Comparative Advertising in Markets with Network Externalities.[*,†]

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Abstract

The present paper investigates the firms' incentives to invest in comparative advertising in a spatially differentiated duopoly market characterized by network externalities. We show that for a wide range of locations, determined by the interaction between the transportation cost, the network effects and the effectiveness of advertising, firms have incentives to invest in comparative advertising with their investment levels to be positively related to the transportation cost and negatively related to the network effects. Further, comparing the equilibrium results of our model with the benchmark case without advertising activities and the case without network externalities, we show that the firms' location distance increases in the presence of network externalities, while it decreases in the presence of comparative advertising.

Keywords Comparative Advertising; Network Effects, Location.

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1 Introduction

Advertising and its' effects in markets that are characterized by consumption externalities have attracted lately the interest of the academic community, since in such market the consumers' decisions to purchase a product do not depend only on the product's physical characteristics but it depends also on the number of the agents that use the same product (Katz and Sharipo, 1985; Veblen, 1899). Thus, the firms' advertising apart from its' informative and persuasive role, it can be further used as a device in order to alter the consumers' expectations about the products' network size in the market (Pastine and Pastine, 2002; Clark and Hostmann, 2005; Pastine and Pastine, 2011). However, and though the economic literature has extensively analyzed the coordination role of advertising in the aforementioned markets, the existing literature has ignored so far the effects that more aggressive forms of advertising, such as comparative advertising, "the form of advertising that compares rivals brands on objectively measurable attributes or price, and identifies the rival brand by name, illustration or other distinctive information"[1], could have in markets characterized by network externalities.

In the present paper we investigate the firms' incentives to invest in comparative advertising in a spatially differentiated duopoly characterized by networks externalities and the effects of such investments on the market's outcomes. The idea that drives our paper can be illustrated taking for example the Orange's comparative advertising campaign in 1994, "On average, Orange users save more than 20£ per month, compared to the Vodafone's and Cellnet's equivalent tariffs". Clearly, the aforementioned advertising campaign promotes the superiority of the Orange's tariffs against the Vodafone's and the Cellnet's tariffs. That means that, it targets to increase the consumers' valuation over the Orange's product, while, at the same time, it targets to decrease the consumers' valuation over both the Vodafone's and the Cellenet's products. However, given that the products are characterized by consumption externalities, even if the message of the comparative advertising is true, the consumers that are exposed to such comparative advertising messages are going to evaluate also the expected network size of each product, since they are willing to participate in the most widely used network. Thus, a number of questions arise with regard to the firms' incentives to invest in comparative advertising when the market is characterized by network externalities. In particular, in the present paper we

[1]Statement of policy regarding comparative advertising, Federal Trade Commission, Washington, D.C., August13, 1979.

aim to address the following three questions: First, are there any firms' incentives to invest in comparative advertising when the market is characterized by network effects? Second, how do the use of comparative advertising and the network externalities affect the firms' locations? Third, how do the firms' location, comparative advertising and network externalities affect the market performance?

We consider a spatially differentiated duopoly market characterized by network externalities where the firms strategically use the comparative advertising in order to present their product as superior to the rivals' one. In particular, the use of comparative advertising has a dual effect, it increases the consumers' valuation for the positively advertised product, while, at the same time, it decreases the consumers' valuation for the rival firm's product. The "net effect" that comparative advertising has on the consumers' demand, crucially depends on product differentiation, or, else, on the distance between the two firms. That means that, the closer the firms are located to each other, or, in other words, the closer substitutes the products are, the higher is the denigrating effect of the comparative advertising. A four stage game is analyzed where the timing of the game is given as follows. In the first stage, the firms decide their location in the Hotteling line. In the second stage, the firms choose their investment levels in comparative advertising. In the third stage, the firms compete by setting their prices. In the final stage, the consumers after observing the firms' location, advertising and prices allocate themselves to one of the two networks.

We show that for a sufficiently high transportation cost the firms has strong incentives to undertake comparative advertising activities, since each firm is willing by investing in comparative advertising to obtain a competitive advantage over the rival firm in the market via the denigrating effect that the comparative advertising has over the rival's firm product. Further, we demonstrate that in equilibrium firms are symmetrically located within the edges of the linear city. In more details, a wide range of firms' location choices is available depending on the transportation cost, the consumers' valuation over the network externalities and the effectiveness of the comparative advertising. In addition, we show that the location distance between the firms is negatively connected to the transportation cost, while it is positively connected to the network externalities and the effectiveness of advertising. Thus, we reconfirmed that, as standard in the literature, the higher the transportation cost is, the closer to each other the firms locate. On the contrary, the presence of network externalities in a market intensifies the market competition and thus, it leads firms to locate further apart.

Moreover, we show that the firms' comparative advertising expenditures are positively connected to the transportation cost, while they are negatively connected to the network effects. Intuitively, as the consumers' valuation over the network size increases, the firms will choose to locate further apart. The latter leads firms to decrease their expenditures on comparative advertising, since a comparison between unrelated products is less effective. The opposite holds, as the transportation cost increases.

Regarding the effects of the comparative advertising on the market outcomes, we show that the firms' expenditures on comparative advertising intensify the market competition and thus, the equilibrium prices and profits when firms invest in comparative advertising are always lower than those obtained in the benchmark case without firms' advertising activities. Thus, comparative advertising can be characterized as "wasteful advertising", since it leads to a prisoners dilemma situation where firms are worse off in terms of profitability.[2] Further, comparing the equilibrium firms' locations, advertising investments, prices and profits in markets that are characterized by network externalities to the respective ones in markets without networks externalities, we show that the firms' location difference and the equilibrium comparative advertising expenditures are higher under the former case, while the equilibrium prices and profits are lower. This is so, due to the fiercer market competition that the presence of network externalities generates.

Our paper is related to the existing literature that examines the role of advertising in markets that are characterized by consumption externalities and has its origins in the seminal work of Pastine and Pastine (2002) that show that in such markets the firms' advertising, apart from its informative and persuasive role, could form as a device to coordinate the consumers' expectations over the purchasing decisions of other consumers. In the same vein, Clark and Hostmann (2005) extend the Pastine and Pastine (2002) work by investigating the coordination role of advertising when the firms' advertising levels are not fully observable to the consumers and show that the firms can use advertising to coordinate the consumers purchases even if advertising levels are unobservable. More recently, Pastine and Pastine (2011), examine whether advertising can serve as a coordination device for products of different quality. They show that, when the products are of sufficiently different quality, the low quality firm has incentives

[2] The term of wasteful advertising was first introduced by Pigou 1924, in order to describe the prisoners' dilemma that arize when the competing firms invest equal efforts in advertising in order to attract the favor of the public and none of the firms gains anything at all.

to advertise more and thus, in the equilibrium the low quality product will be often the most popular. However, all of the aforementioned literature has focused on the coordination role of advertising in markets with consumption externalities while, it has ignored the effects that more aggressive forms of advertising could have in such markets. Closer to the present paper, Kretschmer and Rosner (2010) investigate how the network externalities affect the firms' persuasive advertising expenditures and show that when the firms are symmetrically located, their advertising expenditures are not affected by the networks effects. In contrast, they suggest that if a firm has an initial location advantage its' advertising spending increase in network effects. However, Kretschmer and Rosner (2010) in order to study how the network effects interact with the locational advantage, have assumed that firms' location are exogenously given. In the present paper we relax this assumption and we investigate the firms incentives to invest in comparative advertising in markets with network effects when the firms' locations, advertising expenditures and prices are determined endogenously.

