Regulation of Farming Activities: An Evolutionary Approach[1]

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Abstract

Farming activity is modeled under an intervention policy regime, combining the environmental requirements of the Council Nitrates Directive (91/676/EEC) and the compensatory provisions of the second pillar of the Common Agricultural Policy. The optimizing behavioural rule along with the evolutionary rule is employed in order to model the individual farmer's decision making, regarding compliance or not with regulatory provisions. The impact of these different behavioral rules on the selection of monitoring effort and thus on the compliance incentives of a population of farmers is examined. Analysis indicated that if monitoring effort is chosen arbitrarily or optimally based on the accustomed full rationality assumption then the population adopts a monomorphic behavior in the long-run, involving either full or noncompliance with the Directive's provisions. A polymorphic behavior involving partial compliance of the population also arises if the dynamic model of optimal monitoring is constrained by replicator dynamics which represent the imitation rules. It is evident, thus, that the number and the type of the equilibrium steady-states is affected by the assumption regarding the behavioral rule adopted by regulated agents. Finally, the dynamics of the population of compliant farmers is also assessed under accumulation of monitoring capital indicating identical properties.

Keywords: Nitrates Directive, agri-environmental programs, monitoring effort, monitoring capital, rationality, optimal behavioral rule, replicator dynamics, imitation.

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1 Introduction

Excessive nitrogen (N) surpluses from mineral fertilizers and animal manure, appear to be a major pollutant in many European underground and surface watersheds,[2] posing a threat to the environment and the human health.[3] To provide a general level of protection for all waters against nitrate pollution, the European Council established in 1991 the Nitrates Directive (91/676/EEC), defining a series of codes of good agricultural practice (EC, 1991).[4] The non-point-source characteristics,[5] however, of agricultural pollution pose a substantial problem in the effective regulation of reported water pollution problems. The inability of regulatory authorities to directly observe individual decisions (i.e. nitrogen usage) provides the farmers incentives to deviate from statutory requirements and retain nitrogen usage at the unregulated profit maximizing levels, with the associated adverse consequences. To ensure that regulated farmers comply with statutory nitrogen performance standards and that foreseen sanctions are imposed on those deviating so that compliance is further enforced, Member States are required to incorporate a substantial monitoring mechanism in their policy design.[6] It is evident thus, that the effectiveness of the existing regulatory policies to induce restricted usage of nitrogen input is heavily dependent on the ability of the monitoring and enforcement mechanism to provide adequate compliance incentives, and thus implement the Nitrates Directive.

The purpose of the present paper is to examine the effectiveness of a monitoring and enforcement mechanism to induce in the long-run a large population of homogeneous farmers to comply with the statutory requirements of a regulatory regime under different assumption regarding the way that farmers choose to comply or not with regulation. The examined regulatory regime falls into the

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[2] Excessive concentrations of nutrients result in the eutrophication of slow flowing rivers, lakes, reservoirs and coastal areas, appearing through the proliferation of algal bloom (Huhtala and Laukkanen, 2004; Isik, 2004; Owen et al., 1998; Pau Vall and Vidal, 2006). There is evidence that at least 30-40% of rivers and lakes show eutrophication symptoms or bring high N fluxes to coastal waters and seas. The agricultural origin of such fluxes accounts for 50 to 80% of total N inputs to EU waters (EC, 2002).

[3] N exposure is responsible for the blue-baby syndrome (methemoglobinemia) in infants and gastric cancer in adults (Fleming and Adams, 1997; Abler and Shortle, 1995; Johnson et al., 1991).

[4] These codes concern mostly issues of land application. They are mandatorily implemented through action programs either through out the territory of Member States or at specific zones vulnerable to nitrates pollution (i.e. NVZs). For further details see Axes II and III (EC, 1991).

[5] A pollution problem is called as NPS problem if there is uncertainty on the regulator's behalf about the location of the decision makers (polluters) and the degree of each agent's responsibility in the aggregate pollution. In short the origins of this uncertainty can either be attributed to stochastic influences affecting fate and transport of pollutants, the great number of sources of pollution emissions that can be either static (farms, households) or mobile (vehicles), and/or the regulator's inability to infer individual emissions from ambient pollution levels or inputs used (Xepapadeas, 1995).

[6] For instance, an environment agency has the task of undertaking occasional random spot-checks, visiting farms and inspecting the operation field as well as the field records (DEFRA, 2004). Guidelines for the monitoring referred to in Articles 5 and 6 may be drawn up in accordance with the procedure laid out in Article 9 of the Nitrates Directive (EC, 1991).

category of public voluntary environmental programs[7] and involves a combination of "carrot" financial inducements provided through the agri-environmental programs of the second pillar of the communal agricultural policy (CAP) and "stick" legal binding features of the action programs[8] of the European Council Nitrates Directive (91/676/EEC).[9]

In our approach, and in contrast to the majority of the enforcement literature, farmers do not necessarily adopt an optimizing behavioral rule in their decision to comply or not with the suggested nitrogen usage constraint, but may follow evolutionary rules modeled by imitation dynamics.[10] Most economic models assume that agents are "infinite in faculties", they act "as if" unboundedly rational (Conlisk, 1996). If farmers are characterized by full rationality then they adopt optimizing behavioral rules and they behave as though they had all the necessary information when they decide about complying or not.[11] In such a case farmers have full knowledge of the structure of payoffs and after comparing the payoff that each strategy entails they define their optimal response to the regulation. This response is maintained across time and space if there is no modification of the policy parameters by the regulator. On the other hand, under bounded rationality agents "*are no longer assumed to be mathematical prodigies with access to encyclopedic mannuals written by omniscient game theorists*" (Binmore, 1992).[12] Farmers cannot choose their individual strategy in an optimal manner, and their decision about whether to comply or not is adapted to the information revealed via their interaction over time. We assume that such passive decision making is based on the imitation of the better-off performing strategy and is modeled by the replicator dynamics, imitation rule. Under such an evolutionary process more successful agents and activities gradually increase their share in the population at the expense of less successful agents and activities (Conlisk, 1996), leading potential agents who have no clear idea what is going on to behavior that may look very rational indeed to a Kibitzer (Binmore, 1992).

Individual compliance incentives, along with the aggregate environmental performance of a given population, are affected by the monitoring undertaken given the homogeneity assumption. An environmental agency that engages into

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[7] Public voluntary agreements are environmental programs entirely developed by a regulatory body. No bargaining is involved between regulated agents and the regulator in the definition of environmental goals and means of achieving them. Polluting agents can only agree or not to adopt the terms of regulation affecting their activities. For further details see Mazurek (1998), Lyon and Maxwell (2002) and Šaeur et al. (2001).

[8] Examples of such action programs are the Nitrogen Management Program (Denmark), Ferti-Mieux Initiative (France) and Prop'eau - Sable pilot project (Belgium) (EC, 2002).

[9] "Carrot" incentives involve total-cost or sharing subsidies, information subsidies, technical assistance and /or public recognition (Mazurek, 1998), while "stick" measures involve the implementation of existing mandatory restriction or the establishment of a new regulation.

[10] For such an exception to the traditional enforcement literature see Xepapadeas (2005).

[11] Rational economic choice involves optimization in the sense that agents consider all possible alternatives and choose the best (Conlisk, 1996).

[12] According to Conlisk (1996), though people are bounded rational, they learn optima through practice and in the end act as if unboundendly rational. Economists just take a shortcut and assume unbounded rationality from the start.

costly and accurate monitoring is considered, where the number of random spot-checks is defined either in an arbitrary way, based on the alternative behavioral compliance rules assumed to be adopted by farmers, or selected optimally by minimizing a social welfare criterion, defined as the sum of monitoring costs and social environmental damages, constrained by the farmers' full or bounded rationality behavioral rules. Under each approach the selection criteria for monitoring effort stimulating long-term compliance of farmers are discussed, allowing comparisons of equilibrium outcomes under different rationality assumptions.

The contribution of this paper consists of the development of a dynamic model of optimal monitoring constrained by an evolutionary imitation behavioral rule. The steady-state equilibrium proportion of complying farmers of this model, as well as the corresponding monitoring effort level is contrasted with the equilibrium proportion resulting from a conventional optimal monitoring model which considers that agents are fully rational. Indeed the main distinction between the two behavioral rules and the main finding of this paper is that under full rationality monomorphic outcomes[13] are the equilibrium outcomes, while under bounded rationality and imitation rules polymorphic outcomes[14] are very likely as evolutionary stable equilibria. In particular, our analysis indicates that if the monitoring effort level is chosen arbitrarily, that is not through an optimal monitoring model, then the characteristics of the equilibrium outcome are unaffected by the assumed behavioral rule regarding farmers' decision about choosing compliance decisions. In such a case the equilibrium outcome is monomorphic, implying either full compliance, or noncompliance with the Directive's provisions. To guarantee full compliance the environmental agency should precommit to a monitoring effort value that is higher than the critical value for which farmers are indifferent between compliance and deviation. The number and the type of equilibrium steady-states determining farmers' compliance are affected if monitoring is chosen optimally. A monomorphic behavior is the steady state outcome if the social welfare criterion is minimized conditional to an optimizing behavioral rule by the farmers, while if the problem is constrained by replicator dynamics representing the passive imitation rule, then the population may adopt either a monomorphic or polymorphic behavior. In the latter case whether the population converges in the long-run to the socially-desired outcome of full compliance, or to an intermediate status characterized by partial compliance, depends on the initial conditions of the problem given the fact that both the monomorphic and polymorphic steady-states satisfy a saddle point property. It is evident that the assumption regarding the farmers' adopted behavioral rule and the way that the environmental agent selects monitoring effort level, affects the long-term behavior of the population of farmers. Finally,

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[13] The long-term equilibrium is monomorphic if the entire population of farmers follows the same strategy and adopts a homogenous behaviour. This implies that either all the farmers comply with the provisions of the Directive or that they all deviate from the action program rules.

[14] A steady state is so-called polymorphic if a heterogeneous strategic behaviour is evolutionary stable in the long-run. This implies that only a proportion of farmers comply with the Directive, while the remaining proportion deviates.

dynamics of the regulated population with regard to compliance with the given regulation are reassessed under the presence of investment in monitoring capital.

2 Modelling Farm Activity under the Nitrates Directive

Consider an agricultural area characterised as a nitrates vulnerable zone[15] consisting of $i = 1, 2, ...v$ small and identical farmers operating under competitive conditions. Individual production choices $\{\mathbf{x}_{ij}, n_i\}$ positively affect crop yields:[16]

$$y_i = f(\mathbf{x}_{ij}, n_i)$$

where $\mathbf{x}_{ij} = (x_{i1}, x_{i2}, ..., x_{im})$ is the vector of agent i's choices among a set of $j = 1, ..., m$ inputs and n_i the employed nitrogen input, either organic or manufactured.

Production is associated with unintended generation of nitrates leaching (N_i) that contaminates underground and surface water resources.[17] At time t aggregate emissions flows $N(t)$ are:

$$N(t) = \sum_{i=1}^{v} N_i(t) = \sum_{i=1}^{v} n_i(t) \tag{1}$$

indicating by an appropriate choice of units a positive, one-to-one relation between individual nitrate leaching $N_i(t)$ and employed nitrogen input $n_i(t)$.

