

Comparative versus Informative Advertising in Oligopolistic Markets. *

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Abstract

The present paper examines endogenously the firms' incentives to invest in informative and comparative advertising, in an oligopolistic market with horizontally differentiated products where competition take place in quantities. We show that, in equilibrium the firms undertake a mix advertising strategy that combines both informative and comparative advertising investments. We further compare our results over the equilibrium market outcomes and the social welfare obtained under the endogenous advertising configuration with the benchmark case, without firms' advertising activities, and the cases of mere informative and mere comparative advertising. We demonstrate that the equilibrium market outcomes, as well as, the welfare alter significantly depending on the type(s) of advertising that firms have available in the market and the degree of the market competition.

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1 Introduction

Comparative advertising, "the form of advertising that compares rivals brands on objectively measurable attributes or price, and identifies the rival brand by name, illustration or other distinctive information",¹ has received lately increased attention by business, academics and policy makers since, this aggressive form of advertising has emerged as a prevalent marketing practice in multiple industries. The advertising wars of Pepsi and Coke, Ducking Donuts and Starbucks, or the advertising campaign of Avis, "We try harder", are only few typical examples that describe the daily consumers' exposure to comparative advertising messages. As Pechmann and Stewart (1990) show at the United States Market the 60 % of all the advertising campaigns contains indirect comparative claims, the 20% contains direct comparative claims and only the remaining 20 % contains no comparative claims. Further, Muehling et al. (1990) suggest that almost the 40% of all advertising content is comparative.² Clearly, the use of comparative advertising along with self promoting non comparative advertising is extensive in the markets, despite the existed inconclusive empirical evidence regarding the effectiveness of comparative ads in increasing the demand of the product that it promotes.³ In particular, contrary to the non comparative advertising, such as, informative advertising that firms use to convey self promoting messages to the consumers (i.e., product information, characteristics, etc.) comparative ads are mainly focused to promote the superiority of a firm's product against the targeted product(s). Thus, comparative advertising give rise to a push-me/pull you effect, that is, a firm is willing via a comparative ad to increase its demand, by promoting its own product and by denigrating the rival's product (Anderson et al, 2009).

The objective of this paper is to explore endogenously the firms' incentives to invest in comparative and informative advertising, when both types of advertising are available in the market, as well as, the effects of these investments in the market outcomes and the social welfare. In particular, we address the following four questions. First, which is the optimal

¹Statement of policy regarding comparative advertising, Federal Trade Commission, Washington, D.C., August 13, 1979.

²The distinction between direct and indirect comparative ads is based on whether or not the competitor is explicitly named or precisely identified by logos and images.

³The empirical evidence so far, suggest that comparative advertising may have either positive or negative effects on the consumers' demand. This is so since, comparative ads tend to be more effective than non comparative in inducing consumers' attention, message and brand awareness, favorable brand attitudes, and thus purchase intentions (Grewal et al., 1997; Jung and Sharon, 2002). On the contrary, apart from legal risks, they may enhance consumers' mistrust and lead to misidentifications of the sponsoring brands, (Goodwin and Etgar, 1980; Wilkie and Farris, 1975; Prasad, 1976; Barone and Miniard, 1999).

firms' decisions over the type(s) of advertising that they are going to undertake in order to promote their product? Second, how does the market's features (i.e., the intensity of the market competition, the effectiveness of advertising technology, etc.) affect the firms' investment levels in the two alternative types of advertising? Third, how the firms' advertising investments affect their market performance? Forth, how does the firms' advertising decisions affect the social welfare?

We consider a duopolistic market with horizontally differentiated products, where a-priori consumers do not possess all the relevant information about the products. Firms have on their set of marketing strategies both informative and comparative advertising. Informative advertising transmits all the relevant information to the mass of the consumers that are previously uninformed about the product's characteristics and helps them to identify the product that matches to their needs. Thus, informative advertising increase consumers' valuation of the advertised product and shifts the firm's demand curve outwards. Comparative advertising presents the "positively" advertised product as superior to the rival's one. Therefore, it increases consumers' valuation for the positively advertised product while, at the same time, it decreases consumers' valuation of the rival firm's product. Firms incur sufficiently high advertising cost both for informative and comparative advertising. The sequence of the moves are as follows. In the first stage of the game the firms decide, independently and simultaneously, upon the type(s) of advertising, as well as, the investment level of each type of advertising that they are willing to undertake in order to promote their product. In the second stage, firms compete in the market by setting their quantities.

We show that in equilibrium firms invest in both informative and comparative advertising (mix advertising strategy). This is so since a mix advertising strategy offers a competitive advantage in contrast to a single one-type advertising strategy because, it allows firms to launch the potential benefits of both types of advertising. That is, firms invest in informative advertising in order to increase directly their demand while, at the same time, they invest in comparative advertising in order to increase their demand by the fall out of the rival's firm demand due to the effect of the rival's denigration. Further, we show that firms' investment levels in each type of advertising alter significantly with the intensity of the market's competition. In particular, we show that the firms' expenditures in comparative advertising are increasing in the degree of market competition while, the informative advertising expenditures are U shaped related with the market competition degree. Intuitively, when the market competition is not

fierce the firms tend to decrease their investment levels in informative advertising in order to reduce their costs of advertising, given that the products are not close substitutes and thus, they are easily regongizable by the consumers. On the contrary, the fierce market competition lead firms to invest more in both types of advertising as an attempt to enforce their market position and obtain a competitive advantage over the rival. Intrestingly enough, we show that within the optimal advertising mix the firms expenditures on comparative advertising always exceed the respectives ones on informative advertising. That means that, in the equilibrium firms always prefer an aggressive advertising mix.

Further, in order to unravel the effects of the firms advertising decisions on the market outcomes and the social welfare we compare the equilibrium results of our basic model, named as the endogenous advertising configuration with the benchmark case where firms do not undertake any advertising activities and the mere informative configuration and the mere comparative advertising configurations where firms have on their set of marketing strategies either informative or comparative advertising. Regarding output production, we demonstrate that the highest firms' output production is obtained under the endogenous advertising configuration. Intuitively, the existence of the two types of advertising in the market, intensifies the firms' advertising competition and therefore, the output competition. In more details, we show that the existence of both types of advertising in a market, lead to higher firms' expenditures on both informative and comparative advertising compared to the respective firms' expenditures either on informative or comparative advertising when in the market exist only a single type of advertising. Thus, given that the output is positively related to each own firm's investments in advertising, the higher firms' investment levels in both informative and comparative advertising under the endogenous advertising configuration, lead to firms' higher output production than under the benchmark, the mere informative and the mere comparative configurations.

As far as the firms' market performance in terms of profitability is being concerned, we show that the firms obtain the highest profits under the mere informative advertising configuration. This is because the self promoting informative advertising shifts the firms' demand curves outwards and thus, increases the firms' market share and profits. Further, we indicate that the firms' profits under the mere comparative and the endogenous advertising configuration are always lower than those of the benchmark case. Clearly, in the mere comparative advertising configuration the firms end up worse off comparing to the benchmark since, the

beneficial effect on profits of each own firm's investment in comparative advertising is neutralized by the diminishing effect of the rival's comparative advertising expenditures. Further, in the endogenous advertising configuration, the positive effect on the firms' profits that the investment in informative advertising have, could not compensate the negative effect on the firms' profits due to the increased output competition and the cost of advertising. Thus, firms' profits under the endogenous advertising configuration are lower than those of the benchmark. The above results highlight that the use of comparative advertising lead firms to a prisoner's dilemma situation where they obtain lower profits comparing with the benchmark case. Last but not least, comparing the firms profits in the endogenous advertising configuration and the mere comparative one, we show that the firms' profits are higher than those of the former case when the degree of market competition is low while, the opposite holds for fierce market competition. The intuition behind this result is based on the following effects. First, the effect of informative advertising in the endogenous advertising configuration that shifts outwards the firms' demand curves and thus, tends to increase the firms profits. Second the diminishing effect on the firms' profits that the high firms' advertising expenditures in the endogenous advertising configuration have, due to the high advertising costs and the increased firms' output production. Obviously, when the market competition is not fierce the first effect dominates the second one and therefore, the firms' profits under the endogenous advertising configuration exceed those of the mere comparative one. The opposite holds when the market competition is fierce.

