

# **Measurement of Consumption efficiency in Price-Quantity Space: A Distance Function Approach**

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## Abstract

*In standard consumer demand analysis, it is implicitly assumed that consumers behave optimally and, thus, efficiently. However, optimality is a restrictive assumption to make for consumers' actual behaviour. This study moves away from this restrictive assumption and develops a theoretical model for the analysis of consumer's inefficiency in price-quantity space. The consumption efficiency measures which are developed allow consumer's efficiency to be studied not only in terms of budget that is wasted (i.e., as in the past attempts to study consumption efficiency in price-quantity space), but also in terms of quantities that are wasted. As regards to the empirical measurement of consumer's efficiency, an approach is proposed under which estimation of a distance function representing consumer's preferences is carried out via treatment of the unobserved utility level as a random error term.*

## 1. Introduction

Standard consumer demand analysis assumes *a priori* that consumers always behave optimally, that is, they do succeed in obtaining maximum utility from given purchased commodities, or they do succeed in choosing the minimum quantities required for the achievement of a utility level. However, optimality is a restrictive assumption to make for consumers' actual behaviour. As Afriat (1988) points out, "The ordinary theory of the consumer is based on utility – and unquestioned efficiency. Even when the utility is granted, perfect efficiency seems an extravagant requirement. The familiar volatilities of real consumers make such intolerance unsuitable." (p. 252). It is then more reasonable to assume that consumers may not behave optimally and employ theoretical and empirical models that accommodate any departure for optimality, and, hence, inefficiency, and allow it to be measured.

The importance of studying inefficiency in consumption lies not only on the fact that optimal behaviour, and hence, efficiency, is a restrictive assumption to make for consumers' actual behaviour. It also lies on the fact that consumer's non-optimal

behaviour has a negative impact on welfare levels. In particular, it has a negative impact on consumer's welfare levels in terms of budget that was wasted and which could have been allocated to the satisfaction of other wants. In addition, over-consumption leads to increased and more industrialised production, which itself fuels over-consumption, through, say, advertising. This circle implies excessive use of natural resources and/or wrong allocation of them in the production of commodities, increased waste from both consumption and production, and a negative impact on social welfare.

The assumption of consumer's non-optimal behaviour can be accommodated in the case of commodities, such as highly perishable foods, meat, fish and agricultural products. In such cases, consumers may be inefficient because they are making rough estimates of the volume of the commodities and the quantity combination of them that are enough for the achievement of some desired utility level: when consumers choose a commodity bundle, they choose it on the basis of their estimates of what commodity combination is the suitable one for their wants. Consumers may also be inefficient because they cannot predict the future exactly: since individuals' every day lives cannot be programmed to the detail, it is not unexpected that a portion of the purchased quantities of the commodities are not consumed but – in the case of highly perishable foods that cannot be stored – are disposed of instead. Or it could be lack of information, awareness and responsibility from the part of consumers with respect to the full social costs of their consumption decisions that lead to excess purchases and spending, and consumption inefficiency. Thus, consumers may purchase a commodity bundle which is non-optimal: they could have bought less of all the commodity quantities (*commodity inefficiency*), thus reducing expenditures, and/or they could have re-allocated their expenditures by choosing a different quantity mix (*allocative inefficiency*), thus reducing expenditures even more.

In this context, the aim of this paper is to propose a theoretical framework for the analysis of consumer's efficiency in price-quantity space. The theoretical model which is developed is based on the simple observation that consumer preferences are commonly defined over the consumption levels and no distinction is being made between the quantities of the commodities purchased and the consumption levels themselves, that is, it is implicitly assumed that the purchased quantities and the consumed quantities are the same. However, if it is assumed that consumers are free to dispose of any unwanted quantities of the commodities they have purchased, then it

becomes possible to define a measure of efficiency of the consumers in their effort to minimise expenditure for commodities. Past attempts to study consumption efficiency in price-quantity space have been based on revealed preference relations or money-metric utility functions in order to construct non-parametric or parametric efficiency indices (Afriat, 1967, 1988; Varian 1982, 1983, 1985, 1990).<sup>1</sup> The focus of these studies, however, is on the examination of the goodness-of-fit of optimising models to actual data by measuring the departure from optimisation. Moreover, what is implied by these efficiency measures is that inefficiency occurs because a portion of the consumers' budget is wasted, and not a portion of the purchased quantities. However, it is this latter assumption that allows the construction of a measure of what we define here as commodity efficiency. Finally, since these models do not allow for the possibility that an observed commodity bundle may also be commodity inefficient, no distinction is being made between what we define as allocative efficiency and *expenditure* (or *overall*) *efficiency*. As a result, the efficiency score that these models assign to consumers may be higher than it should.<sup>2</sup>

Our analysis is carried out under the consumer's expenditure-minimisation framework, and the starting point is the assumption that the consumer's objective is to choose a feasible commodity vector in order to achieve a desirable utility level. Assuming also that the consumer need not make use of all the quantities of the purchased commodities and may dispose of any unwanted quantities of them, the quantities of the purchased commodities may well be higher than the ones required to just attain the desirable utility level, and the consumer may well have chosen an inefficient way of attaining this utility level. This type of efficiency is what we are going to define as commodity efficiency. Another type of efficiency is what we call expenditure, or overall, efficiency, and which we describe as the consumer's ability to avoid wasting expenditures, by minimising the cost of purchased commodities in the achievement of a utility level. A third type of efficiency is allocative efficiency: allocative efficiency is concerned with how close an observed commodity vector is to the expenditure-minimising commodity vector on the same indifference curve. Finally, we show the relation between the three types of efficiency, *i.e.*, the decomposition of expenditure efficiency into commodity efficiency and allocative efficiency.

Econometric estimation of our theoretical model calls for establishment of an appropriate empirical framework which will accommodate consumers' non-optimal

behaviour. In particular, the index which is proposed for the measurement of commodity efficiency is based on a distance function representation of consumer preferences. Computation of the commodity efficiency index requires knowledge of the value of the distance function, which can be acquired through econometric estimation of the latter. However, the difficulty in estimation of a distance function representation of consumer preferences lies on that it is a function, not only of observed commodity quantities, but also of consumer's utility level which is unobserved. In order to illustrate how this knowledge can be acquired, we estimated a translog distance function with a panel data set of British household purchases of milk & yoghurt, fruits, and vegetables. The methodology that is adopted for estimation of the translog distance function lies on treating consumer's unobserved utility level as a random error term. Specifically, treatment of the terms associated with the utility level and the distance as one-sided positive error terms gives rise to a density for the composite error term which resembles the two-tiered frontier estimation framework by Polachek and Yoon (1987, 1996). The estimated distance function can then be used as an index to measure commodity inefficiency.<sup>3</sup> As far as calculation of the measure of allocative efficiency is concerned, knowledge of either the expenditure function or the expenditure-minimising commodity vector is required; standard procedures employed in production efficiency analysis for computing the index of allocative efficiency are employed for computation of the measure of allocative efficiency in consumption. Finally, the measure of expenditure efficiency can be computed with the use of the proposed relation for the decomposition of expenditure efficiency into commodity and allocative efficiency.

