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Abstract

We study the endogenous formation of upstream R&D networks in a vertically related industry. We find that, when upstream firms set prices, the complete network that includes all firms emerges in equilibrium. In contrast, when upstream firms set quantities, the complete network will arise but only if within-network R&D spillovers are sufficiently low, while if R&D spillovers are sufficiently high, a partial network arises. Interestingly, when upstream firms set prices, the equilibrium network maximizes social welfare, while a conflict between equilibrium and socially optimal networks is likely to occur when upstream firms set quantities.

Keywords: Networks, R&D collaboration, upstream firms.

JEL Classification: L13, J50.

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1 Introduction

Over the last three decades, we have witnessed a substantial increase in the number of R&D alliances between firms. Consistent evidence across the United States (Röller et al., 2007), Europe (Kaiser, 2002) and Japan (Branstetter and Sakakibara, 1998) further suggests that firms collaborate in order to share know-how and enhance their technological capabilities.[1] The recent upsurge in R&D alliances shows that hi-tech sector firms increasingly prefer non-equity forms of collaboration, such as R&D networks, relative to traditional, equity forms, such as research joint ventures (RJVs hereafter). For example, Hagedoorn (2002) documents that in the major fields of technology, such as information technology and pharmaceuticals, the number of newly established R&D alliances grew steadily during the 1980s and 1990s, reaching an impressive total share of more than 90%, while the share of RJVs declined to less than 10%.[2]

Further empirical evidence and stylized facts suggest that R&D alliances are often established in the context of vertically related industries. For example, Cloodt et al. (2006) find that, in the computer industry, a substantial number of R&D alliances are formed at the upstream market tier – that is, among firms that do not trade directly with consumers but instead supply key inputs. The principal motivation behind this observation is that individual firms find it rather difficult to develop technological capabilities alone, so they prefer to collaborate with others and pool their know-how. In particular, we have observed the formation of R&D alliances between producers of micro chips, such as Intel, Motorola and Texas Instruments – who are located upstream – and supply their inputs to personal computer manufacturers, such as IBM, Hewlett Packard, Sony, Dell

[1]The proliferation of R&D alliances is a phenomenon that has attracted the attention of policy makers, managers and academics alike. This phenomenon has often been described, for instance, as the "age of alliance capitalism" (e.g. Narula and Dysters, 2004, p. 200) or a "frenzy of deals" between firms (e.g. Caloghirou et al., 2003, p. 546).

[2]The main reason behind this diversity in evolution is that non-equity R&D partnerships, such as R&D networks, allow for greater flexibility, thereby enabling firms to innovate in several and often diverse technological fields. On the contrary, although equity types of alliances, such as RJVs, are effective in limiting the opportunistic behaviour of research partners (Buckley and Casson, 1988), they are more appropriate for less turbulent economic environments (i.e. medium or low-tech sectors), as they require greater time to administer, establish and dissolve (Roijakers and Hagedoorn, 2006).

and Compaq – who are located downstream.

These observations raise a number of questions. First, what R&D network architectures will emerge endogenously between upstream firms? Second, how do the incentives to form R&D collaboration links depend on whether upstream firms set prices or quantities? Third, what are the welfare effects of the equilibrium R&D networks? Finally, in light of the favorable treatment of R&D collaborations in the United States (Hagedoorn et al., 2000) and the European Union (Luukkonen, 2002), can our model yield an insight into issues relevant to policy-making?

To address these questions, we study an endogenous network formation model. We envisage an industry with three upstream and three downstream firms, which are locked in exclusive relations.[3] The input produced by the upstream firms is used by their respective downstream customers to produce a final good. In line with the stylized facts above, the upstream firms seek to reduce their costs by pursuing both process R&D investment and the formation of collaborative links to pool R&D outputs with other firms.

In this environment, when upstream firms decide whether to establish an R&D link between them, they anticipate how this will influence the competitive strength of their respective downstream customers. In turn, a more aggressive downstream firm sells more output and thus can secure more profit for its upstream supplier. To put it slightly differently, upstream firms compete against each other indirectly, *through* their downstream customers.

Our results emerge from comparing the upstream firms' network formation decisions under two alternative assumptions regarding their behavior: setting prices versus setting quantities. As far as the first question above is concerned, we argue that the equilib-

[3] As noted by Milliou and Petrakis (2007) exclusive relations are a common feature of many industries. For instance, auto-makers and suppliers of auto-parts, auto-makers and car dealers, petroleum firms and gasoline stations often carry out their dealings through exclusive contracts. It may also worth noting that exclusive relations often arise due to the presence of switching costs. Switching costs, in turn, are typically observed when upstream firms sell inputs which are tailored for the specific needs of downstream firms. At the same time, upstream and downstream firms may have jointly undertaken irreversible investments that render the costs of trading with alternative partners prohibitively high. This situation is common in the Japanese automobile industry, where downstream firms and their upstream suppliers undertake large fixed investments, such as investments in quality-control training, flexible automation and information flow mechanisms (Helper and Levine, 1992). In turn, such relation-specific investments work toward preventing an upstream-downstream firm pair from breaking up.

rium R&D networks between the upstream firms depend crucially on whether they set prices or quantities as well as on the magnitude of within-network R&D spillovers. More specifically, under a price setting, we show that the complete R&D network that includes all firms emerges in equilibrium. Yet, under a quantity setting, the equilibrium R&D network is ambiguous and depends on the size of within-network spillovers. In particular, if spillovers are sufficiently low, a complete network will arise in equilibrium. However, if spillovers are sufficiently high, the alternative, partial network will be formed that includes two of the firms but excludes the third. Finally, for intermediate levels of within-network spillovers, no network is strongly stable.

The intuition can be explained as follows. Consider a partial network under a price setting. In that case, linked firms sell their inputs at lower prices than the isolated firm because they enjoy greater access to lower costs through R&D. However, because input prices are strategic complements, the decrease in the input prices of the linked firms induces the isolated firm to lower its input price. But this "discount" harms the downstream counterparts of the linked firms by increasing the intensity of competition between themselves. As a result, the linked firms will benefit by bringing the isolated one into the R&D network in order to relax competition between downstream firms. Thus, the complete R&D network emerges endogenously under a price setting.

Under a quantity setting, though, our analysis demonstrates the emergence of the partial R&D network. In contrast to a price setting, the cost advantage of the linked firms implies that they are able to increase their input sales. This leads to a contraction of the input sales of the outsider (isolated) firm – because input quantities are strategic substitutes. Consequently, under certain conditions, we show that linked firms have no incentive to expand their partial R&D network. Thus, the equilibrium network formations might contain *more* R&D links under a price setting relative to a quantity setting, for certain values of the spillover parameter. We conclude that the mode of the upstream firms' behavior – setting prices versus setting quantities – plays an important role in explaining the structure of the equilibrium R&D network.

Regarding the second question posed earlier, our analysis suggests that, in the context of an upstream quantity setting, the incentives of upstream firms to form R&D links are non-monotone with respect to the level of within-network R&D spillovers (i.e. initially decreasing, then increasing). In contrast, under a price setting, the incentives to form links are not influenced by R&D spillovers. We also find that an expansion of an upstream firm's R&D network causes its R&D investment to decline. Despite a lower individual effort, our subsequent analysis reveals that the equilibrium R&D networks secure a generally higher aggregate level of effective R&D than any other network.[4] The reason is that more links imply that firms enjoy greater spillover opportunities, thereby offsetting the negative effect on aggregate effective R&D due to a lower individual effort.

As far as the third question is concerned, we note that while a price setting is likely to induce generally denser R&D networks, it is not apriori clear that this is an optimal choice from a social viewpoint. Here our analysis confirms that, under a price setting, the equilibrium network maximizes social welfare. However, under a quantity setting, we uncover a potential conflict between equilibrium and socially optimal networks. In particular, equilibrium networks might contain fewer R&D links than is optimal from a social viewpoint. Thus, our analysis suggests that the *mode* of the upstream firms' behavior (prices versus quantities) is as important for designing technology policy as the *size* of within-network R&D spillovers.

The paper is structured as follows. In section 2, we place our paper within the context of the relevant literature. Section 3 describes the key ingredients of our model and, section 4, characterizes the upstream firms' decision to form R&D networks both under a price and a quantity setting. Section 5 analyzes the efficiency properties of the different networks in terms of social welfare. Section 6 discusses various aspects of our results, focusing on policy implications and extensions to our model. Finally, section 7 concludes the paper.

[4] In the main body of the paper we slightly qualify this result. That is, when upstream firms set prices, the complete network maximizes the aggregate level of effective R&D, except if within-network spillovers are sufficiently large, in which case the aggregate level of effective R&D is higher in the star than in the complete network.

2 Related literature and contribution

Our paper contributes to two strands of literature. *First*, it contributes to the rapidly-expanding literature on R&D networks. In this strand of literature, a pioneer study is Goyal and Moraga-González (2001), who investigate the interaction between the extent of product market competition and R&D network formation. The authors demonstrate that, in a homogeneous-good market, intermediate levels of collaboration are desirable in terms of industry profits and social welfare but complete networks are stable. When firms compete in independent markets, though, this dilemma disappears: private and collective incentives for R&D collaboration do always coincide under the complete network.

Closer in spirit to our paper is Mauleon, Sempere-Monerris and Vannetelbosch (2008), who extend and enrich the relevant literature by studying the effects of firm-level unions on the stability and efficiency of horizontal R&D networks.[5] They show that, when firms set their own wages, the partial R&D network arises in equilibrium provided that within-network spillovers are sufficiently high. However, in the other polar case where unions set wages, the partial network is no longer stable, and the alternative, complete network, emerges in equilibrium. Moreover, this latter architecture does not Pareto-dominate the corresponding partial network when firms settle wages, and vice versa.