In the absence of any regulatory intervention, farmer i employs the profit maximizing amount of nitrogen (n_o) and obtains the maximum payoff defined as.[18]

$$\begin{aligned} \Pi^o(n_o) \quad &: \quad n_o = \arg\max_n \Pi^o(n) && (2) \\ \Pi^o(n) \quad &= \quad \max_{\mathbf{x}} \left[pf(\mathbf{x}, n) - \mathbf{wx} - w^n n\right] && (3) \end{aligned}$$

where $\mathbf{w} = (w_1, ..., w_m)$ is the vector of input prices, w^n the nitrogen price and p the output price in the competitive market.

In the unregulated case the generated nitrate emissions N^o exceed the socially-desired levels, since the externality is not internalized, a fact that stimulates

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[15] Nitrate vulnerable zones (NVZ) cover about 37% of EU-15 total area (EU, 2003) and are identified as land areas which drain into waters contributing to nitrates pollution. For further details see paragraph 1 of Article 3 (EC, 1991).

[16] It holds $f_x, f_n > 0$ and $f_{xx}, f_{nn} < 0$.

[17] The term nitrates leaching (NO_3) refers to the nitrate removal from the soil by the action of water (Owen et al, 1998). This phenomenon includes both leaching below the crop's roots due to the downward movement of water (percolation) and leaching due to the flow of water over the surface of the land (runoff).

[18] Assuming idendical farmers we drop subscript i to simplify notation.

intervention. Each farmer i is required by the action program of the Nitrates Directive to meet an annual per hectare aggregate nitrogen usage standard ($\bar{n}$):[19]

$$n \leq \bar{n} \tag{4}$$

When the Directive is combined with an agri-environmental program of the second pillar of CAP,[20] the given performance standard becomes stricter and farmers are provided with a subsidy s^n per unit of nitrogen fertilizer used beneath the benchmark n_D^* that goes beyond basic standard $\bar{n}$, in the sense that $\bar{n} > n_D^*$. The compensation payment is:

$$s^n(n_D^* - n)$$

Under such a "*mixed*" policy regime combining the environmental requirements of the Directive and the financial provisions of the rural development regime of CAP, the payoff structure of farmers complying with n_D^* is:

$$\Pi^D(n, n_D^*) = \max_n \left[pf(\mathbf{x}, n) - \mathbf{wx} - w^n n + s^n(n_D^* - n)\right]$$

where the nitrogen application is chosen such that $\Pi^D(n, n_D^*)$ is maximized, or:

$$\Pi^D(n_D) : n_D = \arg\max_n \Pi^D(n, n_D^*) \ s.t. \ n \leq n_D^* \tag{5}$$

We assume that after the subsidy is paid profits are lower, relative to the unregulated case, thus making the compliant farmer worse off than the unregulated farmer $\big(i.e.\ \Pi^D(n_D) < \Pi^o(n_o)\big)$.

Such a profit loss might be averted given the non-point-source characteristics of agricultural pollution. The fact that individual actions (i.e. nitrogen usage) can not be directly observed by a third party provides farmers incentives to keep both the nitrogen application at the profit maximizing level n_o, and the full amount of the subsidy $s^n(n_D^* - n_D)$, by falsely reporting compliance with regulation and nitrate use at the level n_D, without incurring the costs that compliance with the Directive entails. The payoff of such noncompliant behavior, if it remains undetected is:

$$\Pi_1^{nc}(n_o) = \Pi^o(n_o) + s^n(n_D^* - n_D) \tag{6}$$

However, farmers are aware that if detected in deviation from the action program rules then there is an exogenous probability q to be prosecuted and pay a fine $F \in (0, F_{\max}]$ if found guilty of causing nitrate leaching pollution by the Court.[21] Moreover, given that CAP payments are subject to the cross-

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[19] The standard is specified into 250 kg N/ha for livestock manure the first four years of the action program and 170 kg N/ha per year after the first four years, while the limit for manufactured nitrogen fertilizers is dependent on the crop requirements (EC, 1991).

[20] For further details see EU (1998; 2003).

[21] The fine is considered to be a fixed amount. Nevertheless, it may be set to cover the damage caused and the regulator's cost (DEFRA, 2004).

compliance principle,[22] the detected noncompliant farmer faces a reduction or even cancellation of provided payments by the amount:

$$\gamma s^n(n_D^* - n_D)$$

where $\gamma \in (0,1]$ is the cross-compliance reduction rate.

Let p the auditing probability, to be specified more precisely later. If the deviating farmer i is caught, his expected payoff is given by the profit maximizing profits plus the amount of the agri-environmental payment left after the imposition of the cross-compliance penalty, minus the legislatively imposed fine:

$$\begin{aligned}\Pi_2^{nc}(n_o) &= \Pi^o(n_o) + s^n(n_D^* - n_D)(1-\gamma p) - qpF \\ &= \Pi^o(n_o) + s^n(n_D^* - n_D) - p\Upsilon\end{aligned}$$

where $p\Upsilon = p\left[s^n\gamma(n_D^* - n_D) + qF\right]$ represents the total expected penalty for noncompliance with the environmental requirements.

To ensure that the deviating farmer incurs a positive cost, if inspected, and that his payoff is lower than the payoff in both the compliant and unregulated case, the structure of penalties should be such that:

$$\Pi_1^{nc}(n_o) > \Pi^o(n_o) > \Pi^D(n_D) > \Pi_2^{nc}(n_o)$$

Despite the adequate compliance incentives, the final decision to comply or not depends mostly on the inspection probability given the fact that individual nitrogen usage can not be directly observed and thus compliant behavior is not directly verifiable. Let p be the probability that deviating activity is detected by an environmental agency undertaking a number of random inspections:

$$p = p(\beta, \boldsymbol{\omega}_v) \quad with \ \frac{\partial p(\beta)}{\partial \beta} > 0 \text{ and } \frac{\partial^2 p(\beta)}{\partial \beta^2} < 0 \tag{7}$$

where ω_v is a vector of parameters affecting the probability of regulation (i.e. legislative procedures, transaction costs) and β is the monitoring effort (i.e. on the spot visits) required for the realization of an auditing scheme. By (7) the detection probability is increasing in undertaken monitoring effort and displays diminishing returns in β.

Hence, the expected profits of the deviating farmers are:

$$\begin{aligned}\mathcal{E}\Pi^{nc}(n_o) &= \Pi^o(n_o) + s^n(n_D^* - n_D)(1 - p(\beta)\gamma) - p(\beta)qF \qquad (8)\\ &= \Pi^o(n_o) - z(s^n, n_D^*, \gamma, q, F, \beta)\end{aligned}$$

where $z(s^n, n_D^*, \gamma, q, F, \beta) = [p(\beta)qF - s^n(n_D^* - n_D)(1 - p(\beta)\gamma)]$ involves the expected penalty imposed on the detected, noncompliant farmer. The total derivative of the expected penalty z with respect to policy parameters and

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[22] The cross-compliance principle involving partial or full removal of aid in the event of deviation from defined standards (EC, 1999), including compliance with the provisions of ND as foreseen by the 2003 CAP reform (EU, 2003; Aquamedia, 2006).

monitoring effort indicates that the noncompliance penalty increases as the performance standard (n_D^*), the undertaken monitoring effort (β) and the enforcement mechanism (q, F, γ) become stricter, while it decreases as the nitrogen usage subsidy (s^n) increases.[23]

Therefore, the sufficient condition for compliance is:

$$\Pi^D(n_D) > \Pi^o(n_o) - z(\mathbf{g}, \beta)$$

depending on the magnitude of the undertaken monitoring effort β and thus the inspection probability $p(\beta)$ as well as the rest of the policy parameters summarized by the vector $\mathbf{g} = (\gamma, s^n, F, n_D^*, q)$.

3 Implementation of the Directive

Monitoring effort is of crucial importance both for the detection of potential violators and the stimulation of compliance through the enforcement of foreseen sanctions. Consider an environment agency (EA) engaging into costly and perfectly accurate monitoring in order to induce the majority or even the entire population of farmers to comply with the aims of the Directive.[24] The selection of the monitoring effort level β that accomplishes this goal can either be arbitrary, based on the alternative behavioral rules adopted by farmers, or it can be optimal in the sense that it is obtained by minimizing a social welfare criterion conditional to the assumed behavioral rules which involve either full or bounded rationality.

3.1 Arbitrary Regulation Design

Assume that the level of monitoring is chosen arbitrarily based on a behavioral context involving initially full and then bounded rationality.

3.1.1 Fully Rational Compliance Decisions

Given the policy $\{s^n, n_D^*, \gamma, q, F, \beta\}$ the choice between the *optimal compliance decision* (n_D) and the *optimal noncompliance decision* (n_o) depends on the structure of payoffs under each strategy. Fully rational farmers will decide about complying or not by maximizing expected profits. Thus, farmers' optimal response implies:

$$n^* = \begin{cases} n_D & if \quad \Pi^D(n_D) > \mathcal{E}\Pi^{nc}(n_o) \\ n_o & if \quad \Pi^D(n_D) < \mathcal{E}\Pi^{nc}(n_o) \end{cases}$$

Assume that a minimum monitoring effort value $\beta^{\min}$ and thus a minimum inspection probability $p\left(\beta^{\min}\right)$ exists for given values of the rest of the parame-

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[23] It holds $\frac{dz}{ds^n}, \frac{dz}{dn_D^*} < 0$ and $\frac{dz}{dq}, \frac{dz}{dF}, \frac{dz}{d\gamma}, \frac{dz}{d\beta} > 0$.

[24] The EA can not influence the range of the policy parameters $\mathbf{g} = \left(\gamma, s^n, F, n_D^*, q\right)$ but only the level of undertaken monitoring effort (β).

ters, making farmers indifferent between the complying and deviating strategy, in the sense that:

$$p\left(\beta^{\min}\right):\Pi^{D}(n_{D})=\mathcal{E}\Pi^{nc}\left(n_{o}\right) \tag{9}$$

Henceforth, if the undertaken monitoring effort exceeds the minimum value required to induce compliance, then the noncompliance decision is not profit maximizing and farmers' optimum response is $n^{*}=n_{D}$. Individual farmers perceive that the imposition of the noncompliance sanctions is more probable and thus prefer the profit losses that compliance entails rather than the losses involved by detected noncompliance. Given the homogeneity assumption the population of farmers adopts a monomorphic behavior characterized by full compliance in the sense that all the farmers adopt the optimal compliance decision. In the opposite case, if monitoring effort is less than $\beta^{\min}$ then the inspection probability is not high enough to stimulate compliance. The optimal noncompliance decision exists and the optimum response of the population of farmers is $n^{*}=n_{o}$, meaning that no farmer complies with the Directive.

