Endogenous managerial incentive contracts in a differentiated duopoly, with and without commitment

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Abstract

In a differentiated Cournot duopoly, we examine the contracts that firms’ owners use to compensate their managers and the resulting output levels, profits and social welfare. If products are either sufficiently differentiated or sufficiently close substitutes, owners use Relative Performance contracts. For intermediate levels of product substitutability, they use Market Share contracts. When owners do not commit over the types of contracts, each type is an owner’s best response to his rival’s choice. Product substitutability has differential effects on output levels and profits, depending on the configuration of contracts in the industry. Finally, managerial incentive contracts are welfare enhancing if they increase consumers’ surplus.

Keywords: Oligopoly; Managerial delegation; Endogenous contracts.

JEL classification: D43; L21

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1 Introduction

It is well established that in modern firms, where ownership and management are separated (Fama and Jensen, 1983), one of the key aspects of corporate governance codes relates to managerial compensation decision-making (van Witteloostuijn et al., 2007). Within this framework, existing evidence suggests that owners design their managers’ compensation contracts so as to motivate them to gain a competitive advantage in the market (Murphy, 1999; Jensen et al., 2004).

The strategic use of managerial incentive contracts has been introduced in the literature by the path-breaking papers of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987). In this line of research, each owner has the opportunity to compensate his manager with an incentive contract, combining own profits and sales or revenues, in order to direct him to a more aggressive behavior in the market, so as to force the rival firm to reduce output.1 Empirical studies by Jensen and Murphy (1990) and Lambert et al. (1991) report that managerial compensation is often associated with firms’ profits and size, as measured by sales or revenues. In a similar vein, Gibbons and Murphy (1990), along with Joh (1999) and Aggarwal and Samwick (1999) report that a widely used practice among firms’ owners, when designing their managers’ compensation contracts, is to take into account the relative performance of own profits against their rivals’ profits. Miller and Pazgal (2001; 2002; 2005) formalize these arguments with the “relative performance” contracts, where a manager’s compensation is a linear combination of own profits and the relative performance against the rivals’ profits. In this case too, firms end up in a prisoners’ dilemma situation. However, firms’ profits are higher than the respective under contracts combining own profits and sales or revenues. From another perspective, Peck (1988) and Borkowski (1999) mention that market share is highly ranked in managers’ objectives and Jansen et al. (2007) and Ritz (2008) offer formal analyses of contracts combining own profits and own market share.

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1In particular, in the above series of papers, in two-staged oligopoly models, in the first stage of the game, profit-maximizing owners choose compensation schemes for their managers that are linear combinations of own profits and either own sales or own revenues. In the second stage, managers, knowing compensation schemes, compete in quantities. Each owner, when determining his manager’s incentives, has an opportunity to become a Stackelberg leader, provided that the rival owners do not delegate output decisions to managers. In equilibrium, all owners act in the same way at the game’s contract stage and firms end up in a prisoners’ dilemma situation with relatively higher output and lower profits.
The literature, so far, seems to seek for an explanation regarding the emergence of the different types of managerial incentive contracts. In this context, Jansen et al. (2009) find that in a Cournot model with product homogeneity, firms’ owners have dominant strategy to compensate their managers with “relative performance” contracts.

Our paper contributes to the relevant literature by investigating the impact of product substitutability on the types of contracts that firms’ owners choose to compensate their managers, as well as on the resulting output levels, profits and social welfare. In particular, the present paper attempts to address the following four questions. First, what is the effect of product substitutability, and the respective competitiveness in the final good market, on the output levels set by managers and the resulting firms’ profits? Second, how does the degree of product substitutability affect the firms’ owners decisions, regarding the types of contracts that they will choose to compensate their managers? Our third question has been motivated by a careful review of the existing literature in the field of strategic managerial incentive contracts. This literature has been grounded on the assumption that firms’ owners commit over the types of contracts that they choose to compensate their managers. Then, the question that easily arises is whether the results obtained with ex-ante commitment still hold without commitment. The final question that the present paper addresses is relevant to the societal effects of the different managerial incentive contracts.

To address the above questions, we build upon Jansen et al. (2009) framework, with one important departure. We assume that the two competing firms produce differentiated instead of homogeneous products. In this environment, we consider a three-staged game with observable actions: In stage one, each firm’s owner commits to one type of contract to compensate his manager. More specifically, each owner commits to a contract that is a linear combination of own profits and either own revenues (Profits-Revenues contract), or competitor’s profits (Relative Performance contract) or, finally, own market share (Market Share contract). In the second stage of the game, given that the types of contracts have become common knowledge and can not be reset, each owner sets the weight (managerial incentive parameter) between own profits and either own revenues, or competitor’s profits, or own market share. At the final stage, managers compete in quantities.