Our paper, like that of Mauleon et al. (2008), can be thought of as an attempt to develop the literature on R&D networks *vertically*. Yet, we depart from Mauleon et al. (2008) in the following two key respects. First, our focus is different in that we are interested in the network formation decisions of upstream rather than downstream firms. Second, unlike previous studies, we also consider two alternative forms of the upstream firms' behavior – a price setting and a quantity setting. Thus, the principal contribution of this paper relates to the market *tier* where the R&D network is formed as well as the *mode* of the upstream firms' strategic behavior.[6]

[5] Recent studies on horizontal R&D networks also include Westbrock (2010), Zikos (2010), Zu, Dong, Zhao and Zhang (2011) and Zirulia (2011).

[6] It is worth noting that R&D alliances are often followed by mergers and acquisitions (Hagedoorn and Sadowski, 1999). Thus, in a dynamic environment, our analysis can be thought of as focusing on the pre-merger phase, where firms seek to learn about their partners' competencies and quality of research efforts. In this light, our study can also be seen in a broader perspective as complementing the growing

Second, our paper contributes to a sizeable literature on R&D cooperation in oligopoly. While earlier studies focused on one-tier industries,[7] recently, Banerjee and Lin (2001), Attalah (2002) and Ishii (2004) investigated the (ambiguous) incentives to form research joint ventures in vertically related industries. Banerjee and Lin (2001) examine the incentives to establish vertical RJVs under different cost-sharing rules. The authors show that the optimal RJV size is positively correlated with the R&D cost, the gains from innovation and the market size. Attalah (2002) and Ishii (2004) extend the analysis to consider horizontal R&D alliances in addition to vertical ones.

In this paper, we restrict our attention to the endogenous determination of upstream R&D networks when upstream firms set either their input prices or quantities.[8] This allows us to shed some light on the *extent* of inter-firm collaboration decisions. Moreover, unlike the literature on RJVs, which assumes that R&D investments are determined cooperatively, the "network approach" that we follow takes the view that R&D investments are determined non-cooperatively, in private R&D labs.[9] As explained in the Introduction, an R&D network is a non-equity form. Therefore, firms retain their own R&D labs and agree to pool their R&D results by forming collaborative links.[10]

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literature on upstream horizontal mergers including Horn and Wolinsky, (1988), Ziss (1995), Milliou and Petrakis (2007).

[7] See, for example, d' Aspremont and Jacquemin (1988), Poyago-Theotoky (1995), Kamien, Muller and Zang (1992), Suzumura (1992), Qiu (1997) and Amir (2000).

[8] In a similar vein, Kesavayuth and Zikos (2012) study the simultaneous emergence of upstream and downstream R&D networks. Yet we depart from this paper in two important dimensions. First, the framework studied in the present paper is less restrictive, in the sense that we do not require that downstream firms and their input suppliers simultaneously establish horizontal R&D networks. Second, we study the effects of the upstream firms' behavior (setting prices versus setting quantities) on their network formation decisions as well as on market and societal outcomes.

[9] It is well known that R&D collaborations may be terminated early or may not meet the expectations of research partners (see e.g. Narula and Hagedoorn, 1999; Podolny and Page, 1998). In light of this observation, our assumption – standard in the R&D network literature – that collaborating firms individually choose their R&D investments captures precisely a lack of trust between themselves. Our focus on non-cooperative investment behaviour is also consistent with Caloghirou et al. (2003, p. 549), among others, who point out that it is very difficult, "even impossible", to write complete contracts on intangible assets such as R&D investments.

[10] It is worth noting that close in spirit to the network approach is the model of RJV competition in the taxonomy by Kamien et al. (1992), which is an exception to the norm of R&D co-operation.

3 Model

We consider a two-tier industry consisting of three upstream firms and three downstream firms denoted, respectively, by U_i and D_i, with $i = 1, 2, 3$ – see Figure 1, left panel.[11] One could think of the upstream and the downstream firms as being, respectively, input suppliers and final good manufacturers. Downstream firms are endowed with constant returns to scale technologies that transform one unit of input to one unit of output. Moreover, there is an exclusive relation between U_i and D_i.[12] Hence, the input produced by each upstream firm is used by its respective downstream customer to produce a final good.

Each downstream firm D_i faces the following (inverse) demand function:[13]

$$p = a - \sum_{i=1}^{3} q_i. \tag{1}$$

Each downstream firm faces no other cost than the input price (w_i) to its exclusive supplier. Thus trading is conducted through linear wholesale price contracts.[14]

Each upstream firm faces an initially constant marginal cost of production $\bar{c}$, with

[11] This is the smallest number of firms that allows us to study asymmetric networks (i.e. partial and star networks) tractably. We note that, as Goyal and Moraga-González (2001), Mauleon et al. (2008) point out, a general analysis of asymmetric networks would be especially challenging, though we return to this issue in section 6.

[12] This kind of exclusivity is a standard assumption in the vertical relations literature (see e.g. Milliou and Petrakis, 2007, and the references therein). As Milliou and Petrakis (2007) mention, "the latter can result from various sources. For instance, when the upstream firms produce inputs which are tailored for specific final good manufacturers, there may be irreversible R&D investments that create *lockin* effects and high switching costs." In section 6 we also discuss what would happen if we allow for non-exclusive relations.

[13] Linear product demand is a simplifying assumption which is typical in the R&D network literature. We further cast our analysis in the context of a homogeneous-product market in order to allow two empirically relevant – and opposing – forces to drive the firms' network formation decisions: efficiency improvement (a positive effect) that subsequently triggers increased competition between the upstream firms – a negative effect (which operates through the corresponding downstream firms). This trade-off between "cooperation" and "competition" is consistent with empirical evidence reported in OECD (2001).

[14] This assumption allows us to concentrate on the main strategic features of a price and a quantity setting (by sidestepping any additional instruments, i.e. a fixed fee, that may be available on the part of the upstream firms in their dealings with their downstream customers). If, however, upstream firms can use a non-linear pricing scheme that takes the form of a two-part tariff, they will internalise perfectly the profit of their downstream customers, thus yielding predictions very similar to one-tier models of R&D networks (e.g. Goyal and Moraga-González, 2001).

$0 \leq \bar{c} < a$ (see e.g. d' Aspremont and Jacquemin, 1988).[15] Upstream firm i, by investing kx_i^2, $k > 0$ in process R&D can attain unit production cost $\bar{c} - x_i$, where x_i is firm i's own R&D output.[16] For simplicity, we set $k = 1$ which ensures nonnegativity of all variables. Note that the R&D cost function reflects diminishing returns to scale to R&D expenditures. Moreover, each upstream firm can establish collaborative links and further reduce its marginal cost by pooling R&D outputs with other upstream firms.

The "effective" R&D investment, X_i, represents the overall reduction in firm i's marginal cost due to R&D. It is obtained from firm i's own R&D output, x_i, and from the research outputs of firms connected with i, which are partially absorbed depending on the extent of within-network R&D spillovers β, $\beta \in (0, 1]$.

In a triopoly, depending on the R&D links established between the upstream firms, four distinct R&D network architectures may arise – see Figure 1, right panel.

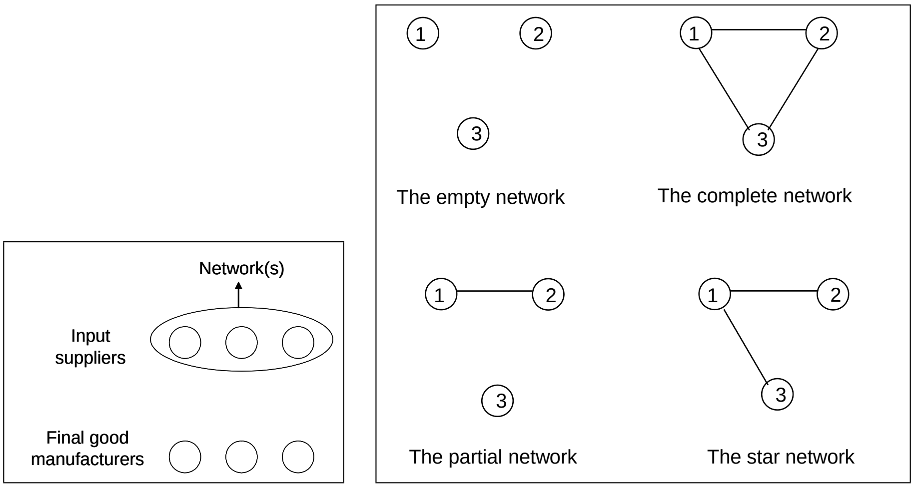

Figure 1: Industry structure (left panel) and networks architectures (right panel)

In the *empty network*, there are no links. Thus the overall marginal cost of each upstream firm is given by:

$$c_i(g^e) = \bar{c} - x_i, \quad i \in \{1, 2, 3\}. \tag{2}$$

[15]We assume that upstream firms face (ex ante) identical marginal costs. These costs are then determined endogenously and, thus, in equilibrium, may differ across firms depending on the exact network architecture as well as the place a firm occupies in it.

[16]Deroian and Gannon (2006) shift the focus from a setting of process R&D to one of product R&D. They show that the latter yields qualitatively similar results with the more common setting of process R&D used by others, at least for sufficiently homogeneous products.

All other network architectures contain links between firms. In the *complete network*, each firm is connected with the other two firms. Marginal costs are thus given by:

$$c_i(g^c) = \bar{c} - x_i - \beta(x_j + x_k), \;\; i \neq j \neq k, \;\; i, j, k \in \{1, 2, 3\}. \tag{3}$$

This implies that each (linked) firm can get access to its partners' R&D stocks at a rate $\beta \in (0, 1]$.[17] Thus, in the complete network, the effective R&D investment of firm i is $X_i = x_i + \beta(x_j + x_k)$.