Moreover, the present paper is related to the emerging economic literature that examines the use of comparative advertising in imperfectly competitive markets. Aluf and Shy (2001) using a Hotteling model, where comparative advertising increases the transportation cost to the rival's product, show that the use of comparative ads weakens price competition by enhancing the degree of product differentiation and leads to higher prices and profits. In a different vein, Barigozzi et al. (2009) examine comparative advertising as a mean to signal quality. In particular, they consider a market where an incumbent whose quality is known faces an entrant whose quality is unknown. The entrant decides whether to use generic advertising, that is a standard money burning to signal quality or comparative advertising, that implies a comparison over the qualities of the two products. They conclude that the comparative advertising can be used to signal quality, while the entrant's incentives to use comparative advertising are determined by the quality of his product and the penalty that he is going to pay if the content of his advertising campaign is manipulative. In a similar vein, Emons and Fluet (2012) examine the signaling role of comparative advertising in a duopolistic market where both firms use comparative advertising to highlight their quality differential and the cost of advertising increases as the firms move away from the truth. Anderson and Renault (2009) considering comparative advertising as a mean through which firms can disclosure information about products' horizontal match characteristics, show that for products of similar quality firms have incentives to advertise only their own goods while, for products of sufficiently different quality, only the low quality firm has incentives to use comparative advertising (if it is legal) in order to survive in the market. In addition, Anderson et al. (2010a,b) empirically investigating the advertising in the US over-the-counter analgetics market, show that almost the half of the advertising expenditures were comparative advertisements. More recently, Alipranti et al. (2013) examining the firms' incentives to invest in comparative and informative advertising in oligopolistic markets with horizontal product differentiation, show that, when firms have on their set of marketing strategies both types of advertising, they invest in mix advertising strategy that compines both informative and comparative advertising. Given the existed theoretical literature on the aspect of comparative advertising, the present paper is novel in two dimensions. First, to the best of our knowledge it is the first that explores the role of comparative advertising in the presence of networks externalities. Second, it is the first that provides results over the effects of strategic comparative advertising into the firms' endogenous locational choices.

Finally, the present paper is related to the literature on networks effects. This literature has its origins on the seminal paper of Veblen (1899) who has first introduce the idea of consumption externalities, by recognizing that the satisfaction obtained by the consumption of a particular good can be affected by the consumption choices of the other consumers. From then and on, an extensive literature on network externalities has been available (see eg. Leibenstein, 1950; Katz and Sharipo, 1985; Economides, 1996; Shy, 2001). Our paper is closer related to the Grilo et al.(2001) who examine price competition in a spatial duopoly model with consumption externalities (classified as conformity or vanity). Similarly, Cintio (2007), investigates price competition in the presence of network effects using a spatial Hotteling model with linear transportation costs and endogenous firms' locational decisions. Our paper differs from these works by introducing into the model strategic comparative advertising. In particular, the present paper provides results on the impact that the firms' comparative advertising expenditures has on the firms' location choices and on the market's outcomes.

The rest of the paper is organized as follows. In Section 2, we present the basic model. In section 3, we adduce the equilibrium analysis of the benchmark case, where firms do not undertake any advertising activities, the case of comparative advertising in markets with network externalities, and the case of comparative advertising in markets without network externalities. Further, the comparison between the three cases is presented in this section. Finally, section 4 concludes. All proofs are demonstrated in the Appendix.

2 The Basic Model

We consider a Hotelling model with product differentiation consisted by two firms, labelled $i = 1, 2$. The firms face a constant marginal cost of production, c, that for sake of simplicity has been normalized to zero (i.e, $c = 0$). The market is populated by a continuum of consumers with mass M, uniformly distributed over the unit-lenght interval. The firms' locations in the Hotelling line, are given respectively by, $y_1 \in [0, 1]$ and $y_2 \in [0, 1]$. Without loss of generality we assume that, $y_2 \geqslant y_1$, that means that, the good 1 is located left to the good 2. Each firms strategically invests in comparative advertising in order to promote its own product and thus, to increase its demand. Following Anderson et al. (2010), comparative advertising has a push-me/pull-you effect, that is, it increases the consumers' valuation of the positively advertised product, while, at the same time, it decreases the consumers' valuation of the targeted product. The "net effect" of the firm i's investment in comparative advertising, X_i, is defined as follows,

$$X_i = x_i - (1 - y_2 + y_1)x_j, \ i, j = 1, 2, \ i \neq j. \tag{1}$$

where, x_i denotes firm i's expenditures on comparative advertising while, the coefficient $(1 - y_2 + y_1)x_j$ reflects the detrimental effect of the rival firm's expenditures on comparative advertising, x_j, and is proportional to the products' differentiation, or else, to to the distance between the firms' locations. Note that, the detrimental effect of comparative advertising is at a minimum when the firms locate at the market endpoints (i.e., $y_2 = 1$, $y_1 = 0$), or else, when the products are independent. On the contrary, the detrimental effect of comparative advertising is at maximum when firms share the same location (i.e., $y_2 = y_1$), or, else, when the product are perfect substitutes.

The market is characterized by network externalities, that means that, the consumers' utility increase with the number of the other individuals consuming the same product. As standard in the literature of spatial models, we assume that each consumer demands at most one unit of product and thus, he participates exclusively in one of the two incompatible networks that exist in the market. Each network is composed by each firm's clients, where, following Grilo et al. (2001), the excess utility that each consumer derives by his participation into the network, is given by, γn_i^e. The parameter γ, with $\gamma \geqslant 0$, measures the network effect while, n_i^e, denotes the number of the consumers that are expected to patronize the firm i's product. Thus, the utility that a consumer located at $\hat{s} \in [0,1]$ derives is given by,

$$U_i = V + X_i - p_i - t(\hat{s} - y_i)^2 + \gamma n_i^e \tag{2}$$

where, V represents the gross intrinsic utility that a consumer derives by the consumption of one unit of product, and it is assumed to be large enough so that the market to be always covered. The parameter p_i is the price for good i, while $t > 0$ is the per unit transportation cost that a consumer incurs when purchases firm i's product. In line with D' Aspermont et al., (1979) in order to guarantee the equilibrium in the price stage, the transportation cost has been assumed to be quandratic. Further, as in Grilo et al. (2001), in order the consumers' expectations about the network size to be fulfilled in the equilibrium, the following condition must be satisfied,

Condition 1. $n_1 = n_1^e = \hat{s}M$ and $n_2 = n_2^e = (1 - \hat{s})M$.