If the structure of the policy regime is not modified over time and the environmental agency precommits to the chosen monitoring effort then the population takes at a given time t a "once and for all" decision, that is retained in the future, implying either full compliance or noncompliance with the statutory environmental requirements of the Directive. This requires that the examined public voluntary agreement has "non-surprise" features in the sence that both the environmental agency and the policy maker[25] offer assurances to regulated agents that they will not change the terms of the agreement (i.e. g,β), in response to changing environmental protection needs (Langpap and Wu, 2004). Therefore, it holds that:

Proposition 1 *If monitoring effort is chosen arbitrarily, based on the assumption that farmers decide about complying or not by using profit maximizing behavior, then the entire population of farmers adopts a monomorphic behavior which persists in the long-run. If* $\beta>\beta^{\min}$ *then the optimum compliance decision* $n^{*}=n_{D}$ *is undertaken and there is full compliance of the population, while if* $\beta<\beta^{\min}$ *the optimum noncompliance decision* $n^{*}=n_{o}$ *is undertaken and there is noncompliance of the population.*

3.1.2 Compliance Decisions under Imitating Behavioral Rules

Under bounded rationality farmers ignore the exact structure of payoffs and form anticipations about the policy impacts. At a given time t the population of farmers is divided in two groups, following different strategies concerning compliance with the Directive. Let $x(t)$ be the proportion of agents adopting the compliant strategy at time t, while $x_{N}(t)$ the remaining proportion deviating from defined standards and retaining the profit maximizing nitrogen usage level n_{o}, with $x(t)+x_{N}(t)=1$.

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[25] The policy maker defines the range of the policy parameters $\mathbf{g}=\left(\gamma,s^{n},F,n_{D}^{*},q\right)$.

The proportion of farmers complying with the Directive evolves in time given the fact that farmers learn the true structure of payoffs via their interaction. This involves that in every time period dt there is a positive probability kdt that a agent i will compare its profits and consequently his strategy, with the corresponding profits and strategy of another randomly chosen agent j.[26] If i perceives that j's profits are sufficiently higher, then he switches his strategy. Under imperfect information concerning the difference in the expected profits of the two strategies, due to uncertainty about the actual auditing probability and possible uncertainty regarding the true cost functions, the probability that farmer i will change strategy increases the higher the profits difference is. Particularly, agent i that did not comply with the directive standard (4) at time t, might decide to switch strategy and comply, by imitating the complying agent if the expected profits $\mathcal{E}\Pi^{nc}$, are less than the profits $\Pi^D(n_D)$ of the complying agent. Hence, the probability that a non-complying farmer will change his strategy and ultimately comply with the Directive is given as:

$$P_{NV}^t = \left\{ \begin{array}{cc} \beta\left[\Pi(n_D) - \Pi^o\left(n_o\right) + z\left(\mathbf{g}, \beta\right)\right] & \text{for } \Pi^D(n_D) > \mathcal{E}\Pi^{nc} \\ 0 & \text{for } \Pi^D(n_D) \leq \mathcal{E}\Pi^{nc} \end{array} \right.$$

The expected proportion of farmers that decide to comply at time $t + dt$ is:

$$\begin{aligned} \mathcal{E}x^{t+dt} &= x^t + \kappa dt x^t \sum_{j=1}^{v} x_N \beta(\Pi^o\left(n_o\right) - \Pi^D(n_D)) \\ \mathcal{E}x^{t+dt} &= x^t + \kappa dt x^t \beta(\Pi^D(n_D) - \bar{\Pi}(n)) \end{aligned}$$

where $\bar{\Pi}(n) = x\Pi^D(n_D) + (1-x)\mathcal{E}\Pi^{nc}(n_o)$ denotes average profits for the whole population.

Since the population of farmers is assumed to be large, $\mathcal{E}x^{t+dt}$ can be replaced by x^{t+dt}. Furthermore, if we subtract from both sides the term x^t, divide by dt and take the limit as $dt \to 0$, an equation describing the motion of the group of compliant agents x over time is derived:

$$\dot{x} = \kappa\delta x^t \left[\Pi^D(n_D) - \bar{\Pi}(n)\right]$$

This is the replicator dynamics equation, indicating that the frequency of the compliance strategy increases when its profits $\Pi^D(n_D)$ are above the average profits $\bar{\Pi}(n)$. Thus proportional imitation rules can be modelled by replicator dynamics. After substituting $\bar{\Pi}(n)$ the replicator dynamics equation is rewritten as:

$$\dot{x} = \kappa\delta x(1-x)[z\left(\mathbf{g}, \beta\right) - \Delta\Pi_D^o] \tag{10}$$

where $\kappa\delta$ are constant factors that affect the rate of adjustment to stationarity and are often set equal to unit without affecting the stability analysis. The expression $[z\left(\mathbf{g}, \beta\right) - \Delta\Pi_D^o] = \Omega(\beta)$ represents the divergence of profit losses

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[26] For details see Schlag (1998), Gindis (2000) and Binmore (1992).

under the deviating and compliant strategy compared to the no regulation case, defined as $z\left(\mathbf{g},\beta\right)=\left(\Pi^{o}(n_{o})-E\Pi^{nc}(n_{o})\right)$ and $\Delta\Pi_{D}^{o}=\left(\Pi^{o}\left(n_{o}\right)-\Pi^{D}(n_{D})\right)$.

By setting $\dot{x}=0$ in (10) we obtain the steady states of the replicator dynamic. It follows that in the long-run the population of farmers converges to a monomorphic critical point characterized either by full compliance $(x_{1}^{*}=1)$ or noncompliance $(x_{2}^{*}=0)$ with the Directive. To show this consider the stability condition:

$$\frac{d\dot{x}}{dx}=(1-2x)\Omega(\beta) \tag{11}$$

This condition implies that full compliance is the evolutionary stable steady state, in the sense that $\frac{d\dot{x}}{dx}\left.\right|_{x_{1}^{*}=1}<0$, if the divergence of profit losses $\Omega(\beta)$ is positive. The mechanism operates in the following way. Assume that there is a critical value of monitoring effort $(\tilde{\beta})$ setting the profit loss divergence equal to zero and thus making farmers indifferent between the considered strategies:

$$\tilde{\beta}:\Omega(\tilde{\beta})=0$$

This critical value is similar to the minimum monitoring effort value $\beta^{\min}$ defined under unbounded rationality, and behaves as a bifurcation parameter since the sign of Ω and thus the stability of the steady states, depend on the magnitude of the undertaken monitoring effort β relative to the critical value $\tilde{\beta}$. Therefore the imposition of the more costly noncompliance sanctions, which are reflected in $z\left(\mathbf{g},\beta\right)$, becomes more likely if the undertaken monitoring effort β, exceeds the critical value $\tilde{\beta}$. In such a case the profit losses that compliance entails are preferred to losses involved by detected noncompliance, inducing in the long run the entire population of farmers to adopt the optimum compliance decision $n^{*}=n_{D}$, in the sense that $\lim_{t\to\infty}x=1$.

Therefore, when the environmental agency precommits itself to an announced fixed monitoring effort β it holds:

Proposition 2 *If monitoring effort is chosen arbitrarily based on the assumption that farmers decide about complying or not by following proportional imitation rules then the population of farmers converges always to a monomorphic steady state. If* $\beta>\tilde{\beta}$ *then the share of compliant farmers increases over time resulting eventually into full compliance* $x_{1}^{*}=1$ *with the Directive, while if* $\beta<\tilde{\beta}$ *then the proportion of complying farmers diminishes over time resulting into noncompliance* $x_{2}^{*}=0$.

The total differential of $\Omega(\mathbf{g},\tilde{\beta})=0$ with respect to the policy parameters $\mathbf{g}$ indicates that given the costs of monitoring effort the target of full compliance can be attainable via the realization of less monitoring effort if the "mixed" policy is characterized by a laxer performance standard (n_{D}^{*}), an increased rural development subsidy (s^{n}) and / or a stricter enforcement mechanism (q,F,γ), as it can be seen by:

$$\frac{d\tilde{\beta}}{dn_D^*} = -\frac{p(\tilde{\beta})s^n\gamma}{dz/d\beta} < 0 \quad \text{and} \quad \frac{d\tilde{\beta}}{ds^n} = -\frac{(n_D^* - n_D)p(\tilde{\beta})\gamma}{dz/d\beta} < 0$$

$$\frac{d\tilde{\beta}}{dq} = -\frac{dz/dq}{dz/d\beta} < 0, \; \frac{d\tilde{\beta}}{dF} = -\frac{dz/dF}{dz/d\beta} < 0 \quad \text{and} \quad \frac{d\tilde{\beta}}{d\gamma} = -\frac{dz/d\gamma}{dz/d\beta} < 0$$

This implies that under the proper design of the policy parameters $\mathbf{g}$ the range of monitoring effort values β that induce full compliance can become wider, allowing the environmental agency to achieve full compliance by committing to a lower monitoring effort value and thus incurring less monitoring expenses. In this sense there is trade-off between the different policy instruments for attaining full compliance.

Under the replicator dynamics imitation rule the aggregate nitrate emissions are affected by the decisions to comply with the Directive. Therefore equation (1) is further specified as:

$$N = v\{xn_D + (1-x)n_o)\} \tag{12}$$

It is notable that if the environmental agency chooses the monitoring effort value (or inspection probability) based on observations of compliance fraction x and / or aggregate emissions N (or equivalently the aggregate nitrogen input usage n), then the inspection probability (7) with joint dependence on compliance status x and stocks would be:

$$p(t) = p\left(\beta\left(x, N\right), \boldsymbol{\omega}_c\right) \quad \textit{with} \; \frac{\partial p}{\partial x} < 0 \text{ and } \frac{\partial p}{\partial N} > 0$$

Under such a generalised inspection probability, the replicator dynamic equation (10) is redefined as:

$$\dot{x} = \kappa\delta x(1-x)[z\left(\mathbf{g}, \beta\left(x, N\right)\right) - \Delta\Pi_D^o]$$

where the associated stability condition is:

$$\frac{d\dot{x}}{dx} = (1-2x)\Omega(\beta\left(x, N\right)) + x(1-x)\frac{\partial p}{\partial \beta}\frac{\partial \beta}{\partial x}\Upsilon$$

Under an inspection probability dependent on (x, N), the replicator dynamic equation defines two monomorphic equilibrium points, $x_1^* = 1$ and $x_2^* = 0$. Nevertheless, there is a potential third equilibrium point $x_3^* \in (0,1)$, which satisfies the equilibrium condition $\dot{x} = 0$ and involves partial compliance of the regulated population. This steady state is determined by a critical pair of compliance fraction and aggregate emissions that sets divergence of profit losses equal to zero, defined as $\left(\hat{x}, \hat{N}\right) : \Omega\left(\beta\left(\hat{x}, \hat{N}\right)\right) = 0$. The existence of the polymorphic steady state and thus the type of the prevailing equilibrium under this generalized case, depends on the magnitude of the critical pair $\left(\hat{x}, \hat{N}\right)$ and the monomorphic pair values, (x_1^*, N_1^*) and (x_2^*, N_2^*).

Given the assumptions that $\frac{\partial\beta}{\partial x} < 0$ for $x \in [0,1]$ and $\frac{\partial\beta}{\partial N} > 0$, it holds $\Omega > 0$ that for any $(x, N) < \left(\hat{x}, \hat{N}\right)$ and that $\Omega < 0$ for any $(x, N) > \left(\hat{x}, \hat{N}\right)$. Hence, if the critical pair $\left(\hat{x}, \hat{N}\right)$ lays in the interval $(0,1)$, in the sense that $(x_1^*, N_1^*) > \left(\hat{x}, \hat{N}\right) > (x_2^*, N_2^*)$, the associated stability conditions are:

$$\frac{d\dot{x}}{dx}\left|_{x_1^*=1}\right., \frac{d\dot{x}}{dx}\left|_{x_1^*=1}\right. > 0 \text{ and } \frac{d\dot{x}}{dx}\left|_{\in(0,1)}\right. < 0$$

involving that both monomorphic equilibria are unstable, while the polymorphic equilibrium x_3^* is stable. In the opposite case that $\left(\hat{x}, \hat{N}\right)$ lies outside the interval $(0,1)$, meaning that either $\left(\hat{x}, \hat{N}\right) > (x_1^*, N_1^*) > (x_2^*, N_2^*)$ or $(x_1^*, N_1^*) > (x_2^*, N_2^*) > \left(\hat{x}, \hat{N}\right)$, the population converges to a monomorphic steady state involving either full or no com-pliance of the population with the Directive.