In the *partial network*, there is only one collaborative link. To fix ideas (and w.l.o.g.) assume that firms 1 and 2 maintain this link. The ensuing marginal costs in this network structure are:

$$\begin{aligned} &\text{Insiders:} \qquad c_i(g^p) = \bar{c} - x_i - \beta x_j, \;\; i \neq j, \quad i, j \in \{1, 2\}, \\ &\text{Outsider:} \qquad c_3(g^p) = \bar{c} - x_3. \end{aligned} \tag{4}$$

Finally, in the *star network*, like the partial network, there are two types of firms: a hub and two spokes. The hub has two links, one with each of the two spoke firms. In turn, the spokes have a direct link with the hub as well as an indirect link with each other. To capture this relatively large distance within the network between spoke firms, we assume that they can benefit from each other's R&D stock at a rate $\frac{\beta}{2}$ (Mauleon et al., 2008).[18] Let firm 1 be the hub and firms 2 and 3 be the spokes. Ensuing marginal

[17] Goyal and Moraga-González (2001) focus instead on public spillovers. They assume that when a link is formed, partner firms can fully benefit from each other's R&D stock, i.e. $\beta = 1$. In addition, non-collaborating (or indirectly connected) firms can benefit from the R&D stocks of collaborating firms, but at a lower rate, which is assumed equal to β, $\beta \in [0, 1)$. We note that both spillover processes yield analogous predictions regarding the equilibrium R&D network formations (see Mauleon et al., 2008; Goyal and Moraga-González, 2001).

[18] We assume that within-network R&D spillovers depend on the distance between a pair of collaborating firms, i and j. In turn, this distance is measured by the number of links, $t(ij)$, in the shortest path between i and j. This means that, if firms i and j are directly linked, then $t(ij) = 1$; while if i and j are spoke firms, who are indirectly linked via the hub, then $t(ij) = 2$. We set $t(ij) = \infty$ to denote the absence of a path between the pair of firms i and j. Thus, following Mauleon et al. (2008), in a network g, the overall marginal cost of producing the input for firm U_i is given by $c_i(g) = \bar{c} - x_i - \beta \left[\frac{x_j}{t(ij)} + \frac{x_k}{t(ik)}\right]$. We note that the idea of spillovers decreasing with increasing distance has also been used in related contexts. For instance, Piga and Poyago-Theotoky (2005) develop a Hotelling-type model, where spillovers are location-specific; that is, the further apart firms are located the less they can benefit from each other's efforts in R&D.

costs are:

$$\begin{aligned} \text{Hub:} \qquad & c_1(g^s) = \bar{c} - x_1 - \beta(x_i + x_j),\ i \neq j, \quad i,j \in \{2,3\}, \\ \text{Spokes:} \qquad & c_i(g^s) = \bar{c} - x_i - \beta x_1 - \frac{\beta}{2} x_j. \end{aligned} \tag{5}$$

Given the different R&D network architectures that may arise, the profits of an upstream firm U_i and a downstream firm D_i are, respectively:

$$\pi_{U_i}(g) = [w_i(g) - c_i(g)]\, q_i(g) - [x_i(g)]^2\,; \text{ and} \tag{6}$$

$$\pi_{D_i}(g) = [p(g) - w_i(g)]\, q_i(g). \tag{7}$$

Note that $w_i(g) - c_i(g)$, i.e. the difference between the input price and the production cost of firm U_i, captures U_i's profit margin per unit of input sold to firm D_i. Similarly, $p(g) - w_i(g)$ reflects D_i's profit margin per unit of final good sold to consumers.

3.1 Sequence of moves

We consider the following four-stage game. In the first stage (*R&D network formation*), the upstream firms choose simultaneously their R&D links. Four conceivable R&D network architectures may arise from this stage (Figure 1, right panel). In the second stage (*R&D selection*), conditional on the network structure, upstream firms decide simultaneously and independently their R&D investments, so as each individual firm to maximize its profits. In the third stage (*upstream price/quantity selection*), the upstream firms choose simultaneously either their wholesale quantities or prices. In the last stage (*downstream competition*), the downstream firms choose their output levels.[19]

The sequencing of moves, which is standard in the R&D network literature, reflects that the selection of collaborative links (stage 1) is a strategic long-run decision for the

[19] We retain the assumption that the product market is characterized by Cournot competition, which is typical in existing R&D network models.

upstream firms.[20] This is consistent with anecdotal evidence that the establishment of R&D alliances requires "strong commitment" from the participating firms (Hagedoorn, 2002, p. 479). The sequencing of moves further captures that the choice of the upstream firms' R&D investments (stage 2) is a longer-run decision than the exact level of their input prices or quantities (stage 3). This is because R&D activity is inherently uncertain and thus may require a relatively long time to come into fruition; while input prices or quantities can be changed more often and more easily, responding to changes in market conditions.

3.2 Equilibrium concepts

We solve the game backwards from stage 4 (downstream competition) to stage 2 (R&D selection). Then we turn to the first stage for which we obtain the set of "stable" networks. To this end we use two well-established equilibrium concepts – "pairwise stability" and "strong stability".

Following Jackson and Wolinsky (1996), a network is *pairwise stable* if no firm has an incentive to delete unilaterally one of its R&D links and no pair of firms want to add a new link between them (with one benefiting strictly and the other at least weakly). If networks can be ordered in the following way {empty, partial, star, complete}, then pairwise stability permits deviation to a 'neighboring' network architecture.

Pairwise stability considers deviations by one pair of firms at a time.[21] This suggests that if we enrich the network formation process to encompass deviations by a coalition of firms, then it may no longer be the case that the same network architectures will materialize in equilibrium. Indeed, it may well be the case that a pairwise stable network is no longer strongly stable. More specifically, we say that a network is *strongly stable* – a concept due to Jackson and Van de Nouweland (2005) – if it survives all possible changes in the number of its links by any coalition of agents, because at least one member of the

[20] Indeed, in stage 1 the upstream firms anticipate the subsequent effects of their network formation decisions on R&D investments, input prices/quantities and output quantities.

[21] Pairwise stability can be seen as a necessary condition for stability (see Jackson and Wolinsky, 1996; Goyal and Moraga-González, 2001).

coalition would be worse off and 'block' the deviation. This constitutes a refinement of the set of pairwise stable networks.

4 Equilibrium R&D networks

In this section we derive the equilibrium of the entire game. Thus we proceed to solve stage 1, the R&D network formation stage, by applying the concepts of pairwise stability and strong stability.

Exploiting the symmetries across firms, we adopt the following notation for equilibrium profits throughout:

Π^E denotes a firm's profits in the empty network;

Π^I denotes the profits of an insider (linked) firm in the partial network;

Π^O denotes the profits of outsider (isolated) firm in the partial network;

Π^H denotes the hub firm's profits in the star network;

Π^S denotes a spoke firm's profits in the star network; and

Π^C denotes a firm's profits in the complete network.

4.1 An upstream price setting

The following Proposition characterizes the upstream firms' decision to form R&D links under a price setting.

Proposition 1 *When upstream firms set prices, the complete network is the unique pairwise stable and strongly stable network.*

For all levels of spillovers β within a network, the upstream firms' profits are ranked as follows:[22]

$$\Pi^H > \Pi^C > \Pi^I > \Pi^S > \Pi^E > \Pi^O. \tag{8}$$

[22] Equilibrium outcomes for R&D investments and profits are reported in Appendix A. Relevant plots are also available on request.

The ranking above implies that a firm in the empty network earns less than a spoke in the star ($\Pi^S > \Pi^E$), but more than the isolated firm in the partial ($\Pi^E > \Pi^O$) – because the latter, outsider firm, is in a weaker competitive position vis-à-vis its rivals. Likewise, the hub in the star network earns more than any of the firms in the complete network ($\Pi^H > \Pi^C$). Finally, notice that $\Pi^C > \Pi^I$: a firm in the complete network performs better than an insider (linked) firm in the partial network.

The intuition behind this last condition can be explained as follows. From stage 3 (upstream price selection) of our game, the reaction functions for input prices from the viewpoint of a linked firm i and the isolated firm are, respectively:

$$w_i(w_j, w_3; x_i, x_j) = \frac{1}{6}[a + 3c + w_j + w_3 - 3(x_i + \beta x_j)], \;\; i \neq j, \quad i, j \in \{1, 2\}, \qquad (9)$$

$$w_3(w_i, w_j; x_3) = \frac{1}{6}(a + 3c + w_i + w_j - 3x_3). \qquad (10)$$

These reaction functions suggest that input prices between the linked firms and the isolated one are strategic complements – that is, $\partial w_i/\partial w_3 > 0$ and $\partial w_3/\partial w_i > 0$.

Consider a partial R&D network, where two of the firms are linked and one is isolated. In that case, the linked firms, who enjoy superior access to lower costs through R&D, are able to set lower input prices. The decrease in the input prices of the linked firms induces the isolated firm to lower its input price – because input prices are strategic complements. But this harms the downstream counterparts of the linked firms by increasing the intensity of competition between themselves. Thus, the linked firms will benefit by expanding their partial R&D network in order to relax competition between downstream firms. Putting this last result slightly differently, a deviation from the partial to the complete network is profitable for all firms involved, both the insiders and the outsider, because $\Pi^C > \Pi^I$ and $\Pi^C > \Pi^O$.

Using the profit ranking (8), Proposition 1 is then proved as follows. To show that the complete network is pairwise stable, we require that $\Pi^C > \Pi^S$: a firm in the complete network earns more than a spoke in the star network. From (8) we observe that this inequality always holds and ensures that no firm will unilaterally sever one of its links to become a spoke in the star network. Intuitively, $\Pi^C > \Pi^S$ arises because the spoke firms in the star network suffer a cost disadvantage relative to the hub, whereas in the complete network all firms are identical. Therefore, the complete network is pairwise stable for all β, as Proposition 1 reports. This also implies that the star network is not itself pairwise stable.

We proceed to show that the complete network is the unique pairwise stable network. To this end, we note that, in the empty network, a pair of firms can improve their competitive position by forming an R&D link because $\Pi^I > \Pi^E$. This gives rise to a partial network, which includes two of the firms but excludes the third. Next, contemplating a deviation from the partial to the star network, from (8) we observe that $\Pi^S > \Pi^O$ (a spoke in the star earns more the outsider in the partial) and $\Pi^H > \Pi^I$ (the hub in the star earns more than an insider in the partial). This implies that the partial network is not pairwise stable. Thus, the complete network is the only candidate for strong stability.