The rest of the paper is organised follows. The next section provides a detailed presentation of the proposed theoretical model: the notions of commodity, allocative, and expenditure efficiency are described, measures for these types of inefficiency are derived, and the decomposition of expenditure efficiency into commodity and allocative efficiency is also illustrated. The empirical methodology that is employed for the estimation of the proposed measures of consumption efficiency is presented in Section 3. Sections 4 and 5 provide a description of the data, and an analysis of the empirical results, respectively. Finally, the last section summarises and concludes.

## 2. Theoretical Framework

Standard consumer theory assumes that consumer's preferences satisfy a number a number of properties (*axioms of choice*). Specifically, consumer's preferences are assumed to be reflexive, complete, transitive, and continuous, so that a continuous utility function exists that represents these preferences, and also that they are non-satiated (or, strongly monotone or, weakly monotone), and (strictly) convex. However, before proceeding to the analysis of measurement of consumer inefficiency, expressions for the non-satiation axiom of choice – alternative to the ones commonly used in consumer theory – are needed. Firstly, let the consumption space be represented by the non-negative Euclidean  $N$ -orthant, that is,  $Q^N = \{\mathbf{q} \in \mathbb{R}^N : \mathbf{q} > \mathbf{0}^N\}$ , where  $\mathbf{q} = (q_1, \dots, q_N)$  is a vector of commodity quantities,  $\mathbb{R}^N$  is Euclidean  $N$ -space, and  $\mathbf{0}^N$  is a  $N$ -dimensional zero vector.<sup>4,5</sup> Also, let the *consumption requirement set* be the set of commodity vectors which are feasible for each utility level  $u$ , that is,  $L(u) = \{\mathbf{q} \in Q^N : U(\mathbf{q}) \geq u\}$ , where  $U(\mathbf{q})$  is the direct utility function.  $L(u)$  is assumed to be a closed, convex and continuous set, and it is also assumed to satisfy the properties of *strong*, or *weak*, *non-satiation*, defined as (Russell, 1998):

- (i) *Strong Non-Satiation*: for every  $\mathbf{q}, \mathbf{q}' \in Q^N$ , if  $\mathbf{q} \geq \mathbf{q}'$  and  $\mathbf{q}' \in L(u)$ , then  $\mathbf{q} \in L(u)$ ,
- (ii) *Weak Non-Satiation*: for all  $\mathbf{q} \in Q^N$ , if  $\mathbf{q} \in L(u)$ , then  $\lambda \mathbf{q} \in L(u)$  for  $\lambda \geq 1$ .

Defined in terms of the consumption requirement set, the definitions of strong and weak non-satiation state explicitly what is implicit in the usual definitions of strong and weak monotonicity:<sup>6</sup> if a vector  $\mathbf{q}$  can generate utility  $u$ , then so can a vector with more of at least one commodity (or more of all commodities) than  $\mathbf{q}$ . Put this way, strong and weak non-satiation imply that the consumer can freely dispose of any spare amount of commodities. As a result, room is left for consumer inefficiency to be defined.

Having provided alternative representations of the non-satiation axiom of choice, we may proceed to the analysis of consumer inefficiency. As mentioned in the introductory section, if we make the assumption that the consumer need not make use

of all the quantities of the purchased commodities and may dispose of any unwanted quantities of them, then the quantities of the purchased commodities may well be higher than the ones required to just attain a desirable utility level  $u$ , and the consumer may well have chosen an inefficient way of attaining  $u$ . We will use the term *commodity efficiency* in order to describe the consumer's ability to avoid wasting any quantities of the purchased commodities, by minimising quantity purchases in the achievement of a target utility level. The extent of such inefficiency can be measured by the following Debreu-Farrell-like (Debreu, 1951; Farrell, 1957) measure

$$CE(u, \mathbf{q}) = \min \{ \zeta : \zeta \mathbf{q} \in L(u) \} . \quad (1)$$

The proposed measure of commodity efficiency calls a reference commodity vector commodity efficient if, when radially contracted, it no longer attains the given utility level  $u$ . Let the indifference curve associated with utility level  $u$  be defined as  $I(u) = \{ \mathbf{q} \in L(u) : \lambda \mathbf{q} \notin L(u) \forall \lambda < 1 \}$ , that is, the set of those feasible commodity vectors which, when scaled down along a ray radiating from the origin, they become incapable of generating utility  $u$ . It is obvious then that the measure of commodity efficiency calls a reference commodity vector commodity efficient if it is an element of the indifference curve associated with the utility level  $u$ . However, when preferences do not satisfy strong monotonicity, this measure of efficiency may assign the same efficiency score to different commodity inefficient commodity vectors.<sup>7</sup> In order to provide a stricter standard for measuring commodity efficiency, we will introduce a notion similar to that of input efficient subsets in production theory, the *commodity efficient subsets*, which we define as

$$Eff(u) = \{ \mathbf{q} \in L(u) : \mathbf{q}' < \mathbf{q} \Rightarrow \mathbf{q}' \notin L(u) \} . \quad (2)$$

That is, the commodity efficient subsets are the sets of those feasible commodity vectors which, when scaled down along any ray, they become incapable of generating utility  $u$ . Commodity efficient subsets are subsets of the indifference curves and, as such, they represent stricter benchmarks for measuring commodity efficiency: if a feasible commodity vector is commodity efficient against  $Eff(u)$ , then it is also

commodity efficient against  $I(u)$ , but not *vice versa*. Nevertheless, a non-radial measure of commodity efficiency is required if we are to attribute the property of commodity efficiency only to those commodity vectors that are members of commodity efficient subsets.<sup>8</sup>

The measure of commodity efficiency can also be defined in terms of the distance function. Let  $R(U)$  denote the range of  $U$  with its infimum value excluded. The distance function,  $D : R(U) \times Q^N \rightarrow \mathbb{R}_{++}^1$ , is defined as  $D(u, \mathbf{q}) = \max_{\lambda} \{ \lambda > 0 : \mathbf{q}/\lambda \in L(u) \}$ , where  $\mathbb{R}_{++}^1$  denotes the positive Euclidian orthant.<sup>9</sup> Given the properties of  $L(u)$ , the distance function is jointly continuous in  $(u, \mathbf{q})$ , decreasing in  $u$ , and non-decreasing, homogeneous of degree one and concave in  $\mathbf{q}$ . Using the distance function, the consumption requirement set can also be defined as<sup>10</sup>

$$L(u) = \{ \mathbf{q} \in Q^N : D(u, \mathbf{q}) \geq 1 \}. \quad (3)$$

The assumption of weak non-satiation is required for (3) to be equivalent to the definition of  $L(u)$  as  $L(u) = \{ \mathbf{q} \in Q^N : U(\mathbf{q}) \geq u \}$ , that is, for the distance function to be able to completely characterise the consumption requirement set. The measure of commodity efficiency can now be defined in terms of the distance function as

$$CE(u, \mathbf{q}) = \min \{ \zeta : D(u, \zeta \mathbf{q}) \geq 1 \}. \quad (4)$$

