

Testing the linearity of a time series.

Some Monte Carlo and Empirical Tests

By

Efstratios Tserkezos (Corresponding author).
Mathematical Modelling in new Technologies and Economy
Postgraduate Programme.
Applied Mathematics and Physics Department
National Technological Institute of Athens
&
Dikaïos E. Tserkezos
Department of Economics
University of Crete
Gallos, Rethymno 74100
Crete
Phone: 28310 77415
Fax: 28310 77416
dtsek@aias.gr

Abstract

This letter proposes a simple test for the linearity of a time series. We compare the small and large samples properties of the suggested test via Monte Carlo techniques with well known time domain linearity tests. Our results suggest that the suggested test over performs the power of the other competitive tests in small samples.

Key words: *Testing nonlinearity, Hinich portmanteau bicorrelation test, Keenan, Mcleodi-Li tests, ARCH and Luukkonen LST Test.*

Rethymno 2008.

I. Introduction

Many time-series encountered in practice exhibit characteristics that cannot be appropriately described by linear Gaussian models. For example, a Gaussian linear process $\{Z_t\}$ is time reversible, i.e., the distribution of $\{Z_{t_1}, \dots, Z_{t_N}\}$ is the same as that of $\{Z_{t_N}, \dots, Z_{t_1}\}$. Deviations from this property are suggested by time-series where the number of ascending periods (i.e., periods with increasing observations) is systematically different from the number of descending periods¹.

In this type of situations, it is often necessary to relax the linear assumption, i.e., it is necessary to consider nonlinear models for the analysis of a time-series. Consequently, it is clear that, before modelling any time-series, it may be necessary to test it for linearity.

This letter presents a method to detect nonlinear dependencies in time series. It is a bicorrelation nonparametric test based on a Box-Cox transformation. Using Monte Carlo results we compare the properties of the suggested test with other four well known tests for such cases. These tests are the *The Hinich Portmanteau Bicorrelation Test*(1996), *McLeod-Li Test*(1983), *Test for nonadditivity Keenan* (1985), *ARCH Test*(1982) and the *Luukkonen LST 1988 Test*. According to our simulation results the suggested test is superior to the other tests in small and large samples.

The plan of the paper is as follows. In Section 2 we present the suggested tests. In Section 3, we present some nonlinear models and the Monte Carlo experiments. Conclusions are given in Section 4.

¹ The well-known series of the Wolf sunspot numbers is a good example of this pattern. Gaussian linear processes do not exhibit sudden bursts of very large amplitude at irregular time epochs. This sort of behaviour is commonly seen in seismological data, in particular explosion and earthquake data.

2. The Suggested Linearity Test.

The suggested test is based on the Box-Cox² transformations of the original series³ using the following relations:

$$Q(\lambda) = n(n+2) \sum_{k=1}^K (n-k)^{-1} \rho(\lambda)_k^2 \sim \chi^2 \text{ with } (K) \quad (1)$$

$$\rho(\lambda)_k = \frac{\text{Cov}(Z(\lambda)_t, Z(\lambda)_{t+k})}{\sqrt{V(Z(\lambda)_t)}\sqrt{V(Z(\lambda)_{t+k})}} = \frac{\text{Cov}(Z(\lambda)_t, Z(\lambda)_{t-k})}{\sqrt{V(Z(\lambda)_t)}\sqrt{V(Z(\lambda)_{t-k})}} \quad (2)$$

$$Z(\lambda)_t = \frac{Z_t^\lambda - 1}{\lambda} \quad (3)$$

$$-1 \leq \lambda \leq 1 \quad (4)$$

According to the relations (1)-(4) in order to apply the suggested test the only we have to do, is conduct an iterative procedure for $-1 \leq \lambda \leq 1$ and to estimate the autocorrelation function (2) and the Box-Pierce⁴ statistic (2). We choose the value of λ for the highest significant value of the Box-Pierce statistic.