Throughout the paper, we restrict our attention on markets where the network effects are not too strong compared to the product differentiation, that means that, the consumers evaluate more the products' features than the beneficial effect that their participation in the network generates. Thus, in order to exclude from our analysis upward sloping demands that lead to multiple subgame perfect equilibria the following assumption should holds, [3]

$$t(y_2 - y_1) - \gamma M > 0 \tag{3}$$

The cost of comparative advertising it is assumed to be quadratic, $C(x_i) = bx_i^2$, $i = 1, 2$, where, the parameter b reflects the effectiveness of the advertising technology. The higher the b is, the less effective is the advertising technology and therefore, the higher the required expenditures by firms are, in order to obtain a given shift on the consumers' demand.

We consider a four stage game with the following timing. In the first stage, firms decide their location. In the second stage, firms choose their investments in comparative advertising. In the third stage, firms compete by setting their prices. In the last stage, consumers after observing the firms' location, advertising and prices, allocate themselves to one of the two networks. We solve the game backwards by employing the Subgame Perfect Nash Equilibrium (SPNE) solution concept.

[3] For an extended analysis of multiple subgame equilibria under conformity, see Grilo et al. 2001

3 Equilibrium Analysis

3.1 The Benchmark Case

We begin our analysis by briefly presenting the benchmark case where in a market that is characterized by network externalities, firms do not have on their set of strategies advertising (i.e., $x_i = 0$, $i = 1, 2$ and, $X_i = 0$, $i = 1, 2$) and thus, they compete by choosing their locations in the Hotteling line and their prices. [4]

Using, $U_0^n = U_1^n \Leftrightarrow V - p_1 - t(\hat{s}^n - y_1)^2 + \gamma n_1 = V - p_2 - t(\hat{s}^n - y_2)^2 + \gamma n_2$ and solving for $\hat{s}^n$, we obtain that the location of the indifferent consumer in the benchmark case is given by,

$$\hat{s}^n = \frac{(p_2 - p_1) + t(y_2^2 - y_1^2) - \gamma M}{2[t(y_2 - y_1) - \gamma M]} \tag{4}$$

Thus, substituting (4) into the firms' profits, $\Pi_i = p_i n_i(p_1, p_2)$ and solving the game backwards, we have that in the equilibrium the firms' locations, prices and profits are given, respectively, by,

$$y_1^n = -\frac{1}{4}\ ,\ y_2^n = \frac{5}{4} \tag{5}$$

$$p_1^n = p_2^n = p^n = \frac{3t}{2} - \gamma M \tag{6}$$

$$\Pi_1^n = \Pi_2^n = \Pi^n = \frac{(3t - 2\gamma M)M}{4} \tag{7}$$

Observe here that, in line with Serfes and Zacharias (2011) in a market where two competeting networks exist, the firms locate on the opposite sides of the market (i.e., maximum product differentiation). Intuitively, the existence of networks externalities in the market intensifies the market competition and leads firms to locate further apart, or else, to maximize the products' differentiation. Note also by the equations (6) and (7), that the equilibrium prices and profits are always lower than the standard d' Aspermont et al (1979) equilibrium prices and profits. This is so, due to the fiercer market competition that the existence of network externalities generates. The following Lemma summarizes,

Lemma 1 *i) In a market with network externalities, there exists a subgame perfect Nash*

[4] This configuration reflects also the case where consumers perceive firm's comparative advertising campaing as manipulative and thus, as a non trustworthy source of information (see for details, Wilkie and Farris, 1975; Barone and Miniard, 1999). In this case, the results coinside with the ones obtain in this subsection.

equilibrium where the firms' locations are given as in (5)

ii) The presence of networks externalities in a market increases the market competition and lead to lower prices and profits.

3.2 Comparative Advertising in markets without networks externalities

We turn now to discuss the case where in a spatially differentiated duopoly market without network externalities the firms compete by choosing, their investment levels in comparative advertising, their locations, and their prices. Using $U_0^c = U_1^c$, we have that,

$$V + X_1 - p_1 - t(\hat{s}^c - y_1)^2 = V + X_2 - p_2 - t(\hat{s}^c - y_2)^2 \tag{8}$$

solving the above equation for $\hat{s}^c$, we obtain the location of the indifferent consumer,

$$\hat{s}^c = \frac{(p_2 - p_1) - (X_2 - X_1)}{2t(y_2 - y_1)} + \frac{y_2 + y_1}{2} \tag{9}$$

That means that, the consumers that are located on the $[0, \hat{s}^c]$ interval buy good 1 while, the opposite holds for the consumers that are located on the $[\hat{s}^c, 1]$. As standard in the literature, one can easily observe that an increase in p_2 (respectively, p_1) shifts the position of the marginal consumer closer towards the right end (left, respectively) of the Hotteling line and thus, the demand for firm 2 (firm 1, respectively) increases.

In order to ensure that the location of the indifferent consumer $\hat{s}^c$ belongs in the $[0, 1]$ interval, the following inequality should holds,

$$t(y_2^2 - y_1^2) - (X_2 - X_1) < (p_2 - p_1) < (X_2 - X_1) + t(y_2 - y_1)(2 - y_2 - y_1) \tag{10}$$

The above inequality, specifies the all the possible pairs of prices, comparative advertising expenditures and locations for which the indifferent consumer belongs to the $[0, 1]$ interval and thus, the firms share the market as in (9). Outside of this area one of the two firms captures the entire market.

In the third stage of the game, the firms decide over their price, p_1 and p_2 in order to maximize their profits given respectively by,

$$\Pi_1 = p_1 n_1 - b x_1^2 \text{ with } n_1 = \hat{s}^c M \tag{11}$$

$$\Pi_2 = p_2 n_2 - b x_2^2 \text{ with } n_2 = (1 - \hat{s}^c)M \tag{12}$$

Solving the maximization problems, from the first order conditions, the prices on the third stage of the game are given respectively by,

$$p_1 = \frac{1}{3}[(x_1 - x_2)(2 + y_1 - y_2) + t(y_2 - y_1)(2 + y_1 + y_2)] \tag{13}$$

$$p_2 = \frac{1}{3}[(x_2 - x_1)(2 + y_1 - y_2) + t(y_2 - y_1)(4 - y_1 - y_2)] \tag{14}$$

Observe here that, the equilibrium prices are increasing in the transportation parameter, t, and in the products' differentiation. Further, the equilibrium prices are increasing in the firm i's own comparative advertising expenditures, while they are decreasing in the rival firm's investments in comparative advertising.