We argue that the effect of product substitutability on the output levels set by managers and
the resulting firms’ profits depends crucially on the configuration of contracts. In the symmetric configurations where both owners compensate their managers either with Profits-Revenues or with Relative Performance contracts, output and profits decrease in the degree of product substitutability. The reason is that as products become closer substitutes, the market segment that each firm exploits decreases. A direct consequence is that the output level that each manager sets and the resulting profits decrease as products become closer substitutes. On the contrary, when both managers are compensated with Market Share contracts, the aforementioned negative product differentiation effect is dominated by the positive competition effect according to which, as products become closer substitutes and competition for market share among managers becomes fiercer, each manager tends to increase output. This tends to increase profits too. Regarding the asymmetric configurations, where managers are compensated with contracts of different types, we identify that product substitutability has differential effects on output levels and profits. In particular, the output level set by the manager who is compensated with the advantageous contract, in terms of output expansion and profits, has a U-shaped relation in the degree of product substitutability. Intuitively, as products become closer substitutes, this manager increases the output level in order to exploit the competitive advantage that his contract gives him. On the contrary, the output level set by the rival manager decreases as products become closer substitutes. Firm’s profits follow the same pattern as the output level in each case.

As far as the second question is concerned, we show that the types of contracts that owners choose to compensate their managers depend crucially on the degree of product substitutability. In particular, when products are sufficiently differentiated, owners compensate their managers with Relative Performance contracts. Intuitively, Relative Performance contracts result in a relatively more severe overproduction situation, as compared with the respective of Market Share contracts. This overproduction situation has two effects on profits. On the one hand it tends to increase profits but on the other hand, it tends to decrease them through price decrease. Our analysis suggests that the overproduction’s positive effect on profits is stronger under Relative Performance contracts, rather than the respective under Market Share contracts. Hence, owners compensate their managers with Relative Performance contracts. For intermediate levels of product substitutability, owners choose to compensate their managers with Market Share con-
tracts. This is so because the prisoners’ dilemma situation, characterized by relatively higher output and lower profits, under Market Share contracts is less severe than the respective under Relative Performance contracts. This reasoning is reversed if products are sufficiently close substitutes, in which case owners compensate their managers with Relative Performance contracts. Our findings further suggest that given an owner’s choice, over the type of contract to compensate his manager, his rival’s best response is a contract of the same type.

Then, we examine the case where there is no ex-ante commitment over the types of contracts that owners choose to compensate their managers. In this environment, the following two-staged game is studied: in the first stage, each firm’s owner chooses the type of contract and sets the corresponding managerial incentive parameter. Hence, we assume that the precise contract (the type of contract and the managerial incentive parameter) that each owner sets is not observable by the rival owner, before contract-setting is everywhere completed. In the second stage, managers compete in quantities. Interestingly enough, we find that each type of contract can be an owner’s best response to the rival owner’s choice. The intuitive explanation behind this finding is based on the conditions that must be fulfilled in equilibrium: firstly, since production decisions are taken by managers, in equilibrium, their reaction curves must be intersected. Secondly, the fact that a firm’s owner offers an incentive contract to his manager, as a strategic tool in order to become Stackelberg leader against the rival firm, standard textbook analysis of the Stackelberg model implies that, in equilibrium, there must be tangency between this firm’s isoprofit curve and the rival firm manager’s reaction curve.

Regarding the fourth question, we find that the symmetric use of managerial incentive contracts by firms’ owners is welfare enhancing, compared to the benchmark case of No-delegation, except if owners compensate their managers with Market Share contracts and products are sufficiently close substitutes. We also find that social welfare in the asymmetric configurations of contracts lies between the respective levels in the symmetric configurations. Our analysis also suggests that social welfare decreases in the degree of product substitutability, because of the dominant negative product differentiation effect.

Comparing our results with those obtained in Jansen et al. (2009), who restrict their attention to the case of homogeneous products, the following observations are in order. First, we identify the differential effects of product differentiation on the output levels set by managers
under the different configurations of contracts. Second, we reconfirm Jansen et al. (2009) result that firms’ owners compensate their managers with Relative Performance contracts, when products are perfect substitutes. Moreover, we find that this is an equilibrium whenever products are either sufficiently differentiated or sufficiently close substitutes. On the contrary, for intermediate degrees of product substitutability, we find that owners use Market Share contracts. Third, Jansen et al. (2009) state that whenever an owner compensates his manager with a Relative Performance contract while the rival owner uses either a Profits-Revenues or a Market Share contract, the former owner directs his manager towards the maximum level of aggressiveness and the latter owner’s best response is to direct his manager towards a strict profit-maximizing behavior.\textsuperscript{2} Our analysis suggests that this is an equilibrium situation but only in the polar case of homogeneous products. For the remaining spectrum of product substitutability, we find that the latter owner sets the managerial incentive parameter at a level that directs his manager away from strict profit-maximization. Note also that our paper is a first attempt to evaluate the societal effects of the different managerial incentive contracts. The only exception is van Witteloostuijn et al. (2007) who compare the welfare effects of symmetric profits-sales contracts and symmetric relative performance contracts with the respective in the benchmark case of No-delegation. However, they do not compare social welfare across the different configurations of contracts.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, we study the equilibrium managerial incentive contracts with owners’ commitment and in Section 4, we carry out the respective analysis under no-commitment. Section 5 includes the welfare analysis. Finally, Section 6 concludes.