² Box, G.E.P., Cox, D.R., 1964

³ When negative Z values occur Box-Cox consider the family:

$$Z_t(\lambda) = \begin{cases} [(Z_t + \lambda_1)\lambda_2 - 1] / \lambda_2 & \lambda_2 \neq 0 \\ \log(Z_t + \lambda_1) & \lambda_2 = 0 \end{cases}$$

with $Z_t + \lambda_1 > 0$

⁴ Box, G.E.P. & Pierce A.D.A., 1970

3. Monte Carlo Experiment Results.

Many nonlinear models have been proposed in the literature with the objective of describing different characteristics exhibited by time-series encountered in practice and that cannot be appropriately accounted for by linear processes. The models we will consider in our study represent some of the most important classes of nonlinear processes that have been proposed in the literature. Excellent presentations of these models can be found in Subba Rao and Gabr (1984) and in Tong 1983 and Tong 1990. They are as follows:

For our simulation experiment⁵ we used the specification⁶:

(i) Bilinear (BL) model.
$$Z_t = 0.4 - 0.5Z_{t-1} + 0.6Z_{t-1}a_{t-1} + a_t$$

(ii) Exponential AR
$$Z_t = (0.2 + (0.3 + 0.95Z_{t-1})e^{-0.01Z_{t-1}^2})Z_{t-1} + a_t.$$

(iii) Nonlinear AR
$$Z_t = (0.004Z_{t-1} + a_t)(0.55Z_{t-1}) + a_t.$$

(iv) Nonlinear MA(NLMA)
$$Z_t = a_t - 0.3a_{t-1} + 0.5a_{t-2} + 0.6a_{t-2} - 0.2a_{t-1}^2.$$

(v) Threshold AR(TAR).
$$Z_t = \begin{cases} 2.0 - 0.3Z_{t-1} + 0.5Z_{t-2} + a_t & \text{if } Z_{t-3} \leq 1.5, \\ 2.5 + 0.2Z_{t-1} - 0.5Z_{t-2} - 0.7Z_{t-3} + a_t & \text{if } Z_{t-3} > 1.5. \end{cases}$$

⁵ The computations were performed in RATS 5.00 with a program written by the author. This RATS code program is available on request.

⁶ Wei W.S& Paulo Teles 2000.

The alternative tests used for comparisons are : *The Hinich Portmanteau Bicorrelation Test(1996)*, *McLeod -Li Test(1983)*, *Test for Nonadditivity Keenan (1985)*, *ARCH Test(1982)* and the *Luukkonen LST 1988 Test*.

Under the null hypothesis, 3,000 replications of the basic nonlinear specifications (i)-(v) are generated. Samples of different sizes from 40 to 1500 data were used in order to evaluate the six nonlinearity tests. Before checking for nonlinearity we remove linear dependencies by fitting an AR(p) fit to the simulated series for the five alternative tests. This way we make sure that the rejection of the null hypothesis of pure noise at the specified threshold level is due only to significant nonlinearity. These results are summarized in Table 1.

Table 1. The Power of the six tests for testing linearity.

The Power of the five tests for the specification : $Z_t=0.4-0.5Z_{t-1}+0.6Z_{t-1}a_{t-1}+a_t$

Tests	Number of Available Observations									
	40	100	200	300	400	500	600	700	1200	1500
1. The Hinich Portmanteau Bicorrelation Test	3,4	50,8	69,8	79,7	86,0	89,8	91,8	93,8	95,2	96,1
2. McLeod -Li Test	8,6	71,2	93,1	98,3	99,2	99,6	99,9	100,0	100,0	100,0
3. Test for nonadditivity Keenan (1985)	0,5	32,0	56,6	72,7	81,2	86,0	88,1	90,0	91,8	93,3
4. ARCH Test	2,6	88,3	98,9	99,8	100	100	100	100	100	100
5. Luukkonen LST 1988 Test	2,3	86,3	99,0	99,9	100,0	100,0	100,0	100,0	100,0	100,0
6. Suggested Test	21,4	80,9	96,9	99,3	99,9	100,0	100,0	100,0	100,0	100,0

Power of the six tests using the specification : $Z_t=(0.2+(0.3+0.95Z_{t-1})e^{-0.01Z_{t-1}^2})Z_{t-1}+a_t$.