Finally, regarding the welfare effects of the firms' advertising investments, we demonstrate that, the consumers surplus is the highest under the endogenous advertising configuration. This is so, since the consumers' surplus follows the same pattern with the firms' output production under the alternative advertising configurations. Further, the total welfare is the highest under the mere informative advertising configuration while, it is the lowest under the mere comparative configuration. Yet, we show that the welfare under the endogenous advertising configuration can be either higher or lower than that of the benchmark case depending on the degree of competition. In particular, the welfare in the endogenous advertising configuration exceed that of the benchmark iff the degree of market competition is relatively low while, the opposite holds iff the degree of market competition is high. Intuitively, when the degree of product substitutability is relatively low, the beneficial effect that the use of advertising has over the consumers' surplus, due to the higher output and better informed consumers,

compensates the detrimental effect that the increased output and advertising competition has over profits. The opposite stands when the degree of product substitutability is relatively high.

Our paper contributes to the recent branch of the economic literature that examines the use of comparative advertising in imperfectly competitive markets. This literature has its origins in Aluf and Shy (2001) where using a Hotelling model, with comparative advertising to increase the transportation cost to the rival's product, show that the use of comparative ads weakens price competition by enhancing the degree of product differentiation and leads to higher prices and profits. In a different vein, Barigozzi et al. (2009), examine comparative advertising as a mean to signal quality, by considering a market where an entrant, whose quality is unknown, decides between the non comparative advertising (i.e, a standard money burning to signal quality) or comparative advertising (i.e., a comparison over the qualities of the products) in order to face an incumbent whose quality is known.⁴ They show that the entrant's incentives to use comparative advertising are close related with the quality of its product and the penalty that the entrant is going to pay if the content of the advertising campaign is manipulative.⁵ Similarly, Emons and Fluet (2008) examine the signaling role of comparative advertising in a duopolistic market where both firms use comparative advertising to highlight their quality differential and the cost of advertising increases as the firms move away from the truth. Further, Anderson and Renault (2009) considered comparative advertising as a mean through which firms' can disclosure information about the horizontal match characteristics of the products and reveal information about the rival's product attributes that the latter might not wish to communicate. They show that when the products are of similar quality firms have incentives to advertise only their own goods and thus, comparative advertising plays no role since, full product information is provided regardless. On the other hand, when the products are of sufficiently different qualities, only the low quality firm has strong incentives to use comparative advertising (if it is legal) in order to reveal the horizontal attributes of both goods and thus, the low quality firm has the opportunity by improving its consumers base to survive in the market.

The present paper contributes to the existing literature in four ways. First, unlike the

⁴The signaling role of advertising is based on the idea that high advertising spending work as a device designed to signal high quality (e.g. Nelson, 1974; Kihlstrom and Riordan, 1984; Milgrom and Roberts, 1986).

⁵They assumed that when the entrant uses comparative advertising, the incumbent has the opportunity to go to the court and obtain gains if the court verdict is that the advertising is manipulative and the entrant's true quality is low.

bulk of the literature that approaches comparative advertising exogenously, we examine the firms' incentives to invest in comparative and informative advertising endogenously by considering the investment level in each type of advertising as a strategic firm's decision. Second, we provide results over the optimal advertising portfolio, or else, the optimal allocation of firms' advertising expenditures between comparative and informative advertising, when both types of advertising are available in the market. Third, considering a duopolistic market with horizontally differentiated products we provide results on the impact of the degree of market competition on the firms' expenditures on each type of advertising. Forth, comparing our results with alternative advertising configuration in the absence of one of the two types of advertising we provide results over the effects that the firms alternative marketing strategies could have on the market performance and the social welfare.

The rest of the paper is organized as follows. In Section 2, we present our basic model. In the section 3, we adduce the equilibrium analysis of the endogenous advertising configuration and we consider the comparisons with the benchmark case without advertising activities, the mere informative and the mere comparative configurations. In section 4, we examine the robustness of our results by extending our model in markets with price competition, advertising cost asymmetries and alternative demand functions. Finally, section 5 concludes. All proofs are demonstrated in the Appendix.

2 The Basic Model

We consider a market consisted by two firms denoted by, $i, j = 1, 2, i \neq j$, each producing one brand of a differentiated good. Firms are profit maximizers and have on their set of marketing strategies both informative and comparative advertising. In the market, there is a unit mass of consumers composed by individuals with homogenous preferences regarding the two goods. The utility function of the representative consumer, following Häckner (2000), is given as follows,

$$U = (\alpha + \mu_i + \kappa_i - \kappa_j)q_i + (\alpha + \mu_j + \kappa_j - \kappa_i)q_j - [q_i^2 + q_j^2 + 2\gamma q_i q_j]/2 + m \quad (1)$$

where, q_i , is the quantity of good i bought by the representative consumer and m is the respective quantity of the "composite good". The parameter $\gamma \in [0, 1]$ denotes the degree of product substitutability, with $\gamma \rightarrow 0$ corresponding to the case of independent goods and $\gamma \rightarrow 1$

to the case of perfect substitute goods. Alternatively, γ can be interpreted as the intensity of competition in the market with higher γ corresponding to higher degree of competition between firms (Vives, 1985).

In line with, Chakrabarti and Haller (2011), firms' expenditures in informative advertising, μ_i , provide to the initially uninformed consumers the necessary information in order to identify the good that covers better their needs. Clearly, given that initially not all of the consumers possess all the relevant information about the product, we assume that informative advertising increases the consumers' realized utility, or in a different setting, it increases the consumers' valuation over the advertised product.⁶ Further as in Anderson et al. (2008), firms' investments in comparative advertising, κ_i , have a dual effect on consumers' valuation, they increase the consumers' valuation of the positively advertised product while on the same time, they decrease the consumers' valuation of the targeted good.

Maximizing the (1) with respect to q_i and q_j we obtain the inverse demand function of the representative consumer that is given by,

$$p_i(q_i, q_j) = \alpha + \mu_i + \kappa_i - \kappa_j - q_i - \gamma q_j \quad (2)$$

where p_i , denotes the price of good i , while the price of the "composite good" has been normalized to unit. Note here that the inverse demand function is positively related to the own firm's expenditures in advertising while, is negatively related to the rival's expenditures in comparative advertising. That is, each firm i 's own investments in informative and comparative advertising shifts its the demand curve outwards while, the firm j 's investments in comparative advertising shifts the firm i 's demand curve inwards.

Firms are endowed with identical constant returns to scale production technologies, with their marginal production cost given by c , $0 \leq c < \alpha$. The cost of each type of advertising is quadratic and separable with diminishing returns of advertising expenditures and is given by, $b(\mu_i^2 + \kappa_i^2)$. Parameter b denotes the effectiveness of the advertising technology on shifting the consumers' demand, with higher b corresponding to a less effective advertising technology and therefore, to higher required expenditures by firms in order to obtain a given shift on consumers demand. As standard in the advertising literature, the convexity assumption, reflects that the

⁶ More details on the definition of informative advertising in our model are provided on the extension section, Informative Advertising.

cost of advertising is increasing in the number of consumers.⁷ Regarding now the separability of costs, a recent strand of managerial literature, considers the fact that the vast advances in media technology have created the need for specialization in different advertising techniques applied by the corresponding agencies. According to Horsky (2006), firms would prefer to use different agencies to promote their product in different channels, based on their specialization.⁸ In our case, given the different handling required for informative and comparative ads, due to the differential perception of consumers, we treat the two kinds of advertising as separate projects with different costs.

