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Abstract
This paper studies oligopolistic markets with differentiated products, with endogenous union structures and quality improvement-R&D investments. In the context of a dynamic game-theoretic analysis we investigate the conditions under which firm-level unions may strategically collude, or not, and the impact of their decisions upon the firms’ incentives to individually spend on R&D investments. We show that, under sufficiently high (low) discount rate and substitutability among the firms' products, an industry-wide union emerges (separate firm-level unions sustain) in the equilibrium, where product quality along with the level of R&D investments are relatively low (high). Moreover, we consider the instance where a benevolent policy maker undertakes the costs of firm-specific R&D investments and finances these costs by indirect taxation. We conclude that in such cases, higher surpluses emerge for the market participants in the equilibrium.

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1. Introduction

Recent literature on Industrial Organization focuses on essays about R&D investments and labor unionization in an oligopolistic product market, with emphasis on the interaction between them. Contributions in this area have mainly focused on a static analysis of the role of union structure (*centralized or decentralized*) on firms’ incentives for R&D investments, with theoretical and empirical studies.

There are supporters that claim that there is a negative correlation between them and consequently centralized wage-setting structure is harmful for the industry, as well as that there is positive correlation as R&D investments increase. The opinions vary, as each one is studying the same subject from a different perspective. It is recognized also, that relevant literature lacks dynamic analysis on the above field.

In order to contribute in this area, we propose a dynamic version of a model that we first introduced in our previous work. More specifically, in the context of a dynamic game-theoretic analysis, we consider a union-dupopoly model with differentiated products and R&D investments on product quality improvement, in where two technologically identical firms compete by independently adjusting their own quantities. As regards the labor market, there are two firm-level unions with monopoly power over the firm-specific wage bargaining. Unions independently decide about their strategy on w-negotiations with their specific firm, by following a decentralized or centralized wage-setting regime. Our model follows a four-stage game with the following sequence of events: At the 1\textsuperscript{st} stage, unions independently decide to proceed with a centralized or decentralized bargaining strategy. At the 2\textsuperscript{nd} stage, firms determine the optimal level of their R&D investments. At the 3\textsuperscript{rd} stage, unions independently set their firm-specific wages, in order either to maximize their own rents, or to maximize joint rents, depending on their decision about their strategy.
at the 1st stage of the game. At the 4th stage, firms independently adjust their own production levels, in order to maximize their own profits (Cournot Competition).

We conclude that under sufficiently high (low) discount rate and substitutability among the firms' products, an industry-wide union emerges (separate firm-level unions sustain) in the equilibrium, where product quality along with the level of R&D investments are relatively low (high).

Moreover, we proceed to research further by developing a new market structure, in where the R&D’s product is a common public good. More specifically, we consider that a benevolent policy maker undertakes the costs of firm-specific R&D investments, finances these costs by indirect taxation and provides firms, without cost, the know-how of product quality improvement. Subsequently, we endogenize the selection of a market structure in our model. One of our main results shows the superiority in terms of Social Welfare of the decentralized wage-setting regime between union structures and the nationalization of R&D between market structures.

The rest of the paper is structured as follows: In Section 2 we present our unionized oligopoly model. In the context of a dynamic game-theoretic analysis, Section 3 demonstrates the conditions under which union centralized wage-setting regime is sustained in equilibrium. In Section 4 and 5, we proceed to the dynamic analysis of two proposed market structures, where the R&D is a private or public good, respectively. Next, in Section 6, our model endogenizes the selection of market structure, while in Section 7 we proceed to the comparative analysis of the proposed market structures. Our results are summarized in Section 8.
2. The Model

We assume a unionized duopoly where the two identical firms produce differentiated goods and investigate in R&D – quality improvement. Each firm faces an inverse linear demand function and following Häckner (2000) its mathematical expression\(^1\) is the following:

\[ p_i(q_i, q_j) = 1 - q_i - \gamma q_j + h x_i \]  

(1)

Where, \( p_i, q_i \) respectively are the price and output of firm \( i \neq j = 1, 2 \) and \( \gamma \in (0,1) \) denotes the degree of product substitutability: As \( \gamma \to 1 \) the firms’ products become more close substitutes. Additionally, \( x_i \) presents the quality of products which is derived from firms’ expenditures on R&D, while \( h \in (0,1) \) is the consumer evaluation of product quality: As \( h \to 0 \) the consumers become completely indifferent about product quality.

We consider that the production technology exhibits constant returns to scale. Moreover, firms’ production requires only labor input\(^2\), with the labor productivity being equal to one, for both firms, namely one unit of labor is needed to produce one unit of product:

\[ L_i = q_i \]  

(2)

Where \( L_i \) and \( q_i \) are the employment and the quantity respectively of firm \( i \).\(^3\)

Firms’ unit transformation cost of labor into product equals the wage rate, denoted by \( w_i \). Firm also proceeds to R&D investments, denoted by \( x_i^2 / 2 \), in order to improve its own product quality. Hence, the profit function of firm \( i \) is defined by:

\[^1\] For simplicity, we assumed that both \( a \) and \( b \) are equal to one.

\[^2\] This is equivalent to a two-factor Leontief technology in which the amount of capital is fixed in the short-run and it is large enough not to induce zero marginal product of labor.

\[^3\] We are aware of the limitations of our analysis in assuming specific functional forms and constant returns to scale. However, the use of more general forms would jeopardize the clarity of our findings, without significantly changing their qualitative character.
\[ \Pi_i = (p_i - w_i)q_i - \frac{x_i^2}{2} \]  

(3)

Regarding the labor market, we assume that is unionized with workers being organized into two separate firm-specific unions. Each firm enters into negotiations exclusively with its own union, over (only) wages (decentralized *Right-to-Manage* bargaining\(^4\)). Moreover, we assume that unions are identical, endowed with monopoly bargaining power during the negotiations with their own firms and may compete or collude by independently adjusting their own wages. In accordance to the above, each union effectively acts as a firm-specific monopoly union by setting the wage, with union, \(i\)’s objective being to maximize the sum of its member rents, as follows:

\[ u_i(w_i, L_i) = w_iL_i \]  

(4)

Where, \(w_i\) is firm \(i\)’s wage rate, provided that union membership is fixed and all members are (or the union leadership treats them as being) identical [see, e.g. Oswald (1982), Pencavel (1991), Booth, (1995)].

We assume an alternative market structure, in where a proposed market policy applies. In particular, we assume that the policy maker proceeds to quality improvement-R&D, as a common public good, by undertaking the costs of those investments and providing the know-how to the industry for free. He finances these costs by indirect consumption taxation on the final product \([TC \leq TR\) (Balance Budget\(^5\)]. Accordingly, the policy maker’s (or social planner’s) cost and revenue functions are presented by the following equations, respectively:

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\(^4\) Right-to-manage literature was initially developed by the British school during the 1980s (Nickell). It implies that the union-firm negotiations agenda includes only the wage rate, which is determined according to the typical Nash Bargaining Maximization.

\(^5\) A government balance budget refers to a budget in which total revenues are equal or greater to total expenditures (no budget deficit). In our case, the balance budget has to do exclusively with our oligopoly industry.
Where \( \frac{x^2}{2} \) denotes the expenditures on R&D in order to improve the industry’s product quality and the factors \( t \) and \( Q \) denote the collected tax per unit of product and the sum of the firms’ output level, respectively.

From the consumers’ perspective, the consumption tax is an additional cost on the purchase price of industry products, as it is presented in the following equation:

\[
p_i^C = p_i + t
\]

Where \( p_i^C \) denotes consumer price of products, which is the sum of the product price received by producers (denoted by factor \( p_i \)) and the consumption tax collected by the Social Planner (denoted by factor \( t \)).

Therefore, by substituting (7) for (1), we get the new inverse linear demand function that firms’ face, which is:

\[
p_i(q_i, q_j) = 1 - q_i - \gamma q_j + hx - t
\]

Throughout the game, market participants discount future payoffs using the Net Present Value (NPV), an essential tool in discounted cash flow analysis. The NPV presents the sum of the present values of all future incoming payoffs, used for discounting the common discount rate \( \delta \), which formula is the following:

\[
\delta = \frac{1}{(1 + r)^t}
\]

Where \( r \) and \( t \) denote the interest rate and the time (game period) of the payoff, respectively.

In the above, we propose three games, one for each potential action the social planner takes, as follows:

\[
TC^{PM} = \frac{x^2}{2}
\]

\[
TR^{PM} = tQ
\]
Under the hypothesis that the social planner decides to not intervene in the market structure, our envisaged four-stage game unfolds as follows:

- At the 1st stage, both unions simultaneously and independently decide whether to collude or to compete at the stage of w-negotiations with their firms.
- At the 2nd stage, firms simultaneously and independently determine the optimal level of their R&D investments, by evaluating on the one hand the increase of their revenues, through increasing their product demand because of their product quality improvement, and on the other hand the cost of these investments.
- At the 3rd stage, if (at the first stage) one or both unions have independently decided to play collusively, they simultaneously and independently set their wages for their own firms, so that each maximize its joint member rents, or maximize its own member rents. If, however, both unions have (at the first stage) independently decided to behave competitively, they both set their own wages in order for each to maximize its own member rents.
- At the 4th stage, each firm simultaneously and independently competes with its rival by adjusting its own quantities, in order to maximize their own profits.

Assuming that the social planner decides to intervene in the market structure by proceeding to quality improvement-R&D by providing the know-how to the industry for free, our envisaged four-stage game unfolds as follows:

- At the 1st stage, the social planner determines the optimal level in terms of Social Welfare of R&D investments and indirect taxation on industry products.
- At the 2nd stage, both unions simultaneously and independently decide whether to collude or to compete in the stage of w-negotiations with their firms.
The 3rd stage and the 4th stage of the present game remain the same with the previously proposed one.

3. Conditions for Union Collusive Play

In an infinitely repeated non-cooperative game setup, we demonstrate the market conditions under which collusion among unions is sustainable in equilibrium, i.e. no union has incentive to unilaterally deviate. We assume trigger strategies on the part of each union. Therefore, collusion is accompanied with threats on the part of each union of punishment, if its rival’s proceed to a single deviation from collusion. The punishment is simply a permanent deviated action, which eventually leads the unions to competitive play for the rest of the periods.

In this section, we set the conditions which have to be satisfied in order to achieve sustainability of union collusive play. The formula of conditions depends on the credibility of union threats in case of a single deviation from collusion. The credibility of union threats is evaluated by their profitability, i.e. if a union’s action of punishment is eventually profitable or not. Thus, union i’s threat is credible, if and only if the following inequality is satisfied:

\[
\frac{\delta}{1-\delta}u_{i,m}^N > \frac{1}{1-\delta}u_{i,dj}^N
\]  

(10)

The first part of inequality presents the discounted union i’s utility when punishing its rival’s single deviation from collusive play, while the second part shows the discounted union i’s utility when it does not punish.

Simplifying the above inequality, we get:

\[
N_i = u_{i,m}^N - u_{i,dj}^N > 0
\]  

(11)

Where, the sign of \( N_i \) denotes the credibility of union i’s threat. Thus, union i’s threat is credible if \( N_i \geq 0 \) and lacks credibility if \( N_i < 0 \).
Taking into consideration the unions’ threat credibility, collusion among unions is a sub game perfect equilibrium configuration in the industry; if and only if unions gain from collusive play more than from deviation by playing competitively.