It is underlined that under an inspection probability defined either as $p(\beta\,(x))$ or $p(\beta(N))$, the long-run behavior of the population is identical to the behavior under the under the generalized inspection probability $p(\beta(x, N))$.[27]

It can be concluded thus that:

Proposition 3 *If monitoring effort is chosen arbitrarily based on the imitation dynamics rule and the state variables of the problem, the regulated population converges either to a polymorphic or monomorphic steady state.*

Finally, it is worth mentioning that under the proper design of the policy parameters the polymorphic steady states can be driven closer to the full compliance steady state.[28]

3.2 Optimal Regulation Design

Even though the unintended generation of nitrates emission flows offers private benefits to individual farmers, their decisions create external costs for the rest of society (Chambers and Quiggin, 1996). Let $D(N)$ be the social damage caused by nitrates leaching that is assumed to be a linear function of aggregate nitrates leaching. Given the assumed direct, one-to-one relation between individual nitrate leaching N_i and nitrogen input n_i, social damages are given by:

$$D(N) = \alpha N = a\sum_{i=1}^{v} n_i = avn$$

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[27] In the case of $p(\beta\,(x))$ there is a critical compliance fraction defined as $\hat{x}$ setting $\Omega = 0$, while in the case of $p(\beta\,(N))$ there is a critical value of aggregate emissions $\hat{N}$ setting $\Omega = 0$ respectively. In each case the type of the evolutionary stable critical point depends on the relation between the critical value and the associated monomorphic values. Hence, the population converges to a polymor-phic steady state either if $x_1^* > \hat{x} > x_2^*$ or $N_1^* > \hat{N} > N_2^*$.

[28] For the analysis of regulation in common pool resources under imitation dynamics, see Xepapadeas (2005).

where α represents the constant marginal damage of aggregate emission flows with $\frac{\partial D(N)}{\partial N} = a > 0$.

Monitoring effort β required to verify compliance is also costly to society since it requires resources. It is usually financed by social funds raised through taxes and furthermore involves transaction costs to the environmental agent. The associated monitoring costs are:

$$m(\beta) \quad with \quad \frac{\partial m(\beta)}{\partial \beta} > 0 \quad and \quad \frac{\partial^2 m(\beta)}{\partial \beta^2} > 0$$

characterized by $\frac{\partial m(0)}{\partial \beta} > 0$.

Consider that the environmental agency selects the monitoring effort level in an optimal way in order to minimize the aggregate social costs SC, defined as the sum of monitoring costs and environmental damages from nitrates leaching:

$$\min_{\beta} SC = \min_{\beta} \{m(\beta) + D(n)\} \tag{13}$$

conditional to the expression of aggregate nitrates emission flows and the behavioral rule considered each time.

3.2.1 Fully Rational Compliance Decisions

If the environmental agent considers that farmers are fully rational then the minimization problem is conditional to the compliance constraint (9). Under this context the problem is static and given by:

$$\begin{aligned}
&\min_{\beta} \{m(\beta) + \alpha v n)\} \\
&\text{s.t.} \\
&\qquad \Pi^D(n_D) = \mathcal{E}\Pi^{nc}(n_o) \\
&\qquad n = \sum_{i=1}^{v} n_i = vn
\end{aligned}$$

The Lagrangean of the problem is:

$$L(\beta, x, \mu) = m(\beta) + \alpha v n + \mu \left\{\mathcal{E}\Pi^{nc}(n_o) - \Pi^D(n_D)\right\}$$

where μ is the Langrangean multiplier denoting the impact of a marginal change in the payoff of complying agents on the value function J^* of aggregate social cost and is considered to represent a marginal cost (*i.e.* $\mu > 0$).

The associated first-order-optimality conditions involve:

$$\frac{\partial L}{\partial \beta} = \frac{\partial m(\beta)}{\partial \beta} - \mu \frac{\partial p(\beta)}{\partial \beta}(qF\gamma + s^n(n_D^* - n_D)) = 0 \quad \text{(14a)}$$

$$\frac{\partial L}{\partial \mu} = \mathcal{E}\Pi^{nc}(n_o) - \Pi^D(n_D) = 0 \quad \text{(14b)}$$

$$\begin{aligned}
\Pi^D(n_D) &= [pf(\mathbf{x}, n) - \mathbf{wx} - w^n n + s^n(n_D^* - n_D)] \\
\mathcal{E}\Pi^{nc}(n_o) &= \Pi^o(n_o) + s^n(n_D^* - n_D)(1 - p(\beta)\gamma) - p(\beta)qF
\end{aligned}$$

In the absence of a budget constrained, the compliance constraint (14b) determines the optimal $\beta : \beta^* = \beta^{\min}$. Then the Lagrangean multiplier μ is determined by (14a) for $\beta = \beta^*$. Since $\partial \mathcal{E}\Pi^{nc}(n_o)/\partial\beta < 0$, if the regulator actually applies monitoring effort $\hat{\beta} = \beta^* + \varepsilon = \beta^{\min} + \varepsilon$, $\varepsilon > 0$, full compliance is attained. This is a 'knife-edge' result induced by the fully rational behavior of the farmers regarding their compliance decisions. If an effective budget constraint of the form $m(\beta) \leq B$ is present, then monitoring effort will be chosen at a level $\bar{\beta}(B) : m(\bar{\beta}) = B$. If $\bar{\beta}(B)$ is less than the minimum value $\beta^{\min}$ required to induce compliance, then the compliance incentives are inadequate and the entire population of farmers ends up adopting the noncompliance decision rule $n^* = n_o$. In the opposite case the compliance strategy is the prevailing strategy and the population is characterized by full compliance with the Directive and the optimum compliance decision $n^* = n_D$ occurs in the long-run.

Proposition 4 *If monitoring effort is selected optimally based on the assumption that farmers decide about complying or not by using profit maximizing behavior, then the population always adopts a monomorphic strategy. If a budget constraint is not effective, then full compliance is attained by choosing* $\hat{\beta} = \beta^{\min} + \varepsilon$. *If a budget constraint is effective, then if* $\bar{\beta}(B) > \beta^{\min}$ *the optimal compliance decision is adopted and the population is characterized by full compliance, while if* $\bar{\beta}(B) < \beta^{\min}$ *the optimal noncompliance decision is adopted and noncompliance emerges.*

3.2.2 Compliance Decisions under Imitating Behavioral Rules

Under imitating behavioral rules modelled by replicator dynamics (10), the regulator's problem is:

$$\min_{\beta} \int_0^{\infty} \exp(-\rho t) \{m(\beta) + \alpha \hat{n}\} \, dt$$

s.t.

$$\dot{x} = x(1-x)[z(\mathbf{g}, \beta) - \Delta\Pi_D^o]$$

$$\hat{n} = \sum_{i=1}^{v} n_i = v[x n_D + (1-x)n_o]$$

The Hamiltonian of the problem is:

$$H(\beta, x, \lambda) = m(\beta) + \alpha v[x n_D + (1-x)n_o] + \lambda x(1-x)\left[z(\mathbf{g}, \beta) - \Delta\Pi_D^o\right] \quad (15)$$

where λ is the associated costate variable reflecting the impact of a marginal change in the proportion of complying agents on minimum discounted social cost J^*:

$$\lambda = \frac{\partial J^*}{\partial x}$$

and represents the dynamic social shadow value of compliance, This value is expected to be negative since an increase in compliance reduce social costs (*i.e.* $\lambda < 0$).

The Pontryagin maximum principle[29] implies for $x \in (0,1)$:[30]

$$\frac{\partial H(\cdot)}{\partial \beta} = \frac{\partial m(\beta)}{\partial \beta} + \lambda x(1-x)\frac{\partial p(\beta)}{\partial \beta}\Upsilon = 0 \text{ if } \beta^* > 0 \quad \text{(16a)}$$

$$\text{if } \frac{\partial H(\cdot)}{\partial \beta} > 0, \beta^* = 0$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H(\cdot)}{\partial x} = \lambda[\rho - (1-2x)\Omega(\beta)] - \alpha v[n_D - n_o] \quad \text{(16b)}$$

The associated Arrow-type transversality conditions imply:

$$\lim_{t\to\infty} \exp(-\rho t)\lambda(t) \geq 0 \quad \text{and} \quad \lim_{t\to\infty} \exp(-\rho t)\lambda(t)x(t) = 0$$

Assume that condition (16a) determines an interior solution (*i.e.* $\beta^* > 0$). From the Implicit Function Theorem the optimal monitoring effort β^* minimizing the discounted aggregate social costs is:

$$\text{(16a)}: \quad \Rightarrow^{*}= \hat{\beta}\left(x, \lambda, \mathbf{g}\right) = \hat{\beta}(x, \lambda, \gamma, s^n, F, n_D^*, q) \quad (17)$$

At the monomorphic steady states the value of β depends on the magnitude of the policy parameters $\mathbf{g}$, while it is independent of the state and costate variables of the problem. Hence, for $x = x_1^* = 1$ or $x = x_2^* = 0$, we assume that $\beta = \beta_i^*, i = 1, 2$ is chosen such that both full compliance and no compliance are conditionally attracting for the replicator dynamic equation.[31] In particular, β_1^* is selected to set the divergence of profit losses $\left[z\left(\mathbf{g}, \beta_1^*\right) - \Delta\Pi_D^o\right] = \Omega(\beta_1^*)$ positive, in the sense that the compliant strategy is preferable in terms of profit losses, so that once the population converges to the full compliance steady state it does not diverge. On the other hand, β_2^* is selected to set the divergence

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[29] The second-order-condition $H_{\beta\beta}(\beta, x, \lambda)$ is positive implying that the optimal β^* minimizes the Hamiltonian $H(\beta, x^*, \lambda)$.

[30] Let $\Upsilon = \left[qF\gamma + s^n(n_D^* - n_D)\right]$.

[31] Both assumptions for the values of $\Omega\left(\beta_1^*\right)$ and $\Omega\left(\beta_2^*\right)$ are necessary for the definition of the stability properties of the monomorphic steady states. For further details see the Appendix.

of profit losses $[z(\mathbf{g}, \beta 2) - \Delta\Pi_D^o] = \Omega(\beta_2^*)$ negative, so as to set profit losses under the deviating strategy preferable to the losses involved by the compliant strategy.