Substituting (13) and (14) into (11) and (12), respectively the profits in the third stage of the game are given by,

$$\Pi_1 = \frac{M[(x_1 - x_2)(2 + y_1 - y_2) + t(y_2 - y_1)(2 + y_1 + y_2)]^2}{18t(y_2 - y_1)} - b x_1^2 \tag{15}$$

$$\Pi_2 = \frac{M[(x_2 - x_1)(2 + y_1 - y_2) + t(y_2 - y_1)(4 - y_1 - y_2)]^2}{18t(y_2 - y_1)} - b x_2^2 \tag{16}$$

In the second stage of the game, firms, taking as given their locations decided in the first stage, choose, independently and simultaneously, their expenditures on comparative advertising in order to maximize their profits given in the (15) and (16).

Solving the maximization problems, from the first order conditions the following best reply functions arise,

$$x_1(x_2) = \frac{M[2 + y_1 - y_2][x_2(2 + y_1 - y_2) - t(y_2 - y_1)(2 + y_1 + y_2)]}{M(2 + y_1 - y_2) - 18bt(y_2 - y_1)} \tag{17}$$

$$x_2(x_1) = \frac{M[2 + y_1 - y_2][(x_2(2 + y_1 - y_2) + t(y_2 - y_1)(y_1 + y_2 - 4)]}{M(2 + y_1 - y_2) - 18bt(y_2 - y_1)} \tag{18}$$

Note here that in order the second order conditions be satisfied, the following condition should holds,

Condition 2. $t > \frac{M(2+y_1-y_2)^2}{18b(y_2-y_1)}$

Further, by the equations (17) and (18), we observe that $\frac{dx_1}{dx_2} = \frac{dx_2}{dx_1} = \frac{M(2+y_1-y_2)^2}{M(2+y_1-y_2)^2-18(y_2-y_1)} <$

0, that means that, the comparative advertising expenditures are being perceived by firms as strategic substitutes.

Solving the system of the best reply functions, the equilibrium comparative advertising expenditures at the second stage are given by,

$$x_1 = \frac{M(2+y_1-y_2)[M(2+y_1-y_2)^2 - 3bt(y_2-y_1)(2+y_1+y_2)]}{6b[M(2+y_1-y_2)^2 - 9bt(y_2-y_1)]} \tag{19}$$

$$x_2 = \frac{M(2+y_1-y_2)[M(2+y_1-y_2)^2 + 3bt(y_2-y_1)(y_1+y_2-4)]}{6b[M(2+y_1-y_2)^2 - 9bt(y_2-y_1)]} \tag{20}$$

Observe here that the firms' equilibrium expenditures on comparative advertising take the highest values, when the firms locate at the endpoints of the linear city (i.e., $y_1 = 0$, $y_2 = 1$), or in other words, when the products are independent, while they take the lowest values when firms share the same location (i.e., $y_1 = y_2$), or in other words, when the products are perfect substitutes. The rationale behind this result is based in the detrimental effect of comparative advertising on the rival's firm demand that is at a minimum when the products are independent, while it is at a maximum when the products are perfect substitutes. In more details, when the products are unrelated, the advertising message is being perceived by the consumers as mere self-promoting advertising. Thus, firms tend to invest more in advertising in order to increase the consumers' valuation over the product and therefore, to increase their demand. In contrast, a comparison between similar products generates strong denigrating effects and thus, firms tend to invest less in comparative advertising.

In the first stage of the game, firms compete by choosing their location in the Hotelling line, y_1 and y_2, with $y_1 + y_2 = 1$. Substituting (19) and (20) into (15) and (16) and solving the maximization problem, we obtain two sets of candidate equilibrium locations that are given as follows,

$$y_1^c = \frac{3\sqrt{bt(8M+9bt)} - 2M - 9bt}{4M} \tag{21}$$

$$y_2^c = \frac{6M + 9bt - 3\sqrt{bt(8M+9bt)}}{4M} \tag{22}$$

and

$$\hat{y}_1^c = -\frac{3\sqrt{bt(8M+9bt)} + 2M + 9bt}{4M} \tag{23}$$

$$\hat{y}_2^c = \frac{6M + 9bt + 3\sqrt{bt(8M+9bt)}}{4M} \tag{24}$$

Notice here that the second order conditions, are being satisfied only by the first set of the candidate equilibrium locations. Thus, the equilibrium firms' locations are given as in the equations (21) and (22). Further, as standard in the literature, we observe that as the transportation cost increases firms tend to locate closer to each other. Notice also here that, since an increase in the transportation cost leads firms to locate closer to each other, the detrimental effect that the firm i's investments in comparative advertising has on the firm j's demand increases.

Further, using the equations, (21), (22), (19), (20), (13), (14), (15) and (16), the equilibrium comparative advertising expenditures, prices and profits are given respectively by,

$$x_1^c = x_2^c = x^c = \frac{\sqrt{bt(8M+9bt)} - 3bt}{4b} \tag{25}$$

$$p_1^c = p_2^c = p^c = \frac{t(4M + 9bt - 3\sqrt{bt(8M+9bt)})}{2M} \tag{26}$$

$$\Pi_1^c = \Pi_2^c = \Pi^c = \frac{t(4M + 9bt - 3\sqrt{bt(8M+9bt)})}{8M} \tag{27}$$

Thus, given the condition 2, for $t > \frac{M}{9b}$, there exist a subgame perfect Nash equilibrium where firms invest in comparative advertising. Further, we observe that the equilibrium firms' comparative advertising expenditures, prices and profits are positively related to the transportation cost parameter, t, while they are negatively connected to the advertising effectiveness parameter, b. The intuitions behind these results are given as follows. Clearly, the higher the transportation cost is, the closer to each other firms locate, or in other words, the lower is the products' differentiation that in turn, lead firms to invest more in comparative advertising in order to capture a competitive advantage over the rival, since the detrimental effect that comparative advertising has on the rival's firm demand increases. Further, with regard to the equilibrium profits two opposing effects stand. On the one hand, as the transportation cost increases, the price competition for market share becomes less fiercer and thus, the profits tend to increase. On the other hand, as the transportation cost increases, the firms' investments in comparative advertising increase that means that, firms incur higher advertising costs and thus, the profits tend to decrease. Clearly, in the equilibrium the beneficial effect of the weaker price competition dominates the detrimental effect of the higher advertising costs and thus, the firms' profits increase as the transportation cost increase. The above analysis is summarized in the following Lemma,

Lemma 2 *i) For $t > \frac{M}{9b}$, there exists a perfect subgame Nash equilibrium where firms invest in comparative advertising.*

ii) The equilibrium comparative advertising expenditures, prices and profits are decreasing in the advertising effectiveness parameter, b, while they are increasing in the transportation cost parameter, t.

