A Supply-Response Model Under Invariant Risk Preferences

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Abstract

In this article we first develop a theoretically consistent supply-response model for producers with invariant preferences facing price risk, and then we empirically apply the model for a group of Cretan olive-oil producers. For doing so, we estimate a *Generalized Leontief* cost function and we use the price distribution historically faced by individual farmers to induce three different representations of price risk corresponding to the second, third and fourth l_p norms. These risk measures are combined with the estimated cost-structure to provide three separate representations of the efficient frontier for the representative producer. Empirical results suggest that, regardless of risk measure used, all farmers curtail production in managing price risk.

Keywords: price risk; invariant risk preferences; producer responses; Cretan olive-oil producers.

JEL Codes: *C33*, *C22*, *D81*, *Q11*.

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Introduction

The earliest studies of producer response to price risk incorporated measures of the mean and the variance of price into otherwise standard empirical supply-response relationships (Behrman, 1968). It was left to Sandmo (1971) to provide a sound theoretical model of producer response to price risk in an expected-utility setting. He showed that risk-averse producers facing price risk would produce less than risk-neutral producers facing the same mean price.

A long line of empirical and theoretical contributions (Batra and Ullah, 1974; Lin, Dean, and Moore, 1974; Just, 1974; Dillon and Scandizzo, 1978; Pope, 1980; Appelbaum and Katz, 1986; Chavas and Holt, 1990, 1996; Coyle, 1992) have followed from these early roots. Perhaps not surprisingly, much of the specific focus was on the effect of price risk on agricultural-supply response. Although specific modelling choices vary, two broad classes of models emerged: models in the Markowitz (1952) and Tobin (1958) tradition that characterize risky decision making in terms of trade offs between *risk* (as measured by variance) and *return* (as measured by the mean); and models hewing more closely to the expected-utility theory.

Few, if any, economic models have had the empirical impact of the Markowitz-Tobin model. For example, even the most naive modern-day stock pickers are familiar with such concepts as a stock's *beta* that are firmly rooted in the Markowitz-Tobin model. But criticisms of the Markowitz-Tobin framework are also familiar. For many, perhaps the most telling is that Markowitz-Tobin is consistent with the expected-utility framework only under restrictive (and unattractive) assumptions on either the *ex post* utility function or on the distribution of returns.

This concern, however, is considerably mitigated by two considerations. Epstein (1986), by adapting Machina's (1982) local-utility function approach, showed that all preference structures satisfying a seemingly innocuous assumption on systematic changes in risk aversion can be characterized by a local mean-variance preference functional. The second arises from the apparent empirical weaknesses in the expected-utility framework captured by the accumulation (and empirical verification of) a variety of behavioral and empirical paradoxes associated with its predictions.

Its strained relationship with the expected-utility framework is only one of the criticisms leveled at the Markowitz-Tobin framework. Another arises from concerns about the appropriateness of the variance (standard deviation) as a measure of risk. Despite its intuitive appeal, the variance is best suited to returns distribution that are normal. The presence of (or distinct preferences over) either skewness or kurtosis in the distribution of returns, however, undermines the suitability of the variance as a measure of risk. Responding to just such concerns, Quiggin and Chambers (2004) introduced the *invariant preference* class. That class generalizes the mean-variance class by allowing for more general risk measures than the variance while preserving its intuitive and analytic tractability by continuing to represent choice in terms of a trade off between *risk* and *return*.

This paper has two goals. First, develop a theoretically consistent, yet empirically tractable, supply-response model for producers with invariant preferences facing price risk. Second, empirically implement that supply-response model for a group of Cretan olive-oil producers. To achieve the second goal, we estimate the cost function for a representative olive-oil producer and a time-series representation of the empirical olive-oil price distribution historically faced by producers. From the latter, we induce a representation of its mean and three separate measures of price risk that correspond, respectively, to the second, third, and fourth l_p norms. These measures of risk and return are then used along with the estimated cost structure to induce three separate representations of the efficient frontier for the representative olive-oil producer.

Model and Assumptions

Random variable space is denoted $\mathbb{R}^{\Omega}$ and is formed by the mappings of the underlying of the underlying state space Ω to the reals. The set of degenerate random variables assuming the same real number in each state of Nature is denoted by $\mathbb{X} \subset \mathbb{R}^{\Omega}$. Producer preferences over random profit, denoted $\tilde{\pi} \in \mathbb{R}^{\Omega}$, are of the invariant class axiomatized by Quiggin and Chambers (2004)

$$W(\tilde{\pi}) = \phi(\mu[\tilde{\pi}], r[\tilde{\pi}]),$$

where $\mu[\tilde{\pi}]$ denotes the expectation of profit, $r[\tilde{\pi}]$ is a real-valued risk or dispersion measure and ϕ is strictly increasing in its first argument and strictly decreasing in its second argument. The risk measure is sublinear (positively linearly homogeneous and convex) with $r[\tilde{\pi}] \geq 0$ for all $\tilde{\pi}$ and $r[0] = 0$. Moreover, it is translation invariant so that adding a nonstochastic constant to any random profit does not change the associated risk:

$$r[\tilde{\pi} + \alpha] = r[\tilde{\pi}] \text{ for all } \alpha \in \mathbb{X}$$

Translation invariance of the risk measure ensures that the risk associated with any nonstochastic or riskless profit $\bar{\pi} \in \mathbb{X}$ is zero because

$$r[\bar{\pi}] = r[0] = 0.$$

Perhaps the most familiar example of such a risk measure is the standard deviation

$$\sigma(\tilde{\pi}) = \left(\mu\left[(\tilde{\pi} - \mu[\tilde{\pi}])^2\right]\right)^{\frac{1}{2}},$$

so that mean-variance preferences are, in fact, a special case of the invariant preference class. Notice, however, that so too are all l_p norms defined in terms of the expectation inner product:

$$l_p(\tilde{\pi}) = (\mu[|\tilde{\pi} - \mu[\tilde{\pi}]|^p])^{\frac{1}{p}}, \qquad p > 2.$$

More generally, any norm defined over random variable space will provide a suitable risk measure.