According to the above, we present three possible cases from which the evaluation of the unions’ threat credibility arises:

A. When both unions’ threat is credible, i.e. \( \{N_i > 0\} \), union collusion is sustained in equilibrium if the following condition is satisfied from both unions:

\[
\frac{u_{ic}^N}{1 - \delta} > u_{idi}^N - u_{dim}^N \left( \frac{\delta}{1 - \delta} \right)
\]

B. When both unions’ threat lacks credibility, i.e. \( \{N_i < 0\} \), union collusion is sustained in equilibrium if the following condition is satisfied from both unions:

\[
\frac{u_{ic}^N}{1 - \delta} > u_{idi}^N \left( \frac{1}{1 - \delta} \right)
\]

C. When union \( i \)’s threat is credible, while its rival’s is not, i.e. \( \{N_i > 0\}, \{N_f < 0\} \), union collusion is sustained in equilibrium if the following conditions are satisfied:

\[
\frac{u_{ic}^N}{1 - \delta} > u_{idi}^N \left( \frac{1}{1 - \delta} \right)
\]

\[
\frac{u_{jc}^N}{1 - \delta} > u_{jdj}^N - u_{jm}^N \left( \frac{\delta}{1 - \delta} \right)
\]

The above findings are summarized in the following Proposition.
**Proposition 1:** Collusion among unions is a sub game perfect equilibrium configuration in the industry, if and only if one of the following conditions is satisfied:

**Condition A:**

\[
\begin{align*}
&\text{If } \begin{cases} N_i = u_{im}^N - u_{idj}^N > 0 \\ N_j = u_{jm}^N - u_{jdi}^N > 0 \end{cases}, \\
&\quad \text{then } \begin{cases} K_i = u_{ic}^N \left( \frac{1}{1-\delta} \right) - u_{id}^N - u_{im}^N \left( \frac{\delta}{1-\delta} \right) > 0 \\ K_j = u_{jc}^N \left( \frac{1}{1-\delta} \right) - u_{jd}^N - u_{jm}^N \left( \frac{\delta}{1-\delta} \right) > 0 \end{cases}
\end{align*}
\]

**Condition B:**

\[
\begin{align*}
&\text{If } \begin{cases} N_i = u_{im}^N - u_{idj}^N < 0 \\ N_j = u_{jm}^N - u_{jdi}^N < 0 \end{cases}, \\
&\quad \text{then } \begin{cases} K_i = u_{ic}^N - u_{id}^N > 0 \\ K_j = u_{jc}^N - u_{jd}^N > 0 \end{cases}
\end{align*}
\]

**Condition C:**

\[
\begin{align*}
&\text{If } \begin{cases} N_i = u_{im}^N - u_{idj}^N > 0 \\ N_j = u_{jm}^N - u_{jdi}^N < 0 \end{cases}, \\
&\quad \text{then } \begin{cases} K_i = u_{ic}^N - u_{id}^N > 0 \\ K_j = u_{jc}^N \left( \frac{1}{1-\delta} \right) - u_{jd}^N - u_{jm}^N \left( \frac{\delta}{1-\delta} \right) > 0 \end{cases}
\end{align*}
\]

We can generalize the conditions for union collusive play with the conditions of any combination of union strategies, by combining the information contained in Proposition 1 and the matrix game that unions deal with at the first stage of the game, as presented in Table 1. Given union’ symmetry, our findings are summarized in Proposition 2.

<table>
<thead>
<tr>
<th>Union i</th>
<th>Collusion</th>
<th>Competition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( u_{ic}, u_{jc} )</td>
<td>( u_{id}, u_{jd} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Union j</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collusion</td>
</tr>
<tr>
<td>Competition</td>
</tr>
</tbody>
</table>

*Table 1: The Matrix Game that unions deal with at the first stage of the game.*
Proposition 2:

(i) Collusion among unions is in a sub game perfect equilibrium, if and only if $K_{i(j)} > 0$ is applied.

(ii) Union competitive play is in a sub game perfect equilibrium, if and only if $N_{i(j)} > 0$ is applied.

(iii) Union Mix of Strategy configuration is in a sub game perfect equilibrium, if and only if $K_{i(j)} < 0$ and $N_{i(j)} < 0$ is applied.

[Proof: See Appendix (A.1)]

The information contained in Proposition 2 can be arranged in a more informative Table, as follows:

<table>
<thead>
<tr>
<th>$N_{i(j)} &gt; 0$</th>
<th>$K_{i(j)} &gt; 0$</th>
<th>Nash Equilibria: Collusion and Competition</th>
<th>Nash Equilibrium: Competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{i(j)} &lt; 0$</td>
<td>$K_{i(j)} &lt; 0$</td>
<td>Nash Equilibrium: Collusion</td>
<td>Nash Equilibrium: Mix of Strategies</td>
</tr>
</tbody>
</table>

Table 2: Conditions, under which the various union strategy combinations, emerge in Nash Equilibrium.

4. Absence of policy maker in market structure (a)

In this section, we determine the equilibrium of the proposed game by using backwards induction, like in standard game-theoretic analysis. Thus, we propose a candidate equilibrium and subsequently validate (or reject) it, by checking for all possible unilateral deviations on the part of the agent(s) who consider such a deviation. Due to symmetry, in our model three candidate equilibria arise, at the first stage of the game: In Subsection 4.1 we propose as candidate equilibrium the union
collusive play in w-negotiations with their firms (e.g., unions independently set the 
wages that maximize the joint rents) and the possible deviation, on the part of any 
union, is to adjust its own wages in order to maximize its own rents given that the 
other union sticks to collusive play. In Subsection 4.2, we propose as candidate 
equilibrium the union competitive play and the possible deviation, on the part of any 
union, is to set its own wages in order to maximize the joint rents, given that the other 
union still behaves competitively. In Subsection 4.3, the proposed candidate 
equilibrium is the one where one union acts collusively by maximizing the joint rents, 
while its rival union behaves as a competitor, and the possible deviations arise by 
unilaterally switching each union’s strategy to its rival’s one.

4.1. Competitive Play \((m)\)

Take as given that the unions’ strategy at the first stage of the game is to behave 
competitively, i.e. to maximize its own rents by independently setting its wages.

At the last stage of the game both firms independently determine their optimal 
output level. Hence, according to (2) and (3), the firm’s \(i\)’s objective is:

\[
\max_{q_i} \left\{ \Pi_i \left\{ q_i \left( 1 - \gamma q_j - w_i + h x_i \right) - \frac{x_i^2}{2} \right\} \right\}
\]

\(\text{(16)}\)

The first order condition \((f.o.c.)\) in (16) provides the reaction function of firm \(i\):

\[
R_i(q_j) = \left( 1 - \gamma q_j - w_i + h x_i \right)/2
\]

\(\text{(17)}\)

Solving the system of both firms’ reaction functions, we get the optimal 
output/employment rules in the candidate equilibrium:

\[
q_i(w_i, w_j) = \frac{2 - \gamma - 2w_i + \gamma w_j + 2hx_i - \gamma hx_j}{(2 - \gamma)(2 + \gamma)}
\]

\(\text{(18)}\)
Notice that firm $i$’s output level is negatively affected by union $i$’s wage rate and rival firm’s R&D investment but it is positively affected by union $j$’s wage rates and its R&D investment.

At the third stage, unions independently determine the wage ($w_i$) that maximizes its rents [given by (4)], taking as given the outcomes of the production game [given in (38)].

$$\max_{w_i} [u_i = w_i q_i] \quad (19)$$

Getting the f.o.c.s we get the unions’ $i \neq j = 1, 2$, wage reaction functions which are as follows:

$$w_i(w_{jm}) = (2 - \gamma + \gamma w_{jm} + 2hx_{im} - \gamma hx_{jm})/4 \quad (20)$$

Observe that wages are strategic complements for the unions, since: $d w_i/d w_j = \gamma / 4 > 0 \ \forall \ \gamma \in (0, 1)$.

The wage outcome ($\omega$) in the candidate equilibrium is derived by solving the system in (20) and is presented by the following equation:

$$w^*_i = \frac{(2 - \gamma)(4 + \gamma) + hx_i(8 - \gamma^2) - 2\gamma hx_j}{(4 - \gamma)(4 + \gamma)} \quad (21)$$

Taking now as given the optimal output/employment rules and the equilibrium wages in (18) and (21), respectively, at the second stage firms simultaneously and independently determine the optimal level of their R&D investment. Thus, substituting (3) for (18) and (21), we get firms’ maximization objective as follows:

$$\max_{x_i} \left[ \Pi_i \left( q_{im} \left( 1 - \gamma q_{jm} - w_{im} + hx_{im} - \frac{x_{im}^2}{2} \right) \right) \right] \quad (22)$$

The f.o.c.s of (32) provides the reaction function of firm $i$ to investments on R&D:

$$\frac{R&D}{im} (x_{jm}) = A_1 \left( 8h(8 - \gamma^2) \left( (4 + \gamma)(2 - \gamma) - 2\gamma hx_{jm} \right) \right) \quad (23)$$
Where $A_1 = \left( (4 + \gamma)(2 - \gamma)(4 - \gamma)^2(2 + \gamma)^2 - 8h^2(8 - \gamma^2) \right)^{-1} > 0 \ \forall \gamma, h \in (0, 1)$.

We derive the (candidate) equilibrium R&D investments by solving the system of f.o.c.s of that maximization, as follows:

$$x_{im}^* = A_1 \left( 8h(8 - \gamma^2) \right) \quad (24)$$

Substituting now (24) and (21) for (18), we get the firms’ output/employment levels in the candidate equilibrium:

$$q_{im}^* = A_1 \left( 2(2 - \gamma)(2 + \gamma)(4 - \gamma)(4 + \gamma) \right) \quad (25)$$

Moreover, we get that the firms’ profits in the candidate equilibrium:

$$\Pi_{im}^* = A_1^2 \left( 4((64 - 20\gamma^2 + \gamma^4)^2 - 8h^2(8 - \gamma^2)) \right) \quad (26)$$

The candidate equilibrium union wages and rents are then derived from Equations (21) and (4), respectively, as follows:

$$w_{im}^* = A_1 \left( (4 - \gamma)(4 + \gamma)(2 - \gamma)^2(2 + \gamma)^2 \right) \quad (27)$$

$$u_{im}^* = A_1^2 \left( 2(4 - \gamma)^2(4 + \gamma)^2(2 - \gamma)^3(2 + \gamma)^3 \right) \quad (28)$$

4.2. **Collusive Play (c)**

In this subsection, suppose that both unions independently choose to behave collusively at the third stage, by maximizing the sum of union rents when entering into negotiations about their wages with their own firms.

Therefore and according to the last stage of the game, in where firms’ Cournot Competition takes place, we get the firms’ reaction functions and the optimal output rules in the candidate equilibrium in Equations (17) and (18), respectively.
At the third stage of the game, given the optimal employment rule in (18), union
\(i\)’s objective is the selection of optimal \(w_i\) in order to maximize the sum of rents of its
members and the competitor firm’s union members:
\[
max_{w_i} \left[ u_i + u_j \left( = w_i q_i + w_j q_j \right) \right]
\] (29)
The union \(i\)’s wage reaction function is derived from the \(f.o.c\) of (29):
\[
w_{ic}(w_{jc}) = \left( 2 - \gamma + 2\gamma w_{jc} + 2hx_{ic} - \gamma hx_j \right) / 4
\] (30)
Solving now the system (30) we get the (candidate) equilibrium wages:
\[
w_{ic}^* = (1 + hx_{ic}) / 2
\] (31)
Now going on to the second stage, firms simultaneously and independently
determine the optimal level of their R&D investment and their objective is given by
substituting (3) for (18) and (31):
\[
max_{x_{ic}} \left[ \Pi_i \left( = q_{ic} \left( 1 - \gamma q_{jc} - w_{ic} + hx_{ic} \right) - \frac{x_{ic}^2}{2} \right) \right]
\] (32)
Taking the \(f.o.c\) of (32) we get the reaction function of firm \(i\) to investments in R&D:
\[
R_{ic}^{R&D}(x_{jc}) = \frac{h \left( 2 - \gamma \left( 1 + hx_j \right) \right)}{(2(2 - h) - \gamma^2)(2(2 + h) - \gamma^2)}
\] (33)
The system solving of (33) provides the (candidate) equilibrium R&D investments:
\[
x_{ic}^* = \frac{h}{(2 - \gamma)(2 + \gamma)^2 - h^2}
\] (34)
Taking into consideration Equations (31), (34) and (18), firms’ output/employment
levels and profits in the candidate equilibrium are derived:
\[
q_{ic}^* = \frac{(2 - \gamma)(2 + \gamma)}{2((2 - \gamma)(2 + \gamma)^2 - h^2)}
\] (35)
\[
\Pi_{ic}^* = \frac{(2 - \gamma)^2(2 + \gamma)^2 - 2h^2}{4((2 - \gamma)(2 + \gamma)^2 - h^2)^2}
\] (36)
From Equations (31) and (4) the candidate equilibrium union wages and rents, respectively, emerge as follows:

\[ w^*_u = \frac{(2 - \gamma)(2 + \gamma)^2}{2((2 - \gamma)(2 + \gamma)^2 - h^2)} \]  
\[ u^*_u = \frac{(2 - \gamma)^2(2 + \gamma)^3}{4((2 - \gamma)(2 + \gamma)^2 - h^2)^2} \]  

4.3. Mix of Strategies \((d_j)\)

Allow now for the candidate equilibrium to be the union’ mix of strategies, in where one union (let union \(j\)) adjusts its own wage competitively, while its rival’s (let union \(i\)’s) strategy is to adjust its own wages collusively.

Thus, at the last stage of the game in where firms’ Cournot competition takes place, we get the optimal output/employment rules in the candidate equilibrium in (18).

