An irreversible investment model with a stochastic production capacity and fixed plus proportional adjustment costs

Yiannis Kamarianakis[a,b], Anastasios Xepapadeas[b]

Abstract

This paper studies the problem of a company which expands its stochastic production capacity in irreversible investments by purchasing capital and faces both fixed and proportional costs. The objective of the company is to find optimal production decisions to maximize its expected total net profit in an infinite horizon. We solve this problem explicitly by applying the theory of stochastic impulse controls.

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[a] Regional Analysis Division, Institute of Applied and Computational Mathematics, Foundation for Research and Technology-Hellas, Vasilika Vouton, GR 71110, Heraklion, Crete, Greece.
[b]Department of Economics, University of Crete, GR 74100, Rethymnon, Crete, Greece.

1. Introduction

This paper examines the problem of a company that aims to expand its stochastic production capacity. Investments in capital for expanding capacity are irreversible in the sense that the company cannot recover the investment by reducing capacity. The company faces fixed and proportional costs for purchasing capital and aims to maximize its expected discounted profits over an infinite horizon.

Dixit and Pindick (1994) provide a review of similar investment problems. Davis *et al.* (1987) where among the first ones to address the issue of optimally determining the timing and size of capacity increases that can be associated with the operation of an investment project in the presence of random economic fluctuations. Kobila (1993) analyzed a model with deterministic capacity in an uncertain market without transaction costs on buying capital. Chiarolla and Haussmann (2003) studied an irreversible investment model in a finite horizon and obtained an explicit solution for a power type production function. Other important contributions include Oksendal (2000), Wang (2003) and Bank (2005). Capacity expansion models in which the installed capacity level can be reduced as well as increased, that is reversible capacity expansion models have been examined by Abel and Eberly (1996) and Guo and Pham (2005).

The presence of fixed costs for purchasing capital requires stochastic impulse control techniques for solving the problem. Here, we adopt the methodological framework presented in Cadenillas and Zapatero (1999), which characterizes the value function as a solution to a system of quasi-variational inequalities (see also Cadenillas, 1999, Suzuki and Pliska, 2004, Cadenillas *et al.,* 2006). Alternatively the problem could have been approached via combining stochastic calculus with standard nonlinear programming techniques as in Alvarez and Virtanen (2004) (see also

Alvarez 2004, Dayanik and Egami, 2004). Pham (2005) solved the problem by assuming only proportional costs for purchasing capital and thus relying on singular stochastic control methods. The optimal strategy in that model involves doing infinitesimal small transactions to avoid that the capacity production process leaves a no transaction region. However, transactions in the real world involve not only proportional but also fixed costs. In the presence of fixed investment costs, these strategies would lead to ruin.

We formulate the impulse control problem in section 2. Section 3 displays some preliminary results for the value function and the admissible expansion strategies. We characterize the value function as the solution to a system of quasi-variational inequalities and solve that system in section 4. The fifth section contains a numerical illustration that depicts how the impulse control band shrinks to the singular control boundary (derived in Pham, 2005) for vanishing fixed cost. Moreover we show that the optimal control actions depend strongly in the discount rate.

2. Problem formulation

Let (*Ω, F, P*) be a complete probability space endowed with a filtration (F_t), which is the *P*-augmentation of the filtration generated by a one-dimensional Brownian motion *W*. We consider a firm producing some output from stochastic capacity production K_t and possibly also from other inputs. The firm can buy capital at any time *t* but faces constant and proportional costs denoted by *C* and *c* respectively. Given an initial capital $k \geq 0$ the firm's capacity production evolves according to the following generalized Ito equation:

$$K_t = k - \int_0^t \delta K_s ds + \int_0^t \gamma K_s dW_s + \sum_{n=1}^{\infty} I_{\{\tau_n < t\}} \xi_n \,, \tag{2.1}$$

where $\delta \geq 0$ is the depreciation rate of the capacity production, $\gamma > 0$ represents its volatility and ξ_n is the (positive) amount of the nth capital purchase. We observe that in the particular case in which there is no control the value of the indicator function is zero, so K is just a geometric Brownian motion.

