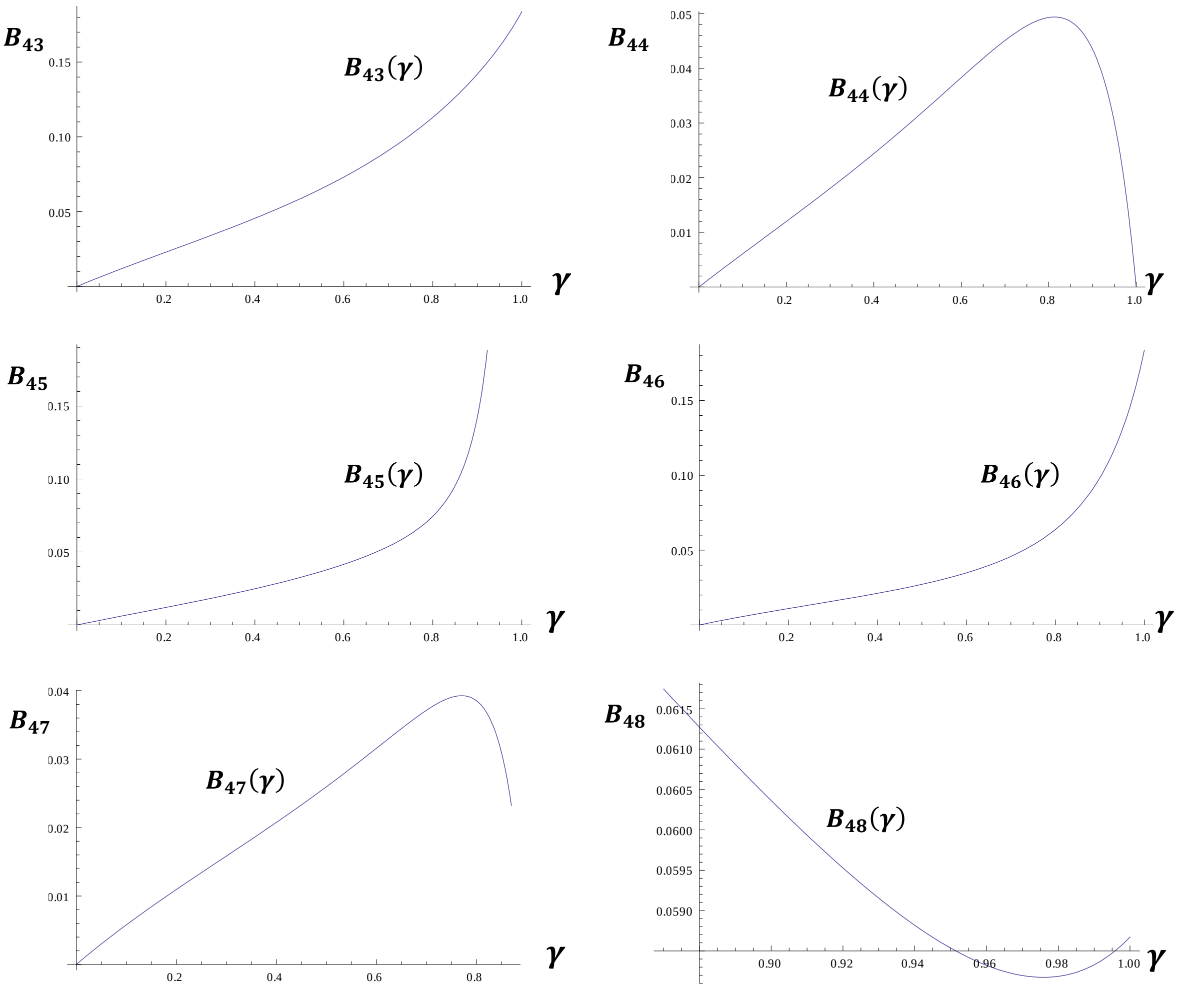
$$UR_{mos_B}^* - UR_c^* = B_{44}(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \tag{A93}$$

$$UR_{mos_C}^* - UR_c^* = \begin{cases} B_{45}(1-w_0)^2 > 0 & \forall \ \gamma \in (0,0.87) \\ \dfrac{\gamma\left(8-\gamma\left((3-\gamma)-4\right)\right)}{8(1+\gamma)(2-\gamma)^2} > 0 & \forall \ \gamma \in (0.87,1) \end{cases} \tag{A94}$$

$$UR_m^* - UR_{mos_B}^* = B_{46}(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \tag{A95}$$

$$UR_m^* - UR_{mos_C}^* = \begin{cases} B_{47}(1-w_0)^2 > 0 & \forall \ \gamma \in (0,0.87) \\ B_{48}(1-w_0)^2 > 0 & \forall \ \gamma \in (0.87,1) \end{cases} \tag{A96}$$

Where $B_{43}, B_{44}, B_{45}, B_{46} > 0 \ \gamma \in (0,1)$, $B_{47} > 0 \ \gamma \in (0,0.87)$ and $B_{48} > 0 \ \gamma \in (0.87,1)$[23]. Their plots are presented below.