According to this Mix of Strategies configuration, at the third stage, the union \(i\)’s and union \(j\)’s reaction functions are presented by Equations (30) and (21), respectively. Solving now the system of union reaction functions, we get the following optimal wages in the candidate equilibrium:

\[ w^*_{idj} = \frac{(2 - \gamma)(2 + \gamma)(1 + hx_i)}{8 - \gamma^2} \]  
\[ w^*_{jdj} = \frac{(2 - \gamma)(4 + \gamma) + hx_j(8 - \gamma^2) - 2\gamma hx_i}{2(8 - \gamma^2)} \]  

At the second stage, firms simultaneously and independently determine the level of their R&D investment that maximizes their own profits [by (3)], given the optimal output and wages rules in candidate equilibrium [in (18) and (39), (40), respectively].
The system solving of f.o.c.s of firm’s profit maximization provides the (candidate) equilibrium R&D investments:

\[ x_{idj}^* = 2A_2(h((2 + \gamma)(8 - \gamma^2)(2 - \gamma^2) - 8h^2)) \] (41)

\[ x_{jdj}^* = 2A_2(2h((2 + \gamma)(4 + \gamma)(2 - \gamma^2) - 4h^2)) \] (42)

Where \( A_2 = \left(2\left((8 - \gamma^2)(4 - \gamma^2)^3 + 4h^2(32 - 12\gamma^2 + \gamma^4) - 8h^4\right)\right)^{-1} > 0 \)

\( \forall \gamma, h \in (0,1) \).

We obtain now the firm-specific output/employment levels and the profits in the candidate equilibrium [by substituting (39), (40), (41) and (42) for (18) and (3)]:

\[ q_{idj}^* = A_2((4 - \gamma^2)((2 + \gamma)(8 - \gamma^2)(2 - \gamma^2) - 8h^2)) \] (43)

\[ q_{jdj}^* = 2A_2((4 - \gamma^2)((2 + \gamma)(4 + \gamma)(2 - \gamma^2) - 4h^2)) \] (44)

\[ \Pi_{idj}^* = A_2^2 \left(\left((4 - \gamma^2) - 2h^2\right)((2 + \gamma)(8 - \gamma^2)(2 - \gamma^2) - 8h^2)\right)^2 \] (45)

\[ \Pi_{jdj}^* = 4A_2^2 \left(\left((4 - \gamma^2) - 2h^2\right)((2 + \gamma)(4 + \gamma)(2 - \gamma^2) - 4h^2)\right)^2 \] (46)

The candidate equilibrium union wages and rents formulae are the following [given in (39), (40) and (4)]:

\[ w_{idj}^* = 2A_2((2 - \gamma^2)(2 + \gamma)^3((2 + \gamma)(2 - \gamma^2) - h^2)) \] (47)

\[ w_{jdj}^* = A_2((2 - \gamma)^2(2 + \gamma)^2((2 + \gamma)(4 + \gamma)(2 - \gamma^2) - 4h^2)) \] (48)

\[ u_{idj}^* = 2A_2^2((2 - \gamma)^3(2 + \gamma)^4((2 + \gamma)(2 - \gamma^2) - h^2)((2 + \gamma)(2 - \gamma^2)(8 - \gamma^2) - 8h^2)) \] (49)

\[ u_{jdj}^* = 2A_2^2((4 - \gamma^2)^3((2 + \gamma)(4 + \gamma)(2 - \gamma^2) - 4h^2)^2) \] (50)
4.4. Equilibrium Analysis: Endogenous Selection of Union Structures

At the first stage of the proposed game, unions simultaneously and independently decide whether to collude or to compete in the stage of w-negotiations with their specific firm (3rd stage). Given our findings in subsections 4.1 – 4.3, unions deal with the following matrix game, in which the payoffs of each union under each pair of union decisions are presented:

<table>
<thead>
<tr>
<th></th>
<th>Union $i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Collusion</td>
</tr>
<tr>
<td>Union $j$</td>
<td>${u_{ic}, u_{jc}}$</td>
</tr>
<tr>
<td>Competition</td>
<td>${u_{id_i}, u_{id_j}}$</td>
</tr>
</tbody>
</table>

Table 3: The Matrix Game that unions deal with at the first stage of the game.

Due to symmetry, the following is applied $(u_{id_i}', u_{id_j}') = (u_{id_i}, u_{id_j})$ and thus the number of candidate equilibria is reduced to three.

According to conventional wisdom in one-shot games, decentralized wage-setting regime emerges in equilibrium, but centralized wage-setting regime brings about a Pareto improving solution for the unions, as it increases the utility of all players (unions). In our dynamic framework, it is reasonable that decentralized wage-setting regime is sustained in equilibrium, as it is in one-shot games, but we further investigate if there is multiplicity of equilibria. In particular, we further investigate the union incentives to play collusively by setting wages in the stage of w-negotiations that maximize the sum of rents of both union members.

It is proved that union incentives to stay in a centralized wage-setting institution is affected positively by firms’ product substitutability and negatively by consumer
evaluation of the product quality. The explanation is simple, as market products tend to be perfect substitutes ($\gamma \to 1$), the total labor demand decreases; the products are targeted at exactly the same market and consequently their total output level is lower. On other hand, as factor $h$ increases ($h \to 1$), the total labor demand increases; firms’ R&D investments are higher, thus consumer demand (market output level) is also higher. Summarizing the above, the higher $\gamma$ is and the lower $h$ is, the lower is the union’s employment, hence the lower are the gains from deviation from union collusive play. Consequently, a stronger discount rate to deter unions deviation from collusion is needed. Our relevant findings are presented in Proposition 3.

**Proposition 3:**

*Under the assumption of the absence of policy maker in market structure, union competitive play in w-negotiations with their specific firm is the only Nash Equilibrium, if product substitutability or discount rates are not sufficiently high, i.e., if $\gamma < \gamma_a(h) (< 0.652)$ or $\delta < \delta_a(h)$. Otherwise, union collusion and competition are both emerging in equilibrium. The Mix of Strategies are never sustained in equilibrium.*

[Proof: See Appendix (A.2)]

In *Figure 1* below, notice that as the products become less close substitutes and consumer evaluation of product quality from R&D increases, collusion among unions becomes more unstable i.e. $\partial \delta_a / \partial \gamma < 0$ and $\partial \delta_a / \partial h > 0$, respectively.
Observe that $\delta_a(\gamma, h) \in (0,1) \forall \gamma < \gamma_a(h)$ and the $\gamma_a(h)$ critical values are depicted below:

![Diagram of Nash Equilibrium: Unions' collusion and competition](image)

**Figure 1:** The $\delta(\gamma, h)$ condition under which union collusion emerges in equilibrium.

Now, we may reasonably make use of the criterion of Pareto optimality as regards union rents, in order to narrow down this multiplicity of equilibria, if possible. According to this criterion, we may select that (those) sub game perfect Nash equilibrium (equilibria) where both unions are better off, by each achieving higher rents in comparison to the remainder ones. Our refined findings are summarized in the following Proposition:

---

6 The mathematical expression of $\gamma_1(h)$ is left out because of its wide extent. They are available by the authors upon request.
Proposition 4: Under the assumption of the absence of policy maker in market structure, union collusive play in w-negotiations is the unique Pareto Optimal Nash equilibrium, as regards rents, if product substitutability and discount rate is sufficiently high, i.e., if $\gamma > \gamma_a(h)(< 0.652)$ and $\delta > \delta_a(h)$.

Otherwise, union competitive play is the unique Nash equilibrium.

[Proof: See Appendix (A.3)]

Figure 3: The $\gamma(h)$ and $\delta(\gamma)$ conditions under which each union's strategy is unique Pareto Optimal Nash equilibrium, as regards rents.

4.5. Welfare Analysis

This section investigates the conditions of product substitutability and discount rate under which equilibrium outcomes, market participant surpluses and Social Welfare are promoted.

Regarding market equilibrium outcomes, i.e. equilibrium output rule, product quality improvement and wage rate under decentralized and the centralized wage-setting regimes [given in (25), (24), (27) and (35), (34), (37), respectively], we get that [given in Proposition 4]:

[21]
Where $A_i = ((4 + \gamma)(2 - \gamma)(4 - \gamma)^2(2 + \gamma)^2 - 8h^2(8 - \gamma^2))^{-1} > 0 \forall \gamma, h \in (0,1)$.  

Our findings from the comparative evaluation of market equilibrium outcomes, i.e. firms’ output level, product quality improvement and union wage rate [given in (51), (52) and (53)] are summarized in Proposition 5.

**Proposition 5:** The firms’ output level, hence union employment, and product quality improvement, hence R&D investments, are always higher under sufficiently low product substitutability or/and discount rate, i.e. if $\gamma < y^a(h)(< 0.652)$ or/and $\delta < \delta^a(h)$, in where decentralized wage-setting regime emerges in equilibrium, i.e. $q_{im} > q_{ic}$ and $x_{im} > x_{ic}$.  

The opposite applies for union wages, i.e. $w_{ic} > w_{im}$.  

[Proof: See Appendix (A.4)]

Regarding now Social Welfare, it is defined to be the sum of participant surpluses, i.e. Consumer Surplus (CS), Producer Surplus (PS) and Union Rents (UR), as follows:

$$SW_s = CS_s + PS_s + UR_s; \ s = c, m$$ (54)

Where $c$ and $m$ denote collusive and competitive equilibria, respectively. The elements of (54) are calculated by the followings equations:
Taking into consideration Proposition 4, equilibrium output rule, product quality improvement and wage rate [given in (51), (52) and (53)], we get that Consumer Surplus (CS), Producer Surplus (PS) and Union Rents (UR) are in equilibrium, as presented below [given in (55), (56) and (57), respectively]:

Consumer Surplus (CS) is

\[ CS = \left\{ \begin{array}{ll}
4 A_1 (1 + \gamma)(4 - \gamma)^2(16 - \gamma^2)^2, & \forall \delta \in (0, \delta_a) or/and \gamma \in (0, \gamma_a) \\
(1 + \gamma)(4 - \gamma)^2 & 4((2 - \gamma)(2 + \gamma)^2 - h^2)^2, \forall \delta \in (\delta_a, 1) and \gamma \in (\gamma_a, 1)
\end{array} \right. \]

Producer Surplus (PS) is

\[ PS = \left\{ \begin{array}{ll}
8 A_1((4 - \gamma)^2(16 - \gamma^2)^2 - 8h^2(8 - \gamma^2)^2), & \forall \delta \in (0, \delta_a) or/and \gamma \in (0, \gamma_a) \\
(4 - \gamma)^2 - 2h^2 & 2((2 - \gamma)(2 + \gamma)^2 - h^2)^2, \forall \delta \in (\delta_a, 1) and \gamma \in (\gamma_a, 1)
\end{array} \right. \]

Union Rents (UR) is

\[ UR = \left\{ \begin{array}{ll}
4 A_1(4 + \gamma)^2(4 - \gamma)^2(4 - \gamma^2)^3, & \forall \delta \in (0, \delta_a) or/and \gamma \in (0, \gamma_a) \\
(4 - \gamma)^2 & 2((2 - \gamma)(2 + \gamma)^2 - h^2)^2, \forall \delta \in (\delta_a, 1) and \gamma \in (\gamma_a, 1)
\end{array} \right. \]

Substituting now (65), (66) and (67) for (61), we get Social Welfare in equilibrium, as it is presented by the following equation:

Social Welfare (SW) is

\[ SW = \left\{ \begin{array}{ll}
4 A_2 A_3^2, & \forall \delta \in (0, \delta_a) or/and \gamma \in (0, \gamma_a) \\
(7 + 3\gamma)(4 - \gamma)^2 - 4h^2 & 4((2 - \gamma)(2 + \gamma)^2 - h^2)^2, \forall \delta \in (\delta_a, 1) and \gamma \in (\gamma_a, 1)
\end{array} \right. \]

Where \( A_3 = (7 + \gamma(1 - \gamma))(64 - 20\gamma^2 + \gamma^4)^2 - 16h^2(8 - \gamma^2)^2 > 0 \) \( \forall \gamma, h \in (0,1) \).
Our findings from the comparative evaluation of market participant surpluses [given in (58), (59) and (60)] and Social Welfare [given in (61)], are summarized in the following Propositions.

**Proposition 6:** Consumer and Producer Surpluses are always higher under sufficiently low product substitutability or/and discount rate, i.e. if $\gamma < \gamma_a(h)(< 0.652)$ or/and $\delta < \delta_a(h)$, where union decentralization of wage bargaining emerges in equilibrium, i.e. $C_{S_m} > C_{S_c}$ and $P_{S_m} > P_{S_c}$

[Proof: See Appendix (A.5)]

**Proposition 7:** Pareto Optimal Union Rents emerge in equilibrium, if and only if $\gamma < \gamma_a(h)(< 0.652)$ or/and $\delta > \delta_a(h)$. Otherwise, unions deal with the well-known paradox of Prisoners’ Dilemma, where the Pareto Optimal solution (centralized bargaining, in this instance) does not emerge in Nash Equilibrium.

[Proof: See Appendix (A.6)]

**Proposition 8:** Social Welfare is always higher under sufficiently low product substitutability or/and discount rate, i.e. if $\gamma < \gamma_a(h)(< 0.652)$ or/and $\delta < \delta_a(h)$, where union decentralized firm-based bargaining system emerges in equilibrium, i.e. $SW_{m} > SW_{c}$.