By substituting (17) into (10) and (16b) the modified Hamiltonian dynamic system (MHDS) in the state-costate space is defined as:

$$\dot{x} = x(1-x)\left[z(\mathbf{g}, \hat{\beta}(x, \lambda, \mathbf{g})) - \Delta\Pi_D^o\right] \tag{18}$$

$$\dot{\lambda} = \lambda\left(\rho - (1-2x)\left[z(\mathbf{g}, \hat{\beta}(x, \lambda, \mathbf{g})) - \Delta\Pi_D^o\right]\right) - \alpha v \Delta\left(n\right)_o^D \tag{19}$$

where $[n_D - n_o] = \Delta\left(n\right)_o^D$. The solution of the MHDS determines the socially-optimal time paths $(x^*(t), \lambda^*(t))$ and the socially-optimal steady state equilibrium point $(x^\infty, \lambda^\infty)$ for the compliance fraction x and its shadow value λ, along with the corresponding socially-optimal monitoring effort path $\beta^*(x^*(t), \lambda^*(t))$.

By setting $\left(\dot{x} = \dot{\lambda} = 0\right)$, two types of possible steady states are determined for the long-run equilibrium compliance fraction:

$$\begin{aligned} &\text{Monomorphic}: \ x_1^* = 1, \quad x_2^* = 0 \\ &\text{Polymorphic}: \ x_3^* \in (0,1) : z(s^n, n_D^*, \gamma, q, F, \hat{\beta}(x, \lambda, \mathbf{g})) = \Delta\Pi_D^o \end{aligned}$$

Monomorphic critical points involve either full compliance (x_1^*) or full noncompliance (x_2^*) with the Directive. They are depicted by two isoclines vertical to the horizontal axis (figure 1).

[Figure 1]

The polymorphic steady state (x_3^*) is characterized by partial compliance and is implicitly defined by an isocline $x(\lambda)|_{\dot{x}=0}$ with the property:

$$\lambda(x)|_{\dot{x}=0} : \Omega(x, \lambda) = \left[z(s^n, n_D^*, \gamma, q, F, \hat{\beta}(x, \lambda, \mathbf{g})) - \Delta\Pi_D^o\right] = 0 \tag{20}$$

All the combinations (x, λ) along the $\lambda(x)|_{\dot{x}=0}$ isocline satisfy $\Omega(x, \lambda) = 0$. For combination (x, λ) outside this isocline $\Omega \neq 0$. For combinations located above the isocline $\Omega > 0$, while combinations located below are characterized by $\Omega < 0$.[32]

The $\lambda(x)|_{\dot{x}=0}$ expression is illustrated by an inverse "U" shaped isocline with maximum at $x = 1/2$, (figure 1) since its slope is:[33]

$$\frac{d\lambda}{dx}|_{\dot{x}=0} = -\frac{\partial\beta^*/\partial x}{\partial\beta^*/\partial\lambda} = \left\{ \begin{array}{lll} \frac{d\lambda}{dx}|_{\dot{x}=0} > 0 & for & x \in (0, 1/2) \\ \frac{d\lambda}{dx}|_{\dot{x}=0} < 0 & for & x \in (1/2, 1) \end{array} \right.$$

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[32] It is assumed that $\Omega(\beta_1^*)$ is positive, while $\Omega(\beta_2^*)$ is negative, so that both full compliance and no compliance are conditionally attracting for the replicator dynamic equation.

[33] This isocline does not intersect with the monomorphic isoclines of (18).

The slope can be interpreted as reflecting the relative variability of the monitoring effort β due to changes in the levels of the state and costate variables.[34]

By setting (19) equal to zero and substituting the steady-state equilibria of (18), the corresponding steady state shadow values of compliance are defined as:

$$\lambda^* \left|_{x_1^*=1} = \frac{\alpha v \Delta (n)_o^D}{\rho + \Omega(\beta_1^*)}, \ \lambda^* \right|_{x_2^*=0} = \frac{\alpha v \Delta (n)_o^D}{\rho - \Omega(\beta_2^*)} \text{ and } \lambda^* \left|_{x_3^* \in (0,1)} = \frac{\alpha v \Delta (n)_o^D}{\rho}\right.$$

By the assumptions for the values of $\Omega(\beta_1^*)$ and $\Omega(\beta_2^*)$ and the fact that $\alpha v \Delta (n)_o^D < 0$, it holds that λ is negative at all steady states and that $\lambda^* \left|_{x_3^* \in (0,1)}\right.$ is greater than $\lambda^* \left|_{x_1^*=1}\right.$ and $\lambda \left|_{x_2^*=0}\right.$. Therefore the monomorphic steady state shadow values λ_1^* and λ_2^* are below the $\lambda(x)|_{\dot{x}=0}$ isocline (see figure 1).

The behavior of the $\dot{\lambda} = 0$ isocline in the (x, λ) space is ambiguous given that the sign of the slope:

$$\frac{d\lambda}{dx}\left|_{\dot{\lambda}=0} = -\lambda \left\{ 2\Omega(\beta) - (1-2x) \frac{\partial p}{\partial \beta} \frac{\partial \beta}{\partial x} \Upsilon \right\} \middle/ \left\{ \rho - (1-2x) \left[\Omega(\beta) + \lambda \frac{\partial p}{\partial \beta} \frac{\partial \beta}{\partial \lambda} \Upsilon \right] \right\}\right. \tag{21}$$

can be determined only in the neighborhood of the steady-states and around the compliance value $(x = 1/2)$ (see figure 1).[35] Whether the $\dot{\lambda} = 0$ curve is continuous or not at the monomorphic values of λ depends on the the assumptions for the optimal values β_1^* and β_2^*. In particular $\lambda_1^*(\beta_1^*)$ and $\lambda_2^*(\beta_2^*)$ are continuity points if:

$$\lim_{x \to 1} \hat{\beta}(x, \lambda, \mathbf{g}) = \beta_1^* \quad \text{and} \quad \lim_{x \to 0} \hat{\beta}(x, \lambda, \mathbf{g}) = \beta_2^* \tag{22}$$

simultaneously occur. However, it is more natural to assume that the hair-line case of (22) is not satisfied and that the monomorphic values of λ are isolated points of $\dot{\lambda} = 0$.

Depending on the shape of $\dot{\lambda} = 0$ and $\lambda(x)|_{\dot{x}=0}$ in the (x, λ) space, the $\dot{\lambda} = 0$ isocline can be depicted in several ways. If the two isoclines do not intersect, then $\dot{\lambda} = 0$ represents either a "U" shaped curve, or a curve with decreasing and increasing parts (figure 2). On the other hand, if $\dot{\lambda} = 0$ intersects the $\Omega(x, \lambda) = 0$ once for $x \in (0, 1)$, then it assigns a fixed value to λ for the monomorphic steady state of (18) and defines a curve with decreasing and / or increasing parts for the remaining x values (see figure 3). Finally, in the

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[34] From the partial derivative $\frac{d\beta^*}{dx} = (-H_{\beta\beta})^{-1} \left[\lambda(1-2x)\frac{\partial p(\beta)}{\partial \beta}\Upsilon\right]$, it is evident that $\frac{d\lambda}{dx} > 0$ for $x \in (0, 1/2)$ and $\frac{d\lambda}{dx} < 0$ for $x \in (1/2, 1)$.

[35] Around the monomorphic equilibrium points the slope is $\frac{d\lambda}{dx}\Big|_{\dot{\lambda}=0}^{x_1^*} > 0$ and $\frac{d\lambda}{dx}\Big|_{\dot{\lambda}=0}^{x_2^*} < 0$. For $x = 1/2$ it holds $\frac{d\lambda}{dx}\Big|_{\dot{\lambda}=0}^{x_2^*} < 0$, while around the polymorphic steady states it can be seen that $\frac{d\lambda}{dx}\Big|_{\dot{\lambda}=0}^{x_3^*} > 0$ if $x_3^* \in (0, 1/2)$ and $\frac{d\lambda}{dx}\Big|_{\dot{\lambda}=0}^{x_3^*} < 0$ if $x_3^* \in (1/2, 1)$. At intermediate compliance values the slope sign is uncertain.

"knife-edge" case where the $\dot{\lambda} = 0$ isocline is symmetric around $x = 1/2$ and intersects the $\Omega(x, \lambda) = 0$ twice, it is defined by an inverse "U" shaped curve (see figure 4).[36]

[Figure 2]
[Figure 3]
[Figure 4]

The intersection of $\dot{\lambda} = 0$ and $\dot{x} = 0$ isocline defines the long-run equilibrium for x and λ. The total number and type of feasible socially-optimal equilibrium steady states (x_i^*, λ_i^*) depends on the shape of $\dot{\lambda} = 0$ and $\lambda(x)|_{\dot{x}=0}$ in the (x, λ) space. If $\dot{\lambda} = 0$ does not intersect with $\lambda(x)|_{\dot{x}=0}$, then the MHDS involves only two monomorphic steady states indicating full or non compliance (figure 2). On the other hand, if they intersect then the system is characterized by the two monomorphic equilibria and one polymorphic critical point (figure 3), given the fact that $\dot{\lambda} = 0$ meets $\lambda(x)|_{\dot{x}=0}$ only once, either at its increasing or decreasing part.[37] In this case if $\dot{\lambda} = 0$ and $\lambda(x)|_{\dot{x}=0}$ intersect at the increasing part of $\lambda(x)|_{\dot{x}=0}$, then the polymorphic rest point involves a small fraction of compliant farmers (*i.e.* $x_3^* \in (0, 1/2)$), while if they intersect at its decreasing part then it involves high proportion of compliant farmers (*i.e.* $x_3^* \in (1/2, 1)$).[38]

Given that monitoring effort is optimally chosen, the MHDS is characterized by multiple equilibria involving either full compliance (x_1^*, λ_1^*), noncompliance (x_2^*, λ_2^*) and/or partial compliance (x_3^*, λ_3^*) with the aims of the Directive. The stability properties of each critical point are examined in detail in Appendix. Stability analysis suggests that the system is characterized by multiple saddle points potentially connected by heteroclinic orbits (see Figure 5a,b). In particular, both monomorphic rest points and the polymorphic steady state involving a high level of compliance (*i.e.* $x_3^* \in (1/2, 1)$) satisfy the saddle point property, implying that for any initial compliance x^0, there exists an initial costate variable λ^0 such that the system converges to one of these steady states as $t \to \infty$. Convergence to a specific monomorphic or a polymorphic state depends on the specific initial compliance state. In this sense the emerging long run steady state exhibits history dependence.

[Figure 5a]
[Figure 5b]

Stability analysis also indicates that depending on the relative magnitude of marginal social benefits and costs steady states may include one stable and

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[36] It is worth mentioning that the $\dot{\lambda} = 0$ isocline cannot intersect with the monomorphic isoclines of (18) in the area defined above the $\lambda(x)|_{\dot{x}=0}$ isocline due to the fact that both $\lambda_1^* < \lambda_3^*$ and $\lambda_2^* < \lambda_3^*$. Furthermore, we exclude hairline cases where the $\dot{\lambda} = 0$ isocline is tangent to the $\Omega(x, \lambda) = 0$ curve.

[37] The slope at the polymorphic steady states is known, since the MHDS can not have multiple polymorphic steady states.