Individual producers face stochastic prices, denoted by $\tilde{p} \in \mathbb{R}^{\Omega \times M}_{++}$, for their outputs, which are denoted $y \in \mathbb{R}^{M}_{+}$. Production is assumed nonstochastic, but output prices realized by farmers are stochastic. If $M \neq 1$, $\tilde{p}$ is interpreted as a vector of M random variables whose inner product with y, denoted $\tilde{p}'y$ is the nonnegative scalar random variable, stochastic revenue. Cost associated with producing the vector of outputs given nonstochastic prices for their N inputs, denoted by $w \in \mathbb{R}^{N}_{++}$, is given by the cost function $c(w, y)$ which is nondecreasing and convex in output and superlinear (positively linearly homogeneous and concave) and nondecreasing in input prices. The cost function is dual to the input correspondence:

$$V(y) = \cap_{w \in \mathbb{R}^{N}_{++}} \left\{ x \in \mathbb{R}^{N}_{+} : w'x \geq c(w, y) \right\}.$$

Invariant Preferences and Optimal Risk Exposure

The producer's problem is

$$\begin{aligned} \max_{y} \{W(\tilde{\pi})\} &:= \max_{y} \{\phi(\mu[\tilde{\pi}], r[\tilde{\pi}])\} \\ &= \max_{y} \left\{ \phi\left(\mu\left[\tilde{p}'y - c(w, y)\right], r\left[\tilde{p}'y - c(w, y)\right]\right)\right\}. \end{aligned}$$

By the translation invariance of r, this reduces to

$$\max_{y} \{W(\tilde{\pi})\} := \max_{y} \left\{ \phi\left(\mu[\tilde{p}]' y - c(w, y), r\left[\tilde{p}'y\right]\right)\right\},$$

where $\mu[\tilde{p}] \in \mathbb{R}^{M}_{+}$ now denotes the M dimensional vector of mean output prices. Because ϕ is strictly increasing in its first argument and strictly decreasing in its second, a convenient decomposition is available:

$$\begin{aligned} \max_{y} \left\{ \phi\left(\mu[\tilde{p}]' y - c(w, y), r\left[\tilde{p}'y\right]\right)\right\} &= \max_{r} \left\{ \phi\left(\max_{y} \left\{ \mu[\tilde{p}]' y - c(w, y) : r\left[\tilde{p}'y\right] \leq r \right\}, r \right) \right\} \\ &= \max_{r} \{\phi(M(\tilde{p}, w, r), r)\}, \end{aligned}$$

where

$$M(\tilde{p}, w, r) := \max_{y} \left\{ \mu[\tilde{p}]' y - c(w, y) : r\left[\tilde{p}'y\right] \leq r \right\},$$

denotes the maximum expected profit associated with a risk exposure of r.

Thus, producers can be viewed as solving their optimization problem in two steps. In the first, for every level of risk exposure, they choose their output mix to maximize the expected profit associated with that level of risk. Regardless of their attitudes towards risk and return, they leave no riskless (as measured by r) opportunities to increase profit unexploited. Just as in mean-variance analysis, the solution to this problem defines an *efficient frontier*, the graph of

$$M(\tilde{p}, w, r)$$

in mean-risk space, tracing out the trade offs between risk and maximal expected returns.

Theorem 1 *$M(\tilde{p}, w, r)$ is nondecreasing and concave in r.*

Proof. Monotonicity is trivial. Let y^o and y' denote the optimal solution to the first-stage problem when r^o and r' are the risk exposures. By the convexity r for $0 < \lambda < 1$,

$$\begin{aligned} r\left(\lambda \tilde{p}' y'\right) + (1-\lambda)\tilde{p}' y^o\right) &\leq \lambda r\left(\tilde{p}' y'\right) + (1-\lambda) r\left(\tilde{p}' y^o\right) \\ &\leq \lambda r' + (1-\lambda) r^o. \end{aligned}$$

By the convexity of c for $0 < \lambda < 1$,

$$\mu[\tilde{p}]'\left[\lambda y' + (1-\lambda y^o)\right] - c\left(w, \left[\lambda y' + (1-\lambda y^o)\right]\right) \geq \lambda M\left(\tilde{p}, w, r^{'}\right) + (1-\lambda) M(\tilde{p}, w, r^o).$$

Thus, $\lambda y' + (1-\lambda y^o)$ is feasible for $\lambda r' + (1-\lambda) r^o$ and offers an expected return at least as large as $\lambda M\left(\tilde{p}, w, r^{'}\right) + (1-\lambda) M(\tilde{p}, w, r^o)$, which establishes the desired concavity. ■

Maximal expected profit is at the point (if one exists) where M assumes a zero slope. Risk-averse individuals choose their optimal risk exposure by equating their marginal rate of substitution between return and risk to the slope of the efficient frontier. Thus, for smooth preferences the individual solves

$$M_r(\tilde{p}, w, r) = -\frac{\phi_2(M(\tilde{p}, w, r), r)}{\phi_1(M(\tilde{p}, w, r), r)} > 0. \tag{1}$$

The right-hand expression measures the individual's marginal rate of substitution between risk, as measured by r, and expected return. Thus, following Epstein (1986), it interpretable as a generalization of the Arrow-Pratt coefficient of risk aversion. If the numerator is zero, the individual is risk neutral. If the numerator is nonzero, the individual is risk averse. Risk-averse individuals always sacrifice some expected return to reduce risk.

Example 1 *Consider the case where $r[\tilde{\pi}] = \sigma(\tilde{\pi})$ and*

$$\phi(\mu[\tilde{\pi}], r[\tilde{\pi}]) = \mu[\tilde{\pi}] - \frac{b}{2}\sigma(\tilde{\pi})^2.$$

Then the producer choses his or her risk exposure so that

$$M_\sigma(\tilde{p}, w, \sigma) = b\sigma.$$

Example 2 *Let $M = 1$ so that $p \in \mathbb{R}^{\Omega}_{++}$. Then*

$$\begin{aligned} M(\tilde{p}, w, r) &= \max\{\mu[\tilde{p}]\, y - c(w, y) : r[\tilde{p}y] \leq r\} \\ &= \max\{\mu[\tilde{p}]\, y - c(w, y) : yr[\tilde{p}] \leq r\} \\ &= \mu[\tilde{p}]\frac{r}{r[\tilde{p}]} - c\left(w, \frac{r}{r[\tilde{p}]}\right). \end{aligned}$$

The Efficient Frontier and Supply Response

We first prove a basic result on the curvature and homogeneity properties of the efficient frontier.

Theorem 2 *$M(\tilde{p}, w, r)$ is positively linearly homogeneous in $(\tilde{p}, w, r)$ and convex in w.*

Proof. By definition for $t > 0$

$$\begin{aligned} M(t\tilde{p}, tw, tr) &= \max_{y}\left\{\mu[t\tilde{p}]'\, y - c(tw, y) : r\left[t\tilde{p}'y\right] = tr\right\} \\ &= t\max_{y}\left\{\mu[\tilde{p}]'\, y - c(w, y) : r\left[\tilde{p}'y\right] = r\right\}, \end{aligned}$$

where the second equality follows by the positive linear homogeneity of $c(w, y)$ and the sublinearity of $r[\tilde{p}'y]$. Because $c(w, y)$ is concave in w :

$$\begin{aligned} \mu[\tilde{p}]'\, y - c\left(\lambda w' + (1-\lambda) w, y\right) &\leq \mu[\tilde{p}]'\, y - \left[\lambda c\left(w', y\right) + (1-\lambda) c(w, y)\right] \\ &= \lambda\left[\mu[\tilde{p}]'\, y - c\left(w', y\right)\right] + (1-\lambda)\left[\mu[\tilde{p}]'\, c - (w, y)\right], \end{aligned}$$

and taking *maxima* for both sides establishes the desired convexity. ■

Finding the efficient frontier involves solving a concave programming problem subject to convex constraints. Hence, first-order conditions are necessary and sufficient to characterize optimal behavior. To that end, some new notation proves convenient. For a real-valued function f defined over random variable space, define the one-sided directional derivative in the direction $\tilde{n} \in \mathbb{R}^{\Omega}$ by

$$f'(\tilde{\pi}; \tilde{n}) := \lim_{\lambda > 0}\left\{\frac{f(\tilde{\pi} + \lambda\tilde{n}) - f(\tilde{\pi})}{\lambda}\right\},$$

and for a real-valued function defined over $\mathbb{R}^{M}$, g, we define the corresponding one-sided directional derivative by

$$g(x; v) := \lim_{\lambda > 0}\left\{\frac{g(x + \lambda v) - g(x)}{\lambda}\right\}.$$

Notice that if random-variable space is finite dimensional, these two concepts coincide and correspond to the natural generalization of a right-hand derivative for a function defined over the real line. In words, the one-sided directional derivatives describe how the function value changes when an arbitrarily small step in the direction $\tilde{n} \in \mathbb{R}^{\Omega}$ ($v \in \mathbb{R}^{M}$) is taken.