[Proof: See Appendix (A.7)]

**5. Policy maker proceeds to R&D investments (b)**

Suppose now that the social planner proceeds to R&D – quality improvement exclusively, as a public good, by providing the know-how to the industry for free. We also assume that the R&D investments are financed by indirect taxation on firms’
products and the balance budget condition applies in the industry. Therefore, firm $i$’s profit function is simplified and defined by:

$$\Pi_i = (p_i - w_i)q_i$$  \hspace{1cm} (62)

In the following Subsections 5.1.-5.3., we propose all the candidate equilibria and then check their validation (or rejection) by analyzing all possible unilateral deviations on the part of the agent(s).

### 5.1 Competitive Play ($m$)

Under the assumption that unions behave competitively at the stage of $w$-bargaining with their firms, by setting independently their wages, at the last stage firms independently set the output level that maximizes their profits. Taking into consideration (8) and (62), firm $i$’s new objective is:

$$\max_{q_i} [\Pi_i = q_i(1 - \gamma q_j - w_i + hx - t)]$$  \hspace{1cm} (63)

Where $x$ denotes the quality of products from social planner’s R&D investments. The f.o.c. of (63) provides firm $i$’s reaction function, as presented by the following equation:

$$R_i(q_j) = \frac{(1 - \gamma q_j - w_i + hx - t)}{2}$$  \hspace{1cm} (64)

Solving the system of both firms’ reaction functions [given in (64)], the optimal output/employment rules in the candidate equilibrium is derived:

$$q_i(w_i, w_j) = \frac{(2 - \gamma)(1 + hx - t) - 2w_i + \gamma w_j}{(2 - \gamma)(2 + \gamma)}$$  \hspace{1cm} (65)

At the third stage, unions independently decide the optimal wage rate ($w_i$). Getting now the f.o.c.s of their revenues’ objective in (19), given the optimal outcome in (65), we take the wage reaction functions:

$$w_{im}(w_{jm}) = \left(\frac{(2 - \gamma)(1 + hx - t) + \gamma w_{jm}}{4}\right)$$  \hspace{1cm} (66)
Thus, the optimal wage rate is derived by solving the system of (66) and is presented below:

\[ w_{lm}^* = \frac{(2 - \gamma)(1 + hx - t)}{4 - \gamma} \]  

(67)

Observe that \( w_{lm}^* > 0 \), if and only if \( 1 + hx - t > 0 \). So we conclude that the proposed oligopoly market structure exists, if and only if the following condition is satisfied:

\[ t < 1 + hx \]  

(68)

We get union \( i \)'s rent and optimal output/employment level in the candidate equilibrium by substituting (67) for (4) and (65), respectively:

\[ u_{lm}^* = \frac{2(2 - \gamma)(2 + hx - t)^2}{(2 + \gamma)(4 - \gamma)^2} \]  

(69)

\[ q_{im}^* = \frac{2(1 + hx - t)}{(2 + \gamma)(4 - \gamma)} \]  

(70)

5.2 Collusive Play (c)

In this section our candidate equilibrium is the union collusive play at the stage of w-negotiations, i.e. they set the wage rate that maximizes the joint rents of its members and the competitor firm’s union members.

At the last stage of the game, firms’ competitive behavior increase their reaction functions and optimal output level, hence employment, as presented in (64) and (65), respectively.

At the third stage, unions enter into w-negotiation with their specific firm and their wage reaction function [by taking the f.o.c. of union \( i \)'s objective function in (29), given the optimal output rules in (65)], is as follows:

\[ w_{tc}(w_{je}) = \left( (2 - \gamma)(1 + hx - t) + 2\gamma w_{je} \right)/4 \]  

(71)
Solving now the system in (71) we get the candidate equilibrium wages:

\[ w_{ic}^* = \frac{(1 + hx - t)}{2} \]  (72)

Substituting now (72) for (4) and (65), we get the union \( i \)’ rent and its employment, hence market outcome, in the candidate equilibrium:

\[ u_{ic}^* = \frac{(1 + hx - t)^2}{4(2 + \gamma)} \]  (73)

\[ q_{ic}^* = \frac{1 + hx - t}{2(2 + \gamma)} \]  (74)

### 5.3 Mix of Strategies \((d_j)\)

We propose the union mix of strategies as candidate equilibrium in this subsection. Let union \( j \) be the one which plays competitively and union \( i \) be the one which plays collusively.

Therefore, at the last stage of the game, where firms’ Cournot completion takes place, we get the optimal output/employment in (65). Thus at the third stage of \( w \)-negotiations, we get the optimal wages by solving the system of union reaction function in (71) and (66), respectively, as follows:

\[ w_{id}^* = \frac{(2 - \gamma)(2 + \gamma)(1 + hx - t)}{8 - \gamma^2} \]  (75)

\[ w_{jd}^* = \frac{(2 - \gamma)(4 + \gamma)(1 + hx - t)}{2(8 - \gamma^2)} \]  (76)

Consequently, the union rents and employment level in the candidate equilibrium are derived by substituting (75) and (76) for (4) and (65):

\[ u_{id}^* = \frac{(2 - \gamma)(1 + hx - t)^2}{2(8 - \gamma^2)} \]  (77)

\[ u_{jd}^* = \frac{(2 - \gamma)(4 + \gamma)^2(1 + hx - t)^2}{2(2 + \gamma)(8 - \gamma^2)^2} \]  (78)
Second Stage: Endogenous Selection of Union Structures

In this stage of the game, unions simultaneously and independently decide on the strategy which follows in the next stage where they enter into negotiations with their specific firm over their wages, i.e. centralized or decentralized w-negotiations. The union matrix game is presented by Table 3 in Subsection 4.4., with the exception that the union readjusted payoffs are given by Subsections 5.1-5.3.

Taking the same path with the endogenous selection of union structures of the previous market’s structure, it is applied that the market product substitutability affects negatively the union incentives for collusive play at the stage of w-negotiations with their specific firm. As market products tend to be perfectly substituted ($\gamma \rightarrow 1$), the lower the union employment is, consequently the lower the gains from deviation from union collusive play are. Therefore, only a strong discount rate is able to deter union deviations from collusive play in w-negotiations. Our relevant findings are summarized in Proposition 9.

**Proposition 9:** Under the presence of a policy maker that proceeds to R&D investments, union competitive play in w-negotiations with their specific firm is the only Nash Equilibrium, if discount rates are not sufficiently high, i.e., $\delta < \delta_b(\gamma)$. Otherwise, union collusion and competition are both emerging in equilibrium. The Mix of Strategies is never sustained in equilibrium.

[Proof: See Appendix (A.4)]

\[
q_{id_j} = \frac{1 + hx - t}{2(2 + \gamma)} \tag{79}
\]

\[
q_{jd_j} = \frac{(4 + \gamma)(1 + hx - t)}{(2 + \gamma)(8 - \gamma^2)} \tag{80}
\]
Where $\delta_b(\gamma) = \frac{(4-\gamma)^2(4-\gamma(2+))}{2(2-\gamma)(32-3\gamma^2)} \in (0.155, 0.5) \forall \gamma \in (0,1)$ and its plot is presented in Figure 4. Notice that as the products become less close substitutes, collusion among unions becomes more unstable i.e. $\partial \delta_b/\partial \gamma < 0$.

**Figure 4:** The $\delta(\gamma)$ conditions under which collusion among unions emerge in equilibrium.

Observe that the game leads to multiplicity of equilibria when $\delta > \delta_b(\gamma)$. It is able to narrow down this multiplicity by inserting the criterion of Pareto optimality as regards union rents. Our refined findings are summarized in Proposition 10.

**Proposition 10:** Under the presence of a policy maker that proceeds to R&D investments, union collusive play in w-negotiations is the unique Pareto Optimal Nash equilibrium, as regards rents, if the discount rate is sufficiently high, i.e., if $\delta > \delta_b(\gamma)$. Otherwise, union competitive play is the unique Nash equilibrium.

[Proof: See Appendix (A.5)]
5.5 First Stage: R&D investments and product taxation

At the first stage of the proposed game, the social planner sets the optimal level of R&D investments (Cost) and indirect taxation (Revenue) in the industry in order to maximize Social Welfare.

In the present market structure, Social Welfare is defined as the sum of total Consumer Surplus (CS), total Producer Profits (PS), total Union Rents (UR) and total revenues from indirect taxation on market products ($TR^{PM}$), minus total expenditures on R&D for product quality improvement ($TC^{PM}$):

$$SW = CS + PS + UR + TR^{PM} - TC^{PM}$$  \hspace{1cm} (81)

Under the assumption of a Balanced Budget, i.e. the R&D investments are exclusively covered from indirect taxation on the final product, we obtain that:

$$TC^{PM}_1 = TR^{PM}_1$$  \hspace{1cm} (82)

Substituting now (82) for (81), we get that the refined social planner’s objective is the following:

$$SW = CS + PS + UR$$  \hspace{1cm} (83)

Where, elements of (83) are calculated by the followings equations:

$$CS = (1 + hx_i)q_i + (1 + hx_j)q_j - \frac{1}{2}(q_i^2 + q_j^2 + 2\gamma q_i q_j) - (p_i + t)q_i - (p_j + t)q_j$$  \hspace{1cm} (84)

$$PS = \sum_{k=i,j} ((p_k - w_k)q_k)$$  \hspace{1cm} (85)

$$UR = \sum_{k=i,j} (w_k q_k)$$  \hspace{1cm} (86)

The problem of the social planner is maximization of Social Welfare by improving the product quality of the industry. This implies R&D investments and indirect taxation on the market products, in order to finance those investments. Therefore, the social planner’s maximization problem is more complex, as investments in R&D have
two effects: a positive one from improving the product quality that improves Social Welfare and a negative one from imposing indirect taxes that create distortions in the market (increase the market price of the product) and lead to a reduction in Social Welfare. The optimal combination of taxation and R&D investments, in terms of market surpluses, is presented in the following Proposition.

**Proposition 11:**

(i) When \( \delta < \delta_b(h) \), then union competitive play emerges in equilibrium and the optimal levels of the Policy Maker’s R&D investments and taxation policy are

\[
x_a^* = \frac{4h}{(2+\gamma)(4-\gamma)-2h^2} \in (0, 0.67) \quad \text{and} \quad t_a^* = \frac{2h^2}{(2+\gamma)(4-\gamma)-2h^2} \in (0, 0.33),
\]

respectively.

(ii) When \( \delta > \delta_b(h) \), then union collusive play emerges in equilibrium and the optimal levels of the Policy Maker’s R&D investments and taxation policy are

\[
x_b^* = \frac{2h}{2(2+\gamma)-h^2} \in (0, 0.67) \quad \text{and} \quad t_b^* = \frac{h^2}{2(2+\gamma)-h^2} \in (0, 0.33), \quad \text{respectively.}
\]

[Proof: See Appendix (A.6)]

Observe that under a low discount rate \( [\delta < \delta_b(h)] \), hence under union competition, both product quality and taxation policy are higher, i.e. \((t_a^*, x_a^*) > (t_b^*, x_b^*)\):

\[
x_a^* - x_b^* = \frac{2\gamma h(2+\gamma)}{(2(2+\gamma)-h^2)((2+\gamma)(4-\gamma)-2h^2)} > 0 \quad (87)
\]

\[
t_a^* - t_b^* = \frac{\gamma h^2(2+\gamma)}{(2(2+\gamma)-h^2)((2+\gamma)(4-\gamma)-2h^2)} > 0 \quad (88)
\]

According to Proposition 11, under each level of discount rate the Social Welfare in equilibrium is defined by:
The optimal union $i$’s wages and firm $i$’s output level is derived by substituting the optimal levels of $(t^*, x^*)$ for (67) and (70), respectively, in the equilibrium:

$$w_i^* = \begin{cases} \frac{4 - \gamma^2}{(2 + \gamma)(4 - \gamma) - 2h^2}, & \forall \delta \in (0, \delta_b) \\ \frac{2 + \gamma}{2(2 + \gamma) - h^2}, & \forall \delta \in (\delta_b, 1) \end{cases}$$  \quad (90)

$$q_i^* = \begin{cases} \frac{2}{(2 + \gamma)(4 - \gamma) - 2h^2}, & \forall \delta \in (0, \delta_b) \\ \frac{1}{2(2 + \gamma) - h^2}, & \forall \delta \in (\delta_b, 1) \end{cases}$$  \quad (91)

Consumer Surplus (CS), Producer Surplus (PS) and Union Rents (UR), given in (84), (85) and (86), respectively, are presented below:

$$CS = \begin{cases} \frac{4(1 + \gamma)}{(2 + \gamma)(4 - \gamma) + 2h^2)^2}, & \forall \delta \in (0, \delta_b) \\ \frac{1 + \gamma}{2(2 + \gamma) - h^2)^2}, & \forall \delta \in (\delta_b, 1) \end{cases}$$  \quad (92)

$$PS = \begin{cases} \frac{8}{((2 + \gamma)(4 - \gamma) + 2h^2)^2}, & \forall \delta \in (0, \delta_b) \\ \frac{2}{2(2 + \gamma) - h^2)^2}, & \forall \delta \in (\delta_b, 1) \end{cases}$$  \quad (93)

$$UR = \begin{cases} \frac{4(2 - \gamma)(2 + \gamma)}{((2 + \gamma)(4 - \gamma) + 2h^2)^2}, & \forall \delta \in (0, \delta_b) \\ \frac{2(1 + \gamma)}{2(2 + \gamma) - h^2)^2}, & \forall \delta \in (\delta_b, 1) \end{cases}$$  \quad (94)
5.6. Welfare Analysis

Taking the same path as Subsection 4.5, in the present section we demonstrate the conditions of discount rate where equilibrium outcomes and market participant surpluses are promoted.