Summarizing our results, we conclude that:

$$UR_m^* > UR_{mos_C}^*, UR_{mos_B}^* > UR_c^* \qquad \forall \ \gamma \in (0,1) \qquad \text{(A97)}$$

A.14 Proof of Proposition 14

By means of market participants surpluses/rents in (37), (38) and (39) and equilibrium output levels [given in (76), (83), (84), (93), (94), (110) and (111)] and wages [given in (75), (81), (82), (91), (92), (108) and (109)] under firms' collusion, competition and mix of strategies configuration, we obtain about Social Welfare that:

[23] The mathematical expressions of $B_{43}, B_{44}, B_{45}, B_{46}, B_{47}, B_{48}$ are left out because of their wide extent. They are available by the authors upon request.

$$SW_m^* - SW_c^* = B_{49}(1-w_0)^2 > 0 \qquad \forall \;\; \gamma \in (0,1) \tag{A98}$$

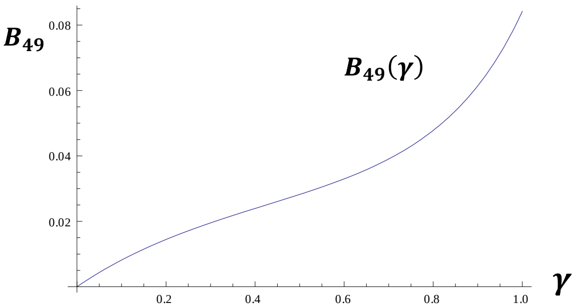
$$SW_{mos_B}^* - SW_c^* = B_{50}(1-w_0)^2 > 0 \qquad \forall \;\; \gamma \in (0,1) \tag{A99}$$

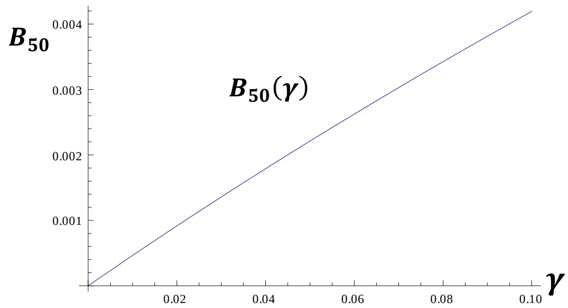
$$SW_{mos_C}^* - SW_c^* = \begin{cases} B_{51}(1-w_0)^2 > 0 & \forall \;\; \gamma \in (0,0.87) \\ \dfrac{28-\gamma\big(32-7\gamma(3-\gamma)\big)}{32(1+\gamma)(2-\gamma)^2} < 0 & \forall \;\; \gamma \in (0.87,1) \end{cases} \tag{A100}$$

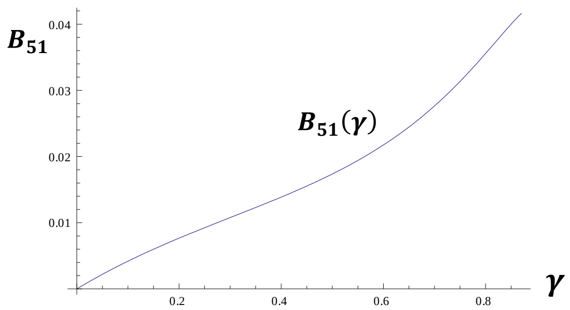
$$SW_m^* - SW_{mos_B}^* = B_{52}(1-w_0)^2 > 0 \qquad \forall \;\; \gamma \in (0,1) \tag{A101}$$

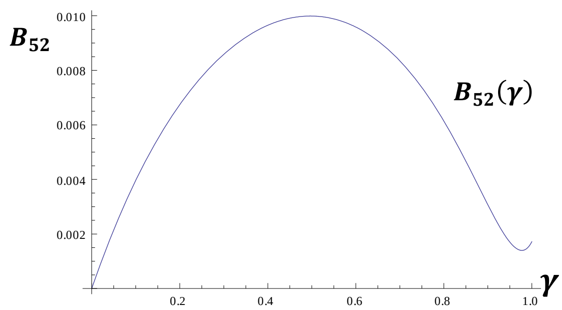
$$SW_m^* - SW_{mos_C}^* = \begin{cases} B_{53}(1-w_0)^2 > 0 & \forall \;\; \gamma \in (0,0.87) \\ B_{54}(1-w_0)^2 > 0 & \forall \;\; \gamma \in (0.87,1) \end{cases} \tag{A102}$$

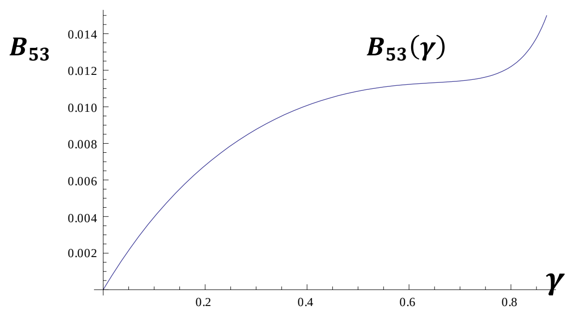
Where $B_{49}, B_{50}, B_{51}, B_{52} > 0$ $\gamma \in (0,1)$, $B_{53} > 0$ $\gamma \in (0,0.87)$ and $B_{54} > 0$ $\gamma \in (0.87,1)$[24]. Their plots are presented below.