2 The Model

Our model builds upon Jansen et al. (2009) framework, with one important departure. We assume that the two competing firms produce differentiated instead of homogeneous products. In particular, we assume that each firm \( i \) faces the following (inverse) demand function:

\textsuperscript{2}See Proposition 1 in Jansen et al. (2009, p. 146).
\[ P_i = 1 - q_i - \gamma q_j, \quad i, j = 1, 2, \ i \neq j, \ 0 < \gamma \leq 1 \]  

(1)

where \( p_i \) and \( q_i \) are, respectively, the price and quantity of firm \( i \)'s product, \( q_i \) is the quantity of its rival’s product, and \( \gamma \) is the degree of product substitutability. Namely, the higher is \( \gamma \), the closer substitutes the two products are.\(^3\)

We further assume that firms have equally efficient production technologies, reflected in constant marginal production costs, i.e. \( c_i = c_j = c < 1 \). Thus, firm \( i \)'s profits are given by:

\[ \Pi_i = (1 - q_i - \gamma q_j - c)q_i \]  

(2)

In this industry, each firm has an owner and a manager. Following Fershtman and Judd (1987), “owner”, is a decision maker whose objective is to maximize the profits of the firm. This could be the actual owner, a board of directors, or a chief executive officer. “Manager” refers to an agent that the owner hires to make real time operating decisions.

Each firm’s owner has the opportunity to compensate his manager by offering to him a “take-it-or-leave-it” incentive contract.\(^4\) In particular, each owner chooses one among three different types of contracts. The first type is the Profits-Revenues (PR) one. Following Fershtman and Judd (1987) and Sklivas (1987), under this type of contract, the incentive structure takes a particular form: the risk-neutral manager \( i \) is paid at the margin, in proportion to a linear combination of own profits and own revenues. More formally, firm \( i \)'s manager will be given incentive to maximize:

\[ U_{i}^{PR} = a_{i}^{PR}\Pi_i + (1 - a_{i}^{PR})R_i \]  

(3)

where \( \Pi_i \) and \( R_i \) are firm \( i \)'s profits and revenues respectively.\(^5\) \( a_{i}^{PR} \) is the managerial incen-

\(^3\)Note that under this demand function, higher product differentiation (lower \( \gamma \)) leads to higher market size.

\(^4\)In the strategic delegation literature, it is a regular assumption that firms’ owners have all the bargaining power during negotiations with their managers, i.e., they offer to their managers “take-it-or-leave-it” incentive contracts (see Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987; and Miller and Pazgal, 2001; 2002; 2005, Jansen et al, 2007; 2009, Ritz, 2008). The only exception is van Witteloostuijn et al. (2007), where the owner and (candidate) manager bargain over the managerial incentive parameter, so as the owner to maximize profits and the manager to optimize bonus.

\(^5\)Following Fershtman and Judd (1987), \( U_{i}^{PR} \) will not be the manager’s reward in general. Since the manager’s reward is linear in profits and sales, he is paid \( A_i + B_i U_{i}^{PR} \) for some constants \( A_i, B_i \), with \( B_i > 0 \). Since he is
tive parameter that is chosen optimally by firm $i$’s owner so as to maximize his profits. We assume that $a_{i}^{TR} \in [0,1]$. Observe that if $a_{i}^{TR} = 1$, manager $i$’s behavior coincides with owner $i$’s objective for strict profit-maximization. If $a_{i}^{TR} < 1$, firm $i$’s manager moves away from strict profit-maximization towards including consideration of sales and thus, he becomes a more aggressive seller in the market.

The second type of contract is the Relative Performance ($RP$) one. Following Miller and Pazgal (2001; 2002; 2005), under this type of contract, firm $i$’s owner compensates his manager putting unit weight on own profits and a weight $-a_{i}^{RP}$ on rival’s profits. Thus, manager $i$’s utility function takes the form:

$$U_{i}^{RP} = \Pi_{i} - a_{i}^{RP}\Pi_{j}$$  (4)

As in Jansen et al. (2007; 2009) we further assume that $a_{i}^{RP} \in [0,1]$. Hence, we do not allow owners to direct their managers towards collusion. If $a_{i}^{RP} = 0$, manager $i$’s behavior coincides with owner $i$’s objective for strict profit-maximization. As $a_{i}^{RP} \to 1$, manager $i$ becomes a more aggressive seller in the market.

The third type of contract is the Market Share ($M$) one. As in Jansen et al. (2007; 2009) and Ritz (2008), under this type of contract, firm $i$’s owner compensates his manager with a contract constituted by a linear combination of own profits and own market share. In this case, manager $i$’s utility function takes the form:

$$U_{i}^{M} = \Pi_{i} + a_{i}^{M}\frac{q_{i}}{q_{i} + q_{j}}$$  (5)

with $a_{i}^{M}$ being the respective managerial incentive parameter optimally chosen by owner $i$ in order to maximize his profits.

In order to examine which types of managerial incentive contracts will firms’ owners choose to compensate their managers, we consider a three-staged game with observable actions: in the first stage, each firm’s owner commits to one among the three different types of contracts. Then, in

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6 In Miller and Pazgal (2002), owner $i$ compensates his manager putting weight of $(1 - a_{i}^{RP})$ on own profits and a weight $a_{i}^{RP}$ on the difference between own profits and the rival firm’s profits, implying that $U_{i}^{RP} = (1 - a_{i}^{RP})\Pi_{i} + a_{i}^{RP}(\Pi_{i} - \Pi_{j})$. This is equivalent to eq. (4).
the second stage of the game, given that the types of contracts have become common knowledge and can not be reset, each owner sets the corresponding managerial incentive parameter \( a_i^D, D : PR, RP, M \). In the third stage of the game, managers compete \( a \) la Cournot. The equilibrium concept employed is the subgame perfect equilibrium.

