Measuring Technical Efficiency in the Stochastic Varying Coefficient Frontier Model

Giannis Karagiannis
(*Department of Economics, University of Macedonia, GREECE*)

Vangelis Tzouvelekas
(*Department of Economics, University of Crete, GREECE*)

Corresponding Author:

Vangelis Tzouvelekas
Dept. of Economics
Faculty of Social Sciences, University of Crete
University Campus, 74100 Rethymno
Crete, GREECE
Tel.: +30-28310-77417; Fax: +30-28310-77406
e-mail: vangelis@econ.soc.uoc.gr

The second author acknowledges the support of a Marie Curie Transfer of Knowledge Fellowship of the European Community's Sixth Framework Programme under contract number MTKD-CT-014288 and of the General Secretariat of Greece under the research project "*PENED 2005*". The usual caveats apply.

Measuring Technical Efficiency in the Stochastic Varying Coefficient Frontier Model

Abstract

Due to the assumption that the best practice methods refer to each input separately instead of the whole set of inputs used by a firm, the benchmark technology as defined in the stochastic varying coefficient frontier model may be infeasible and theoretically improper whenever the maximum response coefficients are not coming from the same production unit. To overcome this problem we suggest an alternative procedure for measuring output-oriented and input-specific technical efficiency inspired from the maximum likelihood formulation of the non-neutral frontier model. The empirical results indicate that there are significant differences between the two procedures in terms of both the estimated efficiency scores (i.e., their means as well as of their frequency distribution) and the ranking of firms.

Keywords: stochastic varying coefficient frontier model, input specific technical efficiency, olive farming, Greece.

JEL Codes: C33, D21, D24.

1. Introduction

In an output oriented manner, technical efficiency is measured as a ratio of realized output to the potential output. The reliability of this measure of technical efficiency depends on how accurately the potential output is measured. It is in general assumed that the potential output is obtained by following the best practice methods, given the technology. This implies in turn that the potential output is determined by the underlying production frontier, given the level of inputs. Since by definition technical efficiency is the discrepancy of the actual (realized) output from the production frontier, its measurement cannot proceed without the estimation of the production frontier.

The estimated frontier depends on the assumptions about the nature and the determinants of best practice methods. The former is related to the question of whether the best practice is a realized method inherent in the data or it may not be realized yet. Consequently, the potential output used to measure technical efficiency may or may not be realized. Up to now in the efficiency measurement literature, all but Kalirajan and Obwona (1994a) have agreed that the frontier results from observed output levels, produced by the firms using the best practice methods.[1] In contrast, Kalirajan and Obwona (1994a) suggested that the potential output need not necessarily be observed in the data at hand. They attempted to justify that by arguing that the best practice method varies from input to input and thus not every firm would be applying all input efficiency. However, it seems more reasonable whatsoever to think of best practice as referring to the whole set of inputs used by a firm instead of each input separately.

On the other hand concerning the determinants of best practice methods, two alternative models have been developed, which are referred to as neutral and non-neutral frontier models. The former assumes that technical efficiency is independent of the levels of input used but is dependent on the method of application of inputs. Thus, even for identical levels of the same inputs, output differs due to differences in the methods of application.[2] In turn the effectiveness of the methods of application is determined by various organizational factors, which are influenced by socioeconomic, demographic etc characteristics that affect the managerial ability of firms. In such a case, the estimated frontier is modeled as a neutral shift of the traditional "average" production function. In contrast, the non-neutral frontier model assumes that both the methods of application of inputs as well as the level of inputs (i.e., scale of operation) determine the potential output and thus, the estimated frontier is modeled as a non-neutral shift of the traditional "average" production function. The non-neutral shift is related to that firms may acquire more information, knowledge and experience with respect to one input's productivity than the other (Huang and Liu, 1994). Apparently, it seems intuitively more appealing to argue that technical efficiency stems from two sources: firm-specific intrinsic characteristics and input levels.