[38] Nevertheless, in the special case that $\dot{\lambda} = 0$ is symmetric around $x = 1/2$ and intersects the $\Omega(x, \lambda) = 0$ twice, the MHDS has two polymorphic critical points, with the same shadow value for λ (figure 4).

one unstable polymorphic steady states, with the stable steady state involving low level of compliance (*i.e.* $x_3^* \in (0, 1/2)$). If the structure of marginal social benefits and costs is such that the trace of the Jacobian determinant $Tr\left(J_3^*\right)$ of the MHDS evaluated at (x_3^*, λ_3^*) :

$$Tr\left(J_3^*\right) = \rho + \frac{\partial p}{\partial \beta} \Upsilon \left\{ x_3^*(1 - x_3^*) \frac{\partial \beta^*}{\partial x} - \lambda_3^*(1 - 2x_3^*) \frac{\partial \beta^*}{\partial \lambda} \right\}$$

is negative, then (x_3^*, λ_3^*) is a stable steady state. Furthermore, depending on the sign of the associated discriminant Δ, this steady state can be a stable focus where the approach path is characterised by oscillations (see figure 6) or a stable node without spilaring trajectories. In the special case that $Tr\left(J_3^*\right) = 0$ and $\Delta < 0$, then the polymorphic steady state is center where the system fluctuates around the rest point.

[Figure 6]

It worths mentioning that when the low compliance steady state is unstable, then there is a possibility that a limit cycle with counterclockwise movement exists around the given critical point (see figure 7). Given that the flow of the vector field (18) - (19) points outwards around the unstable steady state, a limit cycle denoted by L exists if a compact positively invariant region R exists such that the flow of the vector field is pointing inwards on its boundary.[39] In such a case all the (x, λ) combinations along the limit cycle L are stable states and under particular initial conditions the system can be trapped in a low level compliance area characterised by oscillating dynamics.

[Figure 7]

Hence, it can be concluded that:

Proposition 5 *If monitoring effort is chosen optimally based on the assumption that farmers decide about complying or not by following proportional imitation rules, then depending on the initial compliance state* x_0*, the population converges either to a monomorphic steady state involving full compliance* (x_1^*, λ_1^*) *or full noncompliance* (x_2^*, λ_2^*)*, or to a polymorphic steady state involving low or high levels of partial compliance* (x_3^*, λ_3^*)*. Depending on the topological properties of the resulting evolutionary equilibrium point, the approach dynamics can either be monotonic or oscillating.*

The slope of examined isoclines influences the discrepancies between the equilibrium compliance proportions associated with the polymorphic and monomorphic steady states. If the monitoring effort is more sensitive to changes in the compliance fraction (x), or alternatively less sensitive to changes in the shadow value of compliance (λ), then the isocline $\lambda(x)|_{\dot{x}=0}$ becomes steeper and the discrepancy between the polymorphic and monomorphic steady states increases,

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[39] For further details see Xepapadeas (2005). For technical details see Sastry (1999).

leading the polymorphic steady state closer to the central compliance proportion, $x = 1/2$.[40]

The short-run and the steady-state comparative statics analysis indicates that even though a reduced rural development (RD) subsidy (s^n), a lax enforcement mechanism (q, F, γ) and a stringent performance standard (n_D^*) induce a reduction in the short-run socially optimal monitoring effort in the polymorphic compliance range $x \in (0,1)$,[41] their impact on the steady-state monitoring effort value $\beta_i^\infty = \hat{\beta}_i\left(x_i^\infty, \lambda_i^\infty, \mathbf{g}\right)$ is ambiguous and crucially dependent on the relative magnitude of their short-run and long-run impacts on β.[42] Finally, it is worth mentioning that the short-run and steady-state monitoring effort values at the monomorphic compliance values are left unaffected by variations of the policy parameters $\mathbf{g} = (\gamma, s^n, F, n_D^*, q)$.

4 Implementation of the Directive under Accumulation of Monitoring Capital

Apart of the occasional random spot-checks (β), the performance of farmers can be also assessed through a network of sampling stations, monitoring nitrogen in soil, rootzone level, pilot fields and / or small watersheds (EC, 2002). Let I represent investment in monitoring capital in the form of field monitoring systems, laboratory equipment and scientific personnel that accumulates in time defining monitoring capital k. The net capital formation is described by:[43]

$$\dot{k} = I - \delta k \tag{23}$$

where $\delta \geq 0$ is the exponential depreciation rate of monitoring capital k.

The inspection probability (7) is redefined as a positive function of both monitoring effort and accumulated monitoring capital:

$$p\left(t\right) = p\left(\beta, k, \boldsymbol{\omega}_v\right) \quad with\ \frac{\partial p}{\partial k} > 0,\ \frac{\partial^2 p}{\partial k^2} < 0\ and\ \frac{\partial^2 p}{\partial k \partial \beta} > 0$$

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[40] In the opposite case the $\lambda\left(x\right)|_{\dot{x}=0}$ isocline is flatter and the discrepancies between the rest points decrease.

[41] The short-run comparative statics analysis results are summarized in:

Comparative Statics							
	x	λ	γ	s^n	n_D^*	q	F
β^*	?	$-$	$+$	$+$	$+$	$+$	$+$

where for compliance fraction values lying within the range $x \in (0, 1/2)$, it holds that $\frac{d\beta^*}{dx} > 0$, while for $x \in (1/2, 1)$ then $\frac{d\beta^*}{dx} < 0$.

[42] The steady-state comparative statics of the monitoring effort with respect to parameters g are given by:

$$\frac{\partial \beta_i(\infty)}{\partial \mathbf{g}} = \left[\frac{\partial \hat{\beta}_i(\infty)}{\partial x}\frac{\partial x_i^\infty}{\partial \mathbf{g}} + \frac{\partial \hat{\beta}_i(\infty)}{\partial \lambda}\frac{\partial \lambda_i^\infty}{\partial \mathbf{g}}\right] + \frac{\partial \hat{\beta}_i(\infty)}{\partial \mathbf{g}}$$

where $\frac{\partial \hat{\beta}_i(\infty)}{\partial \mathbf{g}}$ and $\left(\frac{\partial \hat{\beta}_i(\infty)}{\partial x}\frac{\partial x_i^\infty}{\partial \mathbf{g}} + \frac{\partial \hat{\beta}_i(\infty)}{\partial \lambda}\frac{\partial \lambda_i^\infty}{\partial \mathbf{g}}\right)$ are the short-run and long-run powers exercised by policy parameters g.

[43] It is considered that due to technological and / or budgetary restrictions the accumulated monitoring capital does not alter the problem of non-point-source pollution into a point-source problem, in the sense that full observability is unattainable.

with diminishing returns both in β and k.[44]

In this context the expected deviating payoff (8) is rewritten as:

$$\mathcal{E}\Pi^{nc}(n_o) = \Pi^o\left(n_o\right) - z(\beta, k, \mathbf{g})$$

Investment in monitoring capital is costly to society since it involves purchase of equipments and potential adjustment costs for the environmental agency. In such a case the function of monitoring cost (CM) is given by a separable function of the form:

$$CM = m(\beta) + bI + q(I)$$

where b is the per unit purchase price of monitoring capital and $q(I)$ the convex adjustment costs including installation costs and personnel training (Xepapadeas, 1992).

It is evident that a farmer's decision to comply or not depends on both the magnitude of monitoring effort and accumulated monitoring capital. This implies that in addition to the level of undertaken monitoring effort, the environmental agent must select the level of realized investment in monitoring capital (I) that ensures that the accumulated monitoring capital k is sufficient enough to stimulate full or at least partial compliance of the regulated population with the aims of the Nitrates Directive.

Under the assumption that there is no binding budget constraint to restrict investment in monitoring capital and given that monitoring effort is chosen to induce the optimum compliance decision, [45] then full compliance of the regulated population is attained if the arbitrarily chosen investment level is set both higher than (i) the value that makes the farmer indifferent between the compliant and deviating strategy, and (ii) the value that retains capital invariant over time $\left(I^{\diamondsuit} : \dot{k} = 0\right)$. Since capital depreciates over time (*i.e.* $\delta > 0$). this policy implies that the accumulated monitoring capital k does not decline over time and that the compliant strategy (n_D) is the optimal decision.

However, given (23), the accumulation of the required monitoring capital is a time-consuming process depending on the size of undertaken investment and the depreciation rate (δ).[46] The notion of time appears in the attainment of the desired long-run behavior. In particular, under the context of full rationality the critical capital level behaves as a bifurcation parameter since for lower values no farmer complies with the Directive's provisions, but as soon as $\hat{k}$ is reached and exceeded there is an automatic switch of all the members of the population to the compliant strategy. Such a direct convergence to the full compliance critical

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[44] Note that complementarity is assumed to exist between the monitoring effort and monitoring capital.

[45] This implies that monitoring effort should be set higher than $\beta^{\min}$ if the full rationality behavioral rule is considered, or $\tilde{\beta}$ under the assumption of bounded rationality.

[46] Even though there is an investment level $(\bar{I})$ that guarantees the instant accumulation of the required capital and thus the direct convergence to the desired long-run equilibrium, such a size of investment is unfeasible due to apparent budgetary and /or technological restrictions.

point is not expected under the bounded rationality context where the dynamics of the farmers' population appear quite differentiated.

Finally, if investment in monitoring capital is selected in an optimal manner, then the regulator pursues the minimum present value of the augmented aggregate social costs by defining the optimal path of both monitoring effort and monitoring investment conditional on aggregate nitrates emission flows, the capital accumulation differential equation (23) and the behavioral rule at each point in time. Under the full rationality behavioural rule there is no modification in the dynamics of the farmers' population since the regulated population retains a monomorphic behavior regarding compliance or not with statutory requirements, which depends on the magnitude of the applied optimum monitoring elements (β^*, I^*) compared to the critical values $\left(\beta^{\min}, \hat{I}\right)$. However, under the imitating behavioral rule the topological properties of the polymorphic steady state involving high compliance levels ($x_3^* \in (1/2, 1)$) are altered, since it no longer satisfies the saddle point property and exhibits identical properties to the low compliance steady state ($x_3^* \in (0, 1/2)$).

5 Conclusions

The non-point-source characteristics of agricultural pollution problems undermine the effectiveness of regulation to induce compliance with environmental considerations, rendering essential an effective monitoring and enforcement mechanism. Both the Council Nitrates Directive (91/676/EEC) and the agri-environmental programs of the second pillar of CAP have incorporated such mechanisms in their policy design in order to verify that regulated farmers are complying with statutory nitrogen usage standards, and that foreseen sanctions are enforced whenever noncompliant behavior is detected. The purpose of the present paper was to examine whether the compliance incentives associated with monitoring and enforcement under 91/676/EEC and the second pillar of CAP, are adequate enough to induce the majority or even the entire population of farmers to restrict, in the long-run, the nitrogen input usage to the level suggested by a regulatory regime combining both the aims of the Directive and the compensation payments of CAP. To do so, we considered a homogeneous population of farmers who in their decision of whether to comply or not with the provisions of regulation, may follow two alternative behavioral rules according to their rationality characteristics. The selection of the monitoring and enforcement scheme by the regulator, is characterized under two assumptions, regarding farmers' compliance: (i) an optimizing behavioral rule which occurs under full rationality and, (ii) an evolutionary imitation rule which occurs under bounded rationality.