Further, to guarantee that all the participants are active in the market under all the configurations considered the following assumption, should hold throughout the paper.

$$b(\gamma) \geq \frac{8 + 4\gamma + \gamma^2}{(\gamma^2 - 4)^2} \quad (3)$$

That assumption implies that the effectiveness of advertising investments is not extremely high and it is in line with Peters (1984) and Bester and Petrakis (1995) who claim that in some cases firms are better off under advertising restrictions.

Thus, firm i 's profits are given by,

$$\Pi_i = (\alpha + \mu_i + \kappa_i - \kappa_j - q_i - \gamma q_j)q_i - cq_i - b(\mu_i^2 + \kappa_i^2) \quad (4)$$

Clearly, the advertising investments of firm i , tend to increase its' profitability while, the costs of own advertising activities, as well as, the rival's comparative advertising expenditures tend to diminish firm i 's profitability.

3 Equilibrium Analysis

In this section we provide the analysis of our basic model named as, endogenous advertising configuration, where in the market firms decide endogenously upon the type(s) and the expenditure levels on each type of advertising and then, compete by setting their outputs. Further,

⁷See for instance, Hamilton (2009), Hernandez-Garcia, (1997), Bester and Petrakis (1996), Grossman and Sharipo, (1984) and Butters (1977).

⁸Arzaghi et al.(2008) mention that advertising agencies in the US have moved from "full service provider" of advertising campaigns to providers of specialized services. Therefore agency compensation has moved from a proportional commission based on final number targeted consumers to "fee for service" provided by each agency.

we briefly analyze three alternative configurations that may occur in a market named as, "no advertising configuration", "mere informative advertising configuration" and "mere comparative advertising configuration", that denote respectively, the case where firms do not advertise their product, the case where firms promote their product via mere informative advertising and the case where firms promote their product via mere comparative advertising.

3.1 No Advertising Configuration.

In this subsection we briefly analyze our benchmark case where, in the market does not exist advertising, i.e., $\mu_i = \mu_j = 0$ and $\kappa_i = \kappa_j = 0$. Thus, the market's outcomes corresponds to the standard Cournot game with horizontally differentiated goods.

Firm i chooses its output q_i , taken as given the rival's decision over the output, q_j , in order to maximize its profits given by,

$$\pi_i = (a - q_i - q_j)q_i - cq_i \quad (5)$$

The first order conditions give rise to the firm i 's reaction function,

$$R_i^N(q_j) = \frac{a - \gamma q_j - c}{2} \quad (6)$$

thus, in the equilibrium firm i 's output, price and profits are given respectively by,

$$q^N = \frac{(a - c)}{2 + \gamma}, \quad p^N = c + \frac{a - c}{2 + \gamma}, \quad \pi^N = \frac{(a - c)^2}{(2 + \gamma)^2} \quad (7)$$

Further, the consumers' surplus and the total welfare are given by,

$$CS^N = (1 + \gamma) \frac{(a - c)^2}{(2 + \gamma)^2}, \quad TW^N = (3 + \gamma) \frac{(a - c)^2}{(2 + \gamma)^2} \quad (8)$$

3.2 Endogenous Advertising Configuration.

In this subsection, we proceed with the analysis of our basic model, where firms have on their set of marketing strategies both informative and comparative advertising.

In the last stage of the game, firm i chooses its output q_i in order to maximize its profits, taking as given the rival's output q_j along with the expenses in advertising $(\mu_i, \mu_j, \kappa_i, \kappa_j)$, decided in the first stage of the game.

The first order conditions of (4), give rise to the firm *i*'s reaction function,

$$R_i^E(q_j) = \frac{\alpha - \gamma q_j - c}{2} + \frac{\mu_i + \kappa_i - \kappa_j}{2} \quad (9)$$

Notice that, comparing $R_i^E(q_j)$ with the reaction function of the benchmark case, $R_i^C(q_j)$, in which only the first part of (9) appears, we observe that firm *i*'s expenditures on informative and comparative advertising (μ_i, κ_i) tend to shift $R_i^E(q_j)$ outwards and thus, tend to increase firm *i*'s equilibrium output production. On the contrary, the rival's firm investment in comparative advertising (κ_j) tends to shift $R_i^E(q_j)$ inwards and therefore, tend to decrease the firm *i*'s equilibrium output.⁹

Solving the system of the reaction functions (9), firm *i*'s equilibrium output and profits in the second stage are given respectively by,

$$q_i^E(\mu_i, \mu_j, \kappa_i, \kappa_j) = \frac{(2 - \gamma)(\alpha - c) + 2(\mu_i + \kappa_i - \kappa_j) - \gamma(\mu_j + \kappa_j - \kappa_i)}{4 - \gamma^2} \quad (10)$$

$$\Pi_i^E(\mu_i, \mu_j, \kappa_i, \kappa_j) = [q_i^E(\mu_i, \mu_j, \kappa_i, \kappa_j)]^2 - b(\mu_i^2 + \kappa_i^2) \quad (11)$$

Observe here that, $\frac{\partial q_i^E}{\partial \mu_i} = \frac{2}{4 - \gamma^2} > 0$, $\frac{\partial q_i^E}{\partial \kappa_i} = \frac{1}{2 - \gamma} > 0$, that means that the firm *i*'s output is positively related to its own investments in advertising. The intuition behind this result comes straightforward from the fact that firm *i*'s advertising investments shifts its demand curve outwards. Further, $\frac{\partial q_i^E}{\partial \kappa_j} = -\frac{1}{2 - \gamma} < 0$, that is firm *i*'s output is negatively connected to the rival's investments in comparative advertising. Intuitively, given the the denigrating effect of comparative ads, firm *j*'s investments in comparative advertising shift firm *i*'s demand curve inwards and thus, it tends to decrease the latter's output production. In addition, we observe that the output is negatively connected to the degree of market competition, γ , and the advertising effectiveness parameter, b , (i.e., $\frac{\partial q_i^E}{\partial \gamma} < 0$, $\frac{\partial q_i^E}{\partial b} < 0$). Clearly, a fierce market competition leads firms to lower output production since, by the equation (9) we have that $\frac{\partial R_i^E(q_j)}{\partial q_j} = -\frac{\gamma}{2}$, that means that, as the market competition increases, the slope of the reaction function increases and therefore, the equilibrium output decreases. Further, the equilibrium output decreases as the advertising technology gets less effective, or in other words, firms incur

⁹Note also that the slope of firm *i*'s reaction curve is, $\frac{\partial R_i^E(q_j)}{\partial q_j} = -\frac{\gamma}{2}$, implying that is an outward and parallel shift of the respective curve in the benchmark case.

higher advertising costs. This is because, a high advertising cost lead firms to decrease their investments in advertising that in turn, given the positive relation between each own firm's advertising expenditures and output production, leads to a reduction on the firms' output.

In the first stage of the game, firm i chooses the expenditure level of each type of advertising (μ_i, κ_i) , taking as given the rival's firm decisions (μ_j, κ_j) , in order to maximize its profits given in (11).