This definition of commodity efficiency shows that there is a close relation between the measure of commodity efficiency and the distance function. In fact, the measure of commodity efficiency is the reciprocal of the distance function. To see this, note that the reciprocal of  $D(\cdot)$  is given by  $(1/D(u, \mathbf{q})) = \min \{ \lambda : \lambda \mathbf{q} \in L(u) \}$ . Then, by (1),

$$CE(u, \mathbf{q}) = 1/D(u, \mathbf{q}). \quad (5)$$



Finally, as revealed by relation (5), the measure of commodity efficiency takes on values in the  $(0,1]$  interval, and, in addition, it satisfies all the properties of radial efficiency indices.<sup>11</sup>

A second type of consumer's efficiency is that of *expenditure* or *overall efficiency*, which we describe as the consumer's ability to avoid wasting expenditures, by minimising the cost of purchased commodities in the achievement of a target utility level. Assuming that consumers face strictly positive commodity prices,  $\mathbf{p} = (p_1, \dots, p_N)$ , and that their objective is to choose that feasible vector of quantities  $\mathbf{q}$  that will minimise the level of total expenditure  $\mathbf{p} \cdot \mathbf{q}$  required to attain a level  $u$ , a measure of expenditure efficiency is given by the ratio of minimum expenditure required for the achievement of  $u$  to actual expenditure, that is,<sup>12</sup>

$$EE(u, \mathbf{q}, \mathbf{p}) = C(u, \mathbf{p}) / (\mathbf{p} \cdot \mathbf{q}), \quad (6)$$

where  $C : R(U) \times \mathbb{R}_{++}^N \rightarrow \mathbb{R}_{++}^1$  is the expenditure function defined by  $C(u, \mathbf{p}) = \min_{\mathbf{q}} \{\mathbf{p} \cdot \mathbf{q} : \mathbf{q} \in L(u)\}$ , or, equivalently,  $C(u, \mathbf{p}) = \min_{\mathbf{q}} \{\mathbf{p} \cdot \mathbf{q} : D(u, \mathbf{q}) \geq 1\}$ .

Given the properties of  $L(u)$ , the expenditure function is jointly continuous in  $(u, \mathbf{p})$ , increasing in  $u$ , and non-decreasing, concave, and positively linearly homogeneous in  $\mathbf{p}$ .

Not all expenditure inefficiency can be attributed to commodity inefficiency, though. This is because even if consumers behave at 100% commodity efficiency they could choose a wrong combination of commodity quantities, given the market prices of the commodities. This type of efficiency is what we call as *allocative efficiency*, and it is concerned with how close a chosen commodity vector  $\mathbf{q} \in I(u)$  is to the expenditure-minimising commodity vector on  $I(u)$ . Allocative efficiency can be measured as the ratio of expenditure efficiency to commodity efficiency, that is,

$$AE(u, \mathbf{q}, \mathbf{p}) = EE(u, \mathbf{q}, \mathbf{p}) / CE(u, \mathbf{q}). \quad (7)$$

Alternatively, the measure of allocative efficiency can be defined as a cost ratio. Recall that  $CE(u, \mathbf{q}) = 1/D(u, \mathbf{q})$ . Multiply and divide  $CE(u, \mathbf{q})$  by actual expenditure,  $\mathbf{p} \cdot \mathbf{q}$ , and then rearrange terms to write  $CE(u, \mathbf{q})$  as

$$CE(u, \mathbf{q}) = \frac{\mathbf{p} \cdot \mathbf{q} / D(u, \mathbf{q})}{\mathbf{p} \cdot \mathbf{q}}, \quad (8)$$

that is, the ratio of the expenditure for the commodity efficient commodity vector to the expenditure for the actual commodity vector. Using (6) and (8), the measure of allocative efficiency can be written as

$$AE(u, \mathbf{q}, \mathbf{p}) = \frac{C(u, \mathbf{p})}{\mathbf{p} \cdot \mathbf{q} / D(u, \mathbf{q})}. \quad (9)$$

Hence, the measure of allocative efficiency is simply the ratio of the minimum expenditure required for attaining  $u$  to the expenditure for the commodity efficient commodity vector.

It is obvious from definition (7) that expenditure efficiency can be decomposed into commodity and allocative efficiency. This decomposition of expenditure efficiency is illustrated in Figure 1 for the case of two commodities. Given commodity prices  $\mathbf{p}^0$ , let  $\mathbf{q}^*$  and  $C(u^*, \mathbf{p}^0) = \mathbf{p}^0 \cdot \mathbf{q}^*$  denote the expenditure-minimising commodity vector and the minimum expenditure required for attaining a target utility level  $u^*$ , respectively. Suppose now that a consumer facing commodity prices  $\mathbf{p}^0$  chooses a commodity bundle which is more than enough to generate  $u^*$ . That is, he/she chooses  $\mathbf{q}^0$ , such that  $\mathbf{q}^0 \in L(u^*)$  and  $\mathbf{q}^0 \notin I(u^*)$ . Suppose also that the consumer does not use the purchased commodities as efficiently as he/she could, so that the actual utility-quantity combination is  $(u^*, \mathbf{q}^0)$ . Then, this consumer is commodity inefficient in that the same utility level  $u^*$  could have been attained with proportionally less of all commodities. In particular, he/she could have chosen the commodity vector  $\mathbf{q}^0 / D(u^*, \mathbf{q}^0)$  which contains a fraction  $OS/OR$  of the quantities  $(q_1^0, q_2^0)$  and it just generates the utility level  $u^*$ . Hence, the degree of the consumer's commodity efficiency is given by  $CE(u^*, \mathbf{q}^0) = OS/OR = [\mathbf{p}^0 \cdot \mathbf{q}^0 / D(u^*, \mathbf{q}^0)] / \mathbf{p}^0 \cdot \mathbf{q}^0 = 1/D(u^*, \mathbf{q}^0)$ . However, in the situation depicted in Figure 1, the commodity efficient commodity vector  $\mathbf{q}^0 / D(u^*, \mathbf{q}^0)$  does not coincide with the expenditure-minimising

vector  $\mathbf{q}^*$ . Hence, even if the consumer in our example had chosen the commodity efficient commodity vector, he/she would still not be expenditure (or overall) efficient: given relative commodity prices as they are reflected in the slope of the tangent at A, the commodity vector  $\mathbf{q}^0/D(u^*, \mathbf{q}^0)$  contains the wrong combination of commodity quantities. This remaining portion of expenditure inefficiency is measured by allocative efficiency which is given by  $AE(u^*, \mathbf{q}^0, \mathbf{p}^0) = 0C/0S = C(u^*, \mathbf{p}^0)/[\mathbf{p}^0 \cdot \mathbf{q}^0/D(u^*, \mathbf{q}^0)]$ . Finally, the expenditure efficiency of the consumer can be measured by the ratio  $0C/0R$ , which is the product of the measures of commodity and allocative efficiency. That is,  $EE(u^*, \mathbf{q}^0, \mathbf{p}^0) = 0C/0R = C(u^*, \mathbf{p}^0)/(\mathbf{p}^0 \cdot \mathbf{q}^0)$ . The decomposition of expenditure efficiency into commodity efficiency and allocative efficiency can now be summarised as follows:

$$\begin{aligned}
EE(u^*, \mathbf{q}^0, \mathbf{p}^0) &= CE(u^*, \mathbf{q}^0) \times AE(u^*, \mathbf{q}^0, \mathbf{p}^0) \\
\Rightarrow \frac{0C}{0R} &= \frac{0S}{0R} \frac{0C}{0S} \Rightarrow \frac{\mathbf{p}^0 \cdot \mathbf{q}^*}{\mathbf{p}^0 \cdot \mathbf{q}^0} = \frac{\mathbf{p}^0 \cdot \mathbf{q}^0/D(u^*, \mathbf{q}^0)}{\mathbf{p}^0 \cdot \mathbf{q}^0} \frac{\mathbf{p}^0 \cdot \mathbf{q}^*}{\mathbf{p}^0 \cdot \mathbf{q}^0/D(u^*, \mathbf{q}^0)} \\
\Rightarrow \frac{C(u^*, \mathbf{p}^0)}{\mathbf{p}^0 \cdot \mathbf{q}^0} &= \frac{1}{D(u^*, \mathbf{q}^0)} \frac{C(u^*, \mathbf{p}^0)}{\mathbf{p}^0 \cdot \mathbf{q}^0/D(u^*, \mathbf{q}^0)}.
\end{aligned}$$