The instantaneous operating profit of the firm is a function $\Pi(K_t)$ of capacity production. In general the production profit function Π is assumed to be continuous in $\Re+$, nondecreasing, concave and C^1 on $(0,\infty)$, with $\Pi(0)=0$ and satisfying the Inada conditions

$$\Pi'(0^+) := \lim_{k \downarrow 0} \Pi'(k) = \infty \quad \text{and} \quad \Pi'(\infty) := \lim_{k \to \infty} \Pi'(k) = 0. \tag{2.2}$$

A typical example arising from the Cobb-Douglas production function leads to a profit function in the form:

$$\Pi(k) = \lambda k^a, \quad \text{with } \lambda > 0, \quad 0 < \alpha < 1. \tag{2.3}$$

In our subsequent steps we adopt this choice for the production function similar to Pham (2005) and Merhi and Zervos (2005).

The firm's objective can now be formulated as follows

Problem 2.1 *The firm aims to maximize discounted profits minus expansion costs over lifetime. In particular, the firm aims to select a pair (T, ξ) that maximizes the functional J defined by*

$$J(k,T,\xi) := E_k\left[\int_0^\infty e^{-rt}\Pi(K_t)dt - \sum_{n=1}^{\infty} e^{-r\tau_n}\left(\left(C + C_1\xi_n\right)I_{\{\xi_n>0\}}\right)I_{\{\tau_n<\infty\}}\right] \tag{2.4}$$

with r representing the discount rate and $C_1=1+c$.

3. Auxiliary results

Admissible strategies

Since we want to maximize the functional J in problem 2.1 we should consider only those strategies for which J is well defined and finite. In order that

$$E_k\left[\sum_{n=1}^{\infty} e^{-r\tau_n}\left(C + C_1\xi_n\right)I_{\{\tau_n<\infty\}}\right] \tag{3.1}$$

be well defined and finite, we need that

$$E\left[\sum_{n=1}^{\infty} e^{-r\tau_n} I_{\{\tau_n<\infty\}}\right] < \infty \text{ and } E\left[\sum_{n=1}^{\infty} e^{-r\tau_n}\xi_n I_{\{\tau_n<\infty\}}\right] < \infty . \tag{3.2}$$

To obtain the inequality on the left-hand-side, we need that

$$\forall T \in [0,\infty): \qquad P\left\{\lim_{n\to\infty}\tau_n \le T\right\} = 0 . \tag{3.3}$$

To obtain the inequality on the right-hand-side, we need that

$$\lim_{T\to\infty} E\left[e^{-rT}K(T+)\right] = 0 \tag{3.4}$$

and

$$E\left[\int_0^{\infty} e^{-rt}K_t dt\right] < \infty . \tag{3.5}$$

The last two conditions are implied from the formula of integration by parts (see, for instance, section VI.38 of Rogers and Williams (1987)) which postulates that for every $0 < s \le t < \infty$,

$$E\left[e^{-rt}K(t+)\right] - E\left[e^{-rs}K(s+)\right] = -(\delta+\lambda)E\left[\int_s^t e^{-ru}K(u)du\right] + E\left[\sum_{n=1}^{\infty} e^{-r\tau_n}\xi_n I_{\{s<\tau_n\le t\}}\right]. \tag{3.6}$$

Note also that in order that $E_k\int_0^{\infty} e^{-rt}\Pi(K_t)dt < \infty$ it suffices that (3.5) holds.

DEFINITION 3.1 [Admissible controls]. We shall say that an impulse control is admissible if the conditions (3.3)-(3.5) are satisfied. We shall denote by A(k) the class of admissible impulse controls.

Bounds for the Value Function

Let us denote by V the value function. That is, for every $k \in (0,\infty)$,

$$V(k) := \sup\{J_1(k;T,\xi);(T,\xi) \in \mathrm{A}(k)\}. \tag{3.7}$$

The following lemma provides bounds for the value function which will be used in the next section.