The first order conditions of the equation (11), give rise to the best reply functions of informative and comparative advertising, given respectively by,

$$\mu_i^E(\mu_j) = \frac{2[(2 - \gamma)(\alpha - c) + (2 + \gamma)(\kappa_i - \kappa_j) - \gamma\mu_j]}{b(4 - \gamma^2)^2 - 4} \quad (12)$$

$$\kappa_i^E(\kappa_j) = \frac{(2 - \gamma)(\alpha - c) - (2 + \gamma)\kappa_j - \gamma\mu_j + 2\mu_i}{[b(2 - \gamma)^2 - 1](\gamma + 2)} \quad (13)$$

Observe here that, $\frac{\partial \mu_i}{\partial \mu_j} < 0$, $\frac{\partial \kappa_i}{\partial \kappa_j} < 0$ and $\frac{\partial \mu_i}{\partial \kappa_j} < 0$, that means that, there exists strategic substitutability between each own firm's investments in advertising and the corresponding values of the rival's firm investments. Clearly, each firm tend to increase its own investments in both types of advertising in order to outweigh the impact of the rival's firm advertising and to remain competitive in the market. Further, $\frac{\partial \mu_i}{\partial \mu_i} > 0$ and $\frac{\partial \kappa_i}{\partial \mu_i} > 0$, that is, each own firm's investments in informative and comparative advertising are strategic complements. That is so because, the two alternative advertising strategies are being perceived as separate advertising tequinques by the firms' that lead firms to increase their investments in both types of advertising in order to extract the maximum of the positive effects that each marketing strategy provides.

Solving the system of the first order conditions, the equilibrium investment levels in informative and comparative advertising are given respectively by,

$$\mu^E = \frac{2(\alpha - c)}{b(2 - \gamma)(2 + \gamma)^2 - 2} > 0 \quad (14)$$

$$\kappa^E = \frac{(\alpha - c)(2 + \gamma)}{b(2 - \gamma)(2 + \gamma)^2 - 2} > 0 \quad (15)$$

Proposition 1 *In the equilibrium, firms invest in both informative and comparative advertising.*

Clearly, in the equilibrium firms undertake a mix advertising strategy in order to extract the benefits of both types of advertising. In other words, firms invest in informative advertising due to its direct effect on increasing the firm's demand while, they invest in comparative advertising in order to be benefit by the indirect effect of the denigration of the rival's product (i.e., the reduction on the rival's firm demand). Further, we observe that the firms' expenditures on comparative advertising are positively related to the degree of market competition, i.e., $\frac{\partial \kappa^E}{\partial \gamma} > 0$. Intuitively, the fierce market competition lead firms to increase their investment levels in comparative advertising as an attempt to enlarge their market shares by obtaining, via the denigration of the rival's product, part of the rival's market share. We also note that, $\frac{\partial \mu^E}{\partial \gamma} < 0$ if $\gamma < \hat{\gamma} \equiv 0.666667$ and $\frac{\partial \mu^E}{\partial \gamma} > 0$ if $\gamma > \hat{\gamma}$, that means that the investment levels in informative advertising are non monotonically (U shaped) related to the degree of market competition. The intuition behind this result is that firms are willing to decrease their investment levels in informative advertising when the market competition is not fierce, or else, when the products are not close substitutes (i.e, the products' attributes can be easily recognized by consumers) in order to reduce the advertising costs that they incur. On the contrary, the fierce market competition intensifies the advertising competition and lead firms to increase their investment levels in informative advertising. Further, we observe that, $\frac{\partial \mu^E}{\partial b} < 0$, $\frac{\partial \kappa^E}{\partial b} < 0$. Thus, the less effective the advertising technology is (i.e., high b), the lower are the expenditures on both types of advertising.

Lemma 1 *i) Equilibrium investments in informative advertising are non-monotonically -U shaped -related to the degree of market competition, γ , while, they are decreasing in the advertising effectiveness parameter, b .*

ii) Equilibrium investments in comparative advertising are increasing in the degree of market competition, γ , while they are decreasing in the advertising effectiveness parameter, b .

Regarding the optimal advertising mix we show that, $\frac{\mu_i^*}{\kappa_i^*} = \frac{2}{(\gamma+2)}$ with, $\frac{\partial \mu_i^*/\kappa_i^*}{\partial \gamma} < 0$. As it follows from the optimal advertising mix, firms in the equilibrium adopt an aggressive advertising behavior, given that the firms' investment levels in comparative advertising always exceed the respective ones of informative advertising, for $\gamma \neq 0$. That means that, even if the market competition is low, or else, the products are not close substitutes (i.e., low γ) firms realize comparative advertising as a credible rival's firm threaten and their best reply to that threaten is to increase the own investment levels in comparative advertising.

As it is clear from the above analysis, as the market competition becomes fiercer, the advertising competition increase and thus, firms invest more in both types of advertising. Intuitively, the increased market competition lead firms to increase their investments in advertising as an attempt to enlarge their market shares, by informing the previously uninformed consumers about the product characteristics and by convincing consumers via comparative advertising messages, that their product is superior than the rival's one.

Substituting equations (14) and (15) into (9) and (4), firms' equilibrium output and profits are given by,

$$q^E = \frac{b(\alpha - c)(4 - \gamma^2)}{b(2 - \gamma)(\gamma + 2)^2 - 2} \quad (16)$$

$$\Pi^E = \frac{(\alpha - c)^2 [b(4 - \gamma^2)^2 - \gamma^2 - 4\gamma - 8]}{[b(2 - \gamma)(\gamma + 2)^2 - 2]^2} \quad (17)$$

From the above equations the following observations are in order. First, the equilibrium output is negatively connected with both the degree of competition, γ , and the advertising effectiveness parameter, b , (i.e., $\frac{\partial q^E}{\partial \gamma} < 0$, $\frac{\partial q^E}{\partial b} < 0$). As standard in the literature, increased market competition leads firms to produce lower output, given the negative slope of the cournot reply function, in our case, $\frac{\partial R_i^E(q_j)}{\partial q_j} = -\frac{\gamma}{2}$. Further, according to Lemma 1 and the analysis after (10), a less effective advertising technology leads to lower advertising investments and thus, to lower equilibrium output. Second, $\frac{\partial \Pi^E}{\partial \gamma} < 0$ and $\frac{\partial \Pi^E}{\partial b} > 0$, thus, the equilibrium profits are negatively connected with the degree of competition, γ while, they are positively connected to the advertising effectiveness. Clearly, the lower the effectiveness of advertising technology is (i.e., high b), the higher is the profitability of firms. The rationale behind the latter result lies on the fact that a less effective advertising technology, or in other words higher cost of advertising, discourage firms from engaging in unnecessary advertising warfare that acts beneficial to their profitability.

Lemma 2 *Equilibrium output and profits are decreasing in the degree of market competition, γ . Equilibrium output is decreasing while, the equilibrium profits are increasing, in the advertising effectiveness parameter, b .*

3.2.1 Welfare Analysis.

In this subsection we analyze the societal effects of the firms' mix advertising strategies.

Total welfare is defined as the sum of consumers and producers surplus:

$$TW = CS + 2\Pi \quad (18)$$

With CS and 2Π corresponding to the consumers surplus and the overall market profits respectively. In particular, the consumer surplus for the representative consumer is given by the following expression:

$$CS^E = (\alpha + \mu_i + \kappa_i - \kappa_j)q_i + (\alpha + \mu_j + \kappa_j - \kappa_i)q_j - \frac{1}{2}(q_i^2 + q_j^2 + 2\gamma q_i q_j) - p_i q_i - p_j q_j \quad (19)$$

Imposing symmetry, $q_i^E = q_j^E = q^E$, $\mu_i^E = \mu_j^E = \mu^E$, $\kappa_i^E = \kappa_j^E = \kappa^E$, $p_i^E = p_j^E = p^E$, the social welfare can be written as,

$$CS^E = (1 + \gamma)[q^E]^2 \quad (20)$$

while, with respect to (19), (20),(16), the total welfare can be written as,

$$TW^E = \frac{(\alpha - c)^2 b [b(3 + \gamma)(4 - \gamma^2)^2 - 16 - 2\gamma(\gamma + 4)]}{[b(2 - \gamma)(\gamma + 2)^2 - 2]^2} \quad (21)$$

Obviously, the consumers' surplus follows the same patterns as the equilibrium output and is decreasing in the degree of market competition (i.e., $\frac{\partial CS^E}{\partial \gamma} < 0$) while, it is increasing in the effectiveness of advertising technology (i.e., $\frac{\partial CS^E}{\partial b} < 0$). Further, regarding the total welfare we show that, it is decreasing in the degree of market competition (i.e., $\frac{\partial TW^E}{\partial \gamma} < 0$) while, it is non monotonically U-shaped related to the effectiveness of advertising technology parameter, b (i.e., $\frac{\partial TW^E}{\partial b} < 0$ iff $\gamma < \check{\gamma}(b)$ and $\frac{\partial TW^E}{\partial b} > 0$ iff $\gamma > \check{\gamma}(b)$, with $\frac{\partial \check{\gamma}}{\partial b} > 0$ and $\check{\gamma}(1.5) = 0.366359$). The intuition behind this result is driven by two opposing effects. First, given the analysis after (15) the equilibrium output, profits and consumers' surplus are decreasing in the degree of market competition, γ , that acts diminishing to the welfare. Second, profits are increasing in the effectiveness of advertising b , that is beneficial to the welfare. Results in equilibrium reveal that the second effect dominates the first for $\gamma < \check{\gamma}(b)$, whilst the opposite holds for $\gamma > \check{\gamma}(b)$.