A comparison of the proposed efficiency measures to Varian's (1990) money-metric measure is suitable before closing this section. Varian (1990) used the money-metric utility function in order to construct a parametric goodness-of-fit measure for violations in optimising behaviour. In particular, his measure is given by the ratio of optimal expenditure (the value of the money-metric utility function) to actual expenditure, and as Varian (1990) states, it serves as a parametric index of overall consumption efficiency. However, money-metric utility functions pick out a commodity vector that makes the consumer as well off as he/she would be consuming a reference vector, *i.e.*, they pick out a commodity vector that lies on the same utility curve as the reference vector. Thus, by comparing the value of the money-metric utility function to actual expenditure, Varian's (1990) index measures, in fact, the overall efficiency of commodity efficient commodity vectors. By leaving no room for commodity inefficiency to arise, and hence for a decomposition of overall efficiency

into commodity and allocative efficiency, Varian's (1990) measure may assign to consumers a higher efficiency score than it should.

### 3. Empirical Model

As mentioned in the introductory section, a translog distance function is estimated in order to compute the commodity efficiency index. The adopted translog distance function is given by<sup>13</sup>

$$\begin{aligned}\ln D_{it}(u, \mathbf{q}) &= \alpha_0 + \sum_j \alpha_j \ln q_{jit} + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \ln q_{jit} \ln q_{kit} + \sum_j \delta_j \ln q_{jit} \ln t + \delta_t \ln t \\ &\quad + \frac{1}{2} \delta_{tt} (\ln t)^2 + \left( \beta_0 + \sum_j \beta_j \ln q_{jit} + \delta_0 \ln t \right) \ln u_{it} + \frac{1}{2} \zeta (\ln u_{it})^2 + \varepsilon_{it} \\ &= f(\ln \mathbf{q}_{it}, t; \mathbf{b}_1) + g(\ln \mathbf{q}_{it}, t; \mathbf{b}_2) \ln u_{it} + \frac{1}{2} \zeta (\ln u_{it})^2 + \varepsilon_{it},\end{aligned}\quad (10)$$

where  $j, k = 1, \dots, N$  is the number of commodities,  $i = 1, \dots, I$  is the number of households,  $t = 1, \dots, T$  is the number of time-periods,  $\mathbf{q} = (q_1, \dots, q_N)$  denotes the commodity vector,  $t$  denotes a time-trend capturing autonomous changes of consumer's preferences over time,  $u_{it}$  denotes the period- $t$  utility level of the  $i$ -th consumer,  $\varepsilon_{it}$  is a random error term which is assumed to be distributed as iid normal with zero mean and variance  $\sigma_\varepsilon^2$ , and  $\alpha_0, \alpha_j, \gamma_{jk}, \delta_j, \delta_t, \delta_{tt}, \beta_0, \beta_j, \delta_0$  and  $\zeta$  are parameters to be estimated. As consumer theory suggests, the distance function should satisfy the restrictions of symmetry and homogeneity, which imply the following restrictions on the parameters of the distance function in (10):  $\gamma_{jk} = \gamma_{kj}$ ,  $\sum_{j=1}^N \alpha_j = 1$ ,  $\sum_{k=1}^N \gamma_{jk} = 0$  for  $j = 1, \dots, N$ ,  $\sum_{j=1}^N \beta_j = 0$ , and  $\sum_{j=1}^N \delta_j = 0$ .

Since the distance function is homogeneous of degree 1 in commodity quantities, imposition of homogeneity through division of commodity quantities by, say,  $q_1$ , yields:

$$\begin{aligned}
-\ln q_{1it} &= \alpha_0 + \sum_j \alpha_j \ln(q_{jit}/q_{1it}) + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \ln(q_{jit}/q_{1it}) \ln(q_{kit}/q_{1it}) \\
&\quad + \sum_j \delta_j \ln(q_{jit}/q_{1it}) \ln t + \delta_t \ln t + \frac{1}{2} \delta_{tt} (\ln t)^2 \\
&\quad + \left( \beta_0 + \sum_j \beta_j \ln(q_{jit}/q_{1it}) + \delta_0 \ln t \right) \ln u_{it} + \frac{1}{2} \zeta (\ln u_{it})^2 - \ln D_{it}(u, \mathbf{q}) + \varepsilon_{it} \\
&= f(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_1) + g(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_2) \ln u_{it} + \frac{1}{2} \zeta (\ln u_{it})^2 \\
&\quad - \ln D_{it}(u, \mathbf{q}) + \varepsilon_{it}.
\end{aligned} \tag{11}$$

As mentioned in the introductory section, the difficulty in estimating a distance function representation of consumer preferences lies on the fact that it is a function not only of purchased quantities, but also of consumer's unobserved utility level. The methodology that is adopted for dealing with this problem concerns treatment and estimation of consumer's unobserved utility level as a one-sided positive error term. Under such a treatment, however, the translog term for utility-squared will not be included in the model since this would complicate the model considerably (the composite error term would be the sum of four different error terms). Treating the distance as a one-sided positive error term too, requires that the error terms associated with utility and distance be preceded by opposite signs in order for them to be discernible in estimation. Consequently, appropriate assumptions must be made for these two terms. Firstly, our theoretical model requires, by construction, that the value of the distance function be greater than or equal to unity, *i.e.*,  $D_{it} \geq 1$ , so that  $\ln D_{it} \geq 0$ . Secondly, as consumer theory suggests, the distance function must be non-decreasing in quantities and decreasing in utility. Thus, monotonicity of the distance function with respect to utility requires that

$$\frac{\partial D_{it}}{\partial u_{it}} = \frac{\partial \ln D_{it}}{\partial \ln u_{it}} \frac{D_{it}}{u_{it}} = g(\ln q_{jit}, t; \mathbf{b}_2) \frac{D_{it}}{u_{it}} < 0, \tag{12}$$

that is, the function  $g(\cdot)$  must be negative. Moreover, since  $u_{it} > 0$ ,  $\ln u_{it}$  can take on any values in the interval  $(-\infty, +\infty)$ . However, since consumer's preferences are

ordinal, no harm is done by normalising utility so that  $u_{it} \in (0,1)$  and  $\ln u_{it} < 0$ . Under these assumptions, we can re-write the model to be estimated as