Turning to strong stability, the firms in the complete network will not jointly deviate to the empty network – by severing all their links – because $\Pi^C > \Pi^E$. Likewise, $\Pi^C > \Pi^I$ implies that two of the firms in the complete network will not force a deviation to the partial network. Therefore, as Proposition 1 states, the complete network emerges endogenously as the unique strongly stable network.

4.2 An upstream quantity setting

We now consider a quantity setting in the upstream market. The following Proposition characterizes the set of stable network structures.

Proposition 2 *When upstream firms set quantities: (i) The complete network is always pairwise stable, and it is strongly stable if within-network spillovers are sufficiently low, $\beta \in [0, \beta^*]$, where $\beta^* \approx 0.33$. (ii) The partial network is pairwise stable and strongly stable if within-network spillovers are sufficiently high, $\beta \in [\beta^{**}, 1]$, where $\beta^{**} \approx 0.95$. (iii) No network is strongly stable if within-network spillovers are intermediate, $\beta \in (\beta^*, \beta^{**})$.*

For all levels of within-network spillovers β, the profits of the upstream firms are ranked as follows:[23]

$$\Pi^H > \Pi^C > \mathbf{\Pi}^I > \Pi^S > \Pi^E > \Pi^O \text{ if } \beta \in (0, 0.33); \tag{11}$$

$$\Pi^H > \mathbf{\Pi}^I > \Pi^C > \Pi^S > \Pi^E > \Pi^O \text{ if } \beta \in (0.33, 0.95); \tag{12}$$

$$\mathbf{\Pi}^I > \Pi^H > \Pi^C > \Pi^S > \Pi^E > \Pi^O \text{ if } \beta \in (0.95, 1]. \tag{13}$$

From (11)-(13) we observe that, under a quantity setting, the relative position of Π^I depends on the level of within-network R&D spillovers. In particular, Π^I is lowest for $\beta \in (0, 0.33)$ and highest for $\beta \in (0.95, 1]$. As we demonstrate in the sequel, the variable position in the ranking of Π^I is crucial for the equilibrium properties of the complete and the partial network as well as the ultimate choice of these network structures themselves. We now elaborate on some aspects of this result.

Our first observation concerns the condition $\Pi^I > \Pi^C$ (if $\beta > 0.33$) – that an insider firm in the partial network can earn more than a firm in the complete network – which contrasts with a price setting, where $\Pi^C > \Pi^I$ for all β; see eq. (8) . Intuitively, it arises because, under a quantity setting, input quantities are strategic substitutes, i.e. $\partial q_i / \partial q_3 < 0$ and $\partial q_3 / \partial q_i < 0$; see eqs. (14) and (15). In particular, from the point of view of a linked firm i and the isolated firm, in stage 2 (upstream quantity selection) of our game, the reaction functions for input quantities are given by:

$$q_i(q_j, q_3; x_i, x_j) = \frac{1}{8}[3(a-c) - 4(q_j + q_3) + 3(x_i + \beta x_j)], \;\; i \neq j, \quad i, j \in \{1, 2\}, \tag{14}$$

$$q_3(q_i, q_j; x_3) = \frac{1}{8}[3(a-c) - 4(q_i + q_j) + 3x_3]. \tag{15}$$

In the partial R&D network, the linked firms can achieve substantially lower costs than the isolated firm due to their access to each other's R&D stock. The cost advantage of the insiders means that they can expand their input sales, which leads to a contraction

[23] Again, equilibrium outcomes are in Appendix A, and relevant plots are available on request.

of the input sales of the outsider firm – because input quantities are strategic substitutes. In turn, the cost advantage of the insiders can be either strong or weak, depending on the extent of within-network R&D spillovers. As a result, when spillovers are relatively high – implying a relatively strong cost advantage – each insider firm in the partial network will earn more than a firm in the complete network, i.e. $\Pi^I > \Pi^C$ if $\beta > 0.33$. In other words, unlike under a price setting, the insider firms in the quantity setting environment will no longer expand their partial R&D network provided that spillovers are sufficiently high.

Our second observation pertains to the condition $\Pi^I > \Pi^H$ (if $\beta > 0.95$): the insiders in the partial network perform better than the hub in the star network. This is a second key condition behind the emergence of the partial R&D network as an equilibrium network formation. The intuition behind $\Pi^I > \Pi^H$ is fairly straightforward. Adding a link to the partial network means that the hub gets access to the R&D stocks of the two spoke firms but also shares its own R&D stock. In the partial network, though, the two linked firms conceal their research outputs from their rival and thus fully internalize their competitive advantage. Consequently, as long as within-network spillovers are sufficiently high, being a linked firm in the partial network is better than being the hub firm in the star network.[24]

Having said this, we next turn to establish part (i) of Proposition 2. Pairwise stability is implied by $\Pi^C > \Pi^S$; this also means that the star network is not pairwise stable. Turning to strong stability, although the three firms in the complete network will not jointly deviate to the empty network, it is the case that two of the firms in the complete network will sever their links with the third firm if $\beta > \beta^* \approx 0.33$, because $\Pi^I > \Pi^C$ in the latter case. Thus, the complete network emerges as a strongly stable network only if $\beta < 0.33$, as part (i) of Proposition 2 states.

We now show part (ii) of the Proposition. We first note that $\Pi^I > \Pi^E$. Further, we have that $\Pi^S > \Pi^O$, so pairwise stability also requires $\Pi^I > \Pi^H$. In turn, the latter condition holds if within-network spillovers are sufficiently high, i.e. $\beta > \beta^{**} \approx 0.95$.

[24] That is, in Figure 1 (right panel), deleting a link from the star network benefits the hub-designate (i.e. firm 1) but harms the spokes-designate (i.e. firm 3).

Therefore, the partial network is pairwise stable when spillovers are sufficiently high, as part (ii) of Proposition 2 reports. Turning to strong stability, from part (i) of the Proposition, we know that $\Pi^I > \Pi^C$ if $\beta > 0.33$. This rules out the possibility that the insider firms in the partial network will each form an R&D link with the outsider, isolated firm. Consequently, the partial network emerges endogenously as a strongly stable architecture when spillovers are sufficiently high, i.e. $\beta > 0.95$.

From the proofs of parts (i) and (ii) of Proposition 2, it follows immediately that no network is strongly stable for intermediate values of the spillover parameter, i.e. $\beta \in (\beta^*, \beta^{**})$. This result relies on the relative ranking of Π^I and Π^C and how this depends on the magnitude of within-network R&D spillovers. Interestingly, we find that there is no 'smooth' transition from the complete to the partial network. This is a non-monotone result highlighting the role played by R&D spillovers in determining the equilibrium network architectures under a quantity setting.

Taken together, Propositions 1 and 2 suggest some additional observations. First, the equilibrium R&D network architectures between the upstream firms depend crucially on whether they set prices or quantities as well as on the magnitude of within-network R&D spillovers. Second, in the context of an upstream quantity setting, the incentives of upstream firms to form collaborative R&D links are non-monotone with respect to the level of within-network spillovers (i.e. initially decreasing, then increasing), whereas under a price setting the incentives to form links are not influenced by spillovers. Third, the equilibrium R&D networks might contain a larger number of R&D links under a price setting than under a quantity setting, which appears to be the case if within-network R&D spillovers are sufficiently large, i.e. $\beta > 0.33$. Interestingly enough, we also find that the preferences of the upstream and downstream firms regarding the formation of R&D networks are largely consistent in the present setting. In particular, when the downstream rather than the upstream firms choose the R&D links, we find that the complete network is the unique pairwise stable network under a price setting. Yet, under a quantity setting, the partial network is pairwise stable provided that spillovers are sufficiently large, i.e. $\beta > 0.46$; while the complete network is pairwise stable for most cases, i.e. $0 < \beta \leq 0.95$.[25]

4.3 R&D investments

In this section, our objective is to investigate how the R&D networks, particularly the strongly stable ones, affect the aggregate level of "effective" R&D.[26] To this end, we begin by analyzing how the different networks affect firm-level R&D investments. The following Proposition summarizes our findings.

Proposition 3 *When upstream firms set prices as well as when they set quantities, an expansion of an upstream firm's R&D network causes its own R&D investment to decline. Moreover, an upstream firm's R&D investment typically declines when the other two firms establish a new R&D link between themselves.*

This result can be explained intuitively in terms of two countervailing effects. When an upstream firm U_i forms a new link, it lowers its own costs by getting access to the R&D stocks of its partners (*efficiency effect*). On the other hand, as a result of this new link, the production costs of partners firms go down as well, which reduces the returns to U_i's initial cost reduction (*competition effect*).[27] As a result, the incentive to exert R&D effort depends on the relative merits of these two effects. It turns out that the competition effect is stronger and thus outweighs the efficiency effect. Therefore, firm U_i will put in a lower R&D effort when it forms new links. This pattern regarding a reduction in a firm's own R&D effort also extends to the case where the other two upstream firms form a new R&D link between them, for this leads to a contraction of the outsider's market share.[28]

These findings highlight the presence of the typical *free-riding* problem (e.g. Kamien et al., 1992) in collaborative R&D activity, according to which the existence of tech-

[25] The formal proof is available from the authors upon request.

[26] As usual, the extent of cost reduction is measured by total "effective R&D" (e.g. d' Aspremont and Jacquemin, 1988; Mauleon et al., 2008). This refers to the total amount of R&D output (or investment/effort) that is applied to production – that is, the sum of a firm's own R&D output and the R&D outputs that it can access through collaborative links.

[27] Recall that, in the present setting, the competition effect between the upstream firms operates through their downstream counterparts.