3.3 Comparative advertising in markets with networks externalities.

We proceed now with the analysis of our basic model where, in a market characterized by networks externalities, firms compete by choosing their location, comparative advertising investments and prices.

Using $U_1(\hat{s}) = U_2(\hat{s})$ we have that,

$$V + X_1 - p_1 - t(\hat{s} - y_1)^2 + \gamma n_1 = V + X_2 - p_2 - t(\hat{s} - y_2)^2 + \gamma n_2 \tag{28}$$

Solving the above equation for $\hat{s}$, we obtain the location of the indifferent consumer,

$$\hat{s} = \frac{(p_2 - p_1) - (X_2 - X_1) + t(y_2^2 - y_1^2) - \gamma M}{2[t(y_2 - y_1) - \gamma M]} \tag{29}$$

that means that, the consumers that are located left in the $[0, \hat{s}]$ interval purchase the good 1, while the consumers that are located in the $[\hat{s}, 1]$ purchase the good 2.

In order to ensure that the location of the indifferent consumer, $\hat{s}$, belongs in the $[0, 1]$ interval, the following assumption should holds,

$$\gamma M - (X_2 - X_1) - t(y_2 - y_1)(2 - y_1 - y_2) < p_1 - p_2 < t(y_2^2 - y_1^2) - (X_2 - X_1) - \gamma M \tag{30}$$

Inequality (30) identifies all the possible price, comparative advertising expenditures and location combinations for which both firms exist in the market. Notice that when, $p_1 - p_2 \leqslant \gamma M - (X_2 - X_1) - t(y_2 - y_1)(2 - y_1 - y_2)$, firm 1 captures the entire market and thus, $n_1 = n$ and $n_2 = 0$ while, the opposite holds when, $p_1 - p_2 \geqslant t(y_2^2 - y_1^2) - (X_2 - X_1) - \gamma M$.

In the third stage of the game, the firms choose their prices, $p_1$and p_2, such as to maximize their profits given by,

$$\underset{p_1}{Max}\ \Pi_1 = p_1 n_1 - b x_1^2 \text{ with } n_1 = \hat{s}M \tag{31}$$

$$\underset{p_2}{Max}\ \Pi_2 = p_2 n_2 - b x_2^2 \text{ with } n_2 = (1-\hat{s})M \tag{32}$$

where, from the first order conditions, the prices in the third stage of the game are given as follows,

$$p_1 = \frac{1}{3}[(x_1 - x_2)(2 + y_1 - y_2) + t(y_2 - y_1)(2 + y_1 + y_2)] - \gamma M \tag{33}$$

$$p_2 = \frac{1}{3}[(x_2 - x_1)(2 + y_1 - y_2) + t(y_2 - y_1)(4 - y_1 - y_2)] - \gamma M \tag{34}$$

Notice here that the prices are decreasing in the network externality parameter, γ. That means that, as the network effect becomes stronger the price competition in the market increases and thus, the equilibrium prices decrease.

Substituting (33) and (34) into (31) and (32) the profits in the third stage of the game are given by,

$$\Pi_1 = \frac{M[3\gamma M + (x_1 - x_2)(2 + y_1 - y_2) + t(y_2 - y_1)(2 + y_1 + y_2)]^2}{18[t(y_2 - y_1) - \gamma M]} - b x_2^2 \tag{35}$$

$$\Pi_2 = \frac{M[3\gamma M + (x_2 - x_1)(2 + y_1 - y_2) + t(y_2 - y_1)(4 - y_1 - y_2)]^2}{18[t(y_2 - y_1) - \gamma M]} - b x_2^2 \tag{36}$$

In the second stage of the game, firms decide, independently and simultaneously, their investment levels in comparative advertising (x_1, x_2) in order to maximize their profits given respectively by, (35) and (36),

Solving the maximization problems, from the first orders conditions the following reaction functions arise,

$$x_1(x_2) = \frac{M(2 + y_1 - y_2)[3\gamma M + x_2(2 + y_1 - y_2) - t(y_2 - y_1)(2 + y_1 + y_2)]}{M[18b\gamma + (2 + y_1 - y_2)^2] - 18bt(y_2 - y_1)} \tag{37}$$

$$x_2(x_1) = \frac{M(2 + y_1 - y_2)[3\gamma M + x_1(2 + y_1 - y_2) + t(y_2 - y_1)(y_1 + y_2 - 4)]}{M[18b\gamma + (2 + y_1 - y_2)^2] - 18bt(y_2 - y_1)} \tag{38}$$

Note here that in order the second orders conditions to be satisfied the following condition should holds,

Condition 3. $t > \frac{M[18b\gamma+(2+y_1-y_2)^2]}{18b(y_2-y_1)}$

Further, by (37) and (38) we observe that $\frac{dx_1}{dx_2} = \frac{dx_2}{dx_1} = \frac{M(2+y_1-y_2)^2}{M[18b\gamma+(2+y_1-y_2)^2]-18bt(y_2-y_1)} < 0$, that means that, the comparative advertising expenditures are being perceived by firms as strategic substitutes.

Solving the system of the reaction functions, the comparative advertising expenditures in the second stage of the game are given by,

$$x_1 = \frac{M(2+y_1-y_2)[M(9b\gamma+(2+y_1-y_2)^2)-3bt(y_2-y_1)(2+y_1+y_2)]}{6b[M(9b\gamma+(2+y_1-y_2)^2)-9bt(y_2-y_1)]} \tag{39}$$

$$x_2 = \frac{M(2+y_1-y_2)[M(9b\gamma+(2+y_1-y_2)^2)+3bt(y_2-y_1)(y_1+y_2-4)]}{6b[M(9b\gamma+(2+y_1-y_2)^2)-9bt(y_2-y_1)]} \tag{40}$$

Comparing now the firms' investment levels in comparative advertising in markets that are characterized by network externalities with those obtained in the absence of network externalities given in (19) and (20) respectively, one can easily observe that the firms' investments in comparative advertising are higher under the former configuration. This is so since, the existence of network externalities intensifies the price market competition and thus, each firm tends to invest more in comparative advertising as an attempt to possess a competitive advantage over the rival and therefore, to increase its' market share.