3 Equilibrium managerial incentive contracts under commitment

In this part of the paper we consider that firms’ owners commit over the types of contracts that they choose to compensate their managers. The payoffs of the different subgames are presented in the following Payoff Matrix.

<table>
<thead>
<tr>
<th></th>
<th>( PR )</th>
<th>( RP )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PR )</td>
<td>( \Pi_1^{pr}, \Pi_2^{pr} )</td>
<td>( \Pi_1^{pr-rp}, \Pi_2^{pr-rp} )</td>
<td>( \Pi_1^{pr-m}, \Pi_2^{pr-m} )</td>
</tr>
<tr>
<td>( RP )</td>
<td>( \Pi_1^{rp-pr}, \Pi_2^{rp-pr} )</td>
<td>( \Pi_1^{rp}, \Pi_2^{rp} )</td>
<td>( \Pi_1^{rp-m}, \Pi_2^{rp-m} )</td>
</tr>
<tr>
<td>( M )</td>
<td>( \Pi_1^{m-pr}, \Pi_2^{m-pr} )</td>
<td>( \Pi_1^{m-rp}, \Pi_2^{m-rp} )</td>
<td>( \Pi_1^{m}, \Pi_2^{m} )</td>
</tr>
</tbody>
</table>

Due to symmetry, the number of candidate equilibria is reduced to six, namely: Symmetric Profits-Revenues Contracts (pr), Symmetric Relative Performance Contracts (rp), Symmetric Market Share Contracts (m), Coexistence of Relative Performance and Profits-Revenues Contracts (rp-pr), Coexistence of Relative Performance and Market Share Contracts (rp-m),

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7 At this point, it is useful to bear in mind two alternative interpretations of the game. According to the first one, following Fershtman and Judd (1987) and Sklivas (1987), an owner hires a manager and directs him through an appropriate incentive contract. The alternative interpretation is the one presented by Miller and Pazgal (2002), where, the problem faced by the owner of each firm is to choose the best type of manager among those that are available, while each manager is committed to behaving in a certain manner by virtue of his personality type. More specifically, in Miller and Pazgal (2002), potential managers take on a continuum of attitudes toward relative performance which is captured by their type, \( \varphi \). However, the difference between Fershtman and Judd (1987) and Miller and Pazgal (2002) is only semantic, since owners have all the bargaining power (by assumption) when setting the contracts.
Coexistence of Market Share and Profits-Revenues Contracts (m-pr).\textsuperscript{8,9}

Let us begin our analysis by investigating the effects of the different contract types on managers’ behavior in the output competition stage of the game. The reaction curve of a PR-compensated manager is given by $q_{iPR}(q_j) = \frac{1}{2}(A - \gamma q_j - a_{iPR}c)$. This implies that manager $i$ considers $a_{iPR}c$ as the marginal cost of production. For $a_{iPR} \in (0, 1]$, this marginal cost is lower than that considered by the owner himself in the benchmark case of No-delegation. Thus, the lower the managerial incentive parameter set by owner $i$, the higher the aggressiveness of his manager and the higher the output level that the latter sets. Note also that the slope of manager $i$’s reaction curve is $\frac{dq_{iPR}}{dq_j} = -\frac{\gamma}{2} - a_{iPR}$. Hence, as $\gamma \to 1$ and products become closer substitutes, manager $i$’s best response to manager $j$ decreases. Moreover, $\frac{dq_{iPR}}{dq_j} = \frac{dq_{iC}}{dq_j}$ suggests that the PR-compensated manager’s reaction curve is an outward and parallel shift of the respective curve in the benchmark case of No-delegation.

Regarding an RP-compensated manager, his reaction curve in the last stage of the game is given by $q_{iRP}(q_j) = \frac{1}{2} \left[A - c - \gamma (1 - a_{iRP}) q_j\right]$, implying that manager $i$ considers $-\gamma (1 - a_{iRP}) q_j$ as the rival manager’s best response. Since $a_{iRP} \in [0, 1]$, $-\gamma (1 - a_{iRP}) q_j \leq -\gamma q_j^C$ suggests that the rival’s best response that an RP-compensated manager anticipates is lower than that anticipated by an owner in case of No-delegation. An immediate consequence is that an RP-compensated manager sets output at a level higher than that set by an owner. Regarding the slope of manager $i$’s reaction curve, it is given by $\frac{dq_{iRP}}{dq_j} = -\frac{1}{2} \gamma (1 - a_{iRP})$, suggesting that as $\gamma \to 1$ and products become closer substitutes, manager $i$’s best response to manager $j$ decreases. Diagrammatically, the manager’s reaction curve is the benchmark’s one rotated through the intercept.

Finally, the reaction curve for an M-compensated manager is given by $\frac{\partial U_M}{dq_i} = A - c - 2q_iM - \gamma q_j + \frac{aMq_i}{(q_iM + q_j)} = 0$. This reaction curve is a third-degree, differentiable and concave

\textsuperscript{8}The first four candidate equilibria (pr, rp, m, rp-pr) were solved analytically for obtaining managerial incentive parameters, quantities and profits. See Appendix A1-A4. The respective results for the last two candidate equilibria (rp-m, m-pr) were obtained through numerical simulations. See Appendix A5-A6. Further details are available from the authors upon request.