$$\begin{aligned} -\ln q_{1it} &= f(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_1) + \left[ -g(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_2) \right] (-\ln u_{it}) - \ln D_{it}(u, \mathbf{q}) + \varepsilon_{it} \\ &= f(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_1) + s_{it}, \end{aligned} \quad (13)$$

where  $s_{it} \equiv \left[ -g(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_2) \right] (-\ln u_{it}) - \ln D_{it}(u, \mathbf{q}) + \varepsilon_{it}$ . Hence, assuming that  $D_{it} \geq 1$  so that  $\ln D_{it} \geq 0$ , and assuming that  $u_{it} \in (0,1)$  so that  $\ln u_{it} < 0$ , we can treat the variables  $\ln D_{it}$  and  $(-\ln u_{it})$  as iid exponentially distributed error terms. In addition, under the assumption that  $g(\ln \mathbf{q}_{it}, t; \mathbf{b}_2) < 0$ , the terms  $\left[ -g(\ln(\mathbf{q}_{it}/q_{1it}), t; \mathbf{b}_2) \right] (-\ln u_{it})$  and  $\ln D_{it}(u, \mathbf{q})$  are preceded by opposite signs and can be discerned in estimation. Another reason for assigning a specific sign to the function  $g(\cdot)$  is that it was necessary for the construction of the density of the composite error term  $s_{it}$  to decide on the sign of the function  $g(\cdot)$ , and since consumer theory suggests that it should be negative for monotonicity of the distance function with respect to utility to hold, we couldn't have chosen otherwise.

In summary, the distributional assumptions we have made for the three random error terms are as follows:

$$\begin{aligned} z_{it} &\equiv (-\ln u_{it}) \sim \text{iid exponential}(\sigma_z, \sigma_z^2), \\ v_{it} &\equiv \ln D_{it} \sim \text{iid exponential}(\sigma_v, \sigma_v^2), \\ \varepsilon_{it} &\sim \text{iid}N(0, \sigma_\varepsilon^2). \end{aligned}$$

Assuming that the  $z_{it}, v_{it}$  and  $\varepsilon_{it}$  are independent with respect to one another, the marginal density of the composite error term is given by

$$\begin{aligned} f(s_{it}) &= \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s_{it}}{\sigma_v}\right) \Phi\left(-\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v}\right) \\ &\quad + \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s_{it}}{\sigma_w}\right) \Phi\left(\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w}\right), \end{aligned} \quad (14)$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function. This density is the same as the density of the composite error term in the two-tiered frontier model of Polachek and Yoon (1987, 1996) with the exception that  $\sigma_w$  is not a parameter to be estimated, but it is defined, instead, as

$$\sigma_w \equiv \left[ -g\left(\ln(\mathbf{q}_{it}/q_{lit}), t; \mathbf{b}_2\right) \right] \sigma_z. \quad (15)$$

The estimators which we propose for commodity efficiency and consumer's utility level, however, are distinctively different from the estimators for the one-sided error terms in the model of Polachek and Yoon (1987, 1996), and are given by

$$\begin{aligned} CE_{it}(u, \mathbf{q}) &= E[D_{it}^{-1} | s_{it}] = E[e^{-v_{it}} | s_{it}] \\ &= \frac{\sigma^*}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s_{it}}{\sigma_w}\right) \frac{1}{f_s(s_{it})} \\ &\quad \times \left\{ \Phi\left(\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w}\right) + \exp\left[\frac{\sigma_\varepsilon^2}{2\sigma^{*2}} + \frac{1}{\sigma^*}\left(s_{it} - \frac{\sigma_\varepsilon^2}{\sigma_w}\right)\right] \Phi\left(-\frac{s_{it}}{\sigma_\varepsilon} + \frac{\sigma_\varepsilon}{\sigma_w} - \frac{\sigma_\varepsilon}{\sigma^*}\right) \right\}, \end{aligned} \quad (16)$$

and

$$\begin{aligned} E[u_{it} | s_{it}] &= \frac{\mu_{it}^*}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s_{it}}{\sigma_v}\right) \frac{1}{f_s(s)} \\ &\quad \times \left\{ \Phi\left(-\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v}\right) + \exp\left[\frac{\sigma_\varepsilon^2}{2\mu_{it}^{*2}} - \frac{1}{\mu_{it}^*}\left(s_{it} + \frac{\sigma_\varepsilon^2}{\sigma_v}\right)\right] \Phi\left(\frac{s_{it}}{\sigma_\varepsilon} + \frac{\sigma_\varepsilon}{\sigma_v} - \frac{\sigma_\varepsilon}{\mu_{it}^*}\right) \right\}, \end{aligned} \quad (17)$$

where

$$\sigma^* = \frac{\sigma_v \sigma_w}{\sigma_v + \sigma_w + \sigma_v \sigma_w}, \quad \mu_{it}^* = \frac{[-g(\cdot)] \sigma_v \sigma_w}{[-g(\cdot)] \sigma_v + [-g(\cdot)] \sigma_w + \sigma_v \sigma_w},$$

and  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

Once the translog distance function has been estimated and the commodity efficiency index has been computed, the value of the expenditure or overall efficiency measure can be computed from relation (7) which gives the decomposition of expenditure efficiency into commodity and allocative efficiency. Calculation of the value of the measure of allocative efficiency is more problematic since it requires knowledge of either the expenditure function or the expenditure-minimising commodity vector. The notion of virtual prices as defined by Grosskopf, Hayes, and Hirschberg (1995) and the procedure developed by Karagiannis, Midmore, and Tzouvelekas (2004) for the derivation of optimal input vectors can also be employed here. In particular, suppose the expenditure function is a linear function of the commodity market prices, that is,

$$C(u, \mathbf{p}) = \sum_{j=1}^N p_j q_j^*, \quad (18)$$

where  $\mathbf{p} = (p_1, \dots, p_N)$  denotes the vector of market prices, and  $q_j^*$  denotes the expenditure-minimising quantity of commodity  $j$ . Dividing relation (18) through by a commodity, say,  $q_1^*$ , yields

$$\frac{C(u, \mathbf{p})}{q_1^*} = p_1 + p_2 \left( \frac{q_2^*}{q_1^*} \right) + \dots + p_N \left( \frac{q_N^*}{q_1^*} \right). \quad (19)$$