Lemma 3.1 The value function V is finite and satisfies: for any $q \in [0,C_1]$,

$$0 \le V(k) \le \frac{\tilde{\Pi}((r+\delta)q)}{r} + kq, \qquad k \ge 0 \tag{3.8}$$

where under the Inada conditions

$$\tilde{\Pi}(z) := \sup_{k\ge 0}[\Pi(k) - kz] < \infty, \qquad \forall z \ge 0 \tag{3.9}$$

defines the Fenchel-Legendre transform of Π.

Proof. For the left part of the inequality one has just to notice that since the value of not performing any capacity expansion is greater than zero, the value function is valued in $[0,\infty]$. The right part of the inequality holds for the singular stochastic control problem (Lemma 1.3.2 in Pham, 2005) and since we can regard the set of impulse controls as a subset of the set of singular stochastic controls[1] we have

[1] The relationship between impulse and singular stochastic control problems is explored in Menaldi and Robin (1983), Menaldi and Rofman (1983) and Oksendal (1999); Alvarez and Virtanen (2004) showed for a class of problems that is similar to ours an inequality analogous to (3.10) holds for the marginal values of the considered stochastic control problems as well.

$$V(k) \leq V_s(k) \tag{3.10}$$

where $V_s(k)$ is the value function of the singular stochastic control problem. □

4. The solution of the QVI

For a function $\phi : [0,\infty) \rightarrow \Re$ we define the *maximum utility operator M* by

$$M\phi(k) := \sup\{\phi(k+\xi) - C - C_1\xi : \xi \in (0,\infty), k \in (0,\infty)\}. \tag{4.1}$$

MV(*k*) represents the value of the strategy that consists in choosing the best immediate capacity expansion and then selecting optimally the times and the amounts of the future control actions. Let us consider the differential operator $\Im$ defined by

$$\Im\psi(k) := \frac{1}{2}\gamma^2 k^2 \frac{d^2\psi(k)}{dk^2} - \delta k \frac{d\psi(k)}{dk} - r\psi(k). \tag{4.2}$$

Now we intend to find the value function and an associated optimal strategy.

Suppose there exists an optimal strategy for each initial point. Then, if the process starts at *k* and follows the optimal strategy, the expected utility associated with this optimal strategy is *V*(*k*). On the other hand, if the process starts at *k*, makes immediately the best immediate intervention, and then follows an optimal strategy, then the expected utility associated with this strategy is *MV*(*k*). Since the first strategy is optimal, its associated expected utility is greater or equal than the expected utility associated with the second strategy. Furthermore, when these two expected utilities are equal, it is optimal to intervene. Hence, $V(k) \geq MV(k)$, with equality when it is optimal to intervene. In the continuation region, that is, when there are not interventions, we must have $\Im V(k) = 0$ (this is an heuristic application of the dynamic programming principle to the problem we are considering). These intuitive observations can be applied to give a characterization of the value function. We formalize this intuition in the next two definitions and theorem.

DEFINITION 4.1 (QVI) *We say that a function* $v:(0,\infty)\rightarrow\Re$ *satisfies the quasi-variational inequalities for Problem 2.1 if for every* $k\in[0,\infty)$:

$$\Im v(k)+\Pi(k)\le 0, \tag{4.3}$$

$$v(k)\ge Mv(k), \tag{4.4}$$

$$(v(k)-Mv(k))(\Im v(k)+\Pi(k))=0. \tag{4.5}$$

Quasi-variational inequalities have been studied, for instance, in Bensoussan and Lions (1984), Perthame (1984a, 1984b) and Baccarin (2004) but the theory developed in those references cannot be applied directly to the above QVI.

A solution *v* of the QVI separates the interval $(0,\infty)$ into two disjoint regions: a continuation region

$$C:=\{k\in(0,\infty): v(k)>Mv(k) \quad \textit{and} \quad \Im v(k)+\Pi(k)=0\}$$

and an intervention region

$$\Sigma:=\{k\in(0,\infty): v(k)=Mv(k) \quad \textit{and} \quad \Im v(k)+\Pi(k)<0\}.$$

From a solution to the QVI it is possible to construct the following stochastic impulse control.

