

On the Growth-Maximizing Allocation of Public Investment

Pantelis Kalaitzidakis and Vangelis Tzouvelekas

(Dept of Economics, University of Crete, Greece)

Abstract

In this paper we present an endogenous growth model to analyze the growth maximizing allocation of public investment among N different types of public capital. Using this general model of public capital formation, we analyze the stability of the long-run equilibrium and we derive the growth-maximizing values of the shares of public investment allocated to the different types of public capital, as well as the growth-maximizing tax rate (amount of total public investment as a share of GDP). The empirical implication of the model is that both the effects of the shares of public investment and the tax rate on the long-run growth rate are non-linear, following an inverse U-shaped pattern.

1. Introduction

Over the last three decades, after the publication of Aschauer's (1989) empirical paper on the productivity of public capital and Barro's (1990) theoretical paper on the effects of government spending on economic growth, the analysis of the macroeconomic effects of public investment has attracted a lot of attention. The theoretical research was mainly focused on showing how public spending and public capital may enhance productivity and promote economic growth.

Indeed public spending enhances productivity through its external effect in the production function of private firms. This effect can be modeled by adding into the production function either the aggregate flow of public spending, following Barro (1990), or the aggregate stock of public capital, as in Turnovsky (1997). A new line of theoretical research recognizes the possibility that different types of public spending (e.g. infrastructure, education, health, military spending) may exert a different effect on growth (Devarajan *et al.*, 1996; Shieh *et al.*, 2002; Kalaitzidakis and Kalyvitis, 2004; Chen, 2006)

In this paper we extend the previous theoretical literature by presenting an endogenous growth model with N different types of productive public capital. Using a general model we analyze the stability of the long-run equilibrium and we derive the growth-maximizing values of

the shares of public investment allocated to the different types of public capital, as well as the growth-maximizing tax rate (amount of total public investment as a share of GDP). The results of our analysis constitute generalizations of the results derived by simpler models which employ only two different types of public capital. The main result of the model is that both the effects of the shares of public investment and the tax rate on the long-run growth rate are non-linear, following an inverse U -shaped pattern. The implications of this result is quite important for the empirical analysis of the effects of public investment on economic growth.

2. A Growth Model with Different Types of Public Capital

2.1 Model Description

We consider a closed economy populated by identical agents who consume and produce a single commodity with no population growth. Labor is supplied inelastically and individuals have identical time-separable utility functions. Firm j produces its output using a Cobb-Douglas technology:

$$Y_j = K_j^\alpha (hL_j)^{1-\alpha}, \quad 0 < \alpha < 1 \quad (1)$$

where K_j denotes the stock of private capital, and L_j the labor use. The productivity of labor is a function of the existing stock of N different types of public, Z_i ($i=1,2,\dots,N$), and aggregate private capital, K , per worker:

$$h = \frac{\left(\prod_{i=1}^N Z_i^{\beta_i} \right)^\gamma K^{1-\gamma}}{L} \quad (2)$$

where $L = \sum_j L_j$, $0 < \beta_i < 1 \quad \forall i$, $\sum_{i=1}^N \beta_i = 1$, and $0 < \gamma < 1$. Private capital depreciates at the constant rate δ_k , *i.e.*,

$$\dot{K} = I - \delta_k K \quad (3)$$

New output may be transformed to any type of capital, but in the case of private capital this process involves adjustment costs, *i.e.*,

$$\Psi(I, K) = \left(1 + \frac{\phi}{2} \frac{I}{K}\right) I \quad (4)$$

where $\phi > 0$ is an adjustment cost parameter. The adjustment cost of private capital is proportional to the rate of investment per unit of installed capital.

The stock of each type of public capital depreciates at the rate δ_i . If G_i denotes gross public investment for public capital i , then the net public capital stock accumulates as:

$$\dot{Z}_i = G_i - \delta_i Z_i \quad \forall i = 1, 2, \dots, N \quad (5)$$

The government finances its total expenditure through tax revenues collected via a tax rate τ , *i.e.*,

$$\sum_{i=1}^N G_i = \tau Y \quad (6)$$

We define the shares of total government expenditure that go towards the i^{th} type of public capital as μ_i :

$$G_i = \mu_i \tau Y \quad \text{with} \quad \sum_{i=1}^N \mu_i = 1 \quad (7)$$

2.2 Model Solution

The representative firm solves the following profit maximization problem:

$$\begin{aligned} \max \int_0^{\infty} e^{-rt} \left[(1-\tau)Y_j - w_j L_j - \left(1 + \frac{\phi}{2} \frac{I_j}{K_j}\right) I_j \right] dt \\ \text{s.t. } \dot{K}_j = I_j - \delta_k K_j \end{aligned}$$