\textsuperscript{9}As a benchmark, we consider the “No-delegation” case where production decisions are taken by firms’ owners. In this case, the reaction function in the output competition stage is $q_i^C = (A - c - \gamma q_j^C)/2$ while equilibrium output, profits and total welfare are $q_i^C = (A - c)/(2 + \gamma)$, $\Pi_i^C = (q_i^C)^2$ and $TW^C = (3 + \gamma)(A - c)^2/(2 + \gamma)^2$ respectively.
function. Diagrammatically, this reaction curve is (slightly) hill-shaped and after a relatively short interval, it turns negatively sloped (Jansen et al., 2007).

Let us now investigate the impact of product differentiation on output levels and profits under the different candidate equilibrium configurations of contracts. The following Proposition summarizes:

**Proposition 1** (i) When both owners compensate their managers either with Profits-Revenues or with Relative Performance contracts, firm i’s output level and profits decrease in $\gamma$.

(ii) When both owners compensate their managers with Market Share contracts, firm i’s output level (profits) increases (decrease) in $\gamma$.

(iii) In the asymmetric configurations of contracts (rp-pr, rp-m, m-pr), firm i’s output level and profits have a U-shaped relation in $\gamma$, with the minimum attained around $\gamma = 0.8$, and firm j’s output and profits decrease in $\gamma$.

According to the first part of Proposition 1, the intuition goes as follows. Recall that the higher the degree of product differentiation (lower $\gamma$), the higher the size of the market and the respective market segment that corresponds to each firm. As $\gamma$ increases, the brands sold become closer substitutes and as a result, the size of the market and the segment that each firm exploits decrease. Hence, as $\gamma \to 1$, this *negative product differentiation effect* becomes more severe and the output level set by manager i decreases too. An immediate consequence is that firm i’s profits also decrease in $\gamma$.

In the symmetric configuration of M contracts, the degree of product substitutability has a twofold impact on output levels. On the one hand, as $\gamma \to 1$ the aforementioned negative product differentiation effect tends to decrease the output level set by each manager. On the other hand, as $\gamma$ increases and products become closer substitutes, competition among managers for gaining higher market share becomes fiercer. Hence, each manager tends to increase output. The latter *positive competition effect* dominates the negative product differentiation effect and thus, as $\gamma \to 1$ the output level set by manager i increases. In turn, this overproduction tends to increase revenues and profits but it decreases prices that subsequently tend to decrease profits. This latter *negative price effect* dominates the *positive output effect* and firm i’s profits decrease in $\gamma$. 
Regarding the asymmetric configurations of contracts, the intuition behind our result goes as follows. Note first that \( q_{i}^{r-p-pr} > q_{j}^{r-p-pr}, q_{i}^{r-p-m} > q_{j}^{r-p-m} \) and \( q_{i}^{m-pr} > q_{j}^{m-pr} \) always hold. The respective inequalities for profits hold too. These inequalities underline the relative competitive advantage (in terms output expansion and profits) between managers compensated with different types of contracts. Hence, Proposition 1(iii) suggests that the output level set by manager \( i \), who exploits the relative competitive advantage against his rival, has a U-shaped relation in \( \gamma \) with the minimum attained around \( \gamma = 0.8 \). Intuitively, for sufficiently differentiated products (low \( \gamma \)), the output level set by both managers decreases in \( \gamma \) because of the negative product differentiation effect. But, as \( \gamma \) increases and products become closer substitutes, manager \( i \) increases the output level in order to exploit the competitive advantage that his contract gives him. Clearly, firms' profits follow the same pattern as the output levels in each configuration of contracts.

Let us now compare the equilibrium output levels under the different configurations of contracts, in order to capture their relative competitiveness. The following Corollary summarizes our findings:

**Corollary 1**

(i) \( q_{i}^{r-pr} > q_{i}^{r-p} > q_{i}^{c} \).

(ii) \( q_{i}^{m} > q_{i}^{c} \), if and only if \( \gamma > 0.666 \).

(iii) \( q_{i}^{r-pr} > q_{i}^{m} \).

(iv) \( q_{i}^{m} > q_{i}^{r-p} \), if and only if \( \gamma > 0.881 \).

(v) In each asymmetric configuration of contracts (\( r-p-pr, r-p-m, m-pr \)), each firm's output level lies between the respective levels in the symmetric configurations.

The following observations are in order. First, the symmetric use of either PR or RP managerial incentive contracts increases output, as compared to the benchmark of No-delegation. The intuition goes as follows. Owner \( i \), by compensating his manager with an incentive contract directs him to a more aggressive behavior in order to force the rival manager to reduce output. Because each owner acts in the same way at the game’s contract stage, firms end up in an overproduction situation.\(^{10}\) Note also that this overproduction increases as products become

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\(^{10}\)These findings are in the spirit of the well-known result in the industrial organization literature, according to which, firms competing in quantities have no incentive to engage in Stackelberg warfare.
closer substitutes (higher $\gamma$) and competition becomes fiercer, i.e. $\frac{d(q'' - q'_i)}{d\gamma} > 0$, $\frac{d(q'' - q'_i)}{d\gamma} > 0$, and $\frac{d(q''_i - q''_i)}{d\gamma} > 0$. Second, the symmetric use of M contracts results in overproduction, as compared to the benchmark of No-delegation, but only if products are sufficiently homogeneous, i.e. $\gamma > 0.666$, and competition among managers for gaining market share is too fierce. Third, regarding the relative severity of the overproduction situation, we find that it is the most fierce in case of PR contracts, while it is more intense under M contracts rather than under RP contracts whenever products are highly substitutable, i.e. $\gamma > 0.881$, and competition is too fierce.