[28] We note that an exception arises in the move from the empty to the partial network when upstream firms set quantities, provided that within-network spillovers are relatively low, i.e. $\beta \in (0, 0.14]$.

nological spillovers allows a firm to free-ride on its partners'/rivals' R&D investments, and thus abstain from own R&D spending. The free-riding problem has also been identified in different contexts in the recent literature on R&D networks (e.g. Goyal and Moraga-González, 2001; Mauleon et al., 2008).

The discussion above points to the following trade-off. On the one hand, more links lead to lower firm-level R&D investments. On the other hand, more links imply that firms enjoy greater spillover opportunities. Can this latter positive effect potentially offset the former negative and thus lead to a higher aggregate level of effective R&D? In other words, one might wonder whether the strongly stable networks, which are typically highly linked formations (recall Proposition 1 and 2), can perform well in terms of aggregate effective R&D.

It can be easily established that the strongly stable architectures generally secure a relatively higher aggregate level of effective R&D.[29] More specifically, when upstream firms set quantities, both the complete and the partial network are effective R&D-maximizing structures. A similar pattern also arises when upstream firms set prices, unless spillovers are sufficiently high. The following Proposition summarizes.

Proposition 4 *(i) When upstream firms set prices, the complete network maximizes the aggregate level of effective R&D, except if within-network spillovers are sufficiently high,* $\beta \in [0.95, 1]$*, in which case the aggregate level of effective R&D is higher in the star network than in the complete network.*

(ii) When upstream firms set quantities, the complete network maximizes the aggregate level of effective R&D if within-network spillovers are relatively low ($\beta < 0.57$*). For intermediate levels of within-network spillovers (*$0.57 \leq \beta < 0.86$*) it is the star network, and for high spillovers (*$\beta \geq 0.86$*) it is the partial network that maximizes the aggregate level of effective R&D.*

Interestingly, Proposition 4 suggests that the level of network-specific aggregate effective R&D depends crucially on whether upstream firms set prices or quantities as

[29] Equilibrium outcomes for effective R&D investments are given in Appendix A. Also, relevant plots are available on request.

well as on the magnitude of within-network R&D spillovers. The intuition is as follows. When β is low, the aforementioned *competition effect* is relatively weak. This means that, under the complete network, the reduction in individual R&D efforts is offset by the spillover-induced information sharing. As a result, the complete network secures the highest aggregate level of effective R&D. As β rises, the *competition effect* becomes more prominent and there is now an incentive for individual firms to reduce further their own R&D efforts. This suggests that asymmetric industry structures, such as the star or the partial network, become more prominent in terms of aggregate effective R&D. Consequently, the number of collaborative links that maximize the aggregate level of effective R&D decline with respect to the spillover parameter, β.

5 Social welfare

In this section we consider the impact of equilibrium R&D networks on social welfare. Our interest is in understanding whether "market forces" governing network formation will lead to an outcome which is also beneficial from a social viewpoint. Clearly, such analysis is important in framing the optimal technology policy for collaborative R&D.

We define social welfare in the standard way as the sum of consumers' surplus, upstream and downstream firms' profits. Social welfare in network g is thus given by:

$$W(g) = \frac{[Q^m(g)]^2}{2} + \sum_{i=1}^{3} \Pi_{U_i}^m(g) + \sum_{i=1}^{3} \Pi_{D_i}^m(g), \tag{16}$$

where $Q^m(g) = \sum_{i=1}^{3} q_i^m(g)$ and m denotes a price setting and a quantity setting in the upstream market. Substituting the relevant expressions for output and firm profits into (16), we obtain social welfare for each of the four R&D networks when upstream firms set prices as well as when they set quantities. In Figures 2 and 3 we then plot welfare levels for the different networks.[30] Under a quantity setting, define $\hat{\beta}$ as the solution to the

[30] We use "Mathematica 8" (see Wolfram, 1999) for the Figures, and set $a = 4$ and $\bar{c} = 2$, which is inconsequential in a qualitative sense (i.e. $a - \bar{c}$ is a scale parameter). Here we plot the equilibrium outcomes for social welfare; while the exact analytical formulas are available on request.

equation $W(g^c) = W(g^s)$, where $\hat{\beta} \approx 0.71$. Figure 3 reveals that $\hat{\beta}$ exists and is unique.

Inspection of Figures 2 and 3 gives us the following key result.

Proposition 5 *(i) When upstream firms set prices, the complete network is always socially optimal. (ii) When upstream firms set quantities, the complete network is the socially optimal structure if $\beta \in [0, \hat{\beta}]$, whereas the star network is socially optimal if $\beta \in [\hat{\beta}, 1]$, where $\hat{\beta} \approx 0.71$.*

Perhaps the simplest way to understand the economic forces involved is to consider each component of social welfare. A key idea is that both consumer surplus and total downstream profits are maximized in the network that yields lowest marginal costs. The reason is that lower marginal costs translate not only into lower input prices, but may also cause product prices to fall as a result. If so, both consumers and downstream producers benefit.

As explained in the previous section (4.3), under a price setting, initially the complete network (if $\beta < 0.95$) and then the star network secures lowest marginal costs. A similar pattern also emerges under a quantity setting: first the complete (if $\beta < 0.57$), then the star (if $0.57 < \beta < 0.86$) and eventually the partial network yields lowest marginal costs. Thus, the number of collaborative links that minimize marginal costs decline with β.

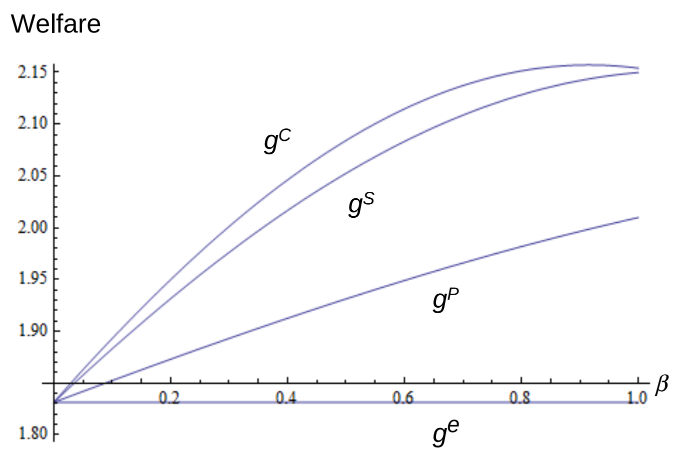

Figure 2: Welfare levels under a price setting

On the other hand, total upstream profits depend not only on the extent of overall cost reduction, but also on the market position of each upstream firm relative to the others.

Thus, under a price setting, we find that the complete network promotes total upstream profits. Intuitively, in the partial network, the cost advantage of the linked firms is eroded by their isolated counterpart, who tends to reduce its input price. Moreover, the complete network contains more R&D links compared to the star or the empty network. As a result, upstream firms earn higher total profits in the complete network than in any other network. Yet, under a quantity setting, we find that apart from the complete network, the star network can maximize total upstream profits – but only if within-network R&D spillovers are sufficiently high, i.e. $\beta > 0.896$.[31,32]

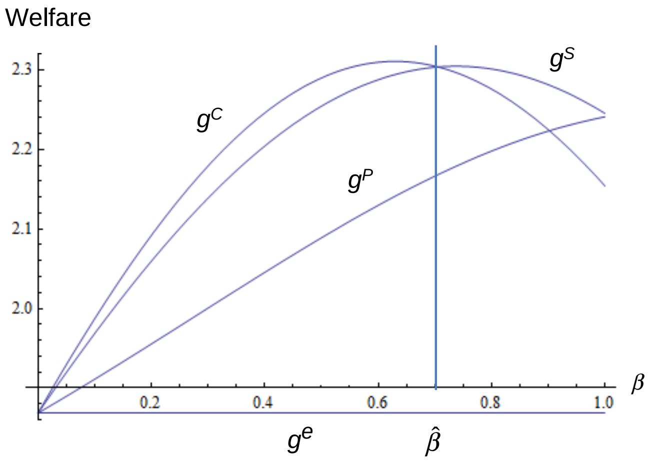

Figure 3: Welfare levels under a quantity setting

The overall effect of network formation on social welfare – reported in Proposition 5 – is determined by the interplay between the aforementioned three forces: (i) consumer surplus, (ii) total downstream profits and (iii) total upstream profits. In particular, as explained previously, (i) and (ii) move in the same direction but (iii) does not necessarily

[31] As it turns out, the ranking of total upstream profits (proof available on request) follows a very similar pattern to social welfare. That is, when upstream firms set quantities, total profits are highest in the complete and then the star network – in other words, first the complete and then the star network is *strongly efficient*. In contrast, when upstream firms set prices, the complete network is the unique industry profit-maximising/strongly efficient architecture. We conclude that total upstream profits and social welfare yield qualitatively similar predictions regarding network efficiency in the present setting.

[32] One might wonder which networks are Pareto efficient in the present setting. We say that network g is Pareto efficient if it is not Pareto dominated by any other network; that is, g is Pareto efficient if there does not exist any other network g' such that $\Pi_i(g') \geq \Pi_i(g)$ for all i, with strict inequality for some i. Applying this definition, it can be easily established (proof available on request) that, under a quantity setting, the complete and star networks are always Pareto efficient, the partial network is Pareto efficient if $\beta \in [0.34, 1]$, and the empty network is never Pareto efficient. In contrast, under a price setting, the complete and the star network are always Pareto efficient, whereas the partial and the empty network are not Pareto efficient.

do so. For example, under a price setting, it turns out that all three forces pull in the same direction if $\beta < 0.95$, whereas for higher β-values the effect (iii) dominates – thus, social welfare is maximized under the complete network. Finally, we note that the intuition under a quantity setting can be explained by following exactly the same logic as under a price setting.