The Lagrangean expression associated with $M(\tilde{p}, w, r)$ is

$$M(\tilde{p}, w, r) = \max_{y,\theta} \left\{ \mu [\tilde{p}]' y - c(w, y) - \theta r [\tilde{p}' y] \right\}, \tag{2}$$

where θ is a non-negative Lagrangean multiplier. Let y^o denote the optimal solution to this problem. Then for any perturbation, v, around that solution it must be true that

$$\mu [\tilde{p}]' v - c'(w, y^o; v) - \theta r' [\tilde{p}' y^o; \tilde{p}' v] \leq 0, \qquad v \in \mathbb{R}^M \tag{3}$$

where $c'(w, y; v)$ now denotes the directional derivative of c in terms of y.[1] If this condition were violated, there would exist a direction of movement away from y that leads to an improvement in the producer's objective function, thus contradicting the optimality of y^o. Therefore, expression (3) is both necessary and sufficient for an optimum.

Taking $v = e_m$, where e_m denotes the m^{th} element of the canonical orthonormal basis, yields the standard first-order condition for the m^{th} output ($m = 1, \ldots, M$).

$$\mu [\tilde{p}_m] - c'(w, y^o; e_m) - \theta r' [\tilde{p}' y^o; \tilde{p}_m] \leq 0,$$

and for an interior solution

$$\mu [\tilde{p}_m] - c'(w, y^o; e_m) = \theta r' [\tilde{p}' y^o; \tilde{p}_m].$$

In words, the producer equates the difference between the expected price of the mth output and its marginal cost to the monetized value of how a small step in the direction of $\tilde{p}_m \in \mathbb{R}^{\Omega}$ affects the riskiness of the production bundle.

Taking $v = y^o$ establishes for an interior solution that

$$\begin{aligned} \mu [\tilde{p}]' y^o - c'(w, y^o; y^o) &= \theta r' [\tilde{p}' y^o; \tilde{p}' y^o] \\ &= \theta r \\ &= M_r(\tilde{p}, w, r)\, r \end{aligned} \tag{4}$$

The second equality here follows from the natural generalization of Euler's Theorem for homogeneous functions to directional derivatives because

$$r [\lambda \tilde{p}' y] = \lambda r [\tilde{p}' y] \Longrightarrow \lim_{\lambda > 0} \left\{ \frac{r [\tilde{p}' y + \lambda \tilde{p}' y] - r [\tilde{p}' y]}{\lambda} \right\} = r [\tilde{p}' y],$$

[1]Because $c(w, y)$ in convex in y and $r[\tilde{\pi}]$ is convex, these directional derivatives almost always exist (Clarke, 1983).

and the third equality follows from the familiar interpretation of the Lagrangean multiplier as the shadow price of r.

The slope of the efficient frontier in risk, therefore, equals the marginal expected profitability of a radial expansion in the optimal output bundle (for that level of risk) divided by the level of risk. By expression (1), it then follows that the producer's marginal rate of substitution between risk and return equals this ratio for the optimal output bundle.

Denote

$$y(\tilde{p}, w, r) \in \arg\max \left\{ \mu[\tilde{p}]' y - c(w, y) - \theta r[\tilde{p}' y] \right\}.$$

and

$$M'(\tilde{p}, w, r; \tilde{q}) := \lim_{\lambda > 0} \left\{ \frac{M(\tilde{p} + \lambda \tilde{q}, w, r) - M(\tilde{p}, w, r)}{\lambda} \right\},$$

for $\tilde{q} \in \mathbb{R}^{\Omega \times M}$.

Applying the envelope theorem to (2) yields

Theorem 3

$$M'(\tilde{p}, w, r; 1^m) = y_m(\tilde{p}, w, r) \tag{5}$$

where $1^m \in \mathbb{R}^{\Omega \times M}$ *satisfies for* $k \in \{1, 2, .., M\}$ *and* $s \in \Omega$

$$1^m_{sk} = \begin{cases} 1 \ \text{if } k = m \\ 0 \ \text{otherwise} \end{cases} \tag{6}$$

for all $s \in \Omega$ *and* $q_{s'k} = 0$ *for* $k \neq m$.

. **Proof.** Applying the envelope theorem gives:

$$\begin{aligned} M'(\tilde{p}, w, r; 1^m) &= \lim_{\lambda > 0} \left\{ \frac{M(\tilde{p} + \lambda 1^m, w, r) - M(\tilde{p}, w, r)}{\lambda} \right\} \\ &= \lim_{\lambda > 0} \left\{ \frac{\mu[\tilde{p} + \lambda 1^m]' y - \mu[\tilde{p}]' y - \theta (r[(\tilde{p} + \lambda 1^m) y] - r[\tilde{p}' y])}{\lambda} \right\} \\ &= y_m - \theta \lim_{\lambda > 0} \frac{(r[(\tilde{p} + \lambda 1^m) y] - r[\tilde{p}' y])}{\lambda} \\ &= y_m - \theta \lim_{\lambda > 0} \frac{(r[\tilde{p}' y + y_m] - r[\tilde{p}' y])}{\lambda} \\ &= y_m - \theta \lim_{\lambda > 0} \frac{(r[\tilde{p}' y] - r[\tilde{p}' y])}{\lambda} \\ &= y_m, \end{aligned}$$

where the penultimate equality follows by the translation invariance of the risk index. ■

The directional derivative $M'(\tilde{p}, w, r; 1^m)$ measures the change in $M(\tilde{p}, w, r)$ associated with a nonstochastic increase in the *mth* stochastic output price. Expression (5), which shows that this directional derivative equals optimal supply of the mth output, generalizes the more familiar *Hotelling's Lemma* from the theory of the competitive firm facing nonstochastic prices.

In perhaps more intuitive terms, notice that a nonstochastic increase in the mth output price leads to a riskless increase in profit. As such, it can be viewed as a *risk-preserving* price change. Thus, the natural generalization of Hotelling's Lemma is that optimal supplies are obtained from the conditional expected-profit function, $M(\tilde{p}, w, r)$, by subjecting it to a risk-preserving price change. In the mean-variance framework, an equivalent result is obtained by taking a partial derivative of the efficient frontier with respect to the mean output price while holding the variance constant.

