

# Public Education and Democracy in a Simple Model of Persistent Inequality

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## Abstract

The paper introduces public education financed by linear taxation into a standard model of persistent inequality. It obtains the straightforward conclusion that agents with income above the average will prefer a positive tax rate. This implies a majority of agents supporting the introduction of public education suggesting that democracy is necessary and sufficient condition for redistribution.

*Key words:* Public education, persistent inequality, democracy

*JEL:* H4

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## 1 Introduction

It is a long established finding in public economics that redistribution through linear taxation implies net benefits for agents below the average income and net costs for agents above it (Romer, 1975; Usher, 1977; Meltzer and Richard, 1981). Agents with income below the average will prefer a positive tax rate and agents with income above it will prefer a zero tax rate. Income inequality reflected in the right skewness of the initial income distribution implies that the median voter will have below average income. Hence democracy will result in income redistribution (Acemoglu and Robinson, 2006).

The paper examines this issue in a particular form of income redistribution that is the public provision of a private good, namely education (Glomm and Ravikumar, 1992; Epple and Romano, 1996). Specifically, the state provides,

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free of charge, a common amount of education per young agent. This is financed by linear taxation on old agents' income. Individuals can acquire private supplements but they cannot opt-out from public education. The single source of agent heterogeneity is parent income.

We introduce this modification into the simple OLG model of persistent inequality proposed by Moav (2002), which is itself a simplified version of the classic paper by Galor and Zeira (1993). A central implication of this type of models is that under credit constraints, initial income inequality will persist resulting in a long run co-existence of rich and poor agents (Nakajima and Nakamura, 2009; Galor, 2011).

Our model obtains the straightforward conclusion that a majority in favour of the introduction of public education will emerge even under symmetric income distribution, i.e. without resorting to the right-skewness of income distribution. In particular we derive relationships between the income class of the agent and the preferences for a positive or a zero tax rate. The next section describes persistent inequality in the presence of public education. Section three presents the class structure of the economy. Section four defines the preferences for the tax rate. The final section concludes.

## 2 The model with public education

### 2.1 Bequests

Consider an OLG economy with agents living for two periods. Agents work and receive income when they are old which is taxed at a rate  $\tau$ . The remaining income is allocated between consumption and bequests to the offspring. Young agents subsist from their parents' consumption and acquire education which determines their efficient units of labour when they turn old. Each parent has a single child, hence population is constant with  $N$  members for each generation.

The old individual born in period  $t - 1$  acquires utility in the next period consuming  $c_t$  and leaving bequests  $b_t$  to the offspring. The utility function is of the "joy of giving" type

$$U_t = (1 - \beta) \log c_t + \beta \log(\bar{\theta} + b_t), \quad 0 < \beta < 1 \quad (1)$$

The individual budget constraint (without borrowing) given a tax rate  $\tau_t$  on income is

$$(1 - \tau_t)I_t = c_t + b_t \quad (2)$$

Where  $I_t$  is the individual's income before taxation

Substitute the constraint (2) into the utility function (1) and maximize with respect to  $b_t$  to obtain

$$b_t = \beta((1 - \tau_t)I_t - \frac{1-\beta}{\beta}\bar{\theta})$$

Since bequests cannot be negative we have the function

$$b_t = b(I_t, \tau_t) = \begin{cases} \beta((1 - \tau_t)I_t - \theta) & \text{for } (1 - \tau_t)I_t > \frac{1-\beta}{\beta}\bar{\theta} \equiv \theta \\ 0 & \text{for } (1 - \tau_t)I_t \leq \frac{1-\beta}{\beta}\bar{\theta} \equiv \theta \end{cases} \quad (3)$$

With  $\theta \equiv \frac{1-\beta}{\beta}\bar{\theta}$  denoting the "poverty threshold" income, below which the agent leaves zero bequests

## 2.2 Human capital formation

Human capital formation (efficient units of labour) depends on previous education spending. This is assumed to be a quasi-concave function of the same form employed in Moav (2002)

$$h_{t+1} = h(e_t) = \begin{cases} 1 + \gamma e_t & \text{for } e_t < \bar{e} \\ 1 + \gamma \bar{e} & \text{for } e_t \geq \bar{e} \end{cases} \quad (4)$$

with  $\bar{e}$  denoting the maximum level of education spending above which the marginal return to human capital is zero.