Our results suggest that the compliance incentives of a given population of farmers are affected by both the rule for selecting monitoring effort by the regulator and the behavioral rules under which farmers decide their compliance strategy. If monitoring effort is chosen by the regulator arbitrarily, based either on farmers' optimizing behavior rule or on farmers' proportional imitation rule, then the entire population of farmers adopts a monomorphic behavior, involving either full compliance or full noncompliance. Full compliance can be guaranteed if the environmental agency precommits to a monitoring effort value set higher than the critical value which makes farmers indifferent between compliance and deviation under both behavioral rules. If monitoring effort level is chosen optimally via the minimization of a social welfare criterion, then the same monomorphic behavior emerges if the problem is constrained by the farmers' optimizing behavioral rule regarding compliance. If the social welfare criterion is minimized conditional to the farmers' imitation behavioral rule regarding compliance, then the population may also adopt a polymorphic behavior involving partial compliance. A further difference between the optimality and imitation behavioral rule is the timing of the occurrence of the long-run behavior. When farmers are fully rational in deciding about their compliance strategy, then there is an immediate switch to full compliance or not, since the population takes a "once and for all" decision. Under the replicator dynamics imitation rule there is a gradual change in the composition of the population, depending on the revealed information via the farmers' interaction over time. Finally, the enforcement problem of the given regulation was reexamined under the presence of an additional choice variable - investment in monitoring capital - indicating identical properties with the previous analysis regarding population dynamics.

The generalized framework developed in the present paper can be further employed for assessing populations' compliance incentives with given environmental regulations in a context of imperfect monitoring, in the sense that individual decisions (i.e. emissions, inputs usage) may not be inferred correctly and a farmer may be erroneously fined.[47] Furthermore, the long-run behavior of farmers's population can be also analyzed under different imitation behavioral rules such as the average profit principle and effective punishment principle, in order to detect potential modifications in the qualitative characteristics of the resulting steady-state equilibriums.[48] Finally, it would be interesting to simultaneously combine in an optimal regulation problem both the optimizing and the imitating behavioral rule for different parts of the population.

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[47] For details see Malik (1993).

[48] For details see Lipatov (2005).

Appendix

The state-costate MHDS (18) - (19) defines three potential socially-optimal equilibrium pairs (x_i^*, λ_i^*), implying either full compliance (x_1^*, λ_1^*), noncompliance (x_2^*, λ_2^*) or partial compliance (x_3^*, λ_3^*) with ND. To characterize the equilibrium type of each steady state the linearization matrix J around each critical point is evaluated, along with their traces $Tr(J) = \frac{d\dot{x}}{dx} + \frac{d\dot{\lambda}}{d\lambda}$, determinants $Det(J) = \frac{d\dot{x}}{dx}\frac{d\dot{\lambda}}{d\lambda} - \frac{d\dot{x}}{d\lambda}\frac{d\dot{\lambda}}{dx}$ and discriminants $\Delta = [Tr(J)]^2 - 4Det(J)$.

The linearization matrix J is in general given by:

$$J_i^* = \begin{bmatrix} (1-2x_i^*)\Omega(\beta_i^*) + x_i^*(1-x_i^*)\frac{\partial p}{\partial \beta}\frac{\partial \beta^*}{\partial x}\Upsilon & x_i^*(1-x_i^*)\frac{\partial p}{\partial \beta}\frac{\partial \beta^*}{\partial \lambda}\Upsilon \\ 2\lambda_i^*\Omega(\beta_i^*) - \lambda_i^*(1-2x_i^*)\frac{\partial p}{\partial \beta}\frac{\partial \beta^*}{\partial x}\Upsilon & \rho - (1-2x_i^*)\Omega(\beta_i^*) - \lambda_i^*(1-2x_i^*)\frac{\partial p}{\partial \beta}\frac{\partial \beta^*}{\partial \lambda}\Upsilon \end{bmatrix}$$

It can be seen that:

- Full compliance (x_1^*, λ_1^*)

In this case the Jacobian matrix is:

$$J_1^* = \begin{bmatrix} -\Omega\left(\beta_1^*\right) & 0 \\ 2\lambda_1^*\Omega\left(\beta_1^*\right) & \rho + \Omega\left(\beta_1^*\right) \end{bmatrix}$$

since $\frac{\partial \beta^*}{\partial x}\big|_{x_1^*}, \frac{\partial \beta^*}{\partial \lambda}\big|_{x_1^*} = 0$. Given that $\Omega(\beta_1^*)) > 0$ the trace and determinant around (x_1^*, λ_1^*) are:

$$Det\left(J_1^*\right) = -\Omega\left(\beta_1^*\right)\left(\rho + \Omega\left(\beta_1^*\right)\right) < 0 \quad \text{and} \quad Tr\left(J_1^*\right) = \rho > 0$$

indicating that the full compliance steady state $(x_1^* = 1, \lambda_1^*)$ satisfies the saddle point property.

- Full noncompliance (x_2^*, λ_2^*)

The Jacobian matrix is:

$$J_2^* = \begin{bmatrix} \Omega\left(\beta_2^*\right) & 0 \\ 2\lambda_2^*\Omega\left(\beta_2^*\right) & \rho - \Omega\left(\beta_2^*\right) \end{bmatrix}$$

since $\frac{\partial \beta^*}{\partial x}\big|_{x_2^*}, \frac{\partial \beta^*}{\partial \lambda}\big|_{x_2^*} = 0$. Hence, given that $\Omega(\beta_2^*)) < 0$ it holds:

$$Det(J_2^*) = -\Omega\left(\beta_2^*\right)\left(\rho + \Omega\left(\beta_2^*\right)\right)) < 0 \quad \text{and} \quad Tr\left(J_2^*\right) = \rho > 0$$

indicating that the non compliance steady state $(x_2^* = 0, \lambda_2^*)$ satisfies also the saddle point property.

- Partial compliance (x_3^*, λ_3^*)

In this case $\Omega(\beta^*(x_3^*, \lambda_3^*)) = 0$, $\frac{\partial\beta^*}{\partial\lambda}\big|_{x_3^*} < 0$ and $\frac{\partial\beta^*}{\partial x}\big|_{x_3^*} > 0$ if $x_3^* \in (0, 1/2)$, while $\frac{\partial\beta^*}{\partial x}\big|_{x_3^*} < 0$ if $x_3^* \in (1/2, 1)$. The Jacobian matrix around the polymorphic steady state is given by:

$$J_3^* = \begin{bmatrix} x_3^*(1-x_3^*)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial x}\Upsilon & x_3^*(1-x_3^*)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial\lambda}\Upsilon \\ -\lambda(1-2x_3^*)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial x}\Upsilon & \rho-\lambda(1-2x)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial\lambda}\Upsilon \end{bmatrix}$$

where the associated trace and determinant are given by:

$$\begin{aligned} Det\,(J_3^*) &= \rho x_3^*(1-x_3^*)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial x}\Upsilon \\ Tr\,(J_3^*) &= \rho + \frac{\partial p}{\partial\beta}\Upsilon\left\{x_3^*(1-x_3^*)\frac{\partial\beta^*}{\partial x} - \lambda_3^*(1-2x_3^*)\frac{\partial\beta^*}{\partial\lambda}\right\} \end{aligned}$$

If $x_3^* \in (0, 1/2)$ then $Det\,(J_3^*) > 0$, while sign of $Tr\,(J_3^*)$ is uncertain. If the structure of marginal social benefits and costs is such that the stability requirement:

$$\rho + x_3^*(1-x_3^*)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial x}\Upsilon < -\lambda_3^*(1-2x_3^*)\frac{\partial p}{\partial\beta}\frac{\partial\beta^*}{\partial\lambda}\Upsilon$$

is satisfied and $Tr\,(J_3^*) < 0$ then (x_3^*, λ_3^*) is a stable steady state and depending on the sign of the discriminant Δ it can be a stable proper node ($if\ \Delta = 0$), a stable improper node ($if\ \Delta > 0$), while if $\Delta < 0$ it is stable focus. In the special case that $Tr\,(J_3^*) = 0$ and $\Delta < 0$ then the polymorphic steady state is center.[49]

If $x_3^* \in (1/2, 1)$ then the polymorphic steady state is a saddle point given that $Det\,(J_3^*) < 0$, while in the special case that $x_3^* = 1/2$ then the dynamic system experiences a nonhyperbolic point.

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[49] For details see Xepapadeas (1997), pages 267-266.

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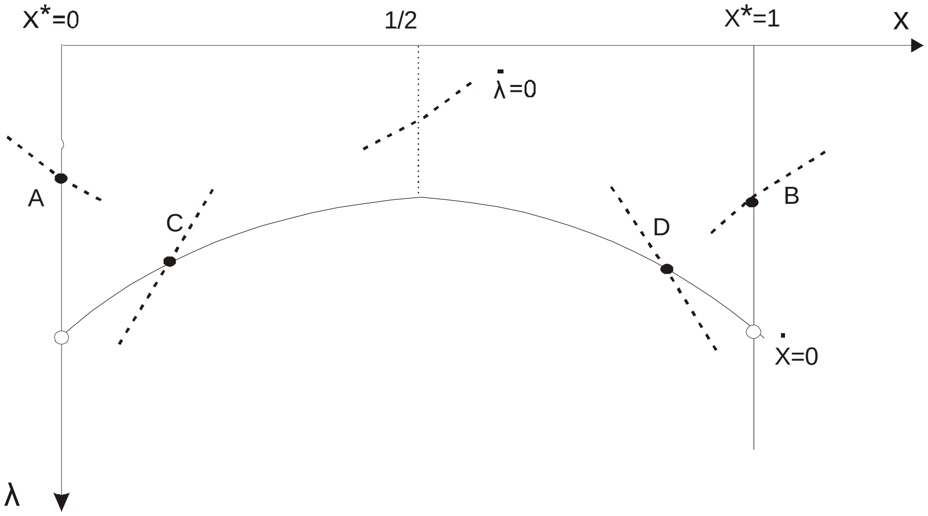

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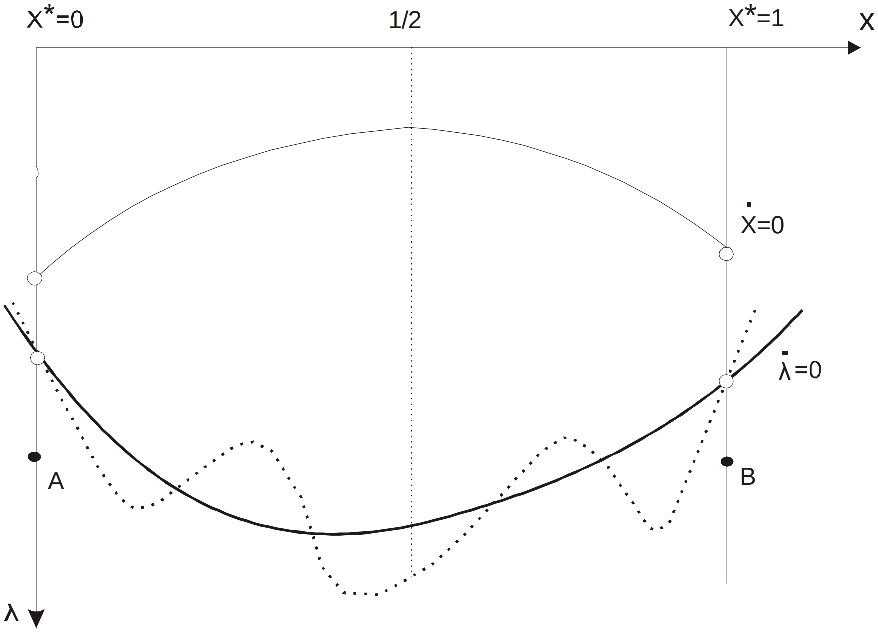