6 Discussion

In this part of the paper we present some findings based on our previous analysis, and also discuss briefly a number of extensions of our model. Taken together Propositions 1, 2 and 5 suggest that when upstream firms set prices, there is a perfect correspondence between market and social incentives for R&D collaboration. This is not necessarily true, though, when upstream firms set quantities – that is, there is a potential conflict between strongly stable and socially optimal networks. Such a conflict between stability and social welfare seems to be prevalent if within-network R&D spillovers are sufficiently high (i.e. $\beta > \beta^* \approx 0.33$): the complete and the star network promote welfare but they don't arise in equilibrium – see Figure 4.

Thus, the key message is:

Proposition 6 *(i) When upstream firms set prices, individual and social incentives for R&D collaboration are always aligned. (ii) When upstream firms set quantities, there is a potential conflict between individual and social incentives if $\beta \in [\beta^*, 1]$, where $\beta^* \approx 0.33$.*

In terms of policy implications, Proposition 6 provides support for a laisser-faire policy if upstream firms set prices. In contrast, it highlights a role for government intervention when upstream firms set quantities and within-networks R&D spillovers are sufficiently high, i.e. $\beta > \beta^*$. In that case, equilibrium R&D networks are under-connected from a social viewpoint. Hence, our model suggests that policy makers should aim at actively promoting R&D networks through the use of an appropriate subsidization policy (e.g. through subsidization of administration and coordination costs incurred by the participating firms).[33] For instance, in this context, the provision of R&D subsidies seems to assume a dual role. It not only supports the expansion of existing R&D networks (thus reducing the likelihood of a conflict between stable and welfare-improving networks), but may also encourage collaborating firms to undertake more R&D investments.[34] In turn, the latter role of technology policy might be relevant particularly as a means of reducing the typical free-riding problem in collaborative R&D activity.

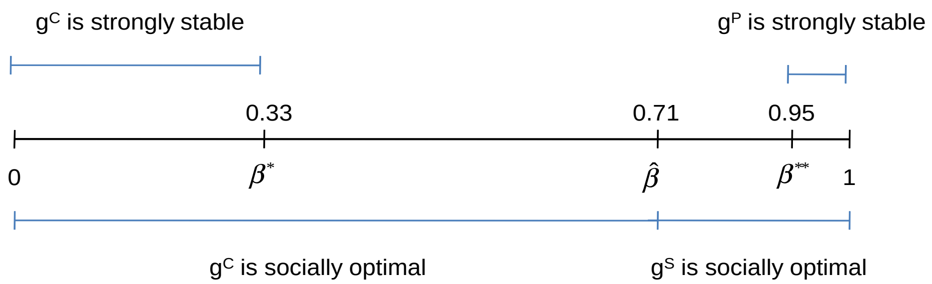

Figure 4: The conflict between individual and collective interests under a quantity setting

As a caveat to the normative conclusions drawn above, we note that our model has several special features that future work might seek to relax. In particular, in our baseline model we have assumed that upstream firms have all the bargaining power to set input prices. In reality however, upstream and downstream firms often negotiate over their input prices. An interesting question is therefore to investigate the role of the bargaining power distribution on R&D network formation.

[33]Subsidization policies for the promotion of R&D networks have been consistently applied in the European Union (E.U.) and Japan. More specifically, the relevant E.U. policy initiatives are based on the establishment of a central science and technology policy as well as the subsidization of R&D networks between country members under the umbrella of the Eureka and Framework Programs for Science and Technology (see Marín and Siotis, 2008). For the relevant policies in Japan, see Branstetter and Sakakibara (1998). On the other hand, the policy initiatives in the United States are near-market oriented and R&D networks are judged on a rule-of-reason basis, where their potential static anticompetitive effects are weighed against their dynamic benefit effects arising from the R&D partnerships (see Hagedoorn et al., 2000).

[34]Taken together, Propositions 4(ii) and 5(ii) suggest that R&D subsidies may help to increase the upstream firms' effective R&D investments when $\beta \in [0.57, 0.71] \cup [0.86, 1]$. Specifically, within the former range of β values, $[0.57, 0.71]$, the star network maximizes the aggregate level of effective R&D, whereas the complete network is socially optimal – thus, R&D subsidies may encourage the upstream firms in the complete network to conduct more R&D. Similarly, within the latter range, $\beta \in [0.86, 1]$, the partial network maximizes aggregate effective R&D but the star network is socially desirable.

We note that the bargaining power distribution reflects the relative importance of the two market tiers, upstream and downstream. Thus, as the bargaining power shifts from upstream input suppliers to their respective downstream customers, the ensuing input prices become naturally lower. In the limiting case that suppliers have no bargaining power at all, their respective customers receive the essential input at cost and thereby suppliers earn zero profits. This in turn implies that suppliers will have no incentive to collaborate in R&D, as there is no scope for further reduction in input prices. Therefore, we can conclude that there exists a bargaining power threshold $\hat{b}$ above which the incentives of suppliers to form R&D links would be sufficiently strong. Then, by continuity, for a bargaining power above $\hat{b}$ our result on the stability of the complete network under a price setting would persist in this variant with bargaining over input prices.

Throughout we have also assumed that each downstream firm has an exclusive relationship with one upstream firm and purchases its input only from that particular upstream supplier. We may now discuss in short the effects of non-exclusive relationships, where each downstream firm can select the cheapest supplier, or each upstream firm may want to contract with several downstream customers. In this modelling variation, the results would be sensitive to the degree of input specificity – namely, the extent to which inputs are tailored for the specific needs of downstream firms. More specifically, when input specificity is zero, suppliers sell perfect substitutes and thus earn zero profits. Notice that the incentives to form R&D links would then vanish altogether under a price setting. In contrast, profits would be positive under a quantity setting, which implies that incentives to form collaborative links would still be present but weaker with non-exclusive than with exclusive relations. This line of reasoning leads us to a similar conclusion as in the case above of bargaining over input prices, for a decrease in the degree of input specificity corresponds to a decrease in bargaining power. That is, a sufficiently high degree of input specificity would be 'required' to relax competition at the upstream market tier in order to restore firms' incentives to form collaborative links.

Finally, in this paper we have considered an industry consisting of three firms at each market tier. As Goyal and Moraga-González (2001) have noted, a complete equilibrium analysis of R&D networks with an arbitrary number of firms is currently beyond reach. However, the simple model employed here allows us to identify the following key mechanism. In essence, under a price setting, the strategic complementarity of input prices harms firms with a large number of links relative to firms with a smaller number of links, who benefit from an increase in the intensity of competition. The intuition works in exactly the opposite direction under a quantity setting: the competitive advantage of firms with a relatively larger number of links is further reinforced – because input quantities are strategic substitutes. Thus, a quantity setting is likely to induce a smaller number of R&D links relative to a price setting, as in our original model.

7 Concluding remarks

Although existing theoretical work has studied extensively R&D networks in one-tier industries, the study of R&D networks in vertically related industries has received only minimal treatment. This paper develops a framework for the endogenous determination of upstream R&D networks that are actually observed in real world industries. Our interest is to understand and analyze, as our framework attempts, whether the upstream firms' network formation decisions depend on whether they set prices or quantities, as well as the implications of the resulting R&D networks for social welfare.

We first examine the endogenous formation of upstream R&D networks. Under a price setting, we show that the complete R&D network emerges in equilibrium. Yet, under a quantity setting, the equilibrium R&D network depends crucially on the size of within-network R&D spillovers. In particular, if spillovers are sufficiently low, a complete network will arise in equilibrium. However, if spillovers are sufficiently high, the partial network – an insider/outsider formation – will arise. This result suggests that the equilibrium R&D networks might contain *more* R&D links under a price setting than under a quantity setting – so long as within-network R&D spillovers are sufficiently high. Thus, the mode of the upstream firms' behavior – setting prices versus setting quantities – plays an important role in explaining the structure of the equilibrium R&D network.

Based on this last finding, our analysis also suggests that in the context of an upstream quantity setting, the incentives of upstream firms to form R&D links are non-monotone with respect to the level of within-network R&D spillovers (i.e. initially decreasing, then increasing). In contrast, under a price setting, the incentives to form links are not influenced by R&D spillovers. The interest behind this result is that it highlights the role of within-network spillovers for the equilibrium properties of the network structures as well as the ultimate choice of each network structure itself under the two different modes of the upstream firms' behavior.

We then turn to a comparison of equilibrium and socially optimal R&D networks. Our focus is on the policy-related question of whether "market forces" will lead to a socially desirable outcome. Here our analysis confirms that equilibrium networks promote social welfare under a price setting. However, under a quantity setting, we uncover a potential conflict: equilibrium networks might contain *fewer* R&D links than is optimal from a social viewpoint provided that within-network R&D spillovers are sufficiently high. We conclude that the *mode* of the upstream firms' behavior (prices versus quantities) is as important for designing technology policy as the *size* of within-network R&D spillovers.

Finally, let us remark that our modelling framework is fairly stylized, so care should be taken in generalizing our conclusions. Despite its obvious simplicity, our model might be useful as a building block that can support further developments in applied theory of industrial economics. We have already discussed potential directions for future research, such as bargaining over input prices and non-exclusive vertical relations. In addition, the joint selection of the contract type (in the dealings between the upstream and downstream firms) and the R&D network structure is an open question in this context. Another issue, which is beyond the scope of present paper but constitutes a promising avenue for future research, is to endogenize the extent of information exchange between collaborating firms.[35]

[35] For studies on endogenous spillovers, though in a different context, see e.g. Cohen and Levinthal (1989); Poyago-Theotoky (1998); Kamien and Zang (2000), Gil-Molto, Georgantzis and Rios (2005).

8 Appendix A

In this section we present the equilibrium outcomes for the different R&D networks. We note that the linearity of the model ensures that second-order conditions are always fulfilled. For the complete, star and partial networks, the equilibrium outcomes are non-negative for all values of the spillover parameter β, $\beta \in (0, 1]$.