In the first stage of the game, firms decide their locations in the Hotelling line (i.e, y_1 and y_2) with, $y_1 + y_2 = 1$. Substituting (39) and (40) into (35) and (36), and solving the maximization problems, we obtain two sets of candidate equilibrium locations given as follows,

$$y_1^* = \frac{3\sqrt{b(8Mt+9bt^2-4\gamma M^2)}-2M-9bt}{4M} \tag{41}$$

$$y_2^* = \frac{6M+9bt-3\sqrt{b(8Mt+9bt^2-4\gamma M^2)}}{4M} \tag{42}$$

and

$$\overset{\frown}{y}_1 = -\frac{3\sqrt{b(8Mt+9bt^2-4\gamma M^2)}+2M+9bt}{4M} \tag{43}$$

$$\overset{\frown}{y}_2 = \frac{3[2M+3bt+\sqrt{b(8Mt+9bt^2-4\gamma M^2)}]}{4M} \tag{44}$$

Notice here that the condition 3 is being satisfied only by the first set of the candidate equilibrium locations. Thus, in the equilibrium the location of firm 1 in the Hotelling line is given by the equation (41), while the location of firm 2 is given by the equation (42).

Further, we observe that, $\frac{\partial y_1^*}{\partial t} > 0$, $\frac{\partial y_2^*}{\partial t} < 0$, $\frac{\partial y_1^*}{\partial \gamma} < 0$, $\frac{\partial y_2^*}{\partial \gamma} > 0$, $\frac{\partial y_1^*}{\partial b} > 0$ and $\frac{\partial y_2^*}{\partial b} < 0$.

Clearly, as standard in the literature, the higher the transportation cost is, the closer to each other the firms locate, or else, the lower the product differentiation is. [5] On the contrary, the higher the network externalities parameter γ, the further apart the firms locate, or else, the higher the product differentiation is. This is so, since as the consumers' valuation over the network externalities increases, the price market competition becomes fiercer that leads firms to locate as far as possible in order to relax the fierce price market competition that the stronger network externalities generate in the market. In addition, one can easily check that the detrimental spillover effect that comparative advertising has on the rival's firm demand, i.e., $sp(y_1^*, y_2^*) = (1 - y_2^* + y_1^*) = \frac{3\sqrt{b(8Mt+9bt^2-4\gamma M^2} - 2M - 9bt}{2M}$, is decreasing in the transportation cost coefficient, t, and in the advertising effectiveness parameter, b, while it is increasing in the network externalities parameter γ, (i.e., $\frac{\partial sp^*}{\partial t} > 0, \frac{\partial sp^*}{\partial b} > 0, \frac{\partial sp^*}{\partial \gamma} < 0$).[6] Intuitively, as the transportation cost increases, the firms tend to locate closer to each other, or else, the products' differentiation decreases. Therefore, the detrimental effect of the comparative advertising on the rival firm's demand increases, since the comparison between similar products becomes more effective. The opposite holds when the consumers' valuation over networks increases. The above discussion is summarized in the following Lemma,

Lemma 3 *i) The higher the transportation cost is, the lower the product differentiation will be and thus, the higher the detrimental effect of comparative advertising will be.*

ii) The higher the valuation of consumers about networks is, the further apart firms will decide to locate and the lower the detrimental effect of comparative advertising will be.

iii) The less effective the comparative advertising is, the closer the firms will choose to locate.

Further, using (41) and (42), the equilibrium comparative advertising expenditures, prices

[5] In particular, the comparative static effect of the transorpation cost parameter t into locational choices is given by:
$\frac{\partial y_1^*}{\partial t} = \frac{\frac{3b(8M+18bt)}{2\sqrt{b(8mt+9b-4\gamma M^2)}} - 9b}{4M} > 0$ $\quad \frac{\partial y_2^*}{\partial t} = \frac{9b - \frac{3b(8M+18bt)}{2\sqrt{b(8mt+9b-4\gamma M^2)}}}{4M} < 0 \frac{\partial y_1^*}{\partial \gamma} = -\frac{3bM}{2\sqrt{b(8Mt+9bt^2-4\gamma M^2)}} < 0, \frac{\partial y_2^*}{\partial \gamma} = \frac{3bM}{2\sqrt{b(8Mt+9bt^2-4\gamma M^2)}} > 0$
and $\frac{\partial y_1^*}{\partial b} = \frac{\frac{3(8Mt+18bt^2-4\gamma M^2)}{2\sqrt{b(8Mt+9bt^2-4\gamma M^2)}} - 9t}{4M} > 0$, $\frac{\partial y_2^*}{\partial b} = \frac{9t - \frac{3(8Mt+18bt^2-4\gamma M^2)}{2\sqrt{b(8Mt+9bt^2-4\gamma M^2)}}}{4M} < 0$

[6] In particular, the comparative static effect of comparative advertising spillover is given by:
$\frac{\partial sp^*}{\partial t} = \frac{\frac{3b(8M+18bt)}{\sqrt[2]{b(8mt+9bt^2-4\gamma M^2)}} - 9b}{2M} > 0, \frac{\partial sp^*}{\partial b} = \frac{\frac{3(8Mt+18bt^2-4\gamma M^2)}{\sqrt[2]{b(8mt+9bt^2-4\gamma M^2)}} - 9t}{2M} > 0, \frac{\partial sp^*}{\partial \gamma} = -\frac{3bM}{\sqrt{b(8Mt+9bt^2-4\gamma M^2)]}} < 0.$

and profits are given, respectively, by,

$$x_1^* = x_2^* = x^* = \frac{\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)} - 3bt}{4b} \tag{45}$$

$$p_1^* = p_2^* = p^* = \frac{t(4M + 9bt - 3\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)}) - 2\gamma M^2}{2M} \tag{46}$$

$$\Pi_1^* = \Pi_2^* = \Pi^* = \frac{1}{8}[t(4M + 9bt - 3\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)}) - 2\gamma M^2] \tag{47}$$

Thus, given the condition 3, in an oligopolistic market that is characterized by networks externalities with, $t > \frac{M[18b\gamma + (2+y_1-y_2)^2]}{18b(y_2-y_1)}$, there exists a subgame perfect Nash equilibrium where, the firms invest positively in comparative advertising. Notice here that, given the condition 3, the extreme case of maximum locational differentiation, $y_1 = 0$ and $y_2 = 1$, is not a Nash equilibrium, that means that, the firms will never choose to locate at the endpoints of the linear city.