Relations (19) and (9) can be used to compute the index of allocative efficiency as follows:

$$AE(u, \mathbf{q}, \mathbf{p}) = \frac{D(u, \mathbf{q})C(u, \mathbf{p})}{(\mathbf{p} \cdot \mathbf{q})} = D(u, \mathbf{q}) \frac{C(u, \mathbf{p})/q_1^*}{(\mathbf{p} \cdot \mathbf{q})/q_1^*}, \quad (20)$$

given that the observed (actual) and the optimal quantity of commodity 1 coincide, that is,  $q_1 = q_1^*$ . The *dual Shephard's lemma* (Shephard, 1953), allows the vector of shadow prices to be derived from partial differentiation of the distance function with respect to quantities. If the reference quantity vector in the definition of the distance function is not the expenditure-minimising one, then the vector of these prices is



interpreted as the vector of shadow prices deflated by shadow expenditure (*i.e.*, virtual prices). On the other hand, if the reference quantity vector in the definition of the distance function is the expenditure-minimising one, then the vector of shadow prices coincides with the expenditure-normalised vector of market prices. Hence, at the expenditure-minimising commodity vector,  $\mathbf{q}^*$ , we have:  $\partial \ln D(u, \mathbf{q}^*) / \partial \ln q_j^* = (\partial D(u, \mathbf{q}^*) / \partial q_j^*) (q_j^* / D(u, \mathbf{q}^*)) = (p_j / C(u, \mathbf{p})) (q_j^* / D(u, \mathbf{q}^*))$ . Using this result, we can derive the following system of  $N-1$  equations

$$\frac{\partial \ln D(u, \mathbf{q}^*) / \partial \ln q_j^*}{\partial \ln D(u, \mathbf{q}^*) / \partial \ln q_1} = \frac{(p_j / C(u, \mathbf{p})) (q_j^* / D(u, \mathbf{q}^*))}{(p_1 / C(u, \mathbf{p})) (q_1 / D(u, \mathbf{q}^*))} = \frac{p_j q_j^*}{p_1 q_1}, \quad j = 2, \dots, N. \quad (21)$$

In the case of the translog input distance function given by relation (11), this system becomes

$$\left( \frac{p_j}{p_1} \right) \left( \frac{q_j^*}{q_1} \right) = \frac{\alpha_j + \sum_{k=2}^N \gamma_{jk} \ln(q_{kit}^* / q_{1it}) + \delta_j \ln t_t + \beta_j \ln u_{it}}{\alpha_1 + \sum_{k=2}^N \gamma_{1k} \ln(q_{kit}^* / q_{1it}) + \delta_1 \ln t_t + \beta_1 \ln u_{it}}, \quad j = 2, \dots, N, \quad (22)$$

where the restrictions of homogeneity and symmetry have been imposed, and the assumption that  $q_1 = q_1^*$  has been made. This system can be solved to obtain the ratios of expenditure-minimising quantities in terms of the observed market prices, the estimated expected value of utility, and the estimated parameters of the distance function. These expenditure-minimising commodity ratios can then be substituted into (19) to derive estimates of  $AE(u, \mathbf{q}, \mathbf{p})$  from relation (20).

Finally, knowledge of the expenditure minimising quantity ratios also allows the derivation of the optimal expenditure shares. The latter can be computed from the following relation:

$$w_i(u, \mathbf{p}) = \frac{p_i q_i^*}{C(u, \mathbf{p})} = \frac{p_i (q_i^* / q_1^*)}{(C(u, \mathbf{p}) / q_1^*)}, \quad (23)$$

The optimal expenditure share for the first commodity can be computed residually, using the adding-up restriction.

#### **4. Data Description**

The data used in the empirical analysis are drawn from a panel of British household data provided by the *TNS* market research institute. The panel provides information on weekly purchases of milk, yoghurt, fruits, and vegetables, from December 2004 to November 2006. The surveyed households reported the volume of and expenditure on the aforementioned products purchased at every shopping trip. For the purposes of the present analysis, the region of London was selected in order to avoid problems associated with the consumption of home-grown agricultural products in rural areas. Since accounting for censoring would add extra complexity to the adopted empirical models, and since our aim is to provide an illustration of the econometric estimation of the proposed model for consumer efficiency, the data on quantities and expenditure were aggregated to monthly figures, and the households selected were the ones that reported positive consumption of all the following three commodity groups: milk & yoghurt, fruits, and vegetables. In particular, the selected sample consists of 884 households in London, which reported positive consumption of all three commodities for a period of 12 months, from July 2005 to June 2006.

The descriptive statistics for the household data are summarized in Table 1. The quantities of fruits and vegetables are measured in kilograms, while the quantities of milk and yoghurt, before aggregation, were measured in litres and kilos, respectively. Aggregation of the quantities for the creation of the milk & yoghurt commodity group was carried out with the use of a *Divisia* index with expenditure shares serving as weights. Finally, expenditure is measured in Pound-Sterling. As shown in Table 1, the average consumption of the three commodities is rather constant during the period of the 12 months, which is to be expected for commodities such as foods.

#### **5. Empirical Results**

The parameter estimates for the two-tiered model were obtained from pooled-data maximum likelihood estimation of the model in equation (13), with the use of the GAUSS software. The model was estimated without the translog constant term. In addition, as is obvious in relation (15), there is a problem with identification of the

standard error  $\sigma_z$  of the random error term which is associated with utility. In order to be able to estimate the parameter  $\sigma_z$ , we normalised  $\beta_0$  to unity.

The maximum likelihood parameter estimates of the model, along with standard errors, are displayed in Table 2. The model was estimated with homogeneity and symmetry imposed, where homogeneity was imposed by division of all quantities by the quantity of milk & yoghurt. Parameters  $a_{MY}$ ,  $\gamma_{MY}$ ,  $\gamma_{MYF}$ ,  $\gamma_{MYV}$ ,  $\beta_{MY}$ , and  $\delta_{MY}$  were computed via the homogeneity and symmetry restrictions, and, their standard errors were approximated by the delta method (see, for example Spanos). As shown in Table 2, 19 out of the 21 parameters were statistically significant. In particular, the standard deviations for the three error terms were statistically significant at the 1% level. The variance of the one-sided error term associated with household's utility is found to be  $\sigma_z^2 = 0.1404$ , whereas the variance of the one-sided error term associated with the distance is found to be  $\sigma_v^2 = 0.0358$ , and that of the normal error term is found to be  $\sigma_\varepsilon^2 = 0.1774$ .

The estimated efficiency indices are presented in Table 3. The estimated mean commodity, allocative, and expenditure efficiency scores were found to be 84.06%, 80.14%, and 67.22%, respectively, during the period July 2005 - June 2006. In particular, the majority of the households in the sample (87%) achieved scores of commodity efficiency between 80 and 90%. 45% of the households achieved scores of allocative efficiency between 70 and 80% and the remaining 55% of the households achieved scores of allocative efficiency between 80 and 90%. Finally, almost all the households (99%) achieved scores of expenditure efficiency between 60 and 70%.

An interpretation of the findings with regard to efficiency scores is suitable here. The central assumption we have made for the development of the proposed efficiency measures is that any unwanted quantities of the purchased commodities can be freely disposed of. This means that the quantities of the purchased commodities, and hence, actual expenditure, may well be higher than the ones required to just attain a target utility level. Using relation (8) which provides a definition of commodity efficiency as a cost ratio, the finding that the estimated mean commodity efficiency was 84.06% during the time-span of the panel indicates that, on average, a 15.94% of the households' budget was wasted, or that households, say, by better planning, could

have decreased their total expenditure by 15.94% and could still have achieved the same utility level with a portion of the purchased commodities. Recall the definition (9) of the measure of allocative efficiency as the ratio of the minimum expenditure required for attaining a target utility level to the expenditure for the commodity efficient commodity vector. Under this definition, the finding that the mean allocative efficiency was 80.14% during the time period covered by the panel indicates that, given some target utility level and the commodity prices that households face, a 19.86% decrease in total expenditure could be made feasible if households had chosen a different combination of commodity quantities. Moreover, since expenditure efficiency is defined as the ratio of minimum expenditure required for the achievement of  $u$  to actual expenditure, a mean expenditure efficiency score of 67.22% indicates that, on average, a 32.78% of the households' budget was wasted due to the presence of commodity and allocative inefficiency.