We proceed with the analysis of the Pareto Optimal Solution, through comparative evaluation, under decentralized and the centralized wage-setting regimes, of market equilibrium outcomes, i.e. equilibrium output rule, product quality improvement and wage rate [given in (91), Proposition 11 and (90), respectively], and market participant surpluses, i.e. Consumer Surplus, Producer Surplus and Union Rents [given by (92), (93) and (94), respectively]. Our findings are summarized in the following Propositions.

**Proposition 12:** The firms’ output level, hence union employment, and product quality improvement, hence R&D investments, are always higher under sufficiently lower discount rate, i.e. if \( \delta < \delta_b(h) \), where decentralized wage-setting regime emerges in equilibrium, i.e. \( q_{im} > q_{ic} \) and \( x_{im} > x_{ic} \).

The opposite applies for union wages, i.e. \( w_{ic} > w_{im} \).

[Proof: See Appendix (A.11)]

**Proposition 13:** Consumer and Producer Surpluses are always higher under sufficiently lower discount rates, i.e. if \( \delta < \delta_b(h) \), where union decentralization of wage bargaining emerges in equilibrium, i.e. \( CS_m > CS_c \) and \( PS_m > PS_c \).

[Proof: See Appendix (A.12)]
**Proposition 14:** Pareto Optimal Unions Rents emerge in equilibrium, if \( \delta < \delta_b(h) \) or \( \gamma < \gamma_b(h) \). 

*Decentralized Bargaining* \( \{ \delta > \delta_b(h) \} \) or \( \gamma > \gamma_b(h) \) \(* Centralized Bargaining* \) when 

\( US_c > US_m \ \forall \ \gamma > \gamma_b(h) \) is applied. Otherwise, unions deal with the well-known paradox of Prisoners’ Dilemma, where the Pareto Optimal solution does not emerge in Nash Equilibrium. 

[Proof: See Appendix (A.13)]

Where \( \gamma_b(h) > 0^7 \ \forall \ h \in (0,1) \) and is depicted below:

![Graph](image)

**Figure 5:** The \( \gamma(h) \) critical values for Union Rents comparison.

As consumer evaluation of the product quality increases, i.e. \( h \to 1 \), and product substitutability decreases, i.e. \( \gamma \to 0 \), affects negatively the \( (u_{ic} - u_{im}) \) differential. The negative impact on utility differential may be such that the union utility differential among competition and collusion can be reversed, i.e. \( u_{im} > u_{ic} \).

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7 The mathematical expression of \( \gamma_c(h) \) is left out because of its wide extent. It is available by the authors upon request.
6. Endogenous Selection of Market Structures

In this Section, we endogenize the market structure by inserting an extra stage on the top of the game, where the social planner decides whether to intervene or not in the market by undertaking R&D investments.

More specifically, the social planners’ selection of one of the two proposed market structures, as presented in Sections 4 – 5, depends on which provides higher Social Welfare. According to the above, the social planner’s maximization problem is presented below:

\[
\max_{SW} \{SW_a, SW_b\} \tag{95}
\]

Where the indexes (a) and (b) denote the market structure where R&D investments are undertaken by firms or the social planner, respectively.

Social Welfare is higher, when:

- Unions set their wage rate, in the stage of w-negotiation with their specific firm, competitively [under decentralized wage-setting regime \((m)\)], rather than collusively [decentralized wage-setting regime \((c)\)]. The explanation is that under market participant competition, the total output level is higher.

\[SW_m > SW_c\]

- The social planner undertakes R&D investments [economic policy \((b)\)], although this implies taxes that create distortions in the industry. The product of R&D, i.e. the know-how of product quality improvement, is a public good and each firm has free access to it. By setting quality improvement as a public good, the market succeeds to save economic resources by investing in R&D in order to improve the quality of all industry products at once, in comparison to the market structure where
R&D is a private good and each firm invests in R&D in order to improve only its own product’s quality.

\[ SW_b > SW_a \]

According to the above results, we conclude that the decentralized wage-setting regime and economic policy \((b)\) is superior as regards Social Welfare.

Taking into consideration now the equilibrium analysis results in Proposition 4 and Proposition 10 and the Social Welfare outcomes in (61) and (89), the social planner demonstrates the optimal market structure by using comparative analysis. Our findings are summarized in the following Propositions:

**Proposition 15:** The market structure that maximizes Social Welfare, and consequently emerges in Nash equilibrium, is that where:

(i) R&D’s product is a public good, namely the social planner proceeds to quality improvement – R&D, if and only if the discount rate is not average or product substitutability is not sufficiently high i.e. \( \delta \notin (\delta_b, \delta_a) \) or \( \gamma \notin (\gamma_{SW}, 1) \).

(ii) R&D’s product is a private good, namely firms proceed to R&D investment, if and only if discount rate is average and product substitutability is sufficiently high, i.e. \( \delta \in (\delta_b, \delta_a) \) and \( \gamma \in (\gamma_{SW}, 1) \).

[Proof: See Appendix (A.14)]

Where \( \gamma_{SW}(h) \in (0.078) \forall h \in (0.087, 1) \)\(^8\) and its plot is presented below:

---

\(^8\) The mathematical expressions of \( \gamma_{SW} \) is left out because of its wide extent. It is available by the authors upon request.
Proposition 16:

The union structure that emerges eventually in Nash equilibrium is:

(i) Decentralized wage-setting regime, under sufficient low discount rate, or average discount rate and high product substitution, i.e. \( \delta \in (0, \delta_b) \) or \( \gamma \in (0, \gamma_{sw}) \).

(ii) Centralized wage-setting regime, under sufficient high discount rate, or average discount rate and low product substitution, i.e. \( \delta \in (\delta_a, 1) \) or \( \gamma \in (0, \gamma_{sw}) \).

[Proof: See Appendix (A.14)]

7. Comparative Results

An important part of our research is to highlight the efficient structure of unions, as well as the efficient structure of the market. The conclusion of our comparative analysis demonstrates the superiority of the competitive formation between unions (decentralized wage-setting regime) and the market structure where R&D’s product is a public good.

Regarding union structure, as it is expected, the competitive form increases Social Welfare, but also employment and product quality improvement. Through union competition, the product quality increases, because firms’ labor cost decreases and this
leads to growth of R&D investments. According to the above, it is easy to conclude that under decentralized bargaining both Consumer and Producer Surpluses increase.

As far as market structure is concerned, Social Welfare is higher when the R&D’s know-how is a public good, namely R&D is the social planner’s work. In this case of market structure, we assumed that taxes are levied on the industry’s products, which is well-known to lead to market distortions. Although there is a negative impact on the market by product taxation, this structure increases all industry outcomes (i.e. output level / employment, product quality / R&D investments and union wages) and all market participant surpluses (i.e. Consumer Surplus, Producer Surplus and Union Rents).

The main reason is that, under the assumption that R&D’s know-how is a public good, the market succeeds to save economic resources by investing in R&D which improves the quality of both products at once. The gains for the economy from R&D investments exceed the losses from market distortions due to product taxation. Consequently, this increases the superiority of the market structure where R&D is a public good. The following Propositions summarize the above results.

**Proposition 17:** Regarding market outcomes, namely firms’ output level (employment), product quality improvement (R&D investments) and union wage rates, the following apply:

- Independently of the market structure, firms’ output level and product quality improvement are always higher under decentralized than centralized wage-setting regime, i.e. \( q_{im} > q_{ic} \) and \( x_{im} > x_{ic} \).

  The opposite applies for union wages, i.e. \( w_{ic} > w_{im} \).

- Given the union structures, firms’ output level, product quality improvement and wages rate are always higher when the know-how of R&D is a public good than when it is a private one, i.e. \( q_{ib} > q_{ia} \), \( x_{ib} > x_{ia} \) and \( w_{ib} > w_{ia} \).

[Proof: See Appendix (A.15)]
Proposition 18: Regarding market participant surpluses, namely Consumer Surplus (CS), Producer Surplus (PS) and Union Rents (UR), the following applies:

- Independently of the market structure, Consumer and Producer Surpluses are always higher under decentralized than centralized wage-setting regime, i.e.
  \[ CS_m > CS_c \text{ and } PS_m > PS_c. \]

- Independently of the market structure, Union Rent is higher under centralized than decentralized wage-setting regime, if product substitutability is high enough, i.e.
  \[ US_{a,c} > US_{a,m} \forall \gamma > \gamma_a(h) \quad \text{and} \quad US_{b,c} > US_{b,m} \forall \gamma > \gamma_b(h). \]

- Given the union structure, all participant surpluses / rents are always higher when the know-how of R&D is a public good than when it is a private one, i.e.
  \[ CS_b > CS_a, \quad PS_b > PS_a \quad \text{and} \quad UR_b > UR_a. \]

[Proof: See Appendix (A.16)]

Where \( \gamma_a(h), \gamma_b(h) > 0 \forall h \in (0,1) \) and their critical values are depicted below:

![Figure 7: The critical values of \( \gamma_a(h) \) and \( \gamma_b(h) \).](image-url)
**Proposition 19:** Regarding Social Welfare, it is higher under decentralized than centralized wage-setting and when the know-how of R&D is a public good than when it is a private one, i.e. \(SW_{b,m} > SW_{b,c}, SW_{a,m} > SW_{a,c}\).

[Proof: See Appendix (A.17)]

8. Concluding Remarks

In the context of a dynamic game-theoretic of duopolistic Cournot competition with differentiated products, the present paper investigates the impact of endogenous unionization structures on firms’ incentives for R&D investments, hence product quality improvement.

Our results show that under sufficiently high (low) discount rate and substitutability among the firms' products, an industry-wide union emerges (separate firm-level unions are sustained) in the equilibrium, where product quality along with the level of R&D investments are relatively low (high).

We proceed to further research by developing a market policy, where a benevolent policy maker undertakes the costs of firm-specific R&D investments, finances these costs by indirect taxation and provides firms, without cost, the know-how of product quality improvement. In particular, the proposed market policy suggests a modulated market structure, where the R&D’s product is a common public good. This field of research is buttoned up by endogenizing the selection of market structure and evaluating comparatively the union structures and market structures, as well. Our findings demonstrate the superiority in terms of Social Welfare of the decentralized wage-setting regime between union structures and the nationalization of R&D between market structures.

Independently of the market structure, the decentralized bargaining increases not only the output level, but also the products quality, hence R&D investments. It leads
to the increment of Consumer and Producer Surpluses. However, the most significant
observation is that under certain conditions of product substitutability and perhaps
discount rates, the competitive union structures may even increase their own rents.

According now to market structure, given the union structures, we conclude that
the nationalization of R&D and its produced know-how, even if it implies taxation
and market distortions, promotes all market outcomes (production level / employment, product quality / R&D investments, union wages) and all industry participant surpluses (Consumer Surplus, Producer Surplus, Union Rents). The main explanation is that through nationalization of R&D, the social planner is able to save economic resources by improving the quality of both products at once. The gains for the economy from R&D investments exceed the losses from market distortions due to product taxation.

The endogenous selection of market structure demonstrates the best selection in
terms of Social Welfare, by comparing the equilibrium outcomes of each market
structure. In fact, the best selection of market structures depends on the degree of
product substitutability and discount rate. In particular, by investigating the endogenous selection of market structures, we obtain that market structures maximize Social Welfare as follows:

- The nationalization of R&D, namely the social planner proceeds to R&D
  investments, if and only if discount rate is not average or product substitutability
  is not sufficiently high, i.e. $\delta \notin (\delta_b, \delta_a) \text{ or } \gamma \notin (\gamma_{sw}, 1)$.
- The privatization of R&D, namely firms proceed to R&D investment, if and only
  if discount rate is average and product substitutability is sufficiently high, i.e.
  $\delta \in (\delta_b, \delta_a) \text{ and } \gamma \in (\gamma_{sw}, 1)$. 