Proposition 1 *For $t > \frac{M}{9b} + \gamma M$, there exists a (subgame-perfect) symmetric location equilibrium where, the firms' locations are given by (41) and (42), while the prices and the comparative advertising expenditures are given respectively by(46) and (45).*

Clearly, given symmetric locations, the firms will choose in the equilibrium the same comparative advertising expenditures and prices. Thus, in line with existing literature (e.g., Tabuchi, 1994; Veendorp and Majeed, 1995; Irmen and Thisse, 1998; Piga and Theotoky, 2001), when firms compete in multi-characteristics space they maximize their differentiation in one dimension (in our case, location), while they minimize their differentiation in all the others (i.e., advertising, price). Further, by (45), (46) and (47), we observe that the equilibrium comparative advertising expenditures, prices and profits are positively related to the transportation parameter, t, while they are negatively related to the network externalities parameter, γ, and the advertising effectiveness parameter, b (i.e., $\frac{\partial x^*}{\partial \gamma} < 0$, $\frac{\partial x^*}{\partial b} < 0$, $\frac{\partial x^*}{\partial t} > 0$, $\frac{\partial p^*}{\partial \gamma} < 0$, $\frac{\partial p^*}{\partial b} < 0$, $\frac{\partial p^*}{\partial t} > 0$, $\frac{\partial \Pi^*}{\partial \gamma} < 0$, $\frac{\partial \Pi^*}{\partial b} < 0$ and $\frac{\partial \Pi^*}{\partial t} > 0$).[7] Interestingly, in contrast to the Kretschmer and Rosner (2010) that suggest that when firms are located symmetrically in the Hotelling line their advertising expenditures are not affected by the networks externalities, we show that the

[7] For the extended presentation see at the Appendix A1.1

firms' comparative advertising expenditures decrease as the consumers' valuation over the network size increases. Intuitively, the presence of the networks externalities intensifies the price market competition and leads firms to locate further apart, or in other words, leads to higher product differentiation. Thus, since the comparison between less related products becomes less effective the firms decrease their expenditures in comparative advertising. The following Proposition summarizes,

Proposition 2 *In a market that is characterized by network externalities, the equilibrium investments in comparative advertising, prices and profits are increasing in the transportation cost parameter, t, while they are decreasing in the network externalities parameter, γ, and the advertising effectiveness parameter, b.*

3.4 Comparison

We turn now to compare the equilibrium outcomes obtained in a market with network externalities where firms undertake comparative advertising activities, with those obtained in a market without network externalities and those of the benchmark case.

Starting with the firms' location choices under the three cases, we show that, $y_1^n < y_1^* < y_1^c$ and $y_2^n > y_2^* > y_2^c$. [8] That means that, the fiercer market competition due to the presence of networks externalities in a market leads firms to locate further apart while, the use of comparative advertising leads firms to locate closer to each other in order to enlarge their market share by targeting the rival product. In more details, the comparison of the benchmark case without advertising activities with our basic model reveals the two alternative forces that drive our basic results. In particular, we observe that, for a high enough transportation cost, the firms' investments in comparative advertising force firms to locate closer to each other. In contrast, the fiercer market competition due to the network externalities force firms to locate further apart. In the equilibrium, we show that $y_1^n < y_1^*$ and $y_2^n > y_2^*$, that means that, the centripetal force of comparative advertising dominates the centrifugal force of networks externalities and leads firms to locate closer to the each other, or alternatively, lead to lower product differentiation.

Regarding the equilibrium prices and profits, comparing (46), (47) with (26), (27) and (6),

[8] For proof see at the Appendix A2.1

(7), we have that $p^n > p^c > p^*$ and $\Pi^n > \Pi^c > \Pi^*$. [9] Obviously, the network externalities in the market along with the use of comparative advertising intensify the price market competition and thus, the prices and the profits in the market decrease. Given the above discussion one can easily observe that the use of comparative advertising leads firms to a prisoner's dilemma situation, where they end up worse off in terms of profitability. Further, we show that, $x_i^* > x_i^c$, that means that, the fiercer market competition due to the presence of network externalities in a market intensifies the comparative advertising competition and lead firms to invest more in comparative advertising.

Proposition 3 *i) The firms' location distance takes the highest value in the benchmark case without firms' advertising activities, the lowest in the comparative advertising without network externalities case, while it lies in between in the comparative advertising with network externalities case.*

ii) The equilibrium prices and profits when firms invest in comparative advertising in markets that are characterized by networks externalities are always lower than those obtained in markets without network externalities and those of the benchmark case without advertising activities.

4 Concluding remarks

The present paper investigates the firms' incentives to invest in comparative advertising in a spatially differentiated duopoly market characterized by network externalities, taking as basic premise that the detrimental effect of each firm's comparative advertising expenditures to the rival's firm demand is proportional to the location distance between firms, or else, to the products' differentiation.

We argue that for a sufficiently high transportation cost the firms have strong incentives to undertake comparative advertising activities. This is so, since each firm is willing to by investigating in comparative advertising to obtain a competitive advantage over the rival firm via the denigrating effect that this type of advertising has on the rival's firm product. Further, we show that in the equilibrium firms are symmetrically located within the edges of the linear

[9] For proof see at the Appendix A2.1

city, with the firms' location distance to be decreasing in the transportation cost, while increasing in the networks externalities and in the effectiveness of the advertising technology. Thus, we reconfirmed that, as standard in the literature, the higher the transportation cost is, the closer to each other the firms locate. In addition, we demonstrate that the presence of network externalities in the market, intensifies the market competition and thus, it leads firms to locate further apart. Regarding the effectiveness of the comparative advertising, our results reveal that the more effective the comparative advertising is, the further apart the firms will choose to locate. This is so, because the firms are willing by increasing the products' differentiation to outweigh the detrimental effect that the rival's comparative advertising generates.

Moreover, we show that the firms' comparative advertising expenditures are positively connected to the transportation cost, while they are negatively connected to the network effects. Thus, contrary to Kretschmer and Rosner (2010) that suggest that, when the firms are symmetrically located, their advertising expenditures are not affected by the networks effects, we demonstrate that firms' investments in comparative advertising decrease as the consumers' valuation over the network size increases. Clearly, as the consumers' valuation over the network size increases, the firms will choose to locate further apart. The latter leads firms to decrease their expenditures in comparative advertising since, a comparison between unrelated products is less effective.

Yet, we demonstrate that the firms' investments in comparative advertising, intensify the market competition and lead to lower prices and profits than those obtained in the benchmark case, without firms' advertising activities. Thus, comparative advertising can be characterized as "wasteful advertising" since it leads to a prisoners dilemma situation where the firms are worse off in terms of profitability. Lastly, we show that the equilibrium firms' investment levels in markets with network externalities always exceed those obtained in markets without network externalities. This is so, due to the fiercer market competition that the presence of network externalities generates.