Lemma 3 *In equilibrium, consumers' surplus is decreasing in both the degree of market competition, γ , and the advertising effectiveness parameter, b . Welfare is decreasing in the degree of market competition, γ , while it is U-shaped related to the advertising effectiveness parameter, b .*

3.3 Mere Informative Advertising Configuration.

In this subsection we consider the case where firms invest only in informative advertising.

¹⁰ That is, $\kappa_i^I = \kappa_j^I = 0$ and thus, firm i 's inverse demand function is given now by, $p_i^I = \alpha + \mu_i - q_i - \gamma q_j$ while, firm i 's profits function is given by, $\Pi_i^I(\cdot) = (\alpha + \mu_i - q_i - \gamma q_j)q_i - b\mu_i^2$.

Employing standard backward induction, we obtain that the firms' equilibrium investment in informative advertising are given by,

$$\mu^I = \frac{\alpha - c}{b(2 + \gamma)^2 - 1} \quad (22)$$

Comparing the equilibrium advertising investment under the mere informative advertising configuration, with the those obtained in the endogenous advertising configuration, the following Proposition derives:

Proposition 2 *Equilibrium investments in informative advertising under the endogenous advertising configuration always exceed those of the mere informative advertising configuration (i.e., $\mu^E > \mu^I$).*

Intuitively, the absence of comparative advertising in the market, relaxes the advertising market competition and thus, leads firms to invest less in informative advertising under the current configuration than under the endogenous advertising one.

Further, the equilibrium output, profits, consumers' surplus and total welfare respectively are given by,

$$q^I = \frac{b(a - c)(\gamma + 2)}{b(2 + \gamma)^2 - 1}, \quad \Pi^I = \frac{b(a - c)^2}{b(2 + \gamma)^2 - 1} \quad (23)$$

$$CS^I = \frac{b^2(1 + \gamma)(a - c)^2(\gamma + 2)^2}{[b(2 + \gamma)^2 - 1]^2}, \quad TW^I = \frac{b[b(\gamma + 2)^2(3 + \gamma) - 2](a - c)^2}{[b(2 + \gamma)^2 - 1]^2} \quad (24)$$

¹⁰This discussion has been motivated by two alternative facts. First, even if the countries legislation framework does not prohibit the use of comparative advertising, firms tend to avoid this aggressive marketing practice because of the high risk to be accused for an attempt to mislead consumers and be prosecuted by the rival to the court (See for instance, Barigozzi and Peitz, 2006, and Barigozzi et al.,2006). In 2000 Papa John's was forced by the court to pay over 468.000\$ in damages to Pizza Hut due to the advertising campaign "Better ingredients. Better pizza" that has been judged as misleading since, such claims can not be proved. Second, the fact that consumers may perceive a firm's comparative advertising campaign as manipulative and thus, as a non trustworthy source of information (Wilkie and Farris, 1975; Barone and Miniard, 1999.)

3.4 Mere Comparative Advertising Configuration

This subsection examines the case in which firms invest only in comparative advertising.¹¹ In other words, $\mu_i = \mu_j = 0$ and thus, firm i 's inverse demand function is given now as, $p_i^K = \alpha + \kappa_i - \kappa_j - q_i - \gamma q_j$ while, firm i 's profits function is given by, $\Pi_i^K(.) = (\alpha + \kappa_i - \kappa_j - q_i - \gamma q_j)q_i - b\kappa_i^2$. Employing standard backward induction, firm i 's equilibrium investments in comparative advertising given,

$$\kappa_i^C = \frac{\alpha - c}{b(4 - \gamma^2)} \quad (25)$$

Comparing the equilibrium advertising investments under the mere comparative advertising configuration with those obtained under the endogenous advertising configuration, the following Proposition derives:

Proposition 3 *Equilibrium investments in comparative advertising under the Endogenous advertising configuration always exceed those of the Mere comparative advertising configuration (i.e., $\kappa^E > \kappa^C$).*

Let us now unravel the reasoning behind this result. Recall the pro-competitive nature of comparative advertising, through increasing own demand and decreasing the rivals demand. By considering (2), under the endogenous advertising configuration, firms could compensate from the losses in their demand due to the comparative advertising investments of the rival, by increasing their demand through investments in informative advertising. Yet, under the current configuration, due to the lack of such compensation mechanism, both firms have the opportunity to restrict the advertising warfare to their benefit, by decreasing their investments in comparative advertising.

The equilibrium results regarding output, profits, consumers' surplus and total welfare are given by,

$$q^C = \frac{a - c}{2 + \gamma}, \quad \Pi^C = \frac{[b(\gamma - 2)^2 - 1](a - c)^2}{b(\gamma^2 - 4)^2} \quad (26)$$

$$CS^C = (1 + \gamma) \frac{(a - c)^2}{(2 + \gamma)^2}, \quad TW^C = \frac{(a - c)^2 [(b(\gamma - 2)^2(\gamma + 3) - 2)]}{b(\gamma^2 - 4)^2} \quad (27)$$

¹¹In this subsection we have exclude informative advertising from firms' set of marketing strategies, assuming that consumers are merely informed about the product characteristics, attributes e.t.c and comparative advertising provides full information in relevance to the rival good in the market.

3.5 Comparative results.

In this subsection we compare our results obtained under the alternative market configurations, in order to examine how the two different types of advertising affect the market outcomes and the social welfare. Starting with the comparison of the equilibrium market outcomes the following proposition derives.¹²

Proposition 4 *(i) Equilibrium output takes the highest value under the Endogenous advertising configuration, the lowest under the Mere comparative advertising configurations that equals that of the benchmark, while it lies in between under the Mere informative advertising configuration ($q^E > q^I > q^C = q^N$).*

(ii) Equilibrium profits under the Mere informative advertising configuration, are always higher than those in the benchmark. Equilibrium profits under the Mere comparative and the Endogenous advertising configuration are always lower than those in the benchmark. Yet, equilibrium profits under the Mere comparative advertising are lower than in the Endogenous advertising one, if the degree of market competition is relatively low, while the opposite holds for relatively high degree of market competition ($\Pi^I > \Pi^N > \max[\Pi^C, \Pi^E]$, $\Pi^C < \Pi^E$ iff $\gamma < \hat{\gamma}(b)$, with $\frac{\partial \hat{\gamma}}{\partial b} > 0$ and $\hat{\gamma}(1.5) = 0.483614$ and $\Pi^C > \Pi^E$ iff $\gamma > \hat{\gamma}(b)$).