DEFINITION 4.2 *Let v be a solution of the QVI. The following stochastic impulse control*

$$\left(T^{v},\xi^{v}\right)=\left(\tau_1^{v},\tau_2^{v},\ldots,\tau_n^{v},\ldots;\xi_1^{v},\xi_2^{v}\ldots,\xi_n^{v},\ldots\right)$$

is called the QVI-control associated with v (if it exists):

$$\tau_1^v := \inf\{t \geq 0 : v(k^v(t)) = Mv(k^v(t))\}$$
$$\xi_1^v := \arg\sup\{v(k^v(\tau_1^v) + \xi) - C - C_1\xi : \xi \in \Re+, k^v(\tau_1^v) + \xi \in \Re+\}$$

and, for every $n \geq 2$:

$$\tau_n^v := \inf\{t \geq \tau_{n-1}^v : v(k^v(t)) = Mv(k^v(t))\}$$
$$\xi_n^v := \arg\sup\{v(k^v(\tau_n^v) + \xi) - C - C_1\xi : \xi \in \Re+, k^v(\tau_n^v) + \xi \in \Re+\}$$

where $\tau_0^v := 0$ *and* $\xi_0^v := 0$.

This means that the capacity expansions occur whenever *v* and *Mv* coincide and their size solve the optimization problem corresponding to *Mv*(*k*).

Korn (1997, Theorem 3.2) has developed a general sufficient condition of optimality for stochastic impulse control problems, and applied it to some examples. In each example, he shows that an admissible control satisfies that sufficient condition, and is therefore optimal. Cadenillas and Zapatero (1999) and Cadenillas *et al* (2006) developed suitable (for their problems) versions of Theorem 3.2 of Korn (1997). The following version is suitable for the application that we consider in this paper; its proof is straightforward by following the arguments in Cadenillas and Zapatero (1999) or Cadenillas *et al.* (2006).

THEOREM 3.1 *Let* $v \in C^1((0,\infty);(0,\infty))$ *be a solution of the QVI and let* $b \in (0,\infty)$ *be such that* $v \in C^2([0,\infty) - \{b\}; \Re+)$. *Suppose there exists* $0 < L < \infty$ *such that v is linear in (0,L). Then, for every* $k \in (0,\infty)$:

$$V(k) \leq v(k).$$

Furthermore, if the QVI-control (T^v, ξ^v) *corresponding to v is admissible, then it is an optimal stochastic impulse control and for every* $k \in (0,\infty)$:

$$V(k) = v(k) = J(k; T^v, \xi^v).$$

We conjecture that there exists an optimal solution (T,ξ) characterized by parameters L, l with $0<L<l<\infty$ such that the optimal strategy is to stay in $[L,\infty)$ and jump to l when reaching the left boundary. That is we conjecture that for every $i \in \mathrm{N}$:

$$\tau_i = \inf\{t > \tau_{i-1} : K_t \notin (L,\infty)\} \tag{4.6}$$

and

$$K_{\tau_{i+}} = K_{\tau_i} + \xi_i = l\left(I_{X_{\tau_i}=L}\right). \tag{4.7}$$

Thus, the value function would satisfy

$$\forall y \in (0,L]: \qquad V(y) = v(l) - C - C_1(l-y). \tag{4.8}$$

If V were differentiable in $\{L,l\}$, then from (3.15) we would get

$$V'(L) = C_1 \tag{4.9}$$

and

$$V'(l) = C_1 \ . \tag{4.10}$$

We also conjecture that the continuation region is the interval (L,∞), so

$$\forall k \in [L,\infty) : \Im v(k) = \frac{1}{2}\gamma^2 k^2 \frac{d^2 v(k)}{dk^2} - \delta k \frac{dv(k)}{dk} - rv(k) + \Pi(k) = 0\ . \tag{4.11}$$