8.1 Complete network

Given the cost structures in eq. (3), in the last stage of the game, each downstream firm D_i chooses its output to maximize its profit given by eq. (7). The equilibrium of this stage game, when upstream firms set either prices or quantities, is:

$$q_i(g^c) = \frac{1}{4}(a - 3w_i + w_j + w_k),$$

Consider first the case in which the upstream firms set quantities. Aggregating the outputs of the downstream firms and rearranging leads to the inverse upstream demand:

$$w(Q) = \frac{1}{3}(3a - 4Q).$$

Given this expression, each upstream firm chooses its output to maximize its profit given by eq. (6). The equilibrium upstream output and price of this stage game under a quantity setting are:

$$q_i(g^c, qs) = \frac{3}{16}\left[(a - \bar{c} + (3 - 2\beta)x_i + (2\beta - 1)(x_j + x_k)\right],$$

$$w(g^c, qs) = \frac{1}{4}\left[a + 3\bar{c} - (1 + \beta)\sum_{i=1}^{3} x_i\right],$$

where the symbol qs denotes the upstream quantity setting. Under a price setting, each upstream firm chooses w_i to maximize its profit given by eq. (6). The equilibrium of this stage game is:

$$w(g^c, ps) = \frac{1}{28}\left[7(a+3\bar{c}) - (15+6\beta)x_i - (3+18\beta)(x_j+x_k)\right],$$

where the symbol ps denotes the upstream price setting. Using the expressions above, at the R&D selection stage, each upstream firm maximizes its profit by choosing its R&D investments x_i. Let $B \equiv 55 - 12\beta + 12\beta^2$ and $C \equiv 409 - 60\beta + 36\beta^2$. The solution to this stage game is:

$$x_U(g^c, qs) = 3(a-\bar{c})(3-2\beta)/B,$$

under a quantity setting, and

$$x_U(g^c, ps) = 3(a-\bar{c})(13-6\beta)/C,$$

under a price setting. We note that R&D investments (or outputs/efforts) are decreasing in the spillover parameter β. Substitutions reveal equilibrium upstream profits and aggregate effective R&D investments ($X_U(g^c) = x_{U_i} + \beta(x_{U_j} + x_{U_k}) = (1+2\beta)x_U$) under both types of upstream firm behavior:[36]

$$\Pi_U(g^c, qs) = 3(a-\bar{c})^2(37+36\beta-12\beta^2)/B^2,$$

$$\Pi_U(g^c, ps) = 3(a-\bar{c})^2(2629+468\beta-108\beta^2)/C^2,$$

$$X_U(g^c, qs) = 9(a-\bar{c})(3-2\beta)(1+2\beta)/B,$$

$$X_U(g^c, ps) = 9(a-\bar{c})(1+2\beta)(13-6\beta)/C. \tag{A1}$$

8.2 Star network

Consider the cost structures in eq. (5). Let $D \equiv 1540 + 3240\beta - 1785\beta^2 + 810\beta^3 - 216\beta^4$ and $F \equiv 256852 + 70032\beta - 27261\beta^2 + 10314\beta^2 - 1944\beta^4$. Following the same procedure

[36] We note that in the main body of the article we have used the shorthand notation Π^i, where $i = E, I, O, H, S, C$, to denote equilibrium profits of the upstream firms.

as we did for the complete network, equilibrium outcomes are shown to be the following:

$$\begin{aligned} x_U^h(g^s, qs) &= 3(a-\bar{c})(84+160\beta-225\beta^2+54\beta^3)/D, \\ x_U^h(g^s, ps) &= 3(a-\bar{c})(13-6\beta)(628+288\beta-81\beta^2)/F, \end{aligned}$$

$$\begin{aligned} x_U^s(g^s, qs) &= 18(a-\bar{c})(14+23\beta-27\beta^2+6\beta^3)/D, \\ x_U^s(g^s, ps) &= 6(a-\bar{c})(26-9\beta)(157+57\beta-18\beta^2)/F, \end{aligned}$$

$$\Pi_U^h(g^s, qs) = 3(a-\bar{c})^2(28+72\beta-27\beta^2)^2(37+36\beta-12\beta^2)/D^2,$$

$$\Pi_U^h(g^s, ps) = 3(a-\bar{c})^2(628+288\beta-81\beta^2)^2(2629+468\beta-108\beta^2)/F^2,$$

$$\Pi_U^s(g^s, qs) = 12(a-\bar{c})^2(7+15\beta-6\beta^2)^2(148+108\beta-27\beta^2)/D^2,$$

$$\Pi_U^s(g^s, ps) = 12(a-\bar{c})^2(157+57\beta-18\beta^2)^2(10516+1404\beta-243\beta^3)/F^2,$$

$$X_U(g^s, qs) = 3(a-\bar{c})(252+856\beta+185\beta^2-810\beta^3+216\beta^4)/D,$$

$$X_U(g^s, ps) = 3(a-\bar{c})(24492+41072\beta-6339\beta^2-10314\beta^3+1944\beta^4)/F. \tag{A2}$$

8.3 Partial network

Consider the cost structures in eq. (4). Letting $G \equiv 385 - 138\beta + 69\beta^2$, $H \equiv 64213 - 7230\beta + 2169\beta^2$, equilibrium outcomes are the following:

$$x_U^l(g^p, qs) = 21(a-\bar{c})(3-\beta)/G, \;\; x_U(g^p, qs) = 9(a-\bar{c})(7-6\beta+3\beta^2)/G,$$

$$x_U^l(g^p, ps) = 471(a-\bar{c})(13-3\beta)/H, \;\; x_U(g^p, ps) = 39(a-\bar{c})(157-30\beta+9\beta^2)/H,$$

$$\begin{aligned}
\Pi_U^l(g^p, qs) &= 147(a-\bar{c})^2(37+18\beta-3\beta^2)/G^2, \\
\Pi_U(g^p, qs) &= 111(a-\bar{c})^2(7-6\beta+3\beta^2)^2/G^2,
\end{aligned}$$

$$\begin{aligned}
\Pi_U^l(g^p, ps) &= 73947(a-\bar{c})^2(2629+234\beta-27\beta^2)/H^2, \\
\Pi_U(g^p, ps) &= 7887(a-\bar{c})^2(157-30\beta+9\beta^2)^2/H^2,
\end{aligned}$$

$$\begin{aligned}
X_U(g^p, qs) &= 3(a-\bar{c})(63+10\beta-5\beta^2)/G, \\
X_U(g^p, ps) &= 3(a-\bar{c})(6123+2750\beta-825\beta^2)/H. \qquad \text{(A3)}
\end{aligned}$$

8.4 Empty network

Given the cost structures in eq. (2), the ensuing equilibrium outcomes are:

$$x_U(g^e, qs) = \frac{9(a-\bar{c})}{55},\ \Pi_U(g^e, qs) = \frac{111(a-\bar{c})^2}{3025},\ X_U(g^e, qs) = \frac{27(a-\bar{c})}{55},$$

$$x_U(g^e, ps) = \frac{39(a-\bar{c})}{409},\ \Pi_U(g^e, ps) = \frac{7887(a-\bar{c})^2}{167281},\ X_U(g^e, ps) = \frac{117(a-\bar{c})}{409}. \qquad \text{(A4)}$$

9 Appendix B

Proof of Proposition 3. When upstream firms set quantities, the result follows directly from the comparisons $x_U(g^e, qs) > x_U^l(g^p, qs) > x_U^h(g^s, qs)$; and $x_U^l(g^p, qs) \gtrless x_U(g^e, qs)$ if $\beta \lesseqgtr 0.14$, $x_U^h(g^s, qs) < x_U^l(g^p, qs)$, $x_U(g^c, qs) < x_U^h(g^s, qs)$. Likewise, under a price setting, it follows from the comparisons $x_U(g^e, ps) > x_U^l(g^p, ps) > x_U^h(g^s, ps)$; and $x_U^l(g^p, ps) < x_U(g^e, ps)$, $x_U^h(g^s, ps) < x_U^l(g^p, ps)$, $x_U(g^c, ps) < x_U^h(g^s, ps)$. Q.E.D.

Proof of Proposition 4. From Proposition 2, we know that when upstream firms set quantities, the complete network is strongly stable if $\beta \leq 0.33$, and the partial network is strongly stable if $\beta \geq 0.95$. Then, for $\beta \leq 0.33$, we have that $X_U(g^c, qs) > X_U(g^s, qs)$, $X_U(g^c, qs) > X_U(g^p, qs)$ and $X_U(g^c, qs) > X_U(g^e, qs)$. Further, for $\beta \geq 0.95$, $X_U(g^p, qs) > X_U(g^c, qs)$, $X_U(g^p, qs) > X_U(g^s, qs)$ and $X_U(g^p, qs) > X_U(g^e, qs)$. We also know from Proposition 1 that when upstream firms set prices, the complete network is the unique strongly stable architecture. Then, for $\beta < 0.95$, we have that $X_U(g^c, ps) > X_U(g^s, ps)$, $X_U(g^c, ps) > X_U(g^p, ps)$ and $X_U(g^c, ps) > X_U(g^e, ps)$. To complete part (ii) of the proof, we establish that, under a price setting, the star network produces a higher level of aggregate effective R&D than the complete network whenever $\beta \geq 0.95$. To this end, we note that $X_U(g^c, ps) < X_U(g^s, ps)$ for $\beta \geq 0.95$. Q.E.D.