1. The Hinich Portmanteau Bicorrelation Test	0,8	36,5	53,3	59,6	66,3	73,7	78,9	86,4	90,5	93,0
2. McLeod -Li Test	16,1	19,8	29,2	43,5	58,0	72,8	82,8	89,6	94,7	97,2
3. Test for nonadditivity Keenan (1985)	0,3	9,1	31,2	51,9	71,1	84,0	91,3	95,4	97,6	98,5
4. ARCH Test	2,7	59,4	79,5	91,7	98	99,5	99,9	100	100	100
5. Luukkonen LST 1988 Test	1,0	37,0	52,7	58,6	63,5	72,3	78,1	86,0	90,6	93,0
5. Suggested Test	99,1	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0

Power of the six tests using the specification : $Z_t=(0.004Z_{t-1}+a_t)(0.55Z_{t-1})+a_t$.

1. The Hinich Portmanteau Bicorrelation Test	1,4	16,4	25,9	32,0	38,3	42,4	47,1	49,0	53,6	56,1
2. McLeod -Li Test	2,1	14,9	32,4	47,1	60,2	71,5	79,6	85,7	90,7	94,2
3. Test for nonadditivity Keenan (1985)	0,4	11,0	14,1	16,8	18,0	17,8	20,2	20,4	20,7	20,6
4. ARCH Test	2	34,2	62,4	78,5	87,6	93,5	96,4	98,5	99,1	99,4
5. Luukkonen LST 1988 Test	1,8	34,1	61,6	76,5	87,1	93,1	96,5	98,1	99,0	99,6
6. Suggested Test	6,6	11,5	15,3	19,8	22,0	24,2	28,1	29,9	34,0	36,6

Power of the six tests using the specification: $Z_t = a_t - 0.3a_{t-1} + 0.5a_{t-2} + 0.6a_{t-2} - 0.2a_{t-1}^2$.

1. The Hinich Portmanteau Bicorrelation Test	1,7	15,7	24,4	30,3	35,7	38,2	39,2	42,1	45,1	46,2
2. McLeod -Li Test	2,4	15,7	32,1	47,5	60,1	72,1	81,4	87,5	92,0	94,9
3. Test for nonadditivity Keenan (1985)	0,3	4,7	6,3	7,5	8,7	9,6	10,9	12,6	13,6	14,9
4. ARCH Test	1,7	17,1	29,8	43,4	55,4	65	72	77	81,5	85,9
5. Luukkonen LST 1988 Test	1,6	17,8	31,6	42,4	53,7	62,5	70,8	77,5	83,1	87,1
6. Suggested Test	24,1	77,2	95,8	99,4	100,0	100,0	100,0	100,0	100,0	100,0

Power of the six tests using the specification:

$$Z_t = \begin{cases} 2,0 - 0,3Z_{t-1} + 0,5Z_{t-2} + a_t & \text{if } Z_{t-3} \leq 1,5, \\ 2,5 + 0,2Z_{t-1} - 0,5Z_{t-2} - 0,7Z_{t-3} + a_t & \text{if } Z_{t-3} > 1,5. \end{cases}$$

1. The Hinich Portmanteau Bicorrelation Test	2,0	34,9	63,7	78,0	89,0	94,2	97,0	99,1	99,6	99,9
2. McLeod -Li Test	5,9	19,6	36,8	51,4	64,6	74,5	82,6	88,6	92,8	95,3
3. Test for nonadditivity Keenan (1985)	0,1	23,3	78,6	76,4	64,1	75,7	81,5	87,0	86,3	86,0
4. ARCH Test	1,6	11,5	19,9	25,6	30,9	36,1	41,3	46,4	51,4	55,1
5. Luukkonen LST 1988 Test	1,0	12,3	16,8	24,6	30,4	35,8	39,6	45,5	50,2	52,5
6. Suggested Test	25,9	93,2	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0

Source: Monte Carlo Experiments.