Let us now unravel the intuitions that drive the above results. Clearly, in the mere informative advertising configuration the beneficial effect of informative advertising over the demand leads to higher firms' output production comparing to the benchmark case. Further, according to the Proposition 2 and Proposition 3, in the endogenous advertising configuration firms' expenditures in advertising are higher than under all the other configurations. Therefore, given the positive relation of the output and the firms' advertising expenditures, the highest output production is obtained when in the market exist both types of advertising. Note also that, in the mere comparative advertising configuration the firms' output production equals that of the benchmark since, the effects of the own and rival's comparative advertising investments neutralize each other in the equilibrium.

As far as the classification of profits is being concerned, we show that the mere informative advertising leads to higher firms' profitability than any other configuration. This is so, because informative advertising enlarge the market share that each firm possess by shifting its demand

¹²For proof see the appendix.

curve outwards. Further, under the mere comparative advertising configuration, we observe that firms' profits are lower than that of the benchmark case since, the beneficial effect on the firm's profitability due to its own investments in the comparative advertising is neutralized by detrimental effect of the corresponding investments of the rival firm. Thus, both firms end up to a prisoners dilemma situation where they conclude to be worst off comparing to the benchmark. Regarding the endogenous advertising configuration, the following observations hold. First, comparing to the benchmark case, it is clear that enhancing effect over the profits of each own firm's investments in advertising could not compensate the negative effects of the rival's investments in comparative advertising, the high advertising expenses (i.e. overinvestment in both types of advertising) and the fierce output competition. Thus, firms performance under the endogenous advertising configuration in terms of profitability is worse than in the benchmark. Second, comparing with the mere comparative configuration we show that $\Pi^C < \Pi^E$ iff $\gamma > \hat{\gamma}(b)$ while, $\Pi^C > \Pi^E$ iff $\gamma < \hat{\gamma}(b)$. The intuition behind this result is based on two alternative effects. On the one hand, under the mere comparative advertising the total advertising investments are lower than under the endogenous advertising configuration, that implies lower advertising costs, and thus higher firms' profitability. On other hand, under the mere comparative advertising configuration, the lack of the beneficial effect that informative advertising expenditures have on the firms' demand and thus, on the firms' profitability tend to decrease the firms' profits comparing to those obtained under the endogenous advertising configuration. Clearly when the market competition is fierce (i.e., $\gamma > \hat{\gamma}(b)$) the first effect dominates the second one, given that the fiercer market competition leads firms to increase their investments in advertising and thus firms' cost saving becomes significant. The opposite holds, when the market competition is relaxed (i.e., $\gamma < \hat{\gamma}(b)$) since, the firms' overall investments in advertising are lower.

Let us now compare the equilibrium societal outcomes among the alternative configuration. The following Proposition summarizes:¹³

Proposition 5 *(i) Equilibrium consumers' surplus takes the highest value under the Endogenous advertising configuration, the lowest under the Mere comparative advertising configuration that equals that of the benchmark, while it lies in between under the Mere informative advertising configuration ($CS^E > CS^I > CS^C = CS^N$).*

¹³For proof see the appendix.

(ii) *Equilibrium total welfare takes the highest value under the Mere informative advertising configuration, the lowest under the Mere comparative advertising one. Total welfare is higher in the Endogenous advertising configuration than in the benchmark iff the degree of market competition is relatively low. The inverse relation holds if the degree of market competition is relatively high ($TW^I > \max[TW^N, TW^E] > TW^C$, while $TW^E > TW^N$ iff $\gamma < \tilde{\gamma}(b)$, with $\frac{\partial \tilde{\gamma}}{\partial b} > 0$ and $\tilde{\gamma}(1.5) = 0.441068$ and $TW^E < TW^N$ iff $\gamma > \tilde{\gamma}(b)$.*

We turn our discussion to the main arguments that drive the above results. Advertising increases consumers' surplus, since it increases the consumers' information about the firms' products, intensifies market competition and leads to higher total output. Clearly, the consumers' surplus follows the same patterns as output, the rationale behind the former is based on the same arguments that lie behind the analysis after Proposition 4.

Regarding welfare it is obvious that the effect of the higher firms' profits in the mere informative advertising configuration, dominates over the effect of the higher consumers' surplus in the endogenous advertising one and the benchmark and thus, $TW^I > TW^N, TW^E$. Considering now the mere comparative case, we have that the welfare is lower than in the benchmark case, due to the lower profits that firms obtain under the former configuration and the equality of the consumers' surplus between two configurations. Further, comparing the mere comparative configuration with the endogenous advertising configuration, we have that the lower level of consumers' surplus dominates over the any positive effects of higher profitability when products are not close substitutes and therefore welfare in the mere comparative advertising configuration is always lower than in the endogenous one. From the above it is clear that, the existence of comparative advertising, creates a prisoner's dilemma situation that in turn leads to lower profits and welfare comparing to the mere informative advertising configuration and the benchmark. This results offers important policy implications, leading to the conclusion that comparative advertising can be characterized as "wasteful advertising" since both firms and consumers can be better off, if this aggressive form of advertising has been prohibited.¹⁴ Regarding the endogenous advertising configuration, from (18), Proposition 4 and the above analysis there are two opposite effects on welfare comparing to the corresponding values in the

¹⁴The term "wasteful advertising" was first introduced by Pigou (1924), in order to describe the prisoner's dilemma which arises when competing firms in a market invest equal efforts in advertising in order to attract the favor of the public from the others. As Pigou first showed this concludes in a prisoner's dilemma where none of the firms gains anything at all.

benchmark: a positive effect due to the higher consumers' surplus and a negative effect due to lower profitability. Results in equilibrium reveal that when the products are poor substitutes the prevailing effect is the first, the opposite holds when the products are close substitutes. Thus, $TW^E > TW^N$ iff $\gamma < \tilde{\gamma}(b)$ and $TW^E < TW^N$ iff $\gamma > \tilde{\gamma}(b)$.

4 Extensions-Discussion.

In this section we examine a number of modifications of the basic model in order to briefly discuss the robustness of our results.¹⁵

4.1 Bertrand competition.

We extend our analysis by examining the robustness of our results under price market competition. That is, each firm i faces a standard linear demand given now by:

$$q_i = [a(1 - \gamma) + \mu_i - \gamma\mu_j + (1 + \gamma)(\kappa_i - \kappa_j) - p_i + \gamma p_j] / (1 - \gamma^2)$$

Keeping all the other modeling specifications fixed, we reconfirm that our main results do not change qualitatively under price market competition. More specifically, we show that in equilibrium firms always invest in both informative and comparative advertising while, all the other outcomes (i.e., firms' profits, consumers surplus, social welfare) follow the same pattern as in the market where the firms competition take place in quantities. Yet, we observe that the Assumption 1 over the advertising effectiveness parameter b , alters to $\bar{b} \geq -8 - \gamma(4 + \gamma(2(\gamma - 1)\gamma - 7)) / (\gamma^2 - 4)^2(\gamma^2 - 1)$ with $\bar{b} > b$ under price competition. Clearly, the effectiveness parameter \bar{b} under price competition is stricter than that under quantity competition. Intuitively, we know from the seminal paper of Singh and Vives (1984) that competition is fiercer and profits are lower under price than under quantity competition. Thus, when firms compete in prices, will require lower effectiveness of advertising technology in order to engage efficiently in advertising investments, than in output competition.

4.2 The n -firm case

¹⁵For each extension discussed below, the detailed analysis is available from the authors upon request.

In this section we extend our analysis to an industry with $n > 2$ firms, that produce differentiated products. Each firm i , $i = 1, 2, \dots, n$, faces an inverse demand function given by, $p_i = \alpha + \mu_i + \kappa_i - K_{-i} - q_i - \gamma Q_{-i}$, where $K_{-i} = \sum_{j \neq i} \kappa_j$, $Q_{-i} = \sum_{j \neq i} q_j$ with all other parameters being defined as in the duopoly case.¹⁶ The rest specifications of the model, as well as, the timing of the game, follows the same pattern as our basic model given in Section 2.