Applying the envelope theorem a second time, while invoking Shephard's lemma, gives

$$\begin{aligned} M_w(\tilde{p}, w, r) &= -c_w(w, y(\tilde{p}, w, r)) \\ &= -x(w, y(\tilde{p}, w, r)) \\ &= -x(\tilde{p}, w, r) \end{aligned} \tag{7}$$

where $x(w, y)$ is the cost-minimizing demand associated with the production of y, and, with a slight abuse of notation, $x(\tilde{p}, w, r)$ represents the optimal input demand associated with a risk-exposure of r.

Together expressions (5) and (7) constitute a supply-response system that is conditional upon the level of risk exposure, r, as chosen by the individual producer. By Theorem 2, the optimal supplies and demands in this conditional supply-response system are invariant to proportional changes in $(\tilde{p}, w, r)$. Moreover, the convexity properties of $M(\tilde{p}, w, r)$ ensure that all the associated derived demands in this system are downward sloping in their own prices. Thus, once the producer's optimal risk exposure is determined from (1), the associated optimal supply and derived demand equations are available from (5) and (7).

An Empirical Illustration

Our empirical application is to a panel of Cretan olive-oil producers drawn from 1999-2004. Our goal is to generate empirical approximations to $M(\tilde{p}, w, r)$ faced by these producers for each of three distinct risk measures.

As candidates for the risk index $r[\tilde{p}]$ we consider three versions of

$$l_\rho(\tilde{p}) = \left(\mu\left[\,|\,\tilde{p} - \mu[\tilde{p}]\,|^\rho\right]\right)^{\frac{1}{\rho}} \quad \text{for} \quad \rho = 2, 3, \text{ and } 4$$

These norms correspond to the second, third, and fourth absolute central moments. Thus, our empirical analysis restricts attention to invariant-symmetric preferences of the type axiomatized by Chambers, Grant, Polak, and Quiggin (2011).

Data on Olive Farming

Our production data are from a survey conducted during 1999 to 2004 by extension specialists of the Greek National Agricultural Research Foundation. The data consist of a balanced panel of 50 olive farms located in the same geographical area in the western part of the island of Crete, Greece. Because the geographic area is quite compact, all the farms operate under quite similar environmental and climatic conditions. In our empirical model we include one output, three variable inputs, one quasi-fixed input, and one environmental variable. The output, measured in kilograms, consists of olive-oil quantities sold off the farm, quantities consumed by the farm household during the crop year, and the portion of output kept by olive mills as a fee for extraction services.

The variable inputs are fertilizers, intermediate inputs, and hired labor. Olive farmers utilize a mixture of fertilizers including nitrate, phosphorous, and potassium fertilizers that are applied after the harvesting season from February to March. The survey contains farm-level data on the quantities used and the expenses paid for these three types of fertilizers. These data were used to derive prices for each type of fertilizer, which were then aggregated into a single Tornqvist fertilizer price index with cost shares as the relevant weights. The intermediate input variable is also an aggregate. It consists of goods and materials used during the crop year, whether purchased outside the farm or withdrawn from beginning inventories. These include fuel and electric power, storage expenses, and irrigation measured in Euros. The survey contains farm level information about current expenditures for these items and unit prices for irrigation water. To construct an aggregate intermediate input price, we used national price indices for fuel, electric power and storage, which were converted to value measures using 2005 prices. These were combined with the data on irrigation water prices to construct an aggregate Tornqvist price index.

Olive-oil producers employ both hired and family labor. Hired-workers are mainly used for harvesting activities. The price of hired labor was computed as the average hourly wage including social security and taxes paid by farmers. The computed hourly wage varies across farms as the demand for hired workers differs significantly during harvesting season (harvesting season usually starts in late October and ends in late January with significant fluctuations depending on the maturity stage among farms). Family labor provided by household members devoted to farming activities was treated as a quasi-fixed input together with capital and land. Capital stock was computed using the perpetual-inventory method as described by Ball *et al.*, (1993) and data on depreciation rates obtained from the Greek Ministry of Agriculture for different farming equipment. Then these three variables are aggregated into a single quasi-fixed input variable using the *Tornqvist* index and their respective cost shares as the relevant weights. The total cost of family labor and land inputs was calculated using off-farm wage rates and land prices published by the Greek Ministry of Agriculture. Finally, an aridity index defined as the ratio of the average temperature in the area where the farm is located over the total precipitation in the same area (Stallings, 1960) was included to capture any differences in climatic conditions across farms that affect their productivity.[2]

[2]Meteorological data for the construction of the aridity index were obtained by the local Meteorological Stations

Summary statistics for all variables used in the empirical analysis are presented in Table 1. Prior to econometric estimation, all variables were converted into indices, with the basis of normalization being the representative olive-oil farm. That representative farm was chosen as the one having the smallest deviation of all variables from the sample means.

Econometric Specification and Estimation of the Cost Function for Olive Farming

The variable-cost function was assumed to take the following modified *Generalized Leontief* form (Guilkey *et al.*, 1983):

$$\begin{aligned} c_{it}\left(w_{it}, y_{it}, x_{it}, d_{it}, t\right) &= y_{it}\sum_j\sum_k \alpha_{jk} w_{hit}^{0.5} w_{kit}^{0.5} + y_{it}^2 \sum_j \alpha_j w_{jit} + \sum_j \beta_j w_{jit} \\ &+ \sum_j \gamma_j w_{jit} t + \sum_j \delta_j w_{jit} x_{it} + \sum_j \zeta_j w_{jit} d_{it} \end{aligned} \tag{8}$$

where i subscripts correspond to the i^{th} farm and t subscripts to the t^{th} year, w_{jit} is the price of the j^{th} variable-input (*i.e.*, fertilizers, hired labor, intermediate inputs), y_{it} is the single farm output (*i.e.*, olive-oil), x_{it} is the aggregate quasi-fixed input (*i.e.*, capital, land, family labor), d_{it} is the aridity index, and t is a time trend to capture the possibility of technical change.

By *Shephard's Lemma*, the associated variable-input demands are given by

$$x_{jit}\left(w_{it}, y_{it}, x_{it}, d_{it}, t\right) = \beta_j + y_{it}\sum_k \alpha_{jk}\left(\frac{w_{kit}}{w_{jit}}\right)^{0.5} + \alpha_j y_{it}^2 + \gamma_j t + \delta_j x_{it} + \zeta_j d_{it}, \quad \forall j \tag{9}$$

The system of variable-input demands in (9) was estimated using the full-information-maximum-likelihood (FIML) method after appending a suitable econometric error structure. The associated likelihood function was maximized using the *Berndt-Hall-Hall-Hausman* (BHHH) algorithm. The FIML estimator has the same asymptotic properties as the three-stage least squares estimator; and with normally distributed disturbances, it is asymptotically efficient (Hausman, 1975).

The estimated parameters for the cost function are reported in Table 2. Table 3 reports estimates of the price elasticities of variable-factor demands and the elasticity of scale (calculated at sample means). All variable inputs are substitutes for one another, and all derived demands are downward sloping in their own prices (as required by theory). All elasticities are of reasonable magnitudes, and both hired labor and materials are quite inelastic relative to the derived demand for fertilizers. The estimated scale elasticity is well less than one. The estimated cost structure, therefore, is consistent with increasing marginal cost.