Two alternative approaches have been used to model non-neutral production frontiers. On the one hand, Kalirajan and Obwona (1994a) developed the stochastic varying coefficient frontier (SVCF) model that related the notion of the non-neutral frontier with cross-sectional and possibly temporal variation in production response coefficients, which include not only the intercept term as in the traditional frontier framework but also the slope coefficients. The idea of slope varying coefficients is consistent with the methods of application of inputs to depend on the level of inputs. On the other hand, Huang and Liu (1994) accommodated the notion of the non-neutral frontier by modeling the one-sided error term measuring technical efficiency as a function of not only the variables affecting the managerial and organizational ability of firms but also of input levels, including interaction terms between them.

Besides conceptual differences, these two non-neutral frontier models require quite different econometric estimation techniques. In particular, Huang and Liu's (1994) model is estimated with maximum likelihood, which necessitate the imposition of particular distributional assumptions regarding the one-sided error term. In contrast, the SVCF model dispenses with this assumption as it can be estimated with generalized least squares by using Hildreth and Houck's (1968) random coefficient regression procedure but the additive error term (appended to account for statistical noise) cannot be distinguished from the randomly varying intercept when only cross-section data are available (Kalirajan and Obwona, 1994b; Tsionas, 2002). Thus, in a cross-sectional setting, SVCF is deterministic frontier model. This is not true however with panel data as it is possible to have a (cross-sectional) random intercept and noise at the same time (Kalirajan, Obwona and Zhao, 1996; Tsionas, 2002)).

Despite its attractiveness as a non-neutral frontier model, SVCF's assumptions about the nature of best practice methods raise doubts about the reliability of the resulting efficiency measures. In particular, it is shown that as long as the best response coefficients are coming from different firms in the sample, which as noted by Kalirajan and Obwona (1994a) is quite likely to happen in empirical applications, the resulting frontier is not well defined in theoretical grounds and infeasible for any sample participant. Consequently, by using it to compute the maximum attainable output yields misleading results regarding both the magnitude of technical efficiency and the ranking of firms according to their efficiency scores. Moreover, Kalirajan and Obwona's (1994a) measure of single factor technical efficiency (defined as the ratio of the actual to the maximum response coefficient for each input) also raises concerns about its appropriateness as an efficiency measure.

In this paper, by relying on stochastic frontier methodology, output-oriented and single-factor technical efficiency measures for the SVCF model are developed that overcome the above shortcomings. The former is adapted from the error component literature and is adjusted accordingly to the stochastic nature of the SVCF model. In that sense, it is conceptually analogous to the measure used in Huang and Liu (1994) non-neutral stochastic frontier model. On the other hand, the proposed single factor measure of efficiency is based on Kopp's (1981) notion of non-radial technical efficiency and it is shown that in the context of the SVCF model it could provide firm-specific estimates even with inflexible production frontiers, such as the Cobb-Douglas. After these adjustments, the SVCF model may be seen as a promising alternative to Huang and Liu (1994) non-neutral frontier model.

The remainder of this paper is organized as follows: the proposed efficiency measures for the SVCF model are presented in the next section. The data concerning a sample of 190 olive-growing farms in Greece during the 1992-93 crop year are described in the third section. Comparative empirical results for the proposed and the Kalirajan and Obwona's (1994a) efficiency measures are discussed in the fourth section. Concluding remarks follow in the last section.