Thus, the profit function of each firm i is now given by,

$$\Pi_i = (\alpha + \mu_i + \kappa_i - K_{-i} - q_i - \gamma Q_{-i})q_i - cq_i - b(\mu_i^2 + \kappa_i^2)$$

Solving the game backwards we obtain the firms' equilibrium level comparative and informative advertising, given, respectively, by,

$$\mu^S = \frac{2(\alpha - c)[2 + \gamma(n - 2)]}{b(2 - \gamma)[(2 + \gamma(n - 1))^2 - 2 - \gamma(n - 2)]}, \quad \kappa^S = \frac{2(\alpha - c)[2 + \gamma(n - 1)]}{b(2 - \gamma)[(2 + \gamma(n - 1))^2 - 2 - \gamma(n - 2)]} \quad (28)$$

Clearly our basic result, given in the Proposition 1, holds independently of the market size. Notice here, that the firms' expenditures in advertising decrease as the number of the firms in the market increase. This is so because, as the market size becomes larger the relative weight of each firm in the market decrease and thus, any potential benefit from the advertising expenditures decrease. Further, note that the optimal advertising mix $\frac{\mu^S}{\kappa^S} = \frac{2 + \gamma(n - 2)}{2 + \gamma(n - 1)}$ with, $\frac{\partial \mu^S / \kappa^S}{\partial n} > 0$. That means that, as the market size increase, the firms substitute the informative advertising with the more aggressive comparative advertising. The intuitions behind this result comes straightforward from the fact that as the market size increases the denigration effect that comparative advertising may have on the rival firms demand increase.

Further, the equilibrium output, profits, consumers' surplus and total welfare respectively are given by,

¹⁶The inverse demand functions are derived by aggregating individual demand functions of individuals who have homogenous preferences regarding the n products. In particular, following Häckner (2000), the utility function of the representative consumer is given by: (??): $U = (\alpha + \mu_i + \kappa_i - \kappa_j) \sum_{i=1}^n q_i - \frac{1}{2} (\sum_{i=1}^n q_i^2 + \gamma \sum_{i=1}^n \sum_{j \neq i}^n q_i q_j) + m$

$$q^S = \frac{b(\alpha - c)(2 - \gamma)[2 + \gamma(n - 1)]}{b(2 - \gamma)[2 + \gamma(n - 1)]^2 - \gamma(n - 2) - 2}, \quad \Pi^S = \frac{b(\alpha - c)^2[b(2 + \gamma)^2(\gamma - 2)^2 - (4 + \gamma)\gamma - 8]}{(2 - b(2 - \gamma)(\gamma + 2)^2)^2} \quad (29)$$

$$CS^S = \frac{1}{2}n[1 + \gamma(n - 1)][q^S]^2, \quad TW^E = CS^S + n\Pi^S \quad (30)$$

As standard in the relevant literature, we observe that, $\frac{\partial q^s}{\partial n} < 0$, $\frac{\partial \Pi^s}{\partial n} < 0$, $\frac{\partial CS^s}{\partial n} > 0$, $\frac{\partial TW^s}{\partial n} > 0$. Thus, as the market size increase each firm's output production and profits decrease while, the consumers' surplus and welfare increase.

4.3 Advertising cost asymmetries.

In the basic model we have assumed that the both informative and comparative advertising have equal marginal cost. However, in reality when firms invest in comparative advertising, with a non negligible profitability, they deal with the risk to be prosecuted to the court by the rival as an attempt of misleading advertising. Thus, in this subsection we relax our assumption over the equality on the marginal cost of both types of advertising, and we consider the case where the marginal cost of comparative advertising (d) exceeds that of informative advertising (b). In other words the effectiveness of comparative advertising is lower given the legal risks of its use. Thus, in this content each firm seeks to maximize its net profits given by $\Pi_i = (\alpha + \mu_i + \kappa_i - \kappa_j - q_i - \gamma q_j)q_i - cq_i - b\mu_i^2 - d\kappa_i^2$. Keeping all the other parameters unchanged, we obtain that even if comparative advertising is more expensive than informative advertising firms' have always strong incentives to invest in both types of advertising. Further, all the other outcomes are in line with those of the basic model.

4.4 Informative Advertising

In this subsection we analyze in more details the effect of informative advertising over the firms' demand. Consider a unit mass of consumers, $\phi < 1$ of which are well informed about the characteristics of both goods and have demand functions $p_i = a + \tau s - q_i - \gamma q_j$, with $\tau = 2$ and $s > 0$ an exogenous increase in the consumer's valuation of the goods. The rest $1 - \phi$ of consumers have imperfect information about these characteristics and believe that τ take values $(-2, -1, 1, 2)$ with equal probabilities. Hence, if they do not receive any advertising messages from the firms, their expected demand functions are: $p_i = a - q_i - \gamma q_j$. Let firm i

launch informative and comparative advertising campaigns with respective intensities μ_i and κ_i , $i = 1, 2$. The latter represent the probability with which an uninformed consumer receives the respective advertising messages from firm i . If an uninformed consumer receives both messages from firm i and does not receive a comparative advertising message from firm j , then he believes that $\tau = 2$ for the firm i 's product. If, however, he receives a comparative advertising message from firm j , then this message nullifies the respective comparative message received by firm i , and as result he believes that $\tau = 1$. If an uninformed consumer receives an informative or a comparative message from firm i and no message from firm j , then he believes again that $\tau = 1$ for the firm i 's product. If, however, he also receives a comparative advertising message from firm j , then this message nullifies the one received from firm i and goes back to being uninformed about firm i 's good characteristics, i.e., $E\tau = 0$. Finally, if the uninformed consumer does not receive any message from firm i and receives a comparative advertising message from firm j , then he believes that $\tau = -1$ for the firm i 's product. Otherwise, he has no additional information about firm i 's product and thus $E\tau = 0$.

It turns out that the expected demand functions of uninformed consumers are: $p_i = a + (\mu_i + \kappa_i - \kappa_j)s - q_i - \gamma q_j$. Then firm i 's demand function is given by:

$$p_i = a + 2\phi s + (1 - \phi)(\mu_i + \kappa_i - \kappa_j)s - q_i - \gamma q_j \quad (31)$$

Then after setting $\hat{a} = a + 2\phi s$ and $\hat{s} = (1 - \phi)s$, (31) takes the form of the demand function assumed in the main model. Assuming that the costs of launching an informative and comparative advertising campaigns of intensity μ_i and κ_i are again separable and quadratic, i.e., $\hat{b}(\mu_i^2 + \kappa_i^2)$, with \hat{b} sufficiently high to guarantee interior solutions, the problem basically reduces the one examined in Section 2.¹⁷

5 Conclusions

In the present paper we investigate the firms' advertising behavior, in an oligopolistic market with horizontal product differentiation, where firms have on their set of marketing strategies both informative and comparative advertising. We show that in equilibrium firms invest in a

¹⁷In a similar way we can obtain the firms' demand functions in case of mere informative, mere comparative and informative plus negative advertising campaigns, which turn out to be of the form already seen in the previous sections.

mix advertising strategy, that combines both informative and comparative advertising, in order to extract the beneficial effect that each type of advertising provides. In addition, we show that the firms' expenditures on each type of advertising crucially depends on the degree of market competition with the firms' expenditures on comparative advertising being positively related to the market competition degree while, the expenditures on informative advertising being U-shaped related to the degree of market competition. Clearly, as the market competition becomes fiercer, firms invest more in both types of advertising as an attempt to enforce their position in the market and obtain a comparative advantage over the rival.