We now turn to the first stage of the game and investigate the types of contracts that owners will choose to compensate their managers. The following Proposition summarizes:

**Proposition 2** When firms’ owners commit over the types of contracts that they choose to compensate their managers:

(i) When products are either sufficiently differentiated ($\gamma \leq 0.242$) or sufficiently close substitutes ($\gamma \geq 0.881$), owners compensate their managers with Relative Performance contracts.

(ii) For intermediate degrees of product substitutability ($0.243 < \gamma < 0.881$), owners compensate their managers with Market Share contracts.

The intuition behind these results goes as follows: When $\gamma < 0.881$, it holds that $q''_i > q'_i$ (Corollary 1(iv)), which in turn implies that $p''_i < p'_i$. This quantity (price) effect suggests that profits under RP-compensated managers tend to be higher (lower) than the respective under M-compensated managers. It proves that for sufficiently differentiated products, i.e. $\gamma \leq 0.242$, the quantity effect on profits is stronger under RP contracts, rather than under M contracts. Hence, owners compensate their managers with RP contracts. For intermediate degrees of product substitutability, i.e. $0.243 < \gamma < 0.881$, it is the price effect that is stronger under RP contracts, rather than under M contracts, and owners choose to compensate their managers with M contracts.

When products are sufficiently close substitutes, i.e. $\gamma \geq 0.881$, it holds that $q''_i > q''_i$. In this case, firms’ owners realize that compensating their managers with M contracts would result in a relatively more severe prisoners’ dilemma situation, characterized by relatively higher output and lower profits, than the respective under RP contracts. In order to avoid this too fierce prisoners’ dilemma situation, owners choose to compensate their managers with RP contracts.
Note also that owners will never choose to compensate their managers with PR contracts. This happens because these contracts result in the most severe prisoners’ dilemma situation (Corollary 1(i), (iii)).

Last, but not least, we find that for each \( \gamma \)-area stated above, no owner has incentives to deviate from the respective symmetric equilibrium configuration of contracts.\(^{11}\) This suggests that given owner \( i \)'s choice, over the type of contract to compensate his manager, owner \( j \)'s best response is a contract of the same type.

4 Equilibrium managerial incentive contracts under no-commitment

So far analysis, as well as the bulk of the received literature in the field of strategic managerial incentive contracts, considers that firms’ owners commit over the types of contracts that they choose to compensate their managers. In this part of the paper we investigate the case where there is no such ex-ante commitment.

We do so by considering a two-staged game with the following timing: in the first stage, each firm’s owner chooses one type of contract to compensate his manager and sets the corresponding managerial incentive parameter. The crucial, yet (due to the symmetric industry) reasonable assumption here is that the precise contract (the type of contract and the managerial incentive parameter) that owner \( i \) sets is not observable by the rival owner, before contract-setting is everywhere completed.\(^{12}\) This implies that each owner can independently shift from one type of contract to another. In the second stage of the game, managers compete à la Cournot. Thus, we propose a configuration of contracts, as a candidate equilibrium, and subsequently check whether or not it survives all possible deviations, at the first stage. If yes, the proposed equilibrium is a sub-game perfect Nash equilibrium.

Let us consider the Symmetric Profits-Revenues Contracts, as a candidate equilibrium. Figure 1 offers the visualization of this candidate equilibrium (point \( E^{PR} \)). Note that in equilibrium,

\(^{11}\)The detailed proof is available from the authors upon request.

\(^{12}\)A crucial assumption of the relevant literature is that delegation is observable. Katz (1991) argues that unobservable contracts have no commitment value at all. Fershtman and Judd (1987) support that even if contracts are not observable, they will become common knowledge when the game is being repeated for several periods. More recently, Kockesen and Ok (2004) argue that to the extent that renegotiation is costly and/or limited, in a general class of economic settings, strategic aspects of delegation may play an important role in contract design, even if the contracts are completely unobservable.
two conditions must be fulfilled: firstly, since production decisions are taken by managers, their reaction curves \( RC_{1}^{PR} \) and \( RC_{2}^{PR} \) must be intersected. Secondly, the fact that firm \( i \)'s owner offers an incentive contract to his manager, as a strategic tool in order to become the Stackelberg leader against firm \( j \), implies that in equilibrium there must be tangency between firm \( i \)'s isoprotit curve \( \Pi^{i} \) and manager \( j \)'s reaction curve \( RC_{j}^{PR} \).