2. Measuring Technical Efficiency in the SVCF Model

Following Kalirajan and Obwona (1994a), let the production frontier of the SVCF model in a cross-sectional setting be approximated by the Cobb-Douglas form:

$$ln\, y_i = \beta_{i0} + \sum_{k=1}^{K} \beta_{ik}\, ln\, x_{ik} \tag{1}$$

where y refers to output produced, i is used to index firms and k to index inputs x, and β corresponds to firm-specific technology parameters to be estimated. Following the random coefficient formulation of the SVCF model, $\beta_{ik} = \beta_k + v_{ik}$ for $k = 0,1,\ldots,K$ where $E\left(v_{ik}\right) = 0$ for all i and k, $E\left(v_i v_j'\right) = \Delta$ for i=j and $E\left(v_i v_j'\right) = 0$ for $i \neq j$. Then, the term $u_i = v_{i0} + \sum v_{ik}\, ln\, x_{ik}$ has a mean value of zero and a covariance matrix given by equation (5) in Kalirajan and Obwona (1994a). In this specification, parameter moments rather than parameters themselves are fixed. Consistent estimates of parameter moments can be obtained by generalized least squares and individual parameters are calculated by using the mean parameter vector and the estimates of individual v_i (Griffiths, 1972).[3]

Having estimated (1), Kalirajan and Obwona (1994a) followed the tradition of the frontier literature and measured output-oriented technical efficiency by the ratio of actual to potential output, i.e., $TE_i^O = y_i \big/ exp\left(l\, n\, y_i^*\right)$. However, in calculating the potential output that serves as a benchmark, they used the maximum of the estimated values of the response coefficients for each input, which are defined as $\beta_k^* = \max_i\{\beta_{ik}\}$ for $k = 0,1,\ldots,K$. Then, the frontier is given as:

$$ln\, y_i^* = \beta_0^* + \sum_{k=1}^{K} \beta_k^*\, ln\, x_{ik} \qquad (2)$$

The idea behind this formulation is that both the intercept and the slope coefficients for those who are using the best practice methods would be larger than for those who are not following the best practice methods (Huang and Kalirajan, 1997).

There are two equally possible roots for the origin of the maximum response coefficients. On the one hand, it may be argued that not every firm would be applying all the inputs efficiently and thus the maximum response coefficients need not come from a single firm. The main reason for this is that best practice methods vary from input to input. On the other hand, we may argue that a firm which uses same inputs efficiently may also use all inputs efficiently and thus the possibility that all maximum response coefficients may come from the same firm cannot be completely ruled out. The implications of these two possibilities for the measurement of technical efficiency are very different, however.

In the case where all maximum response coefficients are coming from the same firm, (2) represents an apparently well defined frontier and it can be used to provide reasonable estimates of technical efficiency as well as a consistent ranking of firms according to their efficiency scores. However, when the maximum response coefficients are coming from different firms in the sample, which as was noted by Kalirajan and Obwona (1994a) is quite likely to happen in empirical applications, two problems arise. *First*, the frontier described by (2) might not be feasible for any sample participant, implying that none of the firms in the sample operates with full efficiency.[4] For a deterministic frontier model, this contradicts with the cornerstone assumption in efficiency measurement literature, namely that efficiency is a relative concept measured with reference to observed best practice outcomes and a benchmark that is determined by some peer firms in the sample.[5] *Second*, the resulting frontier in (2) may not be well defined in the sense that it violates certain theoretical properties. Consequently, the estimated technical efficiency scores are inconsistent.

The implications of these problems are illustrated further by the following two examples. The first was initially mentioned by Kalirajan and Obwona (1994a) and refers to the case where all firms in the sample exhibit (or are enforced to exhibit) constant return to scale. Then if the maximum response coefficients are coming from different firms we cannot ruled out the possibility that the frontier is characterized by increasing returns to scale. But then the best practice output might not be feasible if all firms had to have constant returns to scale. The same argument also applies to cases where all firms are characterized by decreasing returns to scale. Thus in general if all firms in the sample exhibit either decreasing or constant returns to scale and the maximum response coefficient are coming from different firms, there is no guarantee that the resulting frontier will also exhibit the same scale structure.