[41]
Even if, given the union structures, the nationalization of R&D promotes Social Welfare, it is not the unique best solution. The reason is that under average discount rates $[\delta \in (\delta_b, \delta_a)]$ and high product substitutability $[\gamma \in (\gamma_{sw}, 1)]$, unions have incentives for centralized bargaining when there is nationalization of R&D. The opposite applies when there is privatization of R&D, i.e. unions have incentives for decentralized bargaining. Then, the comparative analysis demonstrates that Social Welfare is higher under the second market case, because the competitive formation of union structures emerge in equilibrium.
Appendix

A.1 Proof of Proposition 2

Unions deal with the following matrix game that presents the payoffs of each union’s strategy, i.e. collusive or competitive play, given its rival’s one:

<table>
<thead>
<tr>
<th>Union ( i )</th>
<th>( Collusion )</th>
<th>( Competition )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Collusion )</td>
<td>((E1)) ({u_{ic}, u_{jc}})</td>
<td>((E2)) ({u_{id}, u_{jd}})</td>
</tr>
<tr>
<td>( Competition )</td>
<td>((E2)) ({u_{id}, u_{jd}})</td>
<td>((E3)) ({u_{im}, u_{jm}})</td>
</tr>
</tbody>
</table>

*Table 4: The Matrix Game that unions deal with at the first stage of the game.*

Due to symmetry, it is applied that \( (u_{id}, u_{jd}) = (u_{id}, u_{id}) \) and thus the number of candidate equilibria is reduced to three, which are denoted by (E1), (E2) and (E3) in the above matrix game.

We now check the condition under which each of the candidates equilibriums may be sustainable in Nash equilibrium, where no union has incentives to deviate by unilaterally switching its own strategy, given that its rival does not:

- (E1) is the one where unions independently collude in \( w \)-negotiations with their specific firm and the possible deviation, on the part of any union, is to set its own wage in order to maximize its own rents. Therefore, according to Proposition 1, it emerges in equilibrium if and only if \( K_{i(j)} > 0 \) applies.

- (E2) is when the one union plays collusively, while its rival union plays competitively in \( w \)-negotiations, and the possible deviations arise by switching each union’s strategy to its rival. Consequently, according to Proposition 1 and (11), it emerges in equilibrium if and only if \( K_{i(j)} < 0 \) and \( N_{i(j)} < 0 \) applies.
- (E3) is the one where unions play competitively in w-negotiations and the possible deviation, on the part of any union, is to set its own wages in order to maximize both union rents. So, according to (11), it emerges in equilibrium if and only if $N_{i0} > 0$ applies.

**A.2 Proof of Proposition 3**

At the first stage of the game, the unions deal with the matrix game of Appendix A.1 that presents the payoffs of each union strategies, i.e. collusive or competitive play in w-negotiations (3rd stage) with their specific firms, given its rival’s one.

Due to symmetry, it is applied that $(u_{id}^*, u_{jd}^*) = (u_{jd}^*, u_{id}^*)$ and thus the number of candidate equilibria is reduced to three. Nash equilibrium is the candidate one that no union has incentives to deviate by unilaterally switching its own strategy, given that its rival does not.

According to Proposition 1, the condition type of union collusion depends on the union threat credibility. Substituting equations (28) and (49) for (11), the union threat credibility is applied:

$$N_i = u_{im}^* - u_{id}^* = B_1 > 0 \quad \forall \; \gamma, h \in (0,1) \quad (A1)$$

The 3D plot of factor $B_1$ is presented below.

---

9 The mathematical expressions of $B_1$ is left out, because it was complicated to be shaped as closed forms.
Taking into consideration (A1), collusive play among unions is sustained in equilibrium if and only if the **Condition A** of Proposition 1 is satisfied. Thus, by means of (28), (38) and (50), we obtain:

\[ K_i = u_{ic} \left( \frac{1}{1 - \delta} \right) - u_{idi} - u_{im} \left( \frac{\delta}{1 - \delta} \right) \]  

(A2)

Due to factor \( K_i(y, h, \delta) \)’s complicated formula, its sign cannot be determined. Additionally, as factor \( K_i(y, h, \delta) \) is dependent on three variables it cannot be represented diagrammatically. However, we determine the sign of factor \( K_i(y, h, \delta) \) using the Bolzano’s Theorem\(^{10}\). In particular, we firstly find the critical value of \( \delta \) and after we define \( K_i(y, h, \delta) \)’s sign in intervals:

\[ K_i(y, h, \delta) = 0 \iff \delta(y, h) = \delta_a(y, h) \]  

(A3)

Where \( \delta_a(y, h) \in (0,1) \) \( \forall \gamma < \gamma_a(h) \)\(^{11}\) with the critical values of \( \delta_a(y, h) \) and \( \gamma_a(h) \) are depicted in Figure 1 and Figure 2 of the main text, respectively. Taking now specific values for variables \( \delta, \gamma, h \) and checking the \( K_i(y, h, \delta) \)’s sign, we obtain that:

\[ K_i(y, h, \delta) > 0 \quad \forall \gamma \in (\gamma_a, 1) \text{ and } \delta \in (\delta_a, 1) \]  

(A4)

\[ K_i(y, h, \delta) < 0 \quad \forall \gamma \in (0, \gamma_a) \quad \text{or } \gamma \in (\gamma_a, 1) \text{ and } \delta \in (1, \delta_a) \]  

(A5)

Summarizing the above results, by virtue of (A1), (A4) and (A5), we conclude that:

- According to (A1), (A5) and Proposition 2, when \( \gamma \in (0, \gamma_a) \) or \( \gamma \in (\gamma_a, 1) \) and \( \delta \in (1, \delta_a) \), then union competition \( E_3 \) emerges in Nash Equilibrium.

\(^{10}\) Bolzano (1817) proved that if a continuous function defined in an interval is sometimes positive and sometimes negative, it must be 0 at some point.

\(^{11}\) The mathematical expressions of \( \delta_a(y, h) \) and \( \gamma_a(h) \) are left out because of their wide extent. They are available by the authors upon request.
According to (A1), (A4) and Proposition 2, when \( \gamma \in (\gamma_a, 1) \) and \( \delta \in (\delta_a, 1) \), then both union competition (E3) and collusion (E1) emerges in Nash Equilibrium.

A.3 Proof of Proposition 4

By means of union utility level under collusion and competition [given in (28) and (38), respectively], we obtain:

\[
\begin{align*}
\mu_{ic} &< \mu_{im} & \forall \gamma \in (0, \gamma_a) \quad \text{(A6)} \\
\mu_{ic} &> \mu_{im} & \forall \gamma \in (\gamma_a, 1) \quad \text{(A7)}
\end{align*}
\]

Where \( \gamma_a(h) \in (0,0.652) \quad \forall \ h \in (0,1) \) and is depicted in Figure 2 of the main text.

According to (A6), (A7), Proposition 3 and the criterion of Pareto Optimality, we conclude that:

- When \( \gamma \in (\gamma_a, 1) \) and \( \delta \in (\delta_a, 1) \), then union collusive play in \( w \)-negotiations is the unique Pareto Optimal Nash equilibrium, as regards rents.
- Otherwise, i.e. when \( \gamma \notin (\gamma_a, 1) \) and \( \delta \notin (\delta_a, 1) \), union competitive play is the unique Nash equilibrium, and consequently the Pareto Optimal one.

A.4 Proof of Proposition 5

Assuming the absolute symmetry of unions and their strategies, the following about equilibrium outcome under centralized or decentralized wage-setting regimes, i.e. firms’ output level, product quality improvement and union wages is applied:

\[
\begin{align*}
q_{is} &= q_{js} = q_s \quad \text{(A8)} \\
x_{is} &= x_{js} = x_s \quad \text{(A9)} \\
w_{is} &= w_{js} = w_s \quad \text{(A10)}
\end{align*}
\]
Where \( s = c, m \)

By means of equilibrium outcomes formulae [given in (51), (52) and (53)], we obtain that:

\[
q_m - q_c = B_2 \left( 128 + \gamma(64 - \gamma(2 + \gamma)(20 - \gamma^2) - 4h^2) \right) > 0 \tag{A11}
\]

\[
x_m - x_c = 2B_2 \ h(2 + \gamma)(16 - \gamma(4 + \gamma)) > 0 \tag{A12}
\]

\[
w_c - w_m = B_2(2 + \gamma)\left( (16 - \gamma^2)(2 - \gamma)(2 + \gamma)^2 + 2h^2(16 - s(2 + s)) \right) > 0 \tag{A13}
\]

Where \( B_2 = A_1 \gamma(2 - \gamma(2 + \gamma))/(2(2 - \gamma)(2 + \gamma)^2 - h^2) \) \( \forall \gamma, h \in (0,1) \)

Summarizing now our findings in (A11), (A12) and (A13), we get that:

\[
q^*_m = q^*_c = q^*_c > q^*_i \quad \tag{A14}
\]

\[
x^*_m = x^*_c = x^*_c > x^*_i \quad \tag{A15}
\]

\[
w^*_i = w^*_j \quad \tag{A16}
\]

According to Proposition 4, we conclude that if \( \gamma < \gamma_a(h)(< 0.652) \) or/and \( \delta < \delta_a(h) \), then \( q^* \) and \( x^* \) are higher. The opposite applies for \( w^* \).

### A.5 Proof of Proposition 6

According to (55) and (56), Consumer Surplus and Producer Surplus, under decentralized and centralized wage-setting regimes, are given by the following equation:

\[
CS_s = \frac{1 + \gamma}{4} Q_s^2 \tag{A17}
\]

\[
PS_s = \frac{Q_s^2}{2} - x_{i(j)s}^2 \tag{A18}
\]

Where \( s = c, m \) and \( Q \) is the sum of the firms’ output level.
Regarding Consumer Surplus and given Proposition 5, it is straightforward that:

\[ Q_m > Q_c \implies CS_m > CS_c \quad \forall h, \gamma \in (0,1) \quad (A19) \]

By virtue of Proposition 4 and (59), we obtain the Producer Surplus that:

\[ PS_m - PS_c = B_3 > 0 \quad \forall h, \gamma \in (0,1) \quad (A20) \]

Where, \( B_3 > 0 \forall \gamma, \phi \in (0,1) \)\(^{12}\) and its 3D plot is presented below.

![3D plot of B_3](image)

Summarizing the above results, we get that:

\[ CS_m > CS_c \quad \forall h, \gamma \in (0,1) \quad (A21) \]

\[ PS_m > PS_c \quad \forall h, \gamma \in (0,1) \quad (A22) \]

Consequently, we conclude that if \( \gamma < \gamma_a(h)(< 0.652) \) or/and \( \delta < \delta_a(h) \), then both \( CS \) and \( PS \) are higher.

### A.6 Proof of Proposition 7

Total Union Rents is defined to be the sum of both unions utility, as follows:

\[ UR_s = \sum_{k=i,j} u_{ks} \quad ; \quad s = c, m \quad (A23) \]

Assuming the absolute symmetry of unions and their strategies, it is applied that:

\[ u_{is} = u_{js} = u_s \quad ; \quad s = c, m \quad (A24) \]

Given (A24), we get the following refined Union Rents formula:

\[^{12}\text{The mathematical expression of } B_3 \text{ is left out because of its wide extent. It is available by the authors upon request.}\]
\[ UR_s = 2u_s \quad ; \quad s = c, m \quad \text{(A25)} \]

According now to (A6) and (A7), we easily conclude that:

\[ UR_c < UR_m \quad \forall \gamma \in (0, \gamma_a) \quad \text{(A26)} \]
\[ UR_c > UR_m \quad \forall \gamma \in (\gamma_a, 1) \quad \text{(A27)} \]

Where \( \gamma_a(h) \in (0, 0.652) \ \forall \ h \in (0,1) \) and it is depicted in Figure 2 of the main text.

Combining our results from Proposition 4, (A26) and (A27), we get that:

- If \( \gamma \in (\gamma_a, 1) \) and \( \delta \in (\delta_a, 1) \), centralized wage-setting regime emerges in Nash equilibrium, which is also the Pareto Optimal one.
- If \( \gamma \in (\gamma_a, 1) \) and \( \delta \in (0, \delta_a) \), decentralized wage-setting regime emerges in Nash equilibrium, which is not the Pareto Optimal one, as \( UR_c > UR_m \).
- If \( \gamma \in (0, \gamma_a) \), decentralized wage-setting regime emerges in Nash equilibrium, which is also the Pareto Optimal one.