where r is the interest rate, w is the wage rate, while output price is normalized to one. The optimality conditions imply:

$$w_j = (1-\tau)(1-\alpha) \left(\frac{K_j}{L_j} \right)^{\alpha} h^{1-\alpha} \quad (8a)$$

$$1 + \phi \frac{I_j}{K_j} = q \quad (8b)$$

$$r = \frac{1}{q} \left[\dot{q} + (1-\tau)\alpha \left(\frac{K_j}{L_j} \right)^{\alpha-1} h^{1-\alpha} + \frac{\phi}{2} \left(\frac{I_j}{K_j} \right)^2 \right] - \delta_k \quad (8c)$$

where q is the shadow value of the private capital stock. Equation (8a) equates the wage rate to the value of the marginal product of labor. Given that all firms in the economy will pay the same wage and will hire the same amount of labor and private capital, equation (8a) can be written as $\frac{wL}{Y} = (1-\tau)(1-\alpha)$, implying that the labour share of income remains constant in the long run. As a result, the steady state wage rate and per capita income grow at the same rate. Equation (8b) equates the marginal cost of investment to the shadow value of capital. Finally, equation (8c) equates the interest rate to the rate of return of private capital, net of physical depreciation. The rate of return to private capital consists of three components: the change in its shadow value, the value of its marginal product, and its effect on the cost of investment. In addition, the following transversality condition must hold:

$$\lim_{t \rightarrow \infty} (q e^{-rt} K_j) = 0 \quad (9)$$

Substituting (8b) into (8c), replacing for (2) and aggregating across firms, the optimality

conditions with respect to I and K can be written as:

$$\frac{I}{K} = \frac{q-1}{\phi} \quad (10)$$

$$\dot{q} = (r + \delta_k)q - (1-\tau)\alpha \prod_{i=1}^N \left(\frac{Z_i}{K}\right)^{\beta_i \gamma (1-\alpha)} - \frac{(q-1)^2}{2\phi} \quad (11)$$

From equations (3), (5), (7), and (10), the growth rates of private and the different types of public capital are given by:

$$\frac{\dot{K}}{K} = \frac{q-1}{\phi} - \delta_k \quad (12)$$

$$\frac{\dot{Z}_i}{Z_i} = \mu_i \tau \left(\frac{K}{Z_i}\right) \prod_{i=1}^N \left(\frac{Z_i}{K}\right)^{\omega_i} - \delta_i \quad \forall i = 1, 2, \dots, N \quad (13)$$

given that

$$Y = K^{1-\sum_{i=1}^N \omega_i} \prod_{i=1}^N Z_i^{\omega_i} \quad (14)$$

and $\omega_i = \beta_i \gamma (1-\alpha)$. Defining $z_i \equiv \frac{Z_i}{K}$ and using (12), (13), and (14) we obtain the following system of $N+1$ differential equations given the policy parameters τ and μ_i :

$$\frac{\dot{z}_i}{z_i} = -\delta_i - \frac{q-1}{\phi} + \tau \frac{\mu_i}{z_i} \prod_{i=1}^N z_i^{\omega_i} + \delta_k, \quad \forall i = 1, 2, \dots, N \quad (15)$$

$$\dot{q} = (r + \delta_k)q - (1-\tau)\alpha \prod_{i=1}^N z_i^{\omega_i} - \frac{(q-1)^2}{2\phi} \quad (16)$$

2.3 Steady State

The stationary solution of this system must have at least one solution, in order for output and the

capital stocks, K , and Z_i , to follow a balanced growth path given by $\bar{g}_Y = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{Z}_i}{Z_i}$. From equation (12) and given that at the steady state, output and private capital grow at the same rate, we get:

$$\bar{q} = 1 + \phi(\bar{g}_Y + \delta_k) \quad (17)$$

The steady-state shadow price of private capital is a positive function of the growth rate, as higher growth rates are associated with increased output and profits. The depreciation rate of private capital also affects positively its shadow price because a higher depreciation rate requires a higher shadow price of private capital due to larger associated adjustment costs.

Finally, the transversality condition (9) can be expressed as:

$$\lim_{t \rightarrow \infty} \left(\frac{q}{z_i} e^{-(r-g_Y)t} \right) = 0 \quad (18)$$

which implies that in the steady state the rates of growth of output, private, and public capital must not exceed the real interest rate (Turnovsky, 1997).