To allow for efficient education spending we assume that returns to human capital up to the maximum level  $\bar{e}$  are always higher than returns to physical capital, i.e.

$$w\gamma > 1 + r \tag{A1}$$

### 2.3 Public education

The state provides public education denoted by  $e_t^P$  and the agent can acquire additional units of private education denoted by  $e_t^I$ . We assume an additive function of total education in the form

$$e_t = e_t^P + e_t^I$$

Taxes are imposed on old agents' income at a rate  $\tau_t$  and finance public education for the young. The government follows a *balanced-budget* rule, therefore, with constant population, public education per young agent equals the tax rate times the average income,  $\hat{I}_t$

$$e_t^P = \tau_t \hat{I}_t \tag{5}$$

## 3 Income dynamics and class structure

Individual income is the sum of returns from human and physical capital. The former equals the wage,  $w$ , times the efficiency units of labour and the latter equals the interest rate  $1 + r$ , times the investment in financial assets (which is the bequest received net from private education spending). Both the wage and the interest rate are assumed constant. Hence, individual income in period  $t + 1$  is given by the expression

$$I_{t+1} = w(1 + \gamma e_t) + (1 + r)(b_t - e_t^I) \tag{6}$$

This function implies three income classes in terms of parent income (the superscripts P,M,R denote poor, middle and rich respectively):

The *poor* with income below the poverty threshold level  $\theta$  do not leave nor receive any bequests and do not acquire private education. Hence, poor income is given by

$$I_{t+1}^P = w(1 + \gamma\tau_t\hat{I}_t) \quad (7)$$

Since bequests and private education are the only links between current and future income, the poor who do not leave bequests have no income dynamics. The absence of bequests makes the offspring income independent from the parent, all the young poor receive the same income.

The *middle class* is composed of agents with incomes above  $\theta$  who can afford positive bequests to their children. However these bequests cannot exceed the amount necessary to acquire the maximum education level,  $\bar{e}$ . In other words the middle class offspring receive returns only from human capital but not physical capital.

Income dynamics of the middle class is given by

$$I_{t+1}^M = w(1 + \gamma(\tau_t\hat{I}_t + \beta((1 - \tau_t)I_t^M - \theta))) \quad (8)$$

We assume that

$$\frac{dI_{t+1}^M}{dI_t^M} = w\beta\gamma(1 - \tau_t) > 1 \quad (\text{A2})$$

hence the steady state middle income is unstable , i.e. the middle class exists only in the short run.

The *rich* class includes agents who always acquire the maximum amount of human capital and invest the remaining bequests in financial assets. In fact they are the only owners of physical capital in the economy.

Their income dynamics is

$$I_{t+1}^R = w(1 + \gamma\bar{e}) + (1 + r)[\beta((1 - \tau_t)I_t^R - \theta) - (\bar{e} - \tau_t\hat{I}_t)] \quad (9)$$

We assume

$$\frac{dI_{t+1}^R}{dI_t^R} = \beta(1+r)(1-\tau_t) < 1 \quad (\text{A3})$$

hence there is a stable steady state rich income.

#### 4 Class preferences over the tax rate

Now assume that our model economy starts with a zero tax rate in period  $t-1$  and must decide on the introduction of public education in period  $t$ . Old agents express their preferences for the tax rate considering the effects on their own income as well as on the income prospects of their children. This implies an indirect utility (or value) function that is a weighted average of both incomes. The weight is given by the same parameter  $\beta$  in the utility function (1) that captures the degree of altruism. We denote this function as

$$V(I_t, I_{t+1} | \tau_t) = V(\tau_t) = (1-\beta)(1-\tau_t)I_t + \beta I_{t+1} \quad (10)$$

The preference for the tax rate is reflected on the sign of the derivative  $V'(\tau_t)$ , i.e. a positive sign implies the preference for a positive tax rate whereas a negative sign implies a zero tax rate.

For the *poor* using equation (7) into (10) and differentiating

$$V'(\tau_t) = -(1-\beta)I_t^P + w\beta\gamma\hat{I}_t$$

The first term measures the cost to the parent and the second term the benefit to the offspring

The net effect is positive for

$$w\beta\gamma\hat{I}_t > (1-\beta)I_t^P \rightarrow I_t^P < \frac{w\beta\gamma}{1-\beta}\hat{I}_t$$

that holds always since by assumption (A2)  $w\beta\gamma > 1 \rightarrow \frac{w\beta\gamma}{1-\beta} > 1$  and the poor income is lower than the average.