With regard to households' behaviour over time, it cannot be concluded from the empirical results that the inefficiency scores are diminishing over time. As shown in Table 3, the mean inefficiency scores are rather steady over time. This is to be expected, considering the type of commodities under analysis combined with the short time-span of our panel (12 months): the commodities under analysis are foods that have an important role to play in any average household's diet, and, in addition, the time-span of our panel is too short to allow for changes in households' socio-demographic characteristics which would affect consumption habits and hence efficiency scores.

## **6. Summary and Conclusions**

The aim of this paper has been to propose a theoretical and empirical framework for measuring consumer's efficiency in price-quantity space. In particular, a measure of consumer's expenditure inefficiency is proposed, which can be decomposed into two associated measures of efficiency in consumption, namely, commodity and allocative efficiency, in a manner similar to the one met in production efficiency analysis. The starting point for the empirical measurement of consumer's efficiency is the econometric estimation of a distance function. As the distance function is a function of consumer's unobserved utility level, the empirical approach which is employed tackles this problem by treating and estimating utility as a random error term. The model was applied to a panel data set of British household purchases of highly

perishable foods. Although it seems restrictive to employ highly perishable commodities in order to accommodate the assumption that consumers are free to dispose of any unwanted quantities of the purchased commodities, studying consumer's inefficiency with respect to such type of commodities is important since a significant portion of consumers' budget is allocated to them.

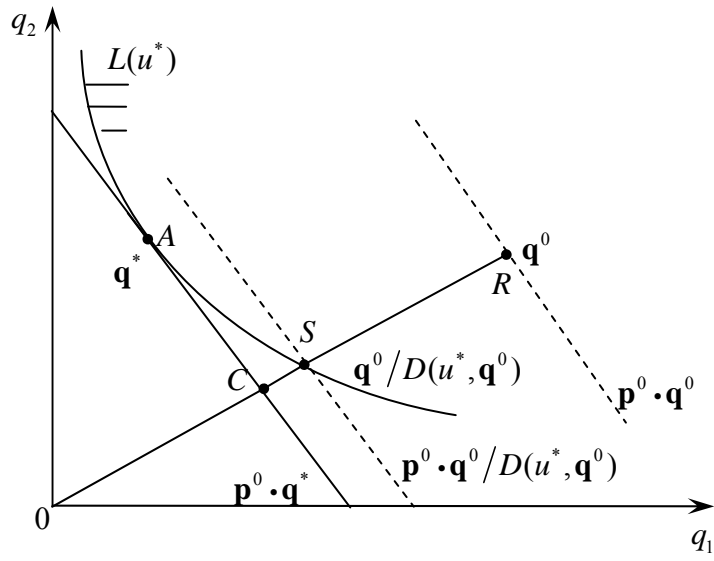
The conclusions derived from the analysis of the empirical results in studies of consumer demand usually direct attention to market and production implications. Our proposed measure of expenditure or overall efficiency can also serve such a goal, but the measures of commodity and allocative efficiency in consumption cannot; they can, however, provide a deeper insight into how expenditure inefficiency arises. None of the three measures can explain in full why consumers are inefficient. For, even if socio-demographic and economic characteristics of the consumers are accounted for in empirical analysis, there are still many characteristics of them, e.g. psychological, that cannot be observed. Nonetheless, the importance of studying inefficiency in consumption lies not only on the fact that optimal behaviour is a restrictive assumption to make for consumers' actual behaviour, or on that changes in consumption efficiency levels may have an effect on products' prices in a competitive market. It also lies on the fact that consumer's non-optimal behaviour has a negative impact on their welfare and on social welfare. Reduction of consumers' inefficiency and mitigation of its negative impact on welfare levels could be accomplished though, say, advertising. If advertising plays an important role in creating and/or sustaining consumers' non-optimal behaviour, then advertising could perhaps be used as a means of awareness raising and initiation of changes in consumers' shopping, purchasing and consumption patterns.

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**Figure 1.** The Decomposition of Expenditure Efficiency





**Table 1.** Summary Statistics of the Data

	Time Period	Quantities			Expenditure		
		Milk & Yoghurt	Fruits	Vegetables	Milk & Yoghurt	Fruits	Vegetables
Mean	Jul. 2005	11.69	9.96	12.3	11.77	18.19	16.78
	Aug. 2005	11.38	9.58	11.54	11.32	16.97	14.94
	Sep. 2005	11.20	8.71	11.36	11.24	14.72	14.16
	Oct. 2005	11.41	8.48	12.17	11.69	13.81	13.88
	Nov. 2005	11.68	8.21	12.43	11.65	12.82	13.95
	Dec. 2005	13.02	8.83	13.16	12.55	13.69	14.36
	Jan. 2006	11.76	7.70	12.63	10.94	12.42	14.29
	Feb. 2006	11.77	8.30	12.06	11.76	13.24	14.22
	Mar. 2006	13.10	9.51	13.44	13.07	15.34	15.96
	Apr. 2006	11.81	8.78	12.29	11.81	13.73	15.04
	May 2006	11.81	9.06	13.09	11.72	14.19	16.29
	Jun. 2006	11.50	9.02	12.48	11.72	15.22	17.09
<hr/>							
Jul. 2005 - Jun. 2006							
	Mean	11.84	8.85	12.41	11.77	14.53	15.08
	Min.	0.15	0.10	0.12	0.23	0.14	0.12
	Max.	109.30	80.01	89.36	72.46	162.53	148.66
	St.Dev.	10.52	6.62	8.88	9.05	11.84	11.38

**Table 2.** Parameter Estimates of the Translog Distance Function

Parameter	Estimate	SE	Parameter	Estimate	SE
$\alpha_{MY}$	0.2684	(0.0094)*	$\delta_F$	0.0198	(0.0096)**
$\alpha_F$	0.3626	(0.0106)*	$\delta_V$	-0.0263	(0.0102)*
$\alpha_V$	0.3690	(0.0114)*	$\delta_T$	-0.0628	(0.0161)*
$\gamma_{MY}$	0.1397	(0.0216)*	$\delta_{TT}$	-0.1051	(0.0200)*
$\gamma_F$	0.2139	(0.0103)*	$\beta_{MY}$	0.0687	(0.0218)*
$\gamma_V$	0.2952	(0.0106)*	$\beta_F$	0.0397	(0.0245)***
$\gamma_{MYF}$	-0.0292	(0.0074)*	$\beta_V$	-0.1084	(0.0263)*
$\gamma_{MYV}$	-0.1105	(0.0079)*	$\delta_0$	0.0124	(0.0281)
$\gamma_{FV}$	-0.1847	(0.0087)*	$\sigma_z$	0.3747	(0.0108)*
$\delta_{MY}$	0.0065	(0.0083)	$\sigma_V$	0.1893	(0.0110)*
			$\sigma_\varepsilon$	0.4212	(0.0110)*

Log-Likelihood -9363.8

Notes: MY refers to milk & yoghurt, F to fruits, and V to vegetables. Asymptotic standard errors in parentheses. \* (\*\*, and \*\*\*) indicate significance level at the 1 (5, and 10) percent.