**Symmetric Profits-Revenues Contracts** is an equilibrium configuration only if no owner has an incentive to unilaterally deviate, in the first stage of the game, by offering to his manager either an RP or an M contract. Of course, such a deviation has to be profitable for the owner. Suppose, for instance, that owner 2 sticks to the PR contract, believing that owner 1 will compensate his manager with the same type of contract. Thus, in the first stage of the game owner 2 sets \( a_{2}^{pr} = \frac{\gamma^{2} - 2c(2+\gamma)}{c(\gamma - 2\gamma - 4)} \). Consider now that owner 1 deviates towards compensating his manager with an RP contract. In this case, owner 1 uses his stage 1 reaction function \( a_{1}^{RP-pr}(a_{2}^{pr}) \) to optimally adjust \( a_{1} \) for his manager’s contract. Thus, in the first stage of the game owner 1 sets \( a_{1d}^{pr} = \frac{\gamma^{2} - 2c(2+\gamma)}{c(\gamma^{2} - 2\gamma - 4)} \). Observe that \( a_{2}^{pr} = a_{1d}^{RP-pr} \). Interestingly enough, the deviant owner 1’s profits will be \( \Pi_{1d}^{pr} = \Pi_{1}^{pr} \), implying that the magnitude of owner 1’s incentive to deviate from the **Symmetric Profits-Revenues** configuration towards compensating his manager with an RP contract, is zero.

Diagrammatically, when owner 1 deviates from the **Symmetric Profits-Revenues** configuration towards compensating his manager with an RP contract, owner 1 directs his manager to the reaction curve \( RC_{1}^{RP} \) and, by readjusting the managerial incentive parameter, optimally readjusts its slope until \( RC_{1}^{RP*} \). However, the manager of the deviant owner sets output at a level equal to the one he would set is the **Symmetric Profits-Revenues** Contracts candidate equilibrium. Intuitively, this is the unique output level that fulfills the equilibrium conditions stated above. The following Proposition summarizes:

**Proposition 3** When firms’ owners do not commit over the types of contracts that they choose to compensate their managers, each type of contract is owner \( i \)'s best response to owner \( j \)'s choice.
Figure 1: Equilibrium managerial incentive contracts under no-commitment

Observe that the two-staged game is characterized by multiplicity of equilibria. Subsequently, the following question arises: which types of the managerial incentive contracts will finally emerge in equilibrium? Using the equilibrium results of Section 3 and employing focal point analysis, we reinforce our arguments stated in Proposition 2.

Note also that the aforementioned equilibrium conditions must hold for all the different types of contracts that owners can offer to their managers, regardless the functional forms of cost and demand and the mode of competition, i.e. Cournot or Bertrand. Thus, assuming no ex-ante commitment over the types of contracts that owners will offer to their managers, even a contract of different type could be an owner’s best response to the rival owner’s choice.

5 Welfare analysis

In this Section we perform a welfare analysis. Social welfare is defined as the sum of consumers’ surplus and firms’ profits:
\[ SW^w = CS^w + \Pi^w, \ w = pr, rp, m, rp - pr, rp - m, m - pr \]  
\[ CS^w = \frac{1 + \gamma}{4} (Q^w)^2 \]

where \( Q^w \) and \( \Pi^w \) is the total industry output and profits respectively.\(^{14}\)

Regarding the symmetric configurations of contracts, one can easily check that \( SW^w > SW^c, \ w = pr, rp, m, \) except if managers are compensated with M contracts and \( \gamma < 0.666 \). The intuition is straightforward. When both managers are compensated either with PR or with RP contracts, consumers’ surplus is higher and firms’ profits are lower than the respective in the benchmark case of No-delegation. Nevertheless, the negative firms’ profits effect is always dominated by the positive consumers’ surplus effect. As a consequence, the symmetric use of either PR or RP contracts is always preferable from the social welfare point of view. When both managers are compensated with M contracts, consumers’ surplus is higher than the respective in the benchmark, if and only if \( \gamma > 0.666 \). On the contrary, firms’ profits are higher but only for intermediate degrees of product substitutability, i.e. \( 0.253 < \gamma < 0.666 \). We reconfirm in this case too that the welfare effects of the symmetric use of M contracts are driven by the consumers’ surplus effect.

Clearly, social welfare in each configuration of contracts follows the pattern that output levels follow. The following Proposition summarizes:

**Proposition 4** (i) \( SW^{pr} > SW^{rp} > SW^c \).

(ii) \( SW^m > SW^c \), if and only if \( \gamma > 0.666 \).

(iii) \( SW^{pr} > SW^m \).

(iv) \( SW^m > SW^{rp} \), if and only if \( \gamma > 0.881 \).

(v) In each asymmetric configuration of contracts (rp-pr, rp-m, m-pr), social welfare lies between the respective levels in the symmetric configurations.

\(^{14}\)Substituting the relevant expressions into eq. (6), we obtain social welfare in the six contract configurations under consideration. More specifically, social welfare for the first four configurations of contracts is given in Appendix B. Regarding the last two configurations, social welfare was obtained numerically, through the simulations’ results, concerning output, presented in Tables 1 and 2 respectively. Further details are available from the authors upon request.
The above Proposition replicates our findings, regarding the comparison of the equilibrium output levels under the different configurations of contracts (see Corollary 1). An immediate consequence is that social welfare in the asymmetric configurations lies between the respective levels in the symmetric configurations (Proposition 4 (v)). We also find that social welfare decreases in the degree of product substitutability. This happens because of the negative product differentiation effect that decreases firms’ profits always; and consumers’ surplus, except if managers are compensated with M contracts and $\gamma < 0.666$.

6 Conclusion

This paper extends the analysis of Jansen et al. (2009) by assuming that the two competing firms produce differentiated instead of homogeneous products. In this context, we have investigated the impact of product substitutability on the types of contracts that firms’ owners choose to compensate their managers, as well as on the resulting output levels, profits and social welfare.