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located throughout the island.

Estimates of the Mean and Risk Measures of Olive-Oil Prices

Recall that in the single output case

$$\begin{aligned} M(\tilde{p}, w, r) &= \max\{\mu[\tilde{p}]\, y - c(w, y) : r[\tilde{p}y] \le r\} \\ &= \max\{\mu[\tilde{p}]\, y - c(w, y) : yr[\tilde{p}] \le r\} \\ &= \mu[\tilde{p}] \frac{r}{r[\tilde{p}]} - c\left(w, \frac{r}{r[\tilde{p}]}\right). \end{aligned}$$

The efficient frontier is thus a function of $\mu[\tilde{p}]$ and $r[\tilde{p}]$. The empirical approach adopted here is to estimate the mean and l_ρ norms of olive-oil prices, and then use these estimates along with expression (8) and the estimated cost function to generate an estimate of the efficient frontier. This entails the specification of a stochastic model for the conditional distribution of olive-oil prices given their past behavior and the estimation of its underlying parameters. These parameter estimates are then used to estimate the moments of the conditional density of prices from which estimates of the l_ρ norms will be derived.

To estimate the stochastic model for the conditional distribution of olive-oil prices, we use monthly data on the price of olive oil for the period 1996-2004 for Crete published by the International Olive-Oil Council (IOOC, 1996-2004). Nominal prices have been deflated to real values using the Consumer Price Index for Greece published by OECD (base year 2005).

A Regime Switching Model for Olive-Oil Prices

Figure 1 shows that the behavior of prices during this period is characterized by an overall declining trend that alternates with some periods of positive jumps in prices. The large sustained jump observed during the 2000-01 period is likely due to bad crop conditions that prevailed at that time in in all major producing countries (Italy, Spain, and Greece) that boosted producer prices worldwide.

To capture nonlinearities present in the olive-oil price data we use a *Markov* switching autoregressive model with two regimes that allows for a regime-dependent intercept, autoregressive and trend parameters, as well as variance. The fact that the autoregressive parameters can change between regimes allows for different dynamics in each regime. For example, one regime could be characterized by a unit root while the other is stationary. Several economic series have been shown to be well represented by *Markov* switching regime models when periods characterized by stationarity alternate with non-stationary ones (see Ang and Bekaert, 2002; Kanas and Genius, 2005). To determine whether unit roots are present in either of the two regimes, we adopt the following MS-ADF (*Markov Switching Augmented Dickey Fuller*) form for our model,

$$p_t = \alpha_1 s_t + \alpha_0 (1 - s_t) + (\beta_1 s_t + \beta_0 (1 - s_t))\, t \quad + \quad (\gamma_1 s_t + \gamma_0 (1 - s_t))\, p_{t-1}$$

$$+ \sum_{j=1}^{q} \left(\phi_{1j} s_t + \phi_{0j} (1 - s_t)\right) \Delta p_{t-j} + \epsilon_t$$

where $\epsilon_t \sim NID\left(0, \sigma_0^2 s_t + \sigma_1^2 (1 - s_t)\right)$ and,

$$s_t = \begin{cases} 0 & \text{if regime 0 holds} \\ 1 & \text{if regime 1 holds.} \end{cases}$$

The variable $s_t = \{0, 1\}$ is unobserved and follows a first-order ergodic *Markov Chain* whose behavior is described by the matrix of transition probabilities $\mathbf{P}$,

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix} \quad \text{with} \quad \sum_{j=0}^{1} p_{lj} = 1, \; p_{lj} \geq 0, \quad \forall j, l = 0, 1$$

In addition, the vector of stable probabilities, $\pi = \begin{bmatrix} \pi_0 \\ \pi_1 \end{bmatrix}$, satisfies $\pi = \mathbf{P}\pi$. Let

$$\begin{aligned} \mu_{0t} &= \alpha_0 + \beta_0 t + \gamma_0 p_{t-1} + \sum_{j=1}^{q} \phi_{0j} \Delta p_{t-j} \\ \mu_{1t} &= \alpha_1 + \beta_1 t + \gamma_{01} p_{t-1} + \sum_{j=1}^{q} \phi_{1j} \Delta p_{t-j}, \end{aligned}$$

and π_0, π_1 be the probability of being in state 0 and 1 respectively, then the conditional density of p_t given its past $\mathbf{P^{t-1}}$ is provided by,

$$\begin{aligned} f\left(p_t | \mathbf{P^{t-1}}\right) &= \pi_0 \, f\left(p_t | s_t = 0, \mathbf{P^{t-1}}\right) + \pi_1 \, f\left(p_t | s_t = 1, \mathbf{P^{t-1}}\right) \\ &= \pi_0 \, f_N\left(p_t, \mu_{0t}, \sigma_0^2\right) + \pi_1 \, f_N\left(p_t, \mu_{1t}, \sigma_1^2\right) \end{aligned} \tag{10}$$

where $f_N\left(p_t, \mu, \sigma^2\right)$ is the density of a univariate normal with mean μ and variance σ^2.

The *MSVARlib-v2.0* developed by Bellone (2005) and OxGauss, both open-source software, were used to estimate the parameters of the model given by (10). The preliminary estimation results of the MS-ADF model suggest that for regime 1 the deterministic trend is not statistically significantly different from zero. The associated t-statistic is 0.45.

The estimation results of the MS-ADF model where the constraint $\beta_1 = 0$ has been imposed are presented in Table 2. The *Schwarz* criterion was used to select the length of the lag q in the autoregressive part and set to $q = 5$. Moreover, testing the null hypothesis of no regime switching against the alternative of regime switching using the non-standard likelihood-ratio (LR) test and the upper bound approach proposed by Davies (1987) results in the rejection of the null hypothesis.

The computed LR test statistic is 36.95, and the upper-bound p-value is 0.0001.

The results in Table 2 show that both regimes are quite persistent ($\hat{p}_{jj} > 0.9,\ j = 0, 1$) but their dynamics are very different. While regime 0 is characterized by a deterministic negative trend and the absence of a unit root, regime 1 (which is more volatile) appears to be characterized by a unit root and no deterministic trend. Note that the MS-ADF t-statistic for the null hypothesis of a unit root in regime 1 is only -1.97. Ang and Bekaert (1998) have shown that in the case of constant transition probabilities the overall process will be covariance stationary as long as the ergodic probability of being in the stationary regime is different from zero although one of the regimes has a unit root. The estimates of the ergodic probabilities π_0, π_1 are 0.44 and 0.56 respectively.