Throughout the paper we have restricted our attention in markets where the network effects are not too strong compared to the product differentiation. In other worlds, we have restricted our attention in markets where the consumers evaluate more the product's characteristics than the network size of the product. Thus, it would be interesting enough to extend our analysis in markets where the consumers evaluate more the product's network size than its' characteristics.

Further, our findings provide some guidelines for future empirical research on the effects of firms' investments in comparative advertising when the markets are characterized by network externalities, that is so far scare. Our analysis give rise to a number of testable hypothesis that should be empirically checked. A first testable hypothesis could be that the firms' investments in comparative advertising may increase competition when the market is characterized by network effects and therefore decrease the prices and the firms' profitability in a given industry. Another testable hypothesis could be that the probability of a firm to engage in comparative advertising is higher in industries with network externalities than the respective one in markets that are not characterized by network externalities.

Appendix

Appendix A1: Extended presentation of equations regarding Proposition 2

A1.1.

Taking into account how the equilibrium firms' investment levels alter with respect to the network externalities parameter, γ, the advertising effectiveness technology parameter, b and transportation cost parameter, t, we have that,

$$\frac{\partial x^*}{\partial \gamma} = -\frac{M^2}{2\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)}} < 0$$

$$\frac{\partial x^*}{\partial b} = -\frac{2Mt - \gamma M^2}{2b\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)}} < 0$$

$$\frac{\partial x^*}{\partial t} = \frac{\frac{b(8M+18bt)}{2\sqrt{b(8Mt+9bt^2-4\gamma M^2)}} - 3b}{4b} > 0$$

Taking into account how the equilibrium prices alter with respect to the network externalities parameter, γ, the advertising effectiveness technology parameter, b and transportation cost parameter, t, we have that,

$$\frac{\partial p^*}{\partial \gamma} = \frac{\frac{6btM^2}{\sqrt{b(8Mt+9bt^2-4\gamma M^2)}} - 2M^2}{2M} < 0$$

$$\frac{\partial p^*}{\partial b} = \frac{t[9t - \frac{3(t(4M+9bt)-2\gamma M^2)}{\sqrt{b(8Mt+9bt^2-4\gamma M^2)}} - 2M^2]}{2M} < 0$$

$$\frac{\partial p^*}{\partial t}=\frac{4M+9bt-3\sqrt{b(8Mt+9bt^2-4\gamma M^2)}+t(9b-\frac{3b(4M+9bt)}{\sqrt{b[t(8M+9bt)-4\gamma M^2}})}{2M}>0$$

Taking into account how the equilibrium firms' profits alter with respect to the network externalities parameter, γ, the advertising effectiveness technology parameter, b and transportation cost parameter, t, we have that,

$$\frac{\partial \Pi^*}{\partial \gamma}=\frac{6btM^2}{8\sqrt{b(8Mt+9bt^2-4\gamma M^2)}}-\frac{M^2}{4}<0$$

$$\frac{\partial \Pi^*}{\partial b}=\frac{1}{8}t[9t-\frac{3(8Mt-4\gamma M^2+18bt^2)}{2\sqrt{b(8Mt+9bt^2-4\gamma M^2)}}]<0$$

$$\frac{\partial \Pi^*}{\partial t}=\frac{4M+9bt-3\sqrt{b(8Mt+9bt^2-4\gamma M^2)}+t(9b-\frac{3b(4M+9bt)}{\sqrt{b[t(8M+9bt)-4\gamma M^2}})}{16M}>0$$

Appendix A2: Proof of Proposition 2

A2.1

We calculate the firms' location difference between the case where the market that is characterized by network externalities, given in (41), and the case where the market is not characterized by network externalities, given in (21),

$$y_1^*-y_1^c=\frac{3[\sqrt{b(8Mt+9bt^2-4\gamma M^2}-\sqrt{bt(8M+9bt)}]}{4M}<0$$

$$y_2^*-y_2^c=\frac{3[\sqrt{bt(8M+9bt)}-\sqrt{b(8Mt+9bt^2-4\gamma M^2}]}{4M}>0$$

From the above equations it can be easily checked that, under assumption 1 and the conditions 2 and 3, for all the given values of t, γ and b, $y_1^*-y_1^c<0$ and thus, $y_1^*<y_1^c$ while, $y_2^*-y_2^c>0$ and thus, $y_2^*>y_2^c$. Further, the firms' location difference between the case where firms invest in comparative advertising and the market is not characterized by network externalities, given in , and the benchmark case, given in,

$$y_1^c-y_1^n=\frac{3\sqrt{bt(8M+9bt)}+6M+9bt}{4M}>0$$

$$y_2^c - y_2^n = -\frac{3\sqrt{bt(8M+9bt)} - M - 9bt}{4M} < 0$$

From the above equations it can be easily checked that under our assumption 1 and the conditions 2 and 3, for all the given values of t, γ and b, $y_1^c - y_1^n > 0$ and thus, $y_1^n < y_1^c$ while, $y_2^c - y_2^n < 0$ and thus, $y_2^n > y_2^c$. Thus, we have that in the equilibrium, $y_1^n < y_1^* < y_1^c$ and $y_2^n > y_2^* > y_2^c$.

We calculate the price differential between the case where the market that is characterized by network externalities, the case where the market is not characterized by network externalities and the benchmark case and we have that,

$$p^* - p^n = \frac{Mt + 9bt^2 - 3t\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)}}{2M} < 0$$

$$p^* - p^c = \frac{3t\sqrt{bt(8M+9bt)} - 3t\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)} - 2M^2 t}{2M} < 0$$

$$p^c - p^n = \frac{2\gamma M^2 + Mt + 9bt^2 - 3t\sqrt{bt(8M+9bt)}}{2M} < 0$$

Thus, under our assumption 1 and conditions 2 and 3 for all the given values of t, γ and b, $p^b > p^c > p^b$ holds.

We calculate the profits differential between the case where the market that is characterized by network externalities, the case where the market is not characterized by network externalities and the benchmark case and we have that,

$$\Pi^* - \Pi^b = \frac{2\gamma M^2 + t(9bt - 2M - 3\sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)})}{8} < 0$$

$$\Pi^* - \Pi^c = \frac{3t(\sqrt{bt(8M+9bt)} - \sqrt{b(8Mt + 9bt^2 - 4\gamma M^2)}) - 2\gamma M^2}{8} < 0$$

$$\Pi^c - \Pi^n = \frac{4\gamma M^2 - 2Mt + 9bt^2 - 3t\sqrt{bt(8M+9bt)}}{8} < 0$$

Thus, under our assumption 1 and conditions 1 and 2 for all the given values of t, γ and b, we have that $\Pi^b > \Pi^c > \Pi^b$.

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