**Table 3.** Frequency Distribution of Commodity, Allocative, and Expenditure Efficiency for the Two-Tiered Frontier Model

	2005						2006					
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
<b>Commodity Efficiency</b>												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	0	1	0	0	0	1	0	0	1	0	1	0
0.5-0.6	2	6	1	4	1	0	1	2	2	2	1	2
0.6-0.7	9	15	15	14	11	20	9	7	27	7	5	9
0.7-0.8	112	124	119	136	128	147	116	102	148	111	120	113
0.8-0.9	761	738	749	730	744	716	758	773	706	764	757	760
>0.9	0	0	0	0	0	0	0	0	0	0	0	0
Mean	0.84	0.84	0.84	0.84	0.84	0.83	0.84	0.84	0.83	0.84	0.84	0.84
<b>Allocative Efficiency</b>												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	0	0	0	0	0	0	0	0	0	0	0	0
0.5-0.6	25	0	2	2	2	4	7	3	11	2	1	1
0.6-0.7	99	0	40	22	33	69	51	18	25	9	98	120
0.7-0.8	221	362	315	402	434	406	430	416	329	269	307	375
0.8-0.9	402	469	460	416	393	362	370	427	469	539	423	350
>0.9	137	53	67	42	22	43	26	20	50	65	55	38
Mean	0.81	0.81	0.81	0.80	0.79	0.79	0.79	0.80	0.81	0.82	0.80	0.78
<b>Expenditure Efficiency</b>												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	19	0	0	1	0	13	8	3	22	7	0	0
0.5-0.6	142	9	75	43	47	122	83	27	27	5	170	162
0.6-0.7	345	605	526	562	627	528	556	597	562	499	452	484
0.7-0.8	351	270	283	278	210	221	237	257	273	373	262	238
0.8-0.9	27	0	0	0	0	0	0	0	0	0	0	0
>0.9	0	0	0	0	0	0	0	0	0	0	0	0
Mean	0.68	0.68	0.67	0.68	0.67	0.66	0.66	0.68	0.68	0.69	0.66	0.66

## Endnotes

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<sup>1</sup> Consumption efficiency has been studied in price-quality space as well. The theoretical framework for consumer demand analysis in a price-quality space dates back to Lancaster (1966), who defined consumer preferences and utility in terms of the characteristics that commodities possess. The latest and more complete advancement in this field is found in the paper of Lee, Hwang, and Kim (2005), who developed a theoretical and empirical framework for measuring the degree of overall consumption efficiency of multi-attribute products in price-quality space. However, the applicability of such a model is restricted by the paucity of data on commodities' quality attributes. Moreover, it seems that a consumption efficiency measure defined in price-quality space is more appropriate for the examination of different varieties of the same commodity. Finally, their model employs theoretical tools found in consumer demand theory (*i.e.*, consumption analysis in price-quality space) in order to measure product efficiency and firm market performance, rather than analyse consumer behaviour.

<sup>2</sup> Varian's (1990) *money metric goodness-of-fit measure* has also a drawback regarding its empirical applications. Varian (1990) illustrated the use of his measure using a Cobb-Douglas direct utility function as the starting point for the derivation of the money-metric utility function. However, it is not easy to derive the money-metric utility function from utility functions that are of a more complex form than the Cobb-Douglas.

<sup>3</sup> Another methodology for estimation of a distance function concerns the use of observable variables as proxies for utility. In particular, Lewbel and Pendakur (2006) invented Implicit Marshallian Demand systems, which are systems of Hicksian demands where utility is substituted by *implicit utility*, a simple function of observables. An application of the approach proposed by Lewbel and Pendakur is found in the paper of Färe, Grosskopf, Hayes, and Margaritis (2008), who used household annual income as a proxy for utility in order to estimate and assess systems of demand equations which are derived from expenditure and benefit functions. The advantage of such an approach is that, once observable variables are used as a proxy for utility, standard frontier-estimation techniques can be used for the estimation of the distance function. Its drawback, however, is that consumer's utility level is

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assumed to be affected by the chosen set of observable variables, while any other factors that may effect consumer's preferences are ignored.

<sup>4</sup> Notation:  $\mathbf{q} \gg 0^N$  means that  $q_i > 0 \quad \forall i$ ;  $\mathbf{q} \geq 0^N$  means that  $q_i \geq 0 \quad \forall i$ ; and  $\mathbf{q} > 0^N$  means that  $q_i \geq 0 \quad \forall i$  and for at least one element  $j$ ,  $q_j \neq 0$ .

<sup>5</sup> We assume that the consumption space is the non-negative Euclidean  $N$ -orthant,  $\mathbb{R}_+^N$ , with its origin excluded. Restricting the consumption space in this way does not affect the generality of our theoretical model. It only affects the way that alternative functional representations of consumer preferences (*i.e.*, the direct utility, indirect utility, expenditure, and distance functions) are defined, the properties they satisfy, and the conditions required for the duality between them to hold. See, for example, Russell (1998), and Blackorby, Primont, and Russell (1978).

<sup>6</sup> The property of strong (weak) monotonicity states that for every  $\mathbf{q}, \mathbf{q}' \in Q^N$ , if  $\mathbf{q} > \mathbf{q}'$  ( $\mathbf{q} \gg \mathbf{q}'$ ) then  $\mathbf{q}$  is strictly preferred to  $\mathbf{q}'$ . Strong (weak) monotonicity implies that the utility function is increasing (non-decreasing) in  $\mathbf{q}$ .

<sup>7</sup> See the example provided by Russell (1998, p. 29).

<sup>8</sup> One could specify commodity efficiency analogues to the non-radial measures of input technical efficiency of production theory, which require that a technically efficient input vector is a member of the input efficient subset. However, these non-radial technical efficiency measures do not satisfy the homogeneity property. Hence, it is to be expected that commodity efficiency analogues to the non-radial measures of input technical efficiency will not satisfy the homogeneity property either. Finally, since homogeneity is considered to be an important property of index numbers, the econometric literature on production efficiency analysis has argued in favour of radial efficiency measures like the Debreu-Farrell measure of technical efficiency.

<sup>9</sup> The infimum value of utility is excluded from the distance function's domain. The reason is that, since the consumption space is assumed not to include its origin, then if the infimum of the range of  $U$  is an element of the range of  $U$ , the maximisation problem in the definition of the distance function will not have a solution when the level of utility is at its infimum value (see, Blackorby, Primont, and Russell, 1978).

<sup>10</sup> For a proof of this in the context of production theory, see Färe and Primont (1995).

<sup>11</sup> For a discussion on the properties of efficiency indices, see, for example, Russell (1998).

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<sup>12</sup> Notation:  $\mathbf{p} \cdot \mathbf{q}$  is the inner product of the two vectors  $\mathbf{p}$  and  $\mathbf{q}$ , that is,

$$\mathbf{p} \cdot \mathbf{q} = \sum_{i=1}^N p_i q_i.$$

<sup>13</sup> This general translog distance function can be found in Diewert (1993, pp. 211-2).