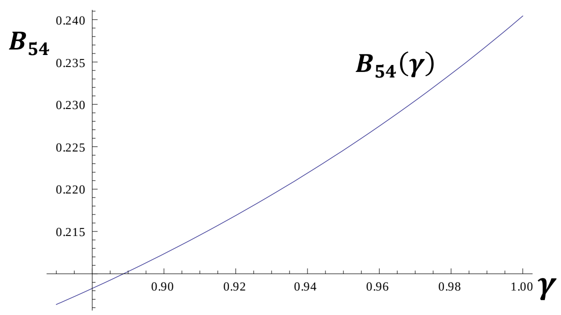












[24] The mathematical expressions of $B_{49}, B_{50}, B_{51}, B_{52}, B_{53}, B_{54}$ are left out because of their wide extent. They are available by the authors upon request.

Summarizing our results, we conclude that:

$$SW_m^* > SW_{mos_C}^*, SW_{mos_B}^* > SW_c^* \qquad \forall \ \gamma \in (0,0.87) \qquad \text{(A103)}$$

$$SW_m^* > SW_{mos_B}^* > SW_c^* > SW_{mos_C}^* \qquad \forall \ \gamma \in (0.87,1) \qquad \text{(A104)}$$

A.15 Proof of Proposition 17

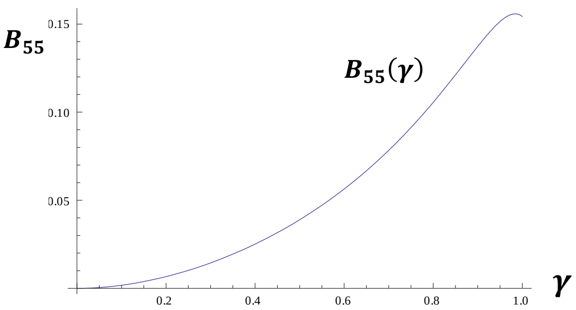
According to, under each firms' mode of competition, market participants' surpluses in (37), (38) and (39) and equilibrium output levels [given in (19), (58),, (81) and (82)] and wages [given in (18), (56), (83) and (84)], given firms' competitive play, we obtain about Social Welfare that:

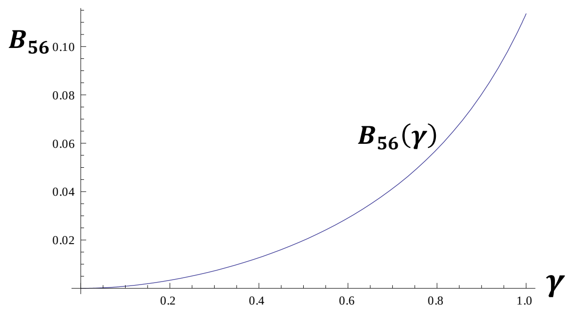
$$SW_B^* - SW_C^* = B_{55}(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \qquad \text{(A105)}$$

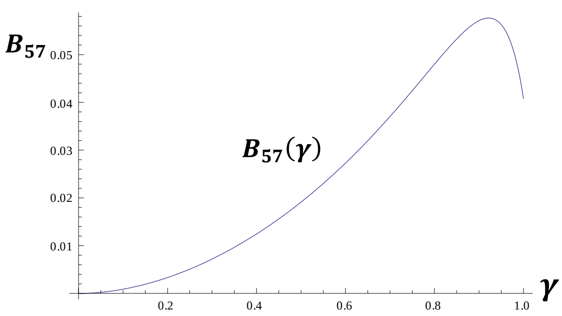
$$SW_{MOS}^* - SW_C^* = B_{56}(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \qquad \text{(A106)}$$

$$SW_B^* - SW_{MOS}^* = B_{57}(1-w_0)^2 > 0 \qquad \forall \ \gamma \in (0,1) \qquad \text{(A107)}$$

Where $B_{55}, B_{56}, B_{57} > 0 \ \gamma \in (0,1)$[25], and their plots are presented below.







Summarizing our results, we conclude that:

$$SW_B^* > SW_{MOS}^* > SW_C^* \qquad \forall \ \gamma \in (0,1) \qquad \text{(A108)}$$

[25] The mathematical expressions of B_{55}, B_{56}, B_{57} are left out because of their wide extent. They are available by the authors upon request.

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