Applying standard methods of ordinary equations, we see that the general solution to (3.18) for Π as in (2.3) is given by

$$v(k) = Ak^{a_1} + Bk^{a_2} + \widetilde{V}_0(k) \tag{4.12}$$

where A, B are unknown constants and

$$a_1 = \frac{2\delta + \gamma^2 + \left(4\delta^2 - 4\delta\gamma^2 + \gamma^4 + 8r\gamma^2\right)^{1/2}}{2\gamma^2} > 1$$
$$a_2 = \frac{2\delta + \gamma^2 - \left(4\delta^2 - 4\delta\gamma^2 + \gamma^4 + 8r\gamma^2\right)^{1/2}}{2\gamma^2} < 0 \quad , \tag{4.13}$$

$$\widetilde{V}_0(k) = E\left[\int_0^\infty e^{-rt}\Pi\left(\widetilde{K}_t\right)dt\right], \tag{4.14}$$

where $\widetilde{K}_t$ represents the uncontrolled geometric Brownian motion process (2.1).

For Π as in (2.3) we have

$$v(k) = Ak^{a_1} + Bk^{a_2} + \frac{\lambda k^a}{\mu r}, \tag{4.15}$$

with

$$\mu = 4\left(\alpha^2\gamma^2 - 2\delta\alpha - 2\alpha\gamma^2 - 4r\right). \tag{4.16}$$

At this point we should observe that if *A*>0 then (3.10) cannot be true[2] (see theorem 1.5.3 in Pham, 2005 for the form of the value function of the associated singular control problem). Moreover for *A*<0, $v'(k) \to -\infty$ as $k \to \infty$ which is also not acceptable, thus *A*=0 should hold.

In summary, we conjecture that the solution is described by (4.6)-(4. 7) and the three unknowns *L, l, B* are a solution to a system of four nonlinear equations

$$h(L) = h(l) - C - C_1(l - L) \tag{4.17}$$

$$h'(L) = C_1 \tag{4.18}$$

$$h'(l) = C_1 \ . \tag{4.19}$$

[2] Dixit (1991) noted that in stochastic impulse control problems that prevent the state process from going too low, but there is no reason to stop it from rising too high (as in our case), then some economic argument such as convergence must be invoked to provide a (necessary for the identification of the value function) boundary condition for the differential equation (4.15) as *K* goes to infinity. In this article, to identify the value function for the impulse control problem we use its boundedness from above from the value function of the associated singular control problem. In stochastic impulse control problems of optimal harvesting/consumption/dividend allocation that just prevent the state process from going too high, researchers required that the value function is zero at a low benchmark level (see Cadenillas, 1999, Cadenillas *et al.* 2006).

where

$$h(x) = Bx^{a_2} + \frac{\lambda x^a}{\mu r}. \tag{4.20}$$

The above are proved rigorously in the following theorem.

THEOREM 4.2 *Let L, l with* $L < l < \infty$ *be a solution of the system of equations (4.17)-(4.19). Let us define the function* $V : (0,\infty) \to \Re$ *by*

$$V(k) := \begin{cases} h(k) & \text{if} \quad L \le k \\ h(k) - C - C_1(l-k) & \text{if} \quad k < L \end{cases}. \tag{4.21}$$

If for every k<L

$$-C_1\delta k - r[h(l) - C - C_1(l-k)] + \Pi(k) < 0 \tag{4.22}$$

then v is the value function of problem 2.1. That is

$$v(k) = V(k) = \sup\{J_1(k;T,\xi);(T,\xi) \in \mathrm{A}(k)\}$$

and the optimal strategy is given by (4.6), (4.7).

Proof. We observe that if V were a solution to the QVI then, according to theorem 4.1, V would be the value function and the optimal strategy would be given by (4.6)-(4.7). Indeed, V is twice continuously differentiable in $(0,L) \cup (L,\infty)$ and once continuously differentiable in L. Furthermore, V is linear in $(0,L)$. In addition, the QVI-control associated with V is admissible, because the trajectory K generated by the QVI-control associated with V behaves like a geometric Brownian motion in each random interval (τ_n, τ_{n+1}) and satisfies $P\{\forall t \in (0,\infty) : k_t \in [L,\infty)\} = 1$. Thus, the conditions (3.3)-(3.5) would be satisfied, and the QVI-control associated to V would be admissible. Hence it only remains to verify that V is a solution to the QVI.