We have identified the differential effects of product substitutability on the output levels set by managers and the resulting firms’ profits, depending on the configuration of contracts in the industry. We have also shown that the types of contracts that owners choose to compensate their managers depend crucially on the degree of product substitutability. In particular, if products are either sufficiently differentiated or sufficiently close substitutes, owners compensate their managers with Relative Performance contracts. For intermediate degrees of product substitutability, owners choose Market Share contracts. When owners do not commit over the types of contracts, each type of contract can be an owner’s best response to the rival owner’s choice. Finally, managerial incentive contracts are welfare enhancing but only when they increase consumers’ surplus.

Our results have been derived in the context of a duopolistic market where competing firms, with equally efficient production technologies, produce differentiated products under a linear demand system. We are of the opinion that a duopolistic market reveals all the essential differences between the different types of contracts. This argument could also be supported by the similarity of findings between a two- and a three-firm industry, in Jansen et al. (2009). We are also aware of the limitations of our analysis, assuming specific functional forms. However, the
equilibrium conditions that drive our results allow us to argue that these results will also hold under general demand and cost functions. The use of more general forms would jeopardize the clarity of our findings, without significantly changing their qualitative character.

Appendix

Appendix A: Equilibrium outcomes for the different configurations of contracts

A1: Symmetric Profits-Revenues Contracts

\[ a_i^{pr} = \frac{\gamma^2 - 2c(2 + \gamma)}{c(\gamma^2 - 2\gamma - 4)}; \quad q_i^{pr} = \frac{2(1 - c)}{4 + 2\gamma - \gamma^2}; \quad \Pi_i^{pr} = \frac{2(2 - \gamma^2)(1 - c)^2}{(\gamma^2 - 2\gamma - 4)^2} \]

A2: Symmetric Relative Performance Contracts

\[ a_i^{rp} = \frac{\gamma}{2 + \gamma}; \quad q_i^{rp} = \frac{(2 + \gamma)(1 - c)}{4(1 + \gamma)}; \quad \Pi_i^{rp} = \frac{(4 - \gamma^2)(1 - c)^2}{16(1 + \gamma)} \]

A3: Symmetric Market Share Contracts

\[ a_i^{m} = \frac{\left[ \frac{2(\sqrt{B} - 17) + \gamma\left(7\sqrt{B} + 19\right) + \gamma\left(\sqrt{B} + 10\right)}{2(6 + \gamma)^2} \right](1 - c)^2}{2(6 + \gamma)^2} \]
\[ B = 1 + \gamma(\gamma + 6) \]
\[ q_i^{m} = \frac{\left(\sqrt{B} + \gamma + 7\right)(1 - c)}{4(6 + \gamma)} \]
\[ \Pi_i^{m} = \frac{\left(\sqrt{B} + \gamma + 7\right)\left[17 - \sqrt{B} - \gamma\left(\sqrt{B} + 4\right)\right](1 - c)^2}{16(6 + \gamma)^2} \]

A4: Coexistence of Relative Performance and Profits-Revenues Contracts

\[ a_1^{rp-pr} = \frac{\gamma\left[\gamma(2 + \gamma) - 4\right]}{\gamma\left[\gamma(2 + \gamma) + 4\right] - 8}; \quad a_2^{rp-pr} = \frac{\gamma^2(\gamma - 1) - c\left[\gamma^2(2 + \gamma) - 4\right]}{c(3\gamma^2 - 4)} \]
\[ q_{1}^{rp-pr} = \frac{[\gamma (2 + \gamma) - 4] (1 - c)}{6\gamma^2 - 8}; \quad q_{2}^{rp-pr} = \frac{[\gamma |2 + \gamma) - 4 - 8| (1 - c)}{4(3\gamma^2 - 4)} \]

\[ \Pi_{1}^{rp-pr} = \frac{(2 - \gamma^2) [\gamma (2 + \gamma) - 4]^2 (1 - c)^2}{8 (4 - 3\gamma^2)^2}; \quad \Pi_{2}^{rp-pr} = \frac{[\gamma |2 + \gamma) - 4 - 8| (2 - \gamma) (1 - c)^2}{48\gamma^2 - 64} \]
# A5: Coexistence of Relative Performance and Market Share Contracts

Table 1 summarizes the simulation results concerning output for this contract configuration.

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A6: Coexistence of Market Share and Profits-Revenues Contracts

Table 2 summarizes the simulation results concerning output for this contract configuration.

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Appendix B: Total Welfare

\[
TW^{PR} = \frac{4[3 - \gamma (\gamma - 1)](A - c)^2}{[\gamma (\gamma - 2) - 4]^2}
\]

\[
TW^{RP} = \frac{(2 + \gamma)(6 - \gamma)(1 - c)^2}{16(\gamma + 1)}
\]

\[
TW^M = \frac{(7 + \gamma + \sqrt{B})[41 - \sqrt{B} - \gamma(\gamma + \sqrt{B})]}{16(\gamma + 6)^2}(1 - c)^2, \quad B = 1 + \gamma(\gamma + 6)
\]

\[
TW^{(rp-pr)} = \frac{[768 + \gamma[-512 + \gamma][-704 + \gamma][352 + \gamma(\gamma - 8)][\gamma(\gamma - 3) - 16]](1 - c)^2}{64(4 - 3\gamma^2)^2}
\]

References