Further, we compare our equilibrium outcomes obtained in the endogenous advertising configuration with alternative market configuration either in the absence of advertising or in the absence of one of the two types of advertising, in order to provide some initial results over the effects that the different firms' marketing strategies have on the market outcomes and the social welfare. We demonstrate that the highest firms' output production is obtained under the endogenous advertising configuration, the lowest under the mere comparative advertising configurations that equals that of the benchmark, while it lies in between under the mere informative advertising configuration. These finding suggest that the existence of both types of advertising in the market, intensifies the market competition and leads to higher output production. Yet, regarding to the firms' market performance in terms of profitability, we show that firms obtain the highest profits under the mere informative advertising configuration while, depending on the degree of market completion they obtain the lowest profits under the mere comparative and the endogenous advertising configuration. These results reveals that the use of comparative advertising give rise to a prisoner's dilemma situation where firms end up to be worse off, in terms of firms profitability, comparing to the mere informative configuration and the benchmark case.

Regarding the societal effects of advertising we argue that the existence of both types of advertising in a market acts beneficially to consumers, since it leads to higher consumers' surplus due to the higher output and the improved information that consumers possess. Further, the total welfare is the highest under the mere informative advertising while, it is the lowest under the mere comparative configuration. In addition, we show that the welfare under the endogenous advertising configuration can be either higher or lower than that of the benchmark case depending on the degree of competition. In particular the welfare in the endogenous advertising configuration exceed that of the benchmark case iff the degree of market competition

is relatively low while, the opposite holds for relatively high market competition degree. These findings suggest that the firms' self promoting informative advertising should be encouraged by the policy makers since, they lead to higher welfare. Finally, we show that our results remain robust when the market competition is conducted in prices, in the market exist more than two firms and the cost of types of advertising is not symmetric.

Our findings provide some guidelines for future experimental research on the firms' incentives to invest in both informative and comparative advertising when subjects have on their set of strategies both types of advertising that could provide credible results over the firms' decision relatively to aggressive advertising marketing strategies and how these affect their market performance in oligopolistic industries. A number of testable hypotheses emerges relatively to our theoretical analysis. For instance, in the presence of both types of advertising in a market, the subjects undertake aggressive advertising strategies with both types of advertising? and if yes, how this affect the competition and therefore, the profitability in the industry. Another testable hypothesis could be that the probability of a firm employing comparative advertising is higher in industries where products are close substitutes.

Appendix

Appendix A1: Proof of proposition 2 Evaluating the difference between the equilibrium investment levels in informative advertising obtained in the endogenous advertising configuration and those of the mere informative advertising configuration we have:

$$\mu^E - \mu^I = \frac{(\alpha - c)2b\gamma(\gamma + 2)^2}{[b(\gamma + 2)^2 - 1][b(\gamma - 2)(\gamma + 2)^2 + 2]} < 0, \text{ for any } \gamma \text{ and } b$$

Appendix A1: Proof of Proposition 3 Evaluating the difference between the equilibrium investment levels in comparative advertising obtained in the endogenous advertising configuration and the those of the mere comparative advertising configuration we have:

$$\kappa^E - \kappa^C = \frac{2(\alpha - c)}{b(\gamma^2 - 4)[2 + b(\gamma - 2)(\gamma + 2)^2]} > 0, \text{ for any } \gamma \text{ and } b$$

Appendix A1: Proof of Proposition 4 Evaluating the difference between the equilibrium output and profits obtained in the endogenous advertising configuration with respective ones obtained in the benchmark case, we have that:

$$q^E - q^N = -\frac{2(a-c)}{(\gamma+2)[b(\gamma-2)(\gamma+2)^2+2]^2} > 0, \text{ for any } \gamma \text{ and } b$$

$$\Pi^E - \Pi^N = -\frac{(\alpha-c)^2[b\gamma(\gamma+2)^2(8+\gamma)+4]}{(\gamma+2)^2[b(\gamma-2)(\gamma+2)^2+2]^2} < 0 \text{ for any } \gamma \text{ and } b$$

Evaluating the difference between the equilibrium output and profits obtained in the endogenous advertising configuration with the respective ones obtained in the mere informative advertising configuration, we have that:

$$q^E - q^I = -\frac{(\alpha-c)b\gamma(2+\gamma)}{[b(\gamma+2)^2-1][b(\gamma-2)(\gamma+2)^2+2]} > 0 \text{ for any } \gamma \text{ and } b$$

$$\Pi^E - \Pi^I = -\frac{b\{(2+\gamma)^2[2b(\gamma^2+2\gamma+2)-2](\alpha-c)\}}{[b(\gamma+2)^2-1][b(\gamma-2)(\gamma+2)^2+2]^2} < 0, \text{ for any } \gamma \text{ and } b$$

Evaluating the difference between the the equilibrium output and profits obtained in the endogenous advertising configuration with the respective ones obtained in the mere comparative advertising configuration we have that:

$$q^E - q^C = -\frac{2(\alpha-c)}{(\gamma+2)[2+b(\gamma-2)(\gamma+2)^2]} > 0, \text{ for any } \gamma \text{ and } b$$

$$\Pi^E - \Pi^C = -\frac{(\alpha-c)^2[4(-1+b^2(\gamma-1)(\gamma^2-4)^2-b(\gamma^3+\gamma^2-12))]}{b(\gamma^2-4)^2[2+b(\gamma-2)(\gamma+2)^2]^2} \text{ with } \Pi^C < \Pi^E \text{ iff } \gamma < \hat{\gamma}(b) \text{ and } \Pi^C > \Pi^E \text{ iff } \gamma > \hat{\gamma}(b)$$

Appendix A1: Proof of Proposition 5 The consumers' Surplus, under each configuration is given by, $CS^N = (1+\gamma)[q^N]^2$, $CS^E = (1+\gamma)[q^E]^2$, $CS^C = (1+\gamma)[q^C]^2$, $CS^I = (1+\gamma)[q^I]^2$. Thus, given that $q^E > q^I > q^C = q^N$ we obtain that, $CS^E > CS^I > CS^C = CS^N$ for all the given values of γ and b .

Total Welfare: Evaluating the difference between the social welfare under the endogenous advertising configuration and that of the benchmark case we have that:

$$TW^E - TW^N = -\frac{4(3+\gamma) + 2b(\gamma+2)^2(3\gamma^2+6\gamma-4)}{(\gamma+2)^2[b(\gamma-2)(\gamma+2)^2+2]^2}$$

From the above equation it can be easily testified that $TW^E > TW^N$ holds for $\gamma < \tilde{\gamma}(b)$

while, $TW^E < TW^N$, for $\gamma > \tilde{\gamma}(b)$.

Evaluating the difference between the social welfare in the endogenous advertising configuration and that of the mere informative advertising we have:

$$TW^E - TW^I = -\frac{(\alpha - c)b(2 + \gamma)^2[2 + b(-16 - \gamma(12 + (3 + \gamma))) + 2b(2 + \gamma^2)(4 + \gamma(\gamma^2 + \gamma + 2))]}{[b(\gamma + 2)^2 - 1]^2[b(\gamma - 2)(\gamma + 2)^2 + 2]^2} < 0, \text{ for any } \gamma \text{ and } b$$

Evaluating the difference between the social welfare in the endogenous advertising configuration and that of the mere comparative advertising, we have that:

$$TW^E - TW^C = -\frac{(\alpha - c)^2 4[(-2 + b^2(\gamma^2 - 4)^2(\gamma^2 + \gamma - 4) - b(\gamma^2(\gamma + 5) - 28))]}{b(\gamma^2 - 4)^2[2 + b(\gamma - 2)(\gamma + 2)^2]^2} > 0, \text{ for any } \gamma \text{ and } b$$

Evaluating the difference between the social welfare in the endogenous advertising configuration and that of the mere comparative advertising we have that:

$$TW^E - TW^C = -\frac{(\alpha - c)^2 4[(-2 + b^2(\gamma^2 - 4)^2(\gamma^2 + \gamma - 4) - b(\gamma^2(\gamma + 5) - 28))]}{b(\gamma^2 - 4)^2[2 + b(\gamma - 2)(\gamma + 2)^2]^2} > 0, \text{ for any } \gamma \text{ and } b$$

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