The dynamics of our economy are described by equations (15), and (16). Under the simplifying assumption that $\delta_k = \delta_i = \delta$, the equilibrium values of q and z_i are determined by the following:

$$\dot{z}_i = 0 \Rightarrow q = 1 + \phi \tau \frac{\mu_i}{z_i} \prod_{i=1}^N z_i^{\omega_i} \quad \forall i = 1, 2, \dots, N \quad (19)$$

$$\dot{q} = 0 \Rightarrow (1 - \tau) \alpha \prod_{i=1}^N z_i^{\omega_i} = (r + \delta) q - \frac{(q - 1)^2}{2\phi} \quad (20)$$

Then, from equation (18) we get that for any two types of public capital holds:

$$\frac{z_m}{z_i} = \frac{\mu_m}{\mu_i} \quad (21)$$

Substituting equation (21) into equations (19) and (20), we get:

$$q = 1 + \phi\tau \left(\frac{\mu_i}{z_i} \right)^{1 - \sum_{i=1}^N \omega_i} \prod_{i=1}^N \mu_i^{\omega_i} \quad \forall i = 1, 2, \dots, N \quad (22)$$

$$(1 - \tau)\alpha \left(\frac{z_i}{\mu_i} \right)^{\sum_{i=1}^N \omega_i} \prod_{i=1}^N \mu_i^{\omega_i} = (r + \delta)q - \frac{(q-1)^2}{2\phi} \quad (23)$$

3. Transitional Dynamics

If we assume the initial values of any two types of public capital stocks satisfy equation (21), then equation (15) implies that equation (21) holds at any instant of time and not only in the steady state (Shieh *et al.*, 2002). This helps us avoid computational complications, since the dynamics of our economy can now be described using the following two differential equations:

$$\frac{\dot{z}_i}{z_i} = -\frac{q-1}{\phi} + \tau \left(\frac{\mu_i}{z_i} \right)^{1 - \sum_{i=1}^N \omega_i} \prod_{i=1}^N \mu_i^{\omega_i} \quad (24)$$

$$\dot{q} = (r + \delta)q - (1 - \tau)\alpha \left(\frac{z_i}{\mu_i} \right)^{\sum_{i=1}^N \omega_i} \prod_{i=1}^N \mu_i^{\omega_i} - \frac{(q-1)^2}{2\phi} \quad (25)$$

It can be shown that the linearized dynamics around the steady-state values (\bar{z}_i, \bar{q}) are represented by:

$$\begin{bmatrix} \dot{z}_i \\ \dot{q} \end{bmatrix} = A \begin{bmatrix} z_i - \bar{z}_i \\ q - \bar{q} \end{bmatrix} \quad (26)$$

where

$$A = \begin{bmatrix} -\frac{q-1}{\phi} \left(1 - \sum_{i=1}^N \omega_i\right) & -\frac{z_i}{\phi} \\ -\frac{1-\tau}{\tau} \frac{q-1}{\phi} \alpha \mu_i \sum_{i=1}^N \omega_i & r + \delta - \frac{q-1}{\phi} \end{bmatrix}.$$

The determinant of the Jacobian is negative, implying that one of the two characteristic roots of the matrix is negative and, therefore, the equilibrium is a saddle point with a downward sloping stable branch. Since the initial value of q is not predetermined, we can choose a unique value that is consistent with the stable manifold.

The dynamics of our economy are shown in Figure 1. Equations (24) and (25) imply that, in the (z_i, q) space, the $\dot{z}_i = 0$ locus slopes downwards, while the $\dot{q} = 0$ locus slopes upwards for $q < 1 + \phi(r + \delta)$ and downwards otherwise. Since the transversality condition (18) implies that the equilibrium occurs at the negative part of the $\dot{q} = 0$ locus, Figure 1 depicts only the negative part of the locus.

4. Steady State Policy Effects

4.1 Change in the Shares of Public Investment

The following proposition demonstrates the condition satisfied by the growth-maximizing shares of public investment.

Proposition 1. *The growth-maximizing shares of public investment satisfy the condition*

$$\mu_i^* = \frac{\omega_i}{\sum_{i=1}^N \omega_i}, \text{ for all } i = 1, 2, \dots, N.$$

Proof. From equation (17) we see that the steady state growth rate of the economy is

$$\bar{g}_Y = \frac{\bar{q}(\mu_1, \mu_2, \dots, \mu_N, \tau) - 1}{\phi} - \delta. \text{ The growth-maximizing shares of public investment are the ones}$$

that maximizes the shadow price of private capital, that is: $\frac{\partial \bar{g}_Y}{\partial \mu_i} = 0 \Rightarrow \frac{\partial \bar{q}(\mu_1, \mu_2, \dots, \mu_N, \tau)}{\partial \mu_i} = 0$

for all $i=1,2,\dots,N$. From equations (22) and (23) we then get that $\mu_i = \frac{\omega_i}{\sum_{i=1}^N \omega_i}$ for all

$i=1,2,\dots,N$.