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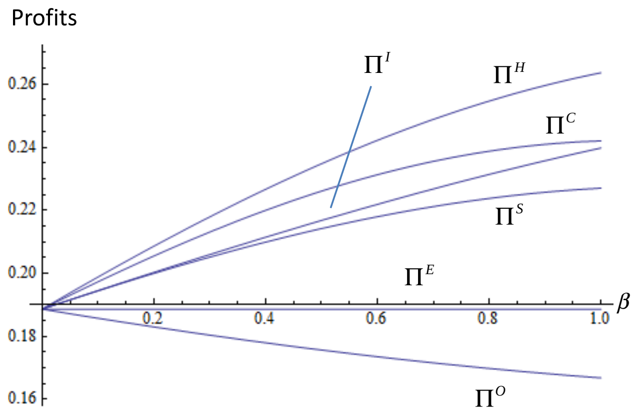
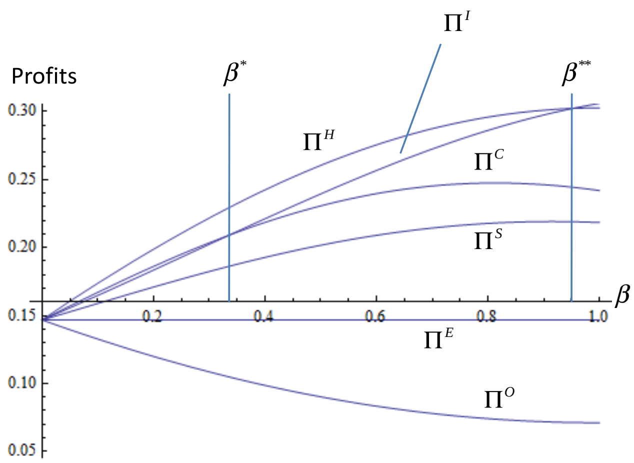
10 Supplementary material: For Referee use only

Figure 1: Firm-Level Profits under a Price Setting[37]



Key: Π^H denotes the hub firm's profits in the star network; Π^I denotes the profits of an insider (linked) firm in the partial network; Π^C denotes a firm's profits in the complete network; Π^S denotes a spoke firm's profits in the star network; Π^E denotes a firm's profits in the empty network; and Π^O denotes the profits of outsider (isolated) firm in the partial network.

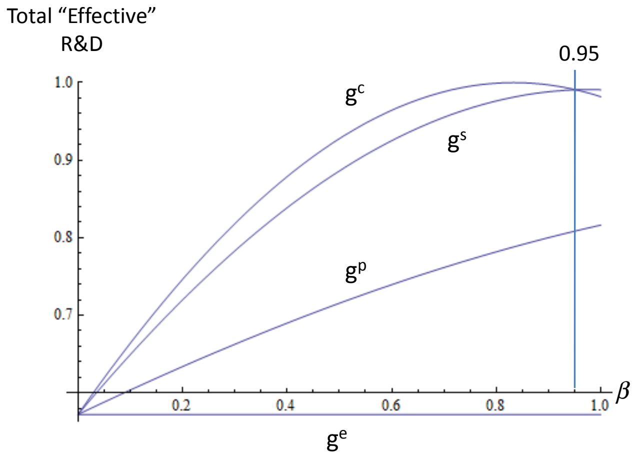
Figure 2: Firm-Level Profits under a Quantity Setting



Key: As Figure 1 above.

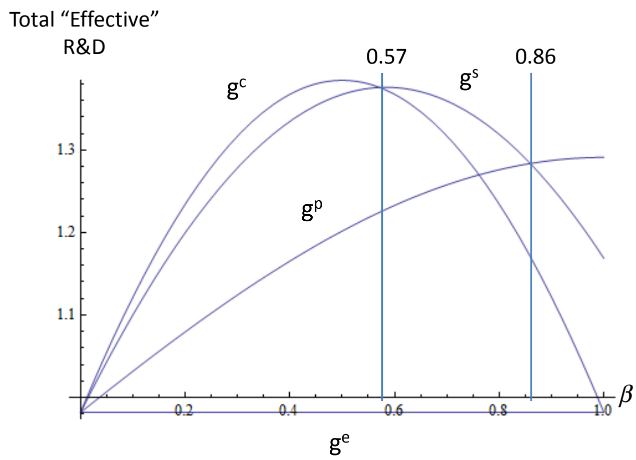
[37]We use "Mathematica 8" (see Wolfram, 1999) for the Figures, and set $a = 4$ and $\bar{c} = 2$, which is inconsequential in a qualitative sense.

Figure 3: Total "Effective" R&D Investment under a Price Setting



Key: g^c denotes total effective R&D investment in the complete network; g^s denotes total effective R&D investment in the star network; g^p denotes total effective R&D investment in the partial network; and g^e denotes total effective R&D investment in the empty network.

Figure 4: Total "Effective" R&D Investment under a Quantity Setting



Key: As Figure 3 above.

Footnote 31: Total upstream profits (strong efficiency).

We say that a network is *strongly efficient* if it secures at least as high a level of aggregate profit as any other network; that is, g is strongly efficient if $\sum_{i=1}^{3} \Pi_i(g) \geq \sum_{i=1}^{3} \Pi_i(g')$ for any other network g'. Application of this definition leads to the following result.

Claim 1 *(footnote 31) (i) Under a quantity setting, the complete network is strongly efficient if* $\beta \in [0, \tilde{\beta}]$, *where* $\tilde{\beta} \simeq 0.896$. *For higher values of the spillover parameter,* $\beta \in [\tilde{\beta}, 1]$, *the star network is strongly efficient.*

(ii) Under a price setting, the complete network is the unique strongly efficient network.

Proof. *Part (i):* We use the equilibrium outcomes in (A1)-(A4) to find total profits in the different network architectures. Let $\tilde{\Pi}_U(g^c, qs) \equiv 3\Pi_U(g^c, qs)$, $\tilde{\Pi}_U(g^s, qs) \equiv \Pi_U^h(g^s, qs) + 2\Pi_U^s(g^s, qs)$, $\tilde{\Pi}_U(g^p, qs) \equiv 2\Pi_U^l(g^p, qs) + \Pi_U(g^p, qs)$ and $\tilde{\Pi}_U(g^e, qs) \equiv 3\Pi_U(g^e, qs)$. We have that $\tilde{\Pi}_U(g^c, qs) > \tilde{\Pi}_U(g^p, qs) > \tilde{\Pi}_U(g^e, qs)$ for all β, $\beta \in (0, 1]$. Also, $\tilde{\Pi}_U(g^s, qs) > \tilde{\Pi}_U(g^p, qs) > \tilde{\Pi}_U(g^e, qs)$ for all β. We next turn to compare total profits under g^c and g^s. This comparison leads to $\tilde{\Pi}_U(g^c, qs) > \tilde{\Pi}_U(g^s, qs)$ for $\beta \leq 0.896$, and $\tilde{\Pi}_U(g^c, qs) < \tilde{\Pi}_U(g^s, qs)$ for $\beta > 0.896$.

Part (ii): Using the equilibrium outcomes in (A1)-(A4), let $\tilde{\Pi}_U(g^c, ps) \equiv 3\Pi_U(g^c, ps)$, $\tilde{\Pi}_U(g^s, ps) \equiv \Pi_U^h(g^s, ps) + 2\Pi_U^s(g^s, ps)$, $\tilde{\Pi}_U(g^p, ps) \equiv 2\Pi_U^l(g^p, ps) + \Pi_U(g^p, ps)$ and $\tilde{\Pi}_U(g^e, ps) \equiv 3\Pi_U(g^e, ps)$. The result follows by noting that $\tilde{\Pi}_U(g^c, ps) > \tilde{\Pi}_U(g^s, ps) > \tilde{\Pi}_U(g^p, ps) > \tilde{\Pi}_U(g^e, ps)$ for all β, i.e. g^c is the unique strongly efficient network. Q.E.D.

Footnote 32: Pareto efficiency.

We say that network g is Pareto efficient if it is not Pareto dominated by any other network; that is, g is Pareto efficient if there does not exist any other network g' such that $\Pi_i(g') \geq \Pi_i(g)$ for all i, with strict inequality for some i. We then establish the following result.

Claim 2 *(footnote 32) (i) Under a quantity setting, the complete and star networks are always Pareto efficient. The partial network is Pareto efficient if* $\beta \in [\bar{\beta}, 1]$, *where* $\bar{\beta} \simeq 0.34$. *The empty network is not Pareto efficient.*

(ii) Under a price setting, the complete network and the star network are always Pareto efficient. The partial network and the empty network are not Pareto efficient.

Proof. *Part (i):* We begin by showing that the empty network g^e is not Pareto efficient. This follows by noting that $\Pi_U(g^c, qs) > \Pi_U(g^e, qs)$, which holds for all $\beta \in (0, 1]$. Next, we show that the partial network g^p is Pareto efficient if $\beta \geq 0.34$. We proceed in two steps. Firstly, consider the following comparisons: $\Pi_U(g^c, qs) > \Pi^l_U(g^p, qs)$ for $\beta < 0.34$, and $\Pi_U(g^c, qs) > \Pi_U(g^p, qs)$ for all β. These comparisons imply that whenever $\beta < 0.34$, a Pareto improvement can be achieved by moving from the partial to the complete network. Hence g^p is a candidate for a Pareto efficient network for $\beta \geq 0.34$. Secondly, we compare firm profits under g^p with the corresponding profits under g^s. We have that $\Pi^h_U(g^s, qs) > \Pi^l_U(g^p, qs)$ for all β; $\Pi^s_U(g^s, qs) < \Pi^l_U(g^p, qs)$ for all β; and $\Pi^s_U(g^s, qs) > \Pi_U(g^p, qs)$ for all β. Hence neither of the two networks, g^p or g^s, Pareto-dominates the other. Combining steps one and two yields the desired result, namely, g^p is Pareto efficient if $\beta \geq 0.34$. Finally, we show that g^c does not Pareto dominate g^s, and vice versa. This follows by noting that $\Pi_U(g^c, qs) < \Pi^h_U(g^s, qs)$ for all β; and $\Pi_U(g^c, qs) > \Pi^s_U(g^s, qs)$ for all β.

Part (ii): Sketch of proof. The proof follows the steps of part (i). The only difference is to show that the partial network g^p is not Pareto efficient. To this end, we show that the complete network g^c Pareto dominates it. That is, $\Pi_U(g^c, ps) > \Pi^l_U(g^p, ps)$ for all β; and $\Pi_U(g^c, ps) > \Pi_U(g^p, ps)$ for all β. Q.E.D.