We observe that

$$\Im V(k)+\Pi(k)=\begin{cases}\Im h(k)+\Pi(k) & if \quad L\le k\\ -C_1\delta k-r\left[h(l)-C-C_1(l-k)\right]+\Pi(k) & if \quad L>k\end{cases}$$

Thus,

$$\Im V(k)+\Pi(k)$$

is equal to zero in $[L,\infty)$ and is negative in (0,L) because of condition (4.22). Hence inequality (4.3) is satisfied. We also note that

$$MV(k)=\begin{cases}h(k)-C & if \quad L<k\\ h(l)-C-C_1(k-l) & if \quad k\le L\end{cases}$$

and observe that

$$\forall k\in[L,\infty): \qquad v(k)-Mv(k)=C>0\,.$$

and

$$\forall k\in:(0,L) \quad v(k)-Mv(k)=h(k)-h(l)+C-C_1(l-k),$$

Thus *v*-*Mv* is equal to zero in the intervention region $(0,L)$ and positive in the continuation region $[L,\infty)$, so inequalities (4.3)-(4.5) are satisfied. Hence *v* is a solution of the QVI and this proves the theorem.□

5. Numerical illustration

In this section, we provide numerical solutions to the nonlinear system (4.15)-(4.17) and conduct sensitivity analysis with respect to the fixed costs and the discount rate via applying the Newton-Raphson algorithm. It should be noted that the system is complex and convergence of the numerical scheme is sensitive to the initial values. Hence, we first found the solutions to a baseline experiment and then, for each perturbation of the parameters, we plugged in as starting values the outcomes of the previous run. MATLAB codes are available upon request from the authors.

First note that according to Pham (2005) the control boundary k_b of the associated singular stochastic control problem is given by the following relation:

$$k_b = \left(\frac{C_1(1-a_2)}{a\theta(a-a_2)} \right)^{\frac{1}{a-1}} \qquad (5.1)$$

with

$$\theta = \frac{1}{r + a\delta + \frac{\gamma^2}{2} a(1-a)} . \qquad (5.2)$$

Now consider the following data:

$$C_1 = 1.1 \quad r = 0.2 \quad \lambda = 1 \quad \delta = 0.1 \quad \gamma = 0.3 \quad a = 0.3$$

and observe that k_b=1.3602 for these values. Table 1 and figure 1 depict the evolution of the control band (*L, l*) for varying levels of the fixed costs parameter *C*. We observe that the boundaries of the impulse control band tend to approach the boundary of the singular stochastic control problem. A perfect fitting regression line (R^2=1) for the width *l-L* of the control band as a function of fixed costs is formulated as:

$$(l-L)^{3.1} = 6.822 \cdot C - 4.24 \cdot C^2 + 1.6 \cdot C^3 \qquad (5.3)$$

where the exponent in (5.3) is found via a Box-Cox procedure.

Next, we keep *C* fixed and equal to 0.5 and solve (4.15)-(4.17) for *r* varying from 0.05 to 0.2. Table 2 and figure 2 depict that the both *l* and *L* decrease for increasing *r* and the same holds for the width *l-L* of the control band. In this case, a perfect fitting regression line (R^2=1) for the width *l-L* of the control band is formulated as follows:

$$(l-L)^{-2.1} = 0.0033 - 0.38 \cdot r + 9.03 \cdot r^2 + 30.19 \cdot r^3$$

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Table 1. Control bands for varying levels of fixed costs.