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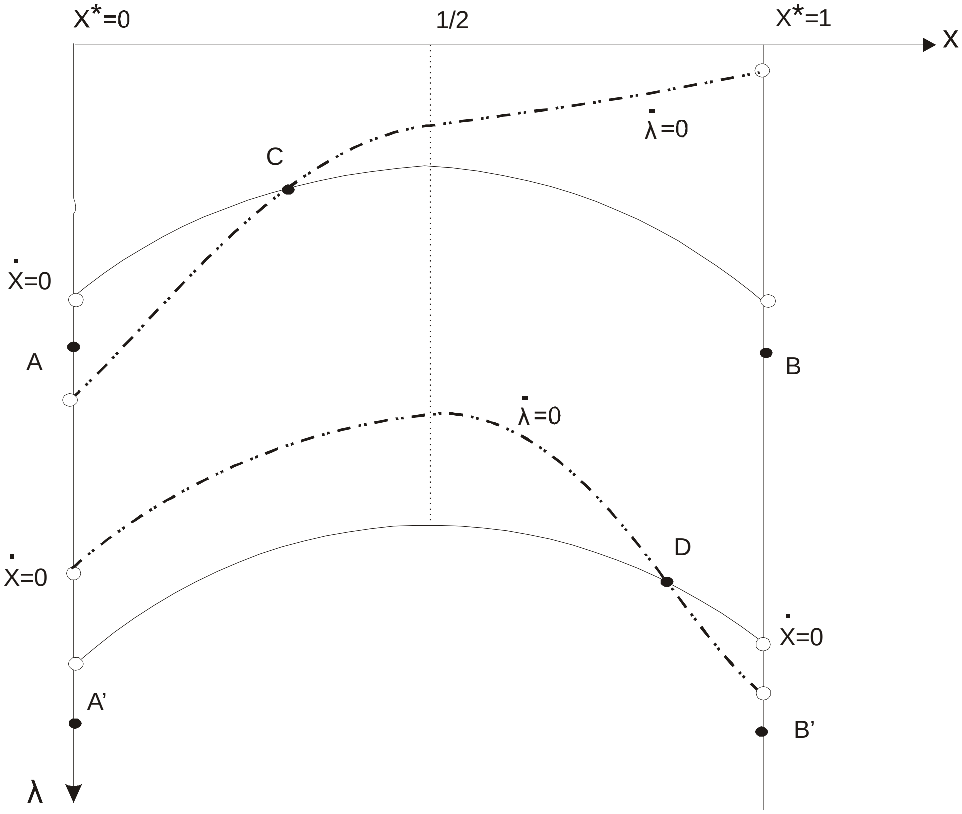

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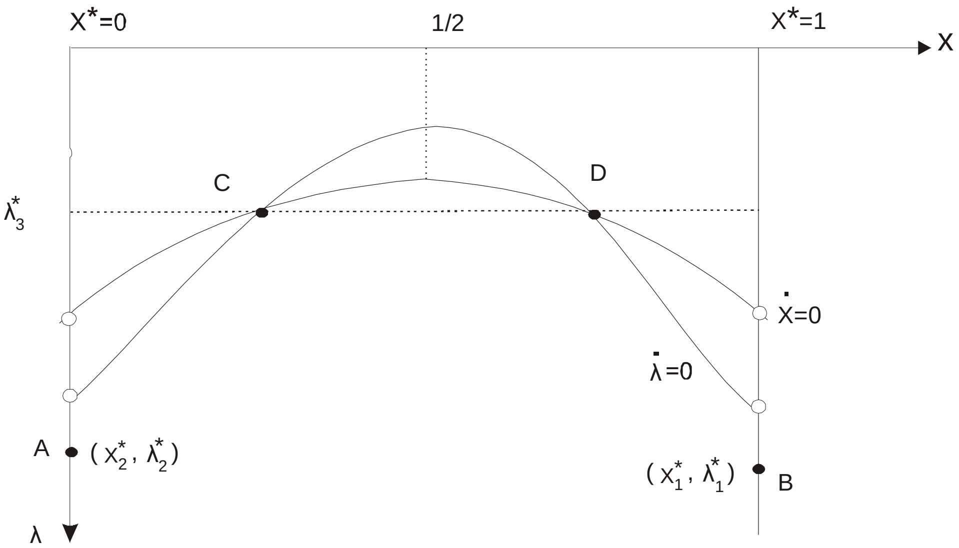

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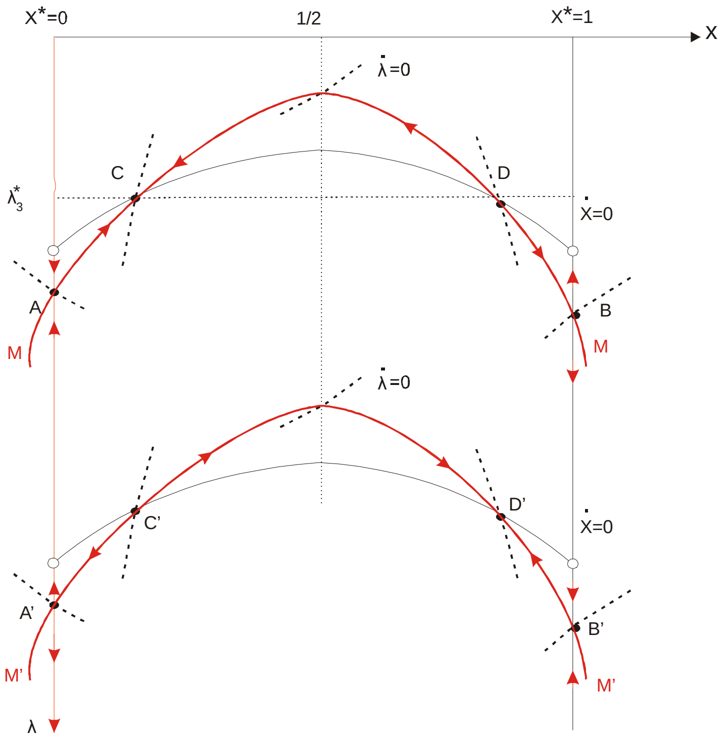

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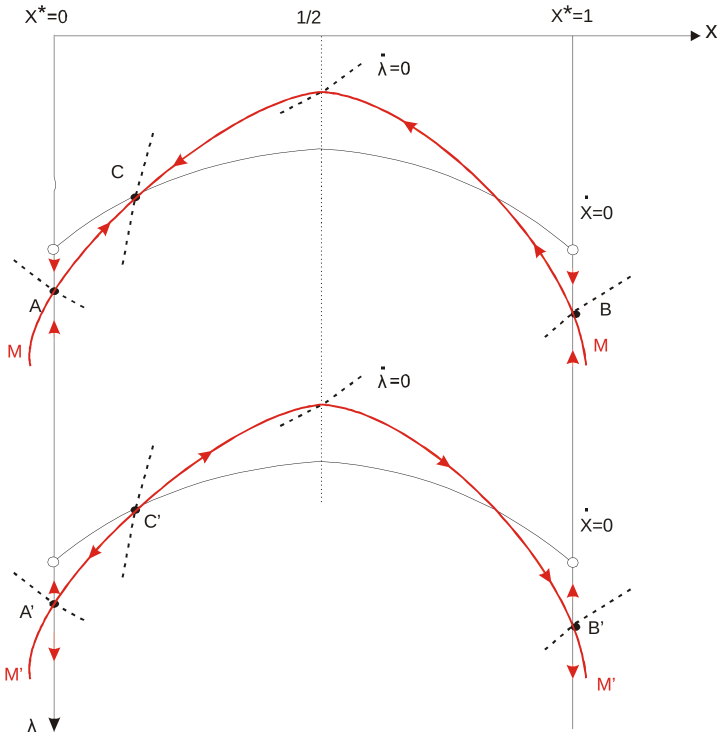

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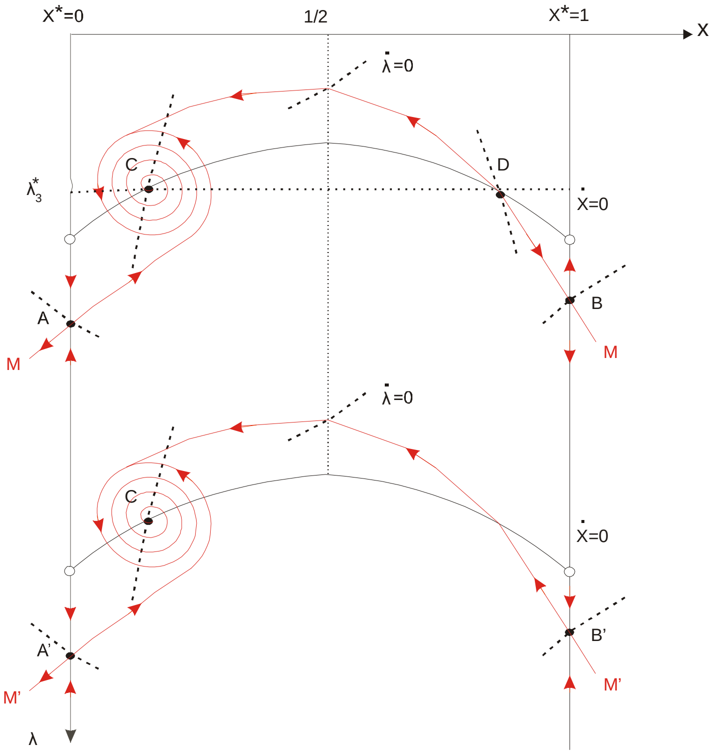

Figure 6

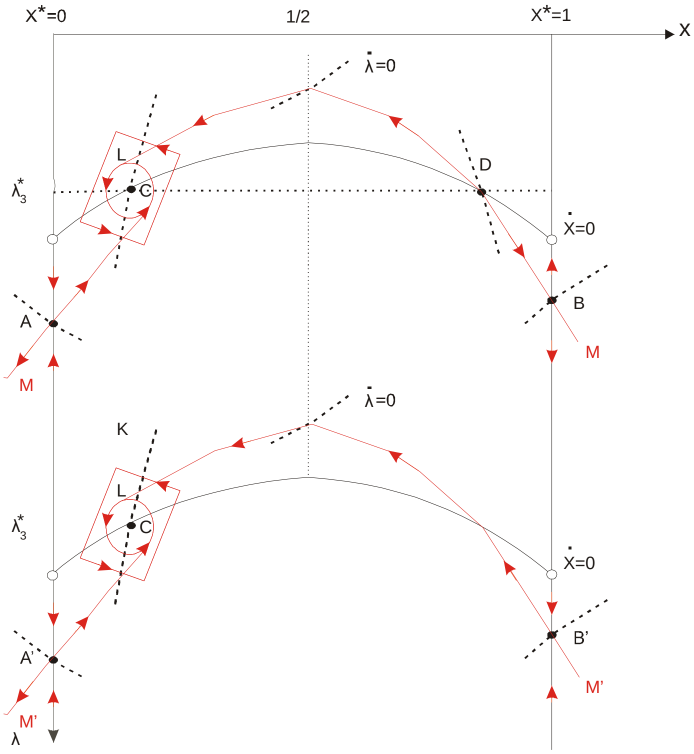
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Figure 7