Central Absolute Moments of Olive-Oil Prices

To estimate the l_ρ norms of output prices it is necessary to derive the formulae for the central absolute moments of a finite mixture of normals. Let μ_{jt}, σ_j^2 in (10) be finite for $j = 0, 1$, then the mean of p_t given its past $\mathbf{P^{t-1}}$ is given by $\mu_t = \pi_0\, \mu_{0t} + \pi_1\, \mu_{1t}$. Furthermore, the central moments of the conditional density of p_t given its past $\mathbf{P^{t-1}}$ are given by (see Frühwirth-Schnatter, 2006):

$$E\left((p_t - \mu_t)^m | \mathbf{P^{t-1}}\right) = \sum_{k=0}^{1}\sum_{n=0}^{m} \begin{pmatrix} m \\ n \end{pmatrix} (\mu_{kt} - \mu_t)^{m-n}\, E\left((p_t - \mu_{kt})^n | \mathbf{P^{t-1}}\right) \pi_k \tag{11}$$

Example 3 *Consider the case of* $m = 2, 3, 4$ *then,*

$$var(p_t |\, \mathbf{P^{t-1}}) = \sum_{k=0}^{1} \left(\mu_{kt}^2 + \sigma_k^2\right) \pi_k - \mu_t^2 \tag{12}$$

$$E\left((p_t - \mu_t)^3 | \mathbf{P^{t-1}}\right) = \sum_{k=0}^{1} \pi_k \left((\mu_{kt} - \mu_t)^2 + 3\sigma_k^2\right)(\mu_{kt} - \mu_t) \tag{13}$$

$$E\left((p_t - \mu_t)^4 | \mathbf{P^{t-1}}\right) = \sum_{k=0}^{1} \pi_k \left((\mu_{kt} - \mu_t)^4 + 6(\mu_{kt} - \mu_t)^2 \sigma_k^2 + 3\sigma_k^4\right) \tag{14}$$

When m is even, expression (11) coincides with the central absolute moments, and therefore $l_m(\tilde{p})$ will just be the m^{th} root of (11). For m odd we derive the expression of the central absolute moments in the proposition below (whose proof is in an appendix).

Proposition 1 *Let the density of* p_t *given its past* $\mathbf{P^{t-1}}$ *be given by (10) with* μ_{jt}, σ_j^2 *being finite for* $j = 0, 1$ *and let* Z *be a standard normal random variable with cumulative distribution function* $\Phi(\cdot)$. *Then the central absolute moment of order* m *for* m *odd is given by,*

$$E\left(|\, p_t - \mu_t\,|^m\, |\mathbf{P^{t-1}}\right) = \left(\sum_{n=0}^{m} \begin{pmatrix} m \\ n \end{pmatrix} (\mu_{0t} - \mu_t)^{m-n} \sigma_0^n E\left(Z^n\right)\right.$$

$$
\begin{aligned}
&- \quad 2\sum_{n=0}^{m} \binom{m}{n} (\mu_{0t}-\mu_t)^{m-n}\sigma_0^n E\left(Z^n|Z<\frac{\mu_t-\mu_{0t}}{\sigma_0}\right)\left[\Phi\left(\frac{\mu_t-\mu_{0t}}{\sigma_0}\right)\right]\Bigg)\pi_0 \\
&+ \quad \left(\sum_{n=0}^{m} \binom{m}{n} (\mu_{1t}-\mu_t)^{m-n}\sigma_1^n E\left(Z^n\right)\right. \\
&- \quad 2\sum_{n=0}^{m} \binom{m}{n} (\mu_{1t}-\mu_t)^{m-n}\sigma_1^n E\left(Z^n|Z<\frac{\mu_t-\mu_{1t}}{\sigma_1}\right)\left[\Phi\left(\frac{\mu_t-\mu_{1t}}{\sigma_1}\right)\right]\Bigg)\pi_1
\end{aligned}
$$

For the case that $m=3$ the final expression is given in the example below.

Example 4 *Let $m=3$, then we have,*

$$
\begin{aligned}
E\left(|\, p_t-\mu_t\,|^3\,|\mathbf{P^{t-1}}\right) &= \Bigg((\mu_{0t}-\mu_t)\left[1-2\Phi\left(\frac{\mu_t-\mu_{0t}}{\sigma_0}\right)\right]\left[(\mu_{0t}-\mu_t)^2+3\sigma_0^2\right] \\
&+ \; 2\sigma_0^3\phi\left(\frac{\mu_t-\mu_{0t}}{\sigma_0}\right)\left[\left(\frac{\mu_t-\mu_{0t}}{\sigma_0}\right)^2+2\right]\Bigg)\pi_0 \\
&+ \; \Bigg((\mu_{1t}-\mu_t)\left[1-2\Phi\left(\frac{\mu_t-\mu_{1t}}{\sigma_1}\right)\right]\left[(\mu_{1t}-\mu_t)^2+3\sigma_1^2\right] \\
&+ \; 2\sigma_1^3\phi\left(\frac{\mu_t-\mu_{1t}}{\sigma_1}\right)\left[\left(\frac{\mu_t-\mu_{1t}}{\sigma_1}\right)^2+2\right]\Bigg)\pi_1 \qquad (15)
\end{aligned}
$$

where $\phi(\cdot)$ is the density function of the standard normal and $l_3(\tilde{p})$ will be given by the cube root of the above expression.

The estimates of the *Markov* switching autoregressive model and the above formulas (12), (14) and (15) can be used to obtain estimates of the mean $l_\rho(\tilde{p})$ norm of monthly output prices for $\rho = 2, 3, 4$.

Estimated Efficient Frontiers

Our estimates of the efficient frontier are now derived by substituting our estimates for the conditional price means and different risk measures into our estimated cost structure. Because the production data are annual, we compute an efficient frontier for each year 1999 to 2004 for each of the three risk measures. Each year's efficient frontier is computed by evaluating the estimated cost structure at mean values for the year for the quasi-fixed factor and the environmental variable. Thus, each yearly frontier is for a counter-factual *average farmer*, and not one associated with any particular farmer.

As our estimates of the conditional mean and risk measures for each year, we take the estimates for May of the preceding year, which roughly corresponds to when farmers make their main production decisions. Thus, the estimates for the mean and l_p norms of prices for years 1999-2004 (that will be referred to as year 1-year 6 hereafter) are based on the corresponding estimates for May 1998 up to May 2003.

Table 3 reports the estimates for the mean and three risk measures of prices for the six years. For each year the measure of risk is increasing with p, the order of the l_p norm, this being a direct consequence of the Lyapunov Inequality which states that $(E|X|^r)^{\frac{1}{r}} \leq (E|X|^p)^{\frac{1}{p}}$, for $1 \leq r \leq p$. The table also shows that for 2001 and 2002, which fall into the unit root-higher volatility regime, the risk measures are much higher and differ less across the different norms. This, in turn, will have important implications for our estimates of the efficient frontiers for those periods.

Figures 2 to 7, which depict the efficient frontier for each one of the six years, illustrate. For years 3 and 4 the frontiers under the alternative measures of risk are almost indiscernible because of the close empirical correspondence of these norms for those years. For those years, the efficient frontiers effectively correspond to those associated with the mean-variance approach.

Because the plots for the remaining years share many common features we will elaborate on one of them only. Turning to Figure 2 it can be readily seen that the three frontiers are concave in r and that the maximal expected profit is achieved at higher levels of r as p increases. However, as expected, maximal expected profit, which is associated with the risk-neutral outcome, is virtually identical for each of the three risk measures. The concavity of the frontier is a direct consequence of the fact that variable returns to scale were estimated to be decreasing and Theorem 1.