Unsurprisingly, the poor will always prefer a positive tax rate.

For the *middle* using equation (8) into (10) and differentiating

$$V'(\tau_t) = -(1 - \beta)I_t^M + w\beta\gamma\hat{I}_t - w\beta^2\gamma I_t^M$$

the first term is again the cost to the parent and the second term the benefit to the offspring. The new term is the third that measures the cost (loss in returns to human capital) imposed on the offspring income because of receiving lower bequests.

The net effect is positive (implying preference for a positive tax rate) for

$$I_t^M < \hat{I}_t \frac{w\beta\gamma}{w\beta^2\gamma + 1 - \beta}$$

Since  $w\beta\gamma > 1$  by (A2), it can be shown that this critical income is higher than the average income, i.e.  $\frac{w\beta\gamma}{w\beta^2\gamma + 1 - \beta} > 1$

Note that

$$\frac{w\beta\gamma}{w\beta^2\gamma + 1 - \beta} > 1 \Leftrightarrow w\beta\gamma > w\beta^2\gamma + 1 - \beta \Leftrightarrow w\beta\gamma - w\beta^2\gamma > 1 - \beta \Leftrightarrow w\beta\gamma(1 - \beta) > 1 - \beta \Leftrightarrow w\beta\gamma > 1$$

This implies that *agents with income above the average will prefer a positive tax rate*. This is a crucial result. A standard argument that explains why inequality generates political pressure for redistribution is the right-skewness of the income distribution with the implication that the decisive median voter has lower than average income. The average income, in turn, separates winners from losers from taxation. Here the political pressure for redistribution emerges even with symmetric income distribution. Majority rule will result in a positive tax rate establishing a direct link between democracy and redistribution. *Democracy is a sufficient condition for public education.*

We can also examine the effects of taxation on the *rich* income. Using equation (9) into (10) and differentiating

$$V'(\tau_t) = -(1 - \beta)I_t^R + \beta(1 + r)\hat{I}_t - \beta^2(1 + r)I_t^R$$

Again the first term is the cost to the parent income, the second term is the benefit for the offspring income and the third term is the cost on the offspring income because of receiving lower bequest. Note that cost and benefit for the rich offspring are expressed in terms of returns to physical capital. The amount of public education granted allows the rich agent to save an equal amount from private education spending, reallocate it to physical capital investment and receive the respective return. However, taxation has decreased the total amount of bequests received, the subsequent investment to physical capital and eventually the returns from it.

The net effect is positive for

$$I_t^R < \hat{I}_t \frac{\beta(1+r)}{\beta((1+r)\beta - 1) + 1}$$

that holds only when the rich have lower income than the average, since  $\frac{\beta(1+r)}{\beta((1+r)\beta - 1) + 1} < 1$

Note that

$$\frac{\beta(1+r)}{\beta(\beta(1+r) - 1) + 1} < 1 \iff \beta(1+r) < \beta(\beta(1+r) - 1) + 1 \iff \beta(1+r) - 1 < \beta(\beta(1+r) - 1)$$

and since assumption (A3) holds for  $\tau_t = 0$  we know that  $\beta(1+r) < 1 \rightarrow \beta(1+r) - 1 < 0$

dividing both sides with that negative amount reverses the inequality sign, hence

$$1 > \beta$$

the rich will never prefer a positive tax rate. Hence, as long as the rich hold full political power before the introduction of democracy, public education will not be implemented. *Democracy is a necessary condition for public education.*

Since for the poor  $V'(\tau_t) > 0$  and for the rich  $V'(\tau_t) < 0$ , it is evident that the critical agent will belong to the middle class.

## 5 Conclusions

Building on the standard assumptions of persistent inequality, the paper developed a simple economic mechanism behind a rather strong political economy conclusion: that democracy is the necessary and sufficient condition for redistribution through public education. Actually our results imply that agents with the average income will prefer a positive tax rate  $\tau$ . Hence, ruling out the abnormal case of a strongly left-skewed income distribution, there will be a majority supporting redistribution and, under democracy, public education will emerge.

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