A.7 Proof of Proposition 8

Taking into consideration Proposition 4 and Social Welfare in equilibrium [given in (61)], we get that:

\[ SW_m - SW_c = B_4 > 0 \quad \forall \ h, \gamma \in (0,1) \quad \text{(A28)} \]

Where, \( B_4 > 0 \ \forall \ h, \gamma \in (0,1) \)\(^{13}\) and its 3D plots are respectively the following:

---

\(^{13}\) The mathematical expression of \( B_4 \) is left out because of its wide extent. It is available by the authors upon request.
According to Proposition 4, we conclude that if \( \gamma < \gamma_a(h) (< 0.652) \) or/and \( \delta < \delta_a(h) \), then \( SW \) is higher.

**A.8 Proof of Proposition 9**

At the second stage of the game, unions deal with the matrix game presented in Appendix A.1, except that payoffs of each union are given by equations (69), (73), (77) and (78) in Subsections 5.1-5.3.

Taking the same path as Proposition 3’s proof, firstly we investigate the credibility of union threats \( (N_i) \) and secondly the satisfaction of appropriate factor \( K_i \)’s formula (condition for collusive play) in Proposition 1. According to Proposition 2, combining our results on the credibility of union threats and the conditions for collusive play, we eventually get the Nash equilibrium of the proposed game.

It is derived that union threats are credible, by substituting equations (69) and (77) for (11):

\[
N_i = u_{im}^* - u_{idj}^* = \frac{\gamma^2(2-\gamma)^2(1+hx-t)^2}{2(2+\gamma)(8-\gamma^2)(4-\gamma)^2} > 0 \quad \forall \gamma, h \in (0,1) \quad (A29)
\]

By virtue of (A1), we get that collusive play among unions is sustained in equilibrium if and only if **Condition A** of Proposition 1 is satisfied:

\[
K_i = u_{ic}^* \left( \frac{1}{1-\delta} \right) - u_{idi}^* - u_{im}^* \left( \frac{\delta}{1-\delta} \right) = B_2(1+hx-t)^2 \quad (A30)
\]

Where \( B_2 > 0 \ \forall \ \delta > \delta_b(\gamma) \), with \( \delta_b(\gamma) = \frac{(4-\gamma)(4-\gamma)(2+\gamma)}{2(2-\gamma)(32-3\gamma^2)} \in (0.16, 0.5) \) and its plot is presented in **Figure 4** of the main text, so we get that:

\[
K_i > 0 \quad \forall \ \delta \in (\delta_b, 1) \quad (A31)
\]

\[
K_i < 0 \quad \forall \ \delta \in (0, \delta_b) \quad (A32)
\]

Summarizing the above results, by virtue of (A29), (A31) and (A32), we conclude that:
• According to (A29), (A32) and Proposition 2, when \( \delta \in (0, \delta_b) \), the union competition (E3) emerges in Nash Equilibrium.

• According to (A29), (A31) and Proposition 2, when \( \delta \in (\delta_b, 1) \), both union competition (E3) and collusion (E1) emerges in Nash Equilibrium.

A.9 Proof of Proposition 10

Taking into consideration the union utility level under collusion and competition in (69) and (73), respectively, we get that:

\[
\frac{\gamma^2(1 + hx - t)^2}{2(2 + \gamma)(4 - \gamma)^2} > 0 \quad \forall \quad \gamma \in (0, 1) \tag{A33}
\]

Consequently, by virtue of (A33), Proposition 9 and the criterion of Pareto optimality, we conclude that

• If \( \delta \in (\delta_b, 1) \), union collusive play in w-negotiations is the unique Pareto Optimal Nash equilibrium, as regards rents.

• Otherwise, i.e. if \( \delta \in (0, \delta_b) \), union competitive play is the unique Nash equilibrium.

A.10 Proof of Proposition 11

At the top (1st) stage of the game, the social planner aims to determine the optimal level of R&D investments \([TC^{PM}]\) and indirect taxation \([TR^{PM}]\) that maximize Social Welfare, under the assumption of a Balanced Budget policy \([TC^{PM} = TR^{PM}]\).

Substituting the equations of \(TC^{PM}\) and \(TR^{PM}\) [given in (5) and (6)] in the Balanced Budget equation [given in (82)], we obtain that:

\[
Q = \frac{x^2}{2t} \tag{A34}
\]
Where $Q$ is the sum of the firms’ output level.

For convenience, taking into consideration Proposition 10, we investigate the two potential Nash Equilibria separately as follows:

- **When the discount rate is sufficiently low, i.e. $\delta \in (0, \delta_b)$:**

  If $\delta \in (0, \delta_b)$, then unions’ competitive play emerges in equilibrium and the optimal industry output level is given by the following equation [by (65) and (67)]:

  $$Q^* = \frac{4(1 + h x - t)}{(2 + y)(4 - y)} \quad (A35)$$

  The optimal levels of R&D investments and indirect taxation on market products are derived by substituting the optimal market output rule in (A35) for the Balanced Budget condition in (A34):

  $$t_1 = \frac{1}{4} \left( 2 + 2hx - \sqrt{4 + 2x(4h + 2h^2x - (4 - y)(2 + y)x)} \right) \quad (A36)$$

  $$t_2 = \frac{1}{4} \left( 2 + 2hx + \sqrt{4 + 2x(4h + 2h^2x - (4 - y)(2 + y)x)} \right) \quad (A37)$$

  Consequently, the objective of the social planner is defined by the following maximization problem:

  $$\max_{x,t} [SW(= CS + PS + UR)] \quad (A38)$$

  The above maximization problem gives the following solutions:

  $$\begin{cases}
  t_1^* = \frac{2h^2}{(2 + y)(4 - y) + 2h^2} \in (0,0.33) \\
  x_1^* = \frac{4h}{(2 + y)(4 - y) + 2h^2} \in (0,0.67) \end{cases} \quad (A39)$$

  $$\begin{cases}
  t_2^* = \frac{(2 + y)(4 - y)}{(2 + y)(4 - y) + 2h^2} \in (1,1.33) \\
  x_2^* = \frac{4h}{(2 + y)(4 - y) + 2h^2} \in (0,0.67) \end{cases} \quad (A40)$$
It arises that the solution \((t_1^*, x_1^*)\) is the only acceptable one, as the solution \((t_2^*, x_2^*)\) is rejected, because it does not satisfy the condition of oligopoly existence in (68).

Thence, the Social Welfare in equilibrium is defined by:

\[
SW = \frac{4(7 + \gamma(1 - \gamma))}{((2 + \gamma)(4 - \gamma) + 2h^2)^2}
\]  
(A41)

- **When the discount rate is sufficiently high, i.e. \(\delta \in (\delta_b, 1)\):**

If \(\delta \in (\delta_b, 1)\), then union competition emerges in equilibrium and the optimal industry output level is given by (72) and (74), as follows:

\[
Q^* = \frac{1 + hx - t}{2 + \gamma}
\]  
(A42)

Given now the Balance Budget condition in (A34) and the equilibrium output in (A42), we get the optimal indirect taxation rule on market products:

\[
t_1 = \frac{1}{2} \left(1 + hx - \sqrt{1 + 2hx - x^2(2(2 + \gamma) - h^2)}\right)
\]  
(A43)

\[
t_2 = \frac{1}{2} \left(1 + hx + \sqrt{1 + 2hx - x^2(2(2 + \gamma) - h^2)}\right)
\]  
(A44)

Thus, the social planner’s objective is to define the optimal R&D investments in order to maximize the Social Welfare, given the limitation of tax level [given in (A44)]:

\[
\max_{x_1} [SW(= CS + PS + UR)]
\]
\[
s.t. \quad t = t_1, t_2
\]

(A45)

The solution of the above maximizing problem is presented below:

\[
\begin{align*}
t_1^* &= \frac{h^2}{2(2 + \gamma) - h^2} \in (0, 0.34) \\
x_1^* &= \frac{2h}{2(2 + \gamma) - h^2} \in (0, 0.67)
\end{align*}
\]  
(A46)
\[
\begin{align*}
\begin{cases}
t_2^* &= \frac{2(2 + \gamma)}{2(2 + \gamma) - h^2} \in (1,1.33) \\
x_2^* &= \frac{2h}{2(2 + \gamma) - h^2} \in (0,0.67)
\end{cases}
\end{align*}
\] (A47)

We reject the solution \((t_2^*, x_2^*)\) because it does not satisfy the condition of oligopoly in (68). The accepted solution is \((t_1^*, x_1^*)\), under which Social Welfare in equilibrium is given by the following equation:

\[
SW = \frac{7 + 3\gamma}{(2(2 + \gamma) - h^2)^2}
\] (A48)

A.11 Proof of Proposition 12

Due to the absolute symmetry of unions and their strategies, (A8), (A9) and (A10) are applied in equilibrium outcomes under centralized or decentralized wage-setting regimes. Comparing now the formulae of market equilibrium outcomes in (91), Proposition 11 and (90), we obtain that:

\[
q_m - q_c = \frac{\gamma(2 + \gamma)}{(2(2 + \gamma) - h^2)((2 + \gamma)(4 - \gamma) - 2h^2)} > 0 \quad \forall \ h, \gamma \in (0,1)
\] (A49)

\[
x_m - x_c = \frac{2\gamma h(2 + \gamma)}{(2(2 + \gamma) - h^2)((2 + \gamma)(4 - \gamma) - 2h^2)} > 0 \quad \forall \ h, \gamma \in (0,1)
\] (A50)

\[
w_c - w_m = \frac{\gamma(2 + \gamma)(2 + \gamma - h^2)}{(2(2 + \gamma) - h^2)((2 + \gamma)(4 - \gamma) - 2h^2)} > 0 \quad \forall \ h, \gamma \in (0,1)
\] (A51)

Summarizing now our findings in (A49), (A50) and (A51), we get that:

\[
q_{im}^* = q_{jm}^* = q_m^* > q_{ic}^* = q_{jc}^* = q_c^*
\] (A52)

\[
x_{im}^* = x_{jm}^* = x_m^* > x_{ic}^* = x_{jc}^* = x_c^*
\] (A53)

\[
w_{ic}^* = w_{jc}^* = w_c^* > w_{im}^* = w_{jm}^* = w_m^*
\] (A54)
According to Proposition 11, we conclude that if $\delta < \delta_b(h)$, then $q^*$ and $x^*$ are higher. The opposite applies for $w^*$.

A.12 Proof of Proposition 13

The formulae of Consumer Surplus and Producer Surplus are given in (55) and (56) and their outcomes under decentralized and centralized bargaining are given in (92) and (93), respectively.

According to (55) and (A52), we get about the Consumer Surplus that:

$$Q_m > Q_c \Rightarrow CS_m > CS_c \quad \forall \ h, \gamma \in (0,1) \ (A55)$$

Using now comparative analysis of (93), we obtain about the Producer Surplus that:

$$PS_m - PS_c = \frac{2\gamma(2 + \gamma)(2 + \gamma)(8 - \gamma) - 4h^2}{(2(2 + \gamma) - h^2)((2 + \gamma)(4 - \gamma) - 2h^2)} > 0 \quad \forall \ h, \gamma \in (0,1) \ (A56)$$

Summarizing the above results, we conclude that:

$$CS_m > CS_c \quad \forall \ h, \gamma \in (0,1) \quad (A57)$$

$$PS_m > PS_c \quad \forall \ h, \gamma \in (0,1) \quad (A58)$$

Consequently, we conclude that if $< \delta_b(h)$, then both $CS$ and $PS$ are higher.

A.13 Proof of Proposition 14

The total Union Rents is defined to be the sum of both union utilities, [given in (A23)] and due to absolute symmetry of unions and their strategies, its simplified mathematical expression is presented by the following:

$$UR_s = 2u_s \quad ; \quad s = c, m \quad (A59)$$

By means of union utility levels under centralized and decentralized bargaining [given in (69) and (73), respectively], we get that:
\begin{equation}
 u_{ic} < u_{im} \quad \forall \gamma \in (0, \gamma_b) \tag{A60}
\end{equation}

\begin{equation}
 u_{ic} > u_{im} \quad \forall \gamma \in (\gamma_b, 1) \tag{A61}
\end{equation}

Where \( \gamma_b(h) \in (0,1) \ \forall \ h \in (0,1) \) is depicted in Figure 5 of the main text.

Consequently, we easily conclude that:

\begin{equation}
 UR_c < UR_m \quad \forall \gamma \in (0, \gamma_b) \tag{A62}
\end{equation}

\begin{equation}
 UR_c > UR_m \quad \forall \gamma \in (\gamma_b, 1) \tag{A63}
\end{equation}

Combining our results from Proposition 11, (A62) and (A63) we get that:

- If \( \gamma \in (0, \gamma_b) \) and \( \delta \in (0, \delta_b) \), decentralized wage-setting regime emerges in Nash equilibrium, which is also the Pareto Optimal one.

- If \( \gamma \in (\gamma_b, 1) \) and \( \delta \in (0, \delta_b) \), decentralized wage-setting regime emerges in Nash equilibrium, which is not the Pareto Optimal one, as \( UR_c > UR_m \).