The slope of the efficient frontier in each case corresponds to the difference between the expected price of olives and the marginal cost of olives divided by the appropriate measure of price risk. Thus, it corresponds to a stochastic version of the Lerner index (price minus marginal cost) divided by a measure of price risk. Here, however, the Lerner index is not a measure of monopolistic power. Instead, in equilibrium, it measures each individual's attitudes towards risk and return because it must be equated to the producer's marginal rate of substitution between return and risk to determine the optimal level of production.

Hence, one way to visualize producer responsiveness to risk as captured by each risk measure is to determine where he or she locates on his or her respective efficient frontier. In Figures 8-13, we plot each farmer's return-risk couple relative to the efficient frontier for the average farmer. Most observations fall very close to these representative frontiers but relatively far away from the point of maximal expected profit. Thus, regardless of the risk measure used, our results suggest that olive farmers are, in fact, quite responsive to risk and routinely trade return for a decrease in risk.

Concluding Remarks

This paper develops a theoretically consistent and empirically tractable supply-response model for producers with invariant preferences facing price risk. Versions of that model are estimated for a group of Cretan olive-oil producers and different measures of price risk. Specifically, we estimate the cost function for a representative olive-oil producer and time-series representations of the empirical olive-oil price behavior and use them to induce a representation of its mean and three separate measures of price risk. These measures of risk and return are combined with the estimated cost structure to induce three separate representations of the efficient frontier for the representative olive-oil producer. Our empirical results suggest that, regardless of risk measure chosen, all producers curtail production in managing price risk.

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A Appendix

A.1 Proof of Proposition 1

Proof. The m^{th} central absolute moment of p_t given $\mathbf{P^{t-1}}$ is given by

$$E\left(\mid p_t - \mu_t \mid^m \mid \mathbf{P^{t-1}}\right) = E\left(\mid p_t - \mu_t \mid^m \mid s_t = 0, \mathbf{P^{t-1}}\right)\pi_0 + E\left(\mid p_t - \mu_t \mid^m \mid s_t = 1, \mathbf{P^{t-1}}\right)\pi_1$$

We will derive the expression for the first term on the right-hand side since the derivation is analogous for the second term. If m be odd we can write,

$$\mid p_t - \mu_t \mid^m = (p_t - \mu_t)^m \,\mathbf{1}[p_t \geq \mu_t] + (\mu_t - p_t)^m \mathbf{1}[p_t \leq \mu_t] = (p_t - \mu_t)^m - 2(p_t - \mu_t)^m \left(\mathbf{1}[p_t \leq \mu_t]\right)$$

where $\mathbf{1}\,[.]$ is an indicator function that takes the value 1 if the expression inside the bracket holds and 0 otherwise. Using the binomial expansion we have:

$$\begin{aligned}
E\left(\mid p_t - \mu_t \mid^m \mid s_t = 0, \mathbf{P^{t-1}}\right) &= \sum_{n=0}^{m} \binom{m}{n} (\mu_{0t} - \mu_t)^{m-n} E\left((p_t - \mu_{0t})^n \mid s_t = 0, \mathbf{P^{t-1}}\right) \\
&- 2\sum_{n=0}^{m} \binom{m}{n} (\mu_{0t} - \mu_t)^{m-n} \int_{-\infty}^{\mu_t} (p - \mu_{0t})^n \left[\frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(p-\mu_{0t})^2}{2\sigma_0^2}}\right] dp \\
&= \sum_{n=0}^{m} \binom{m}{n} (\mu_{0t} - \mu_t)^{m-n} \sigma_0^n \int_{-\infty}^{\infty} t^n \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\right] dp \\
&- 2\sum_{n=0}^{m} \binom{m}{n} (\mu_{0t} - \mu_t)^{m-n} \sigma_0^n \int_{-\infty}^{\frac{\mu_t - \mu_{0t}}{\sigma_0}} t^n \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\right] dp
\end{aligned}$$

Taking into account the definition of a truncated normal variable, we can write the above as:

$$\begin{aligned}
E\left(\mid p_t - \mu_t \mid^m \mid s_t = 0, \mathbf{P^{t-1}}\right) &= \sum_{n=0}^{m} \binom{m}{n} (\mu_{0t} - \mu_t)^{m-n} \sigma_0^n E\left(Z^n\right) \\
&- 2\sum_{n=0}^{m} \binom{m}{n} (\mu_{0t} - \mu_t)^{m-n} \sigma_0^n E\left(Z^n \mid Z < \frac{\mu_t - \mu_{0t}}{\sigma_0}\right) \left[\Phi\left(\frac{\mu_t - \mu_{0t}}{\sigma_0}\right)\right]
\end{aligned}$$

where Z is a standard normal random variable and $\Phi(\cdot)$ is its cumulative distribution function. Similarly conditional on regime 1 we have,

$$E\left(\mid p_t - \mu_t \mid^m \mid s_t = 1, \mathbf{P^{t-1}}\right) = \sum_{n=0}^{m} \binom{m}{n} (\mu_{1t} - \mu_t)^{m-n} \sigma_1^n E\left(Z^n\right)$$

$$- \quad 2\sum_{n=0}^{m} \left(\begin{array}{c} m \\ n \end{array} \right) (\mu_{1t} - \mu_t)^{m-n} \sigma_1^n E\left(Z^n | Z < \frac{\mu_t - \mu_{1t}}{\sigma_1} \right) \left[\Phi\left(\frac{\mu_t - \mu_{1t}}{\sigma_1} \right) \right]$$

Therefore the m^{th} central absolute moment can be obtained by substituting the above expressions into the expression of $E\left(| \, p_t - \mu_t \, |^m \, |\mathbf{P^{t-1}}\right)$ above and requires computing the moments of both the truncated and untruncated standard normal up-to-the m^{th} order. Alternatively, one can use the following recurrence relation in order to compute $E\left(| \, p_t - \mu_t \, |^m \, |\mathbf{P^{t-1}}\right)$ and its equivalent for regime 1 to obtain the necessary expressions,

$$\int_{-\infty}^{a} t^n e^{-\frac{t^2}{2}} dt = \left(-a^{n-1} e^{-\frac{a^2}{2}} \right) + (n-1) \int_{-\infty}^{a} t^{n-2} e^{-\frac{t^2}{2}} dt$$

with

$$\int_{-\infty}^{a} t^0 e^{-\frac{t^2}{2}} dt = \Phi(a) \quad \text{and} \quad \int_{-\infty}^{a} t e^{-\frac{t^2}{2}} dt = -\phi(a)$$

where $\phi(\cdot)$ is the density of the standard normal. ■

Tables and Figures

Table 1: Summary Statistics of the Variables

Variable	Mean	Maximum	Minimum
Olive-Oil			
- Quantity (kgs)	21,051	111,168	1,658
- Price (euros)	2.62	3.62	1.63
Hired Labor			
- Quantity (hrs)	578	2,985	48
- Price (euros)	20.33	29.85	13.79
Fertilizers			
- Quantity (kgs)	16,037	87,266	846
- Price (euros)	0.236	0.350	0.113
Intermediate Inputs			
- Total Cost (euros)	2,813	13,587	350
- Price (euros)	3.19	4.41	1.85
Capital (euros)	25,214	142,543	2,341
Land (stremmas[1])	54	202	4
Family Labor (hrs)	403	2,292	35
Aridity Index	0.90	1.70	0.26

[1] one stremma equals 0.1 ha.