4. Conclusions.

In this short paper we present a non linearity test for time series. In order to test its properties in small and large samples we conducted a Monte Carlo experiment with five nonlinear specifications and five alternative linearity tests.

According to our Monte Carlo experiments in small samples the suggested test over performs the power of the alternative tests in the majority of the nonlinear specifications used in the experiment. In large samples the six tests converge although we note a limiting over performance of the suggested test in most cases.

Reference.

- Ashley, R.A., Patterson, D.M. and Hinich, M.J., 1986. A diagnostic test for nonlinear serial dependence in time-series fitting errors. *J. Time Ser. Anal.* 7, pp. 165–178.
- Box, G.E.P., Cox, D.R., 1964. An analysis of transformations. *J. Roy. Statist. Soc. Ser. B* 26, pp.211–243.
- Box, G.E.P. & Pierce A.D.A., 1970. Distributions of Residual Autocorrelation in ARMA time Series.,*Journal of the American Statistical Association* 65.,pp1509-26.
- Brock, W., D. Hsieh and B. Le Baron, 1991. *Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence*, MIT Press, Reading, Mass.
- Brooks, C., 1996, Testing for non-linearity in daily sterling exchanges rates, *Applied Financial Economics* 6, 307 – 317.
- Brooks, C. and M.J. Hinich, 1998, Episodic nonstationarity in exchange rates, *Applied Economics Letters* 5, 719-722.
- Brooks, C. and M.J. Hinich, 2001, Bicorrelations and cross-bicorrelations as tests for nonlinearity and as forecasting tools, *Journal of Forecasting* 20, 181-196.
- Hinich, M.J., 1996, Testing for dependence in the input to a linear time series model, *Journal of Nonparametric Statistics* 6, 205-221.
- Hinich, M.J. and D.M. Patterson, 1985, Evidence of nonlinearity in daily stock returns, *Journal of Business and Economic Statistics* 3, 69-77.
- Hsieh, D.A., 1991, Chaos and nonlinear dynamics: application to financial markets, *Journal of Finance* 46, 1839-1877.
- Keenan, D.M., 1985. A Tukey nonadditivity-type test for time-series nonlinearity. *Biometrika* 72, pp. 39–44.
- Luukkonen R,Saikkonen P and Terasvitra T.,(1988)., Testing Linearity against Smooth TransitionAutoregressive Models.,*Biometrika* 75.,pp-491-500.
- Mcleod and Li W.,(1983).,Diagnostic checking ARMA Time Series Models Using Squared Residual Autocorrelation.,*Journal of Time Series Analysis*.,4.,pp.269-273.

- Opong, K.K., G. Mulholland, A.F. Fox, and K. Farahmand, 1999, The behavior of some UK equity indices: an application of Hurst and BDS tests, *Journal of Empirical Finance* 6, 267-282.
- Ramsey, J.B., 1996, If nonlinear models cannot forecast, what use are they? *Studies in Nonlinear Dynamics and Econometrics* 1(2), 65-86.
- Scheinkman, J. and B. LeBaron, 1989, Nonlinear dynamics and stock returns, *Journal of Business* 62, 311-337.
- Tong, H., 1983. *Threshold Models in Non-linear Time Series Analysis*. Springer, New York.
- Tong, H., 1990. *Non-linear Time Series: A Dynamical System Approach*. Oxford University, Oxford.
- Tsay, R.S., 1986. Nonlinearity tests for time-series. *Biometrika* 73, pp. 461–466.
- Tsay, R.S., 1989. Testing and modeling threshold autoregressive processes. *J. Amer. Statist. Assoc.* 84, pp. 231–240.
- Wei W.S& Paulo Teles, The effects of temporal aggregation on tests of linearity of a time series *Computational Statistics and Data Analysis*, 2000.,pp.91-103.