- If \( \gamma \in (0, \gamma_b) \) and \( \delta \in (\delta_b, 1) \), centralized wage-setting regime emerges in Nash equilibrium, which is not the Pareto Optimal one, as \( UR_m > UR_c \).

- If \( \gamma \in (\gamma_b, 1) \) and \( \delta \in (\delta_b, 1) \), centralized wage-setting regime emerges in Nash equilibrium, which is also the Pareto Optimal one.

**A.14 Proof of Proposition 15 and Proposition 16**

The problem of the social planner is to decide whether to intervene, or not, in the market structure by setting product quality improvement as a public good, i.e. he proceeds to R&D investments and provides the know-how to the industry for free.

Therefore, the social planner’s maximization problem is the following:

\[
\max_{SW} \{ SW_a, SW_b \} \tag{A64}
\]

Where the indexes (a) and (b) denote the market structure where the R&D investments are undertaken by firms’ or the social planner, respectively.
According to (61) and (89), Social Welfare under each market structure is defined by the following equations:

\[
SW_a = \begin{cases}
SW_{a,m} = 4A_3A_1^2, & \forall \delta \in (0, \delta_a) \text{ or } \gamma \in (0, \gamma_a) \\
SW_{a,c} = \frac{(7 + 3\gamma)(4 - \gamma^2)^2 - 4h^2}{4((2 - \gamma)(2 + \gamma)^2 - h^2)^2}, & \forall \delta \in (\delta_a, 1) \text{ and } \gamma \in (\gamma_a, 1)
\end{cases}
(A65)
\]

\[
SW_b = \begin{cases}
SW_{b,m} = \frac{4(7 + \gamma(1 - \gamma))}{(2 + \gamma)(4 - \gamma) - 2h^2}, & \forall \delta \in (0, \delta_b) \\
SW_{b,c} = \frac{7 + 3\gamma}{(2 + \gamma) - h^2}, & \forall \delta \in (\delta_b, 1)
\end{cases}
(A66)
\]

Where \(A_1 = ((4 + \gamma)(2 - \gamma)(4 - \gamma)^2(2 + \gamma)^2 - 8h^2(8 - \gamma^2))^{-1} > 0 \quad \forall \gamma, h \in (0,1)\)

and \(A_3 = (7 + \gamma(1 - \gamma))(64 - 20\gamma^2 + \gamma^4)^2 - 16h^2(8 - \gamma^2)^2 > 0 \quad \forall \gamma, h \in (0,1)\).

The indexes (m) and (c) denote unions’ decentralized and centralized bargaining, respectively.

Due to the complexity of the Social Welfare formula determination, we first arrange the values of Social Welfare among the various equilibria in descending order, so we obtain that:

\[
SW_{b,m} - SW_{b,c} = \frac{\gamma(2 + \gamma)((2 + \gamma)(3 - \gamma)(8 + 3\gamma) + 4h^2(1 + \gamma - h^2))}{(2(2 + \gamma) - h^2)^2((2 + \gamma)(4 - \gamma) - 2h^2)^2} > 0 \quad (A67)
\]

\[
SW_{a,m} - SW_{a,c} = B_4 > 0 \quad (A68)
\]

\[
SW_{b,m} - SW_{a,m} = B_5 > 0 \quad (A69)
\]

\[
SW_{b,c} - SW_{a,c} = B_6 > 0 \quad (A70)
\]

\[
SW_{b,c} - SW_{a,m} = B_7 > 0, \quad \forall \gamma \in (0, \gamma_{sw}) \quad (A71)
\]

Where \(B_7 > \forall \gamma \in (0, \gamma_{sw}) \text{ with } \gamma_{sw} \in (0.078, 1) \quad \forall \delta \in (0.087, 1)\) and its plot is presented by Figure 6 in the main text. Moreover, it is applied that \(B_4 > \forall \gamma \in (0,1)\)
and its 3D plot is presented in subsection A.7, while $B_5$, $B_6 > 0$ and their 3D plots are presented below:\[14\]:

\[\text{while, the descending order of } \gamma(h) \text{ critical schedules is presented below,}\]

\[\begin{align*}
SW_{b,m} > SW_{b,c} &> SW_{a,m} > SW_{a,c}, \quad \forall \gamma \in (0, \gamma_{SW}(h)) \quad (A72) \\
SW_{b,m} > SW_{a,m} &> SW_{b,c} > SW_{a,c}, \quad \forall \gamma \in (\gamma_{SW}(h), 1) \quad (A73)
\end{align*}\]

Arranging now $\delta(\gamma)$ and $\gamma(h)$ critical schedules in descending order, we obtain that:

\[\delta_a(\gamma) - \delta_b(\gamma) = B_8 > 0, \quad \forall \gamma, h \in (0,1) \quad (A74)\]

Where $B_8 > 0 \forall \gamma, h \in (0,1)\[15\]$ and its 3D plot is the following:

While, the descending order of $\gamma(h)$ critical schedules is presented below, mathematically and diagrammatically:

\[\text{The mathematical expression of } B_8 \text{ is left out because of its wide extent. It is available by the authors upon request.}\]

\[\text{The mathematical expressions of } B_a, B_5, B_6, B_7 \text{ and } \gamma_b \text{ are left out because of their wide extent. They are available by the authors upon request.}\]

[58]
Combing now the above information, given in (A72), (A73), (A74) and (A75), we conclude that:

- If $\delta < \delta_a(\gamma)$, the social planner’s optimal selection is to set R&D as a public good and thus decentralized unions in the frame of w-negotiations are formed in equilibrium.

- If $\delta > \delta_a(\gamma)$ or $\delta \in (\delta_b, \delta_a)$ and $\gamma < \gamma_{sw}$, the social planner’s optimal selection is to set R&D as a public good and thus centralized unions in the frame of w-negotiations are formed in equilibrium.

- If $\delta \in (\delta_b, \delta_a)$ and $\gamma > \gamma_{sw}$, the social planner’s optimal selection is to set R&D as a private good and thus decentralized unions in the frame of w-negotiations are formed in equilibrium.

According to our findings, the Social Welfare in equilibrium is defined by:

$$ SW = \begin{cases} 
SW_{b,m} & \forall \delta \in (0, \delta_a) \\
SW_{b,c} & \forall \delta \in (\delta_a, 1) \text{ or } \delta \in (\delta_b, \delta_a), \gamma \in (0, \gamma_{sw}) \\
SW_{a,m} & \forall \delta \in (\delta_b, \delta_a), \gamma \in (\gamma_{sw}, 1) 
\end{cases} \quad (A76) $$
A.15 Proof of Proposition 17

Given our findings on comparative analysis of market outcomes, i.e. output level, product quality improvement and wages between union structures (centralized and decentralized bargaining institution) under each market structure [given in (A14), (A15), (A16) and (A52), (A53), (A54), respectively], we get that:

\[ q^*_{k,m} > q^*_{k,c} \] \hspace{1cm} (A77)
\[ x^*_{k,m} > x^*_{k,c} \] \hspace{1cm} (A78)
\[ w^*_{k,m} > w^*_{k,c} \] \hspace{1cm} (A79)

Where \( k = a, b \) denotes the market structure.

We now proceed to the comparative analysis of market outcomes between market structures \((a)\) and \((b)\), with the given union structures and we get that:

\[ q^*_{b,m} - q^*_{a,m} = A_1 \frac{4h^2(32 - 16\gamma^2 + \gamma^4)}{(2 + \gamma)(4 - \gamma - 2h^2)} > 0 \] \hspace{1cm} (A80)
\[ q^*_{b,c} - q^*_{a,c} = \frac{h^2(2 - \gamma^2)}{(2(2 + \gamma) - h^2)((2 - \gamma)(2 + \gamma)^2 - h^2)} > 0 \] \hspace{1cm} (A81)
\[ x^*_{b,m} - x^*_{a,m} = A_1 \frac{16h^3(8 - \gamma^2) + 4h(4 - \gamma)(2 + \gamma)(48 - 18\gamma^2 + \gamma^4)}{(2 + \gamma)(4 - \gamma - 2h^2)} > 0 \] \hspace{1cm} (A82)
\[ x^*_{b,c} - x^*_{a,c} = \frac{h((2 + \gamma)(3 - \gamma^2) - h^2)}{(2(2 + \gamma) - h^2)((2 - \gamma)(2 + \gamma)^2 - h^2)} > 0 \] \hspace{1cm} (A83)
\[ w^*_{b,m} - w^*_{a,m} = \frac{2h^2(2 + \gamma)(2 - \gamma)(32 - 16\gamma^2 + \gamma^4)}{(2(2 + \gamma) - h^2)((2 - \gamma)(2 + \gamma)^2 - h^2)} > 0 \] \hspace{1cm} (A84)
\[ w^*_{b,c} - w^*_{a,c} = \frac{h^2(2 + \gamma)(2 - \gamma^2)}{(2(2 + \gamma) - h^2)((2 - \gamma)(2 + \gamma)^2 - h^2)} > 0 \] \hspace{1cm} (A85)

Where \( A_1 = \frac{((4 + \gamma)(2 - \gamma)(4 - \gamma)^2(2 + \gamma)^2 - 8h^2(8 - \gamma^2))^{-1}}{0} \forall \gamma, h \in (0,1) \)

Summarizing the above results, we conclude that:
Given our findings on the comparative analysis of market participant surpluses/rents, i.e. Consumer Surplus, Producer Surplus and Union Rents between union structures (centralized and decentralized bargaining institution) under each market structure [given in (A21), (A22), (A26), (A27) and (A57), (A58), (A62), (A63), respectively], we get that:

\[ CS_{k,m}^* > CS_{k,c} \]  \hspace{1cm} (A89)

\[ PS_{k,m}^* > PS_{k,c}^* \]  \hspace{1cm} (A90)

\[ UR_{k,m}^* > UR_{k,c}^*, \quad \forall \quad \gamma < \gamma_k(h) \]  \hspace{1cm} (A91)

Where \( k = a, b \) denotes the market structure.

We follow with the comparative analysis of market participant surpluses/rents between market structures \((a)\) and \((b)\), given the union structures.

According to (55) and (A86), we get the Consumer Surplus by:

\[ q_{b,s}^* > q_{a,s}^* \]  \hspace{1cm} (A86)

\[ x_{b,s}^* > x_{a,s}^* \]  \hspace{1cm} (A87)

\[ w_{b,s}^* > w_{a,s}^* \]  \hspace{1cm} (A88)

Where \( s = m, c \) denotes union structures.

\[ A.16 \text{ Proof of Proposition 18} \]

According to (55) and (A86), we get the Consumer Surplus by:

\[ q_{b,s}^* > q_{a,s}^* \]  \hspace{1cm} (A86)

\[ x_{b,s}^* > x_{a,s}^* \]  \hspace{1cm} (A87)

\[ w_{b,s}^* > w_{a,s}^* \]  \hspace{1cm} (A88)

Where \( s = m, c \) denotes union structures.
\[ PS_{b,c}^* - PS_{a,c}^* = B_{10} > 0 \quad \forall \ h, \gamma \in (0,1) \quad (A94) \]

\[ UR_{b,m}^* - UR_{a,m}^* = B_{11} > 0 \quad \forall \ h, \gamma \in (0,1) \quad (A95) \]

\[ UR_{b,c}^* - UR_{a,c}^* = B_{12} > 0 \quad \forall \ h, \gamma \in (0,1) \quad (A96) \]

Where \( B_9, B_{10}, B_{11}, B_{12} > 0 \ \forall \ \gamma, h \in (0,1) \)\(^{16} \) and their 3D plots are the following:

Summarizing the above results, we conclude that:

\[ CS_{b,s}^* > CS_{a,s}^* \quad (A97) \]

\[ PS_{b,s}^* > PS_{a,s}^* \quad (A98) \]

\[ UR_{b,s}^* > UR_{a,s}^* \quad (A99) \]

Where \( s = m, c \) denotes the union structures.

\(^{16}\) The mathematical expression of \( B_9, B_{10}, B_{11}, B_{12} \) are left out because of their wide extent. They are available by the authors upon request.
A.17 Proof of Proposition 19

Taking into consideration the descending order of Social Welfare under each combination of unions and market structures, given in (A72) and (A73), we obtain that:

\[
SW_{b,s} > SW_{a,s} \quad \forall \ h, \gamma \in (0,1) \quad ; \quad s = c, m \quad \text{(A100)}
\]

\[
SW_{k,m} > SW_{k,c} \quad \forall \ h, \gamma \in (0,1) \quad ; \quad k = a, b \quad \text{(A101)}
\]

Where indexes \( s \) and \( k \) denote union structures (\( c \) or \( m \)) and market structures (\( a \) or \( b \)), respectively.
References