C	L	l	B
0.7	0.3460	1.8079	0.0151
0.6	0.3913	1.8036	0.0213
0.5	0.4443	1.7972	0.0301
0.4	0.5075	1.7878	0.0428
0.3	0.5851	1.7731	0.0618
0.2	0.6858	1.7486	0.0913
0.1	0.8326	1.7005	0.1420
0.05	0.9491	1.6506	0.1852
0.01	1.1297	1.5483	0.2466
0.005	1.1800	1.5136	0.2602
0.001	1.2576	1.4536	0.2762
0.0005	1.2795	1.4351	0.2794
0.0001	1.3136	1.4048	0.2830
0.00005	1.3234	1.3957	0.2837
0.00001	1.3388	1.3811	0.2845
0.000005	1.3433	1.3768	0.2846
0.000001	1.3503	1.37	0.2848

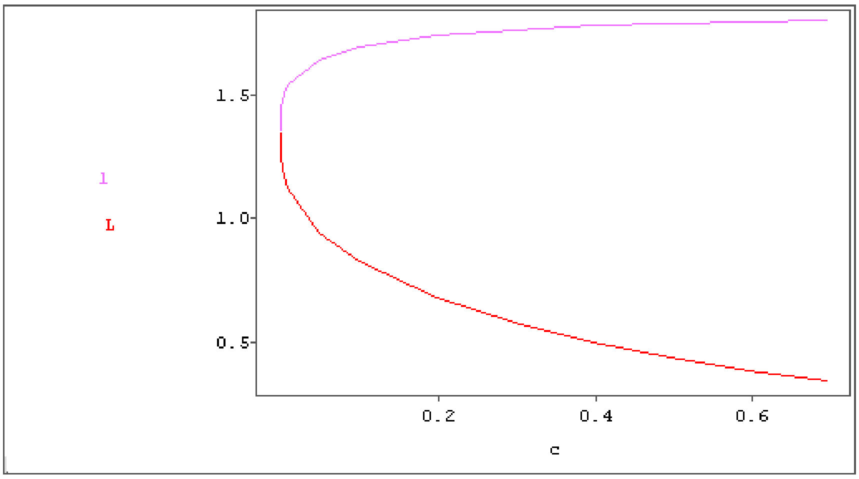

Figure 1. Control bands for varying levels of fixed costs.

Table 2. Control bands for varying levels of the discount rate.

r	L	l	B
0.2	0.4443	1.7972	0.0301
0.19	0.5045	1.9456	0.0489
0.18	0.5753	2.1157	0.0798
0.17	0.6594	2.3124	0.1311
0.16	0.7602	2.5418	0.2169
0.15	0.8826	2.8125	0.3623
0.14	1.033	3.1357	0.6122
0.13	1.2208	3.5275	1.0501
0.12	1.4599	4.0107	1.8355
0.11	1.7711	4.6196	3.2866
0.1	2.1876	5.4071	6.0713
0.09	2.7651	6.4597	11.6893
0.08	3.6033	7.9285	23.8134
0.07	4.897	10.1005	52.5682
0.06	7.0769	13.5903	130.9941
0.05	11.2807	19.9644	398.7536

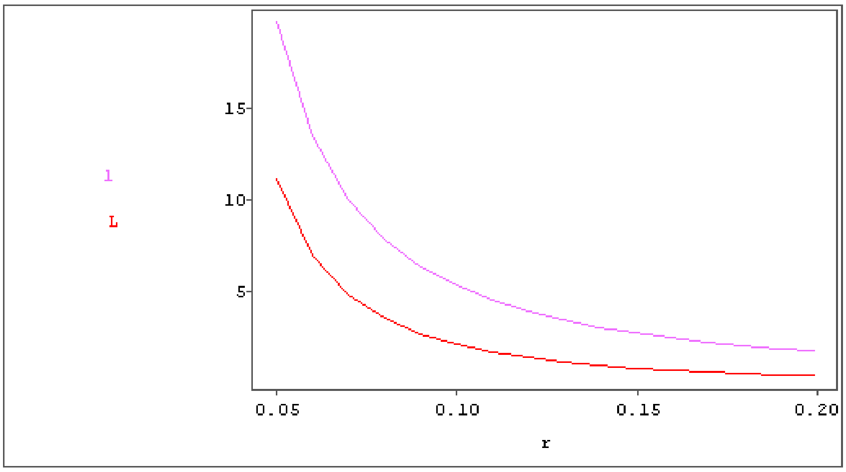

Figure 2. Control bands for varying levels of the discount rate.