Table 2: Parameter Estimates for *Markov* Switching Model of Olive-Oil Prices

Regime 0			Regime 1		
Parameter	Estimate	StdError	Parameter	Estimate	StdError
α_0	5.5469	(0.5455)	α_1	0.4853	(0.2464)
β_0	-0.0059	(0.0007)	β_1	-	-
γ_0	-0.0529	(0.1032)	γ_1	0.9070	(0.0472)
ϕ_{02}	0.3243	(0.0693)	ϕ_{12}	0.2960	(0.1271)
ϕ_{03}	0.0925	(0.0716)	ϕ_{13}	-0.1173	(0.1284)
ϕ_{04}	0.1434	(0.0748)	ϕ_{14}	0.2652	(0.1317)
ϕ_{05}	-0.3243	(0.0791)	ϕ_{15}	-0.2137	(0.1232)
ϕ_{06}	-0.2133	(0.0756)	ϕ_{16}	0.2065	(0.1137)
σ_0^2	0.0007	(0.0002)	σ_1^2	0.0052	(0.0012)
p_{00}	0.9588	(0.0364)	p_{11}	0.9670	(0.0329)
ADF t-stat for $\gamma_0 = 1$:		-10.2025	ADF t-stat for $\gamma_1 = 1$:		-1.9703
LnL			154.442		

Table 3: Annual Estimates of $\mu[\tilde{p}]$ and $l_\rho[\tilde{p}]$ Norms of Olive-Oil Prices (ρ=2,3,4)

Year	$\mu[\tilde{p}]$	$l_2[\tilde{p}]$	$l_3[\tilde{p}]$	$l_4[\tilde{p}]$
1999	4.9734	0.0567	0.0704	0.0824
2000	4.8875	0.0846	0.0955	0.1060
2001	5.1463	0.2775	0.2836	0.2889
2002	5.1000	0.3375	0.3432	0.3482
2003	4.7794	0.0593	0.0726	0.0855
2004	4.6957	0.0599	0.0731	0.0850

Table 4: Parameter Estimates of the *Generalized Leontief* Cost Function for Olive Farms in Crete

Parameter	Estimate	StdError	Parameter	Estimate	StdError
β_L	-24.066	(10.554)	α_{IY}	9.84x10^{-7}	(8.9x10^{-8})
β_F	-283.63	(103.82)	γ_L	0.0027	(0.0017)
β_I	-164.95	(90.479)	γ_F	-0.0139	(0.0095)
α_{LL}	0.0223	(0.0038)	γ_I	-0.0019	(0.0008)
α_{LF}	0.0323	(0.0152)	δ_{LA}	0.0093	(0.0016)
α_{LI}	0.0134	(0.0039)	δ_{FA}	0.0024	(0.0007)
α_{FF}	1.2149	(0.2235)	δ_{IA}	0.0038	(0.0006)
α_{FI}	0.0151	(0.0063)	ζ_L	-0.0200	(0.0203)
α_{II}	0.0993	(0.0140)	ζ_F	-0.3166	(0.0871)
α_{LY}	1.0x10^{-6}	(2.4x10^{-7})	ζ_I	-0.0036	(0.0016)
α_{FY}	9.3x10^{-6}	(1.0x10^{-6})			
LnL			-732.536		

where, L stands for hired labor, F for fertilizers, I for intermediate inputs, A for quasi-fixed input, and Y for output (*i.e.*, olive-oil).

Table 5: Variable-Input Demand Elasticities and Returns-to-Scale

Variable-Input Demands	w_L	w_F	w_I
Hired Labor	-0.0045	0.0020	0.0027
Fertilizers	0.1330	-0.1582	0.0253
Intermediate Inputs	0.0165	0.0022	-0.0187
Returns-to-Scale		0.6433	

where, L stands for hired labor, F for fertilizers, and I for intermediate inputs.

Figure 1: Monthly Real Retail Prices for Olive-Oil in Crete

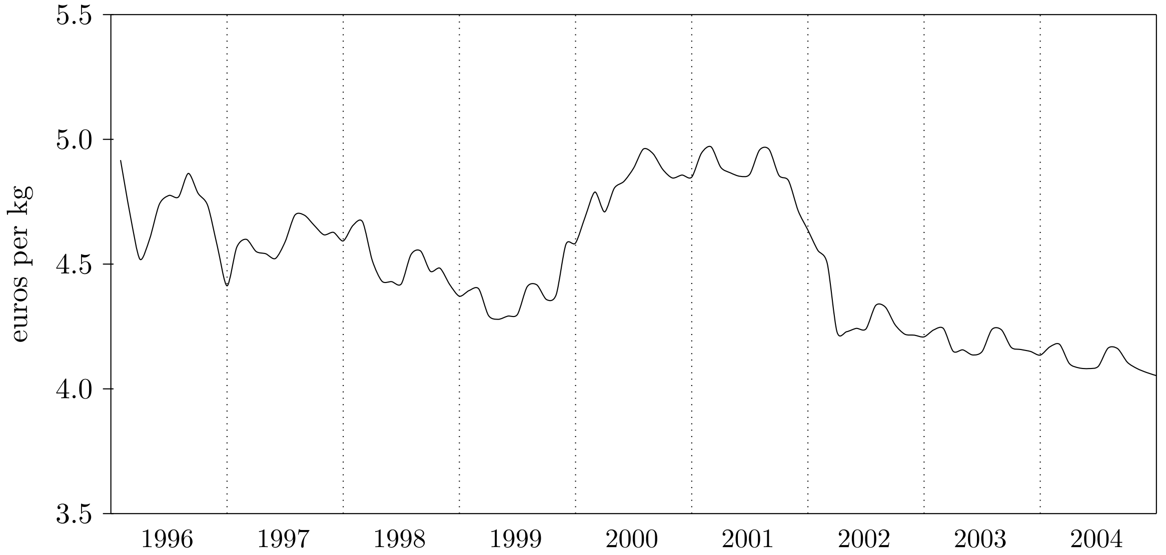

Figure 2: Efficient Frontier for Year 1

Figure 3: Efficient Frontier for Year 2

Figure 4: Efficient Frontier for Year 3

Figure 5: Efficient Frontier for Year 4

Figure 6: Efficient Frontier for Year 5

Figure 7: Efficient Frontier for Year 6

Figure 8: Observed Farm Expected Returns and Risk vs Frontier for Year 1

Figure 9: Observed Farm Expected Returns and Risk vs Frontier for Year 2

Figure 10: Observed Farm Expected Returns and Risk vs Frontier for Year 3

Figure 11: Observed Farm Expected Returns and Risk vs Frontier for Year 4

Figure 12: Observed Farm Expected Returns and Risk vs Frontier for Year 5

Figure 13: Observed Farm Expected Returns and Risk vs Frontier for Year 6