Proposition 1 along with equation (21) imply that, when the shares of public investment are set to their growth maximizing levels, the steady state ratio of any two stocks of public capital is equal to the ratio of the corresponding output elasticities:

$$\frac{z_m}{z_i} = \frac{Z_m}{Z_i} = \frac{\omega_m}{\omega_i} \quad (27)$$

In other words, the growth rate of the economy is maximized when we allocate public investment so that the last dollar invested in any type of public capital has the same contribution to the increase in the aggregate output. We can easily show, using equations (22) and (23), that when $\frac{z_m}{z_i} < \frac{\omega_m}{\omega_i}$, a reallocation of public investment from the i^{th} to the m^{th} type of public capital will raise the steady state growth rate of the economy.

4.2 Change in the Tax Rate

From equations (24) and (25) we can easily see that an increase in τ shifts the $\dot{z}_i = 0$ upwards and to the right, and the $\dot{q} = 0$ locus downwards and to the right. As a result, the equilibrium value of z_i rises, while the effect on q and, consequently, on the growth rate of the economy is ambiguous. Recall that the government affects the growth of the economy through two channels. Taxation affects negatively the marginal product of private capital, while government expenditure increases the productivity of labor. At low values of τ the positive effect of government expenditure dominates, and, hence, the growth rate of the economy rises with the tax rate. At higher tax rates, however, the negative impact of taxation eventually dominates, and the growth rate declines as τ rises. The tax rate that maximizes the growth rate is the one that equates the marginal cost of government expenditure to its marginal benefit.

By differentiating equations (22) and (23) with respect to τ , we find that:

$$\frac{\partial z_i}{\partial \tau} > 0 \quad (28)$$

$$\frac{\partial q}{\partial \tau} \begin{cases} > 0 & \text{if } \tau < \sum_{i=1}^N \omega_i \\ < 0 & \text{if } \tau > \sum_{i=1}^N \omega_i \end{cases} \quad (29)$$

Proposition 2. *The tax rate that maximizes the growth rate of the economy is $\tau^* = \sum_{i=1}^N \omega_i$.*

Proof. The proof is similar to that of Proposition 1.

Proposition 2 simply shows that in the presence of N different types of public capital, the growth maximizing tax rate is equal to the sum of the elasticities of all types of public and human capital in the aggregate production function.

5. Concluding Remarks

The main goal of this paper was to study the growth implications of public capital formation. We developed an endogenous growth model with N different types of productive public capital. We analysed the stability of the long-run equilibrium and we derived the growth-maximizing allocation of public investment among the different types of public capital, as well as the growth maximizing amount of total public investment as a share of GDP (tax rate). Our theoretical findings imply: *i*) the steady-state ratio of any two stocks of public capital is equal to the ratio of the corresponding output elasticities; *ii*) the growth-maximizing tax rate of the economy is equal to the sum of output elasticities of the N types of public capital; *iii*) the effects of both the shares of public investment and the tax rate on the growth rate of the economy are nonlinear, following an inversed U shape pattern.

References

- Aschauer, D., 1998. How big should the public capital stock be? The relationship between public capital and economic growth. *Public Policy Brief*, 43, The Jerome Levy Economics Institute of Bard College.
- Barro, R., 1990. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98, 103-125.
- Barro, R., and Sala-i-Martin, X., 1995. *Economic growth*. McGraw-Hill.
- Chen, B., 2006. Economic growth with an optimal public spending composition. *Oxford Economic Papers*, 58, 123-136.
- Devarajan, S., Swaroop, V., and Zou, H., 1996. The composition of public expenditure and economic growth. *Journal of Monetary Economics*, 37, 313-344.
- Kalaitzidakis, P., and Kalyvitis, S., 2004. On the macroeconomic implications of maintenance in public capital. *Journal of Public Economics*, 88, 695-712.
- Shieh, J., Lai, C., and Chang, W., 2002. The impact of military burden on long-run growth and welfare. *Journal of Development Economics*, 68, 443-454.
- Turnovsky, S., 1997. Public and private capital in an endogenously growing open economy. In Jensen, B., and Wong, K. (eds), *Dynamics, Economic Growth and International Trade*. The University of Michigan Press, Ann Arbor, pp. 171-210.

Figure 1. Dynamics Around the Steady-State