The second example considers the case where a cost rather than a production frontier is used as a benchmark. In a manner analogous to (2), the cost frontier would be constructed by using the minimum (instead of the maximum) response coefficients. If the minimum response coefficients are coming however from different firms in the sample, there is no guarantee that the resulting cost frontier will satisfy the linear homogeneity property even though the individual cost functions are by definition linearly homogeneous in input prices. But if the resulting cost frontier is not linear homogeneous in input prices, the best practice technology described by an equation analogous to (2) is not well defined.[6]

Even though the aforementioned problems are not meet at the empirical results reported by Kalirajan and Obwona (1994a) and Salim and Kalirajan (1999) as the maximum response coefficients are coming for same firm, there are inherent in other studies.[7] For example, Kalirajan and Obwona (1994b), Huang and Kalirajan (1997), Kalirajan and Huang (2001), as well as the present study, found that the maximum response coefficients are coming from different firms.

A different procedure for calculating technical inefficiency scores is proposed in this paper to resolve the above shortcomings of the SVCF model, which relies on the idea that best practice methods refer to the whole set of inputs used by a firm instead of each input separately. Starting with the basic relation that $y_i = f\left(\cdot\right)TE_i^O$, where $f\left(\cdot\right)$ refers to the production frontier, we can rewrite it for the Cobb-Douglas form as:

$$ln\, y_i = \beta_0 + \sum_{k=1}^{K} \beta_k \, ln\, x_{ik} + ln\, TE_i^O \qquad (3)$$

On the other hand, by explicitly considering the random coefficient formulation of (1) it may be written as:[8]

$$ln\, y_i = \beta_0 + \sum_{k=1}^{K} \beta_k \, ln\, x_{ik} + v_{i0} + \sum_{i=1}^{K} v_{ik} \, ln\, x_{ik} \tag{4}$$

Then by comparing (3) and (4) yields:

$$ln\, TE_i^O = v_{i0} + \sum_{i=1}^{K} v_{ik} \, ln\, x_{ik} \tag{5}$$

Notice that (5) is completely analogous to the measure of technical efficiency used by Huang and Liu (1994) in the maximum likelihood formulation of the non-neutral frontier model.

Given the assumptions about v_i, it is clear that the expected value of $\ln TE_i^O$ in (5) is equal to zero implying that the expected value of TE_i^O is equal to one. This means that the estimated values of TE_i^O may be less or greater than one. To ensure that estimated values of TE_i^O are bounded above by one, the following normalization suggested by Schmidt and Sickles (1984) is employed:

$$ln\, \hat{TE}_i^O = \max_i \left\{ \hat{v}_{i0} + \sum_{i=1}^{K} \hat{v}_{ik} \, ln\, x_{ik} \right\} - \left(\hat{v}_{i0} + \sum_{i=1}^{K} \hat{v}_{ik} \, ln\, x_{ik} \right) \tag{6}$$

where ^ denotes estimates values. This normalization amounts to counting the most efficient firm in the sample as fully efficient and to compare efficiency across firms in a consistent manner.

On the other hand, Kalirajan and Obwona (1994a, 199b) used the ratio of the actual to the maximum response coefficients for each input to obtain firm-specific estimates of input-specific technical efficiency. That is,

$$ITE_i^k = \frac{\beta_{ik}}{\max_i \{\beta_{ik}\}} \tag{7}$$

where values less than one indicate inefficiency. The inappropriateness of (7) as an efficiency measure arises from the fact that is based on production elasticities, which following Forsund (1996) are frontier measures. Thus (7) lacks any theoretical foundation for being an appropriate efficiency measure.

Instead Kopp's (1981) notion of ITE_i^k may be used to identify in a theoretically consistent way the technical efficient use of individual inputs. In particular, Kopp's

(1981) measure of ITE_i^k is defined as the ratio of minimum feasible to observed use of each input conditional on the production technology and the observed levels of output and other inputs, i.e., $ITE_i^k = x_{ik}^I / x_{ik}$. The minimum feasible use x_{ik}^I for the k^{th} input coincides with that quantity necessary to ensure technical efficiency without altering the quantities of other inputs and the level of output produced. Then, it is clear that Kopp's (1981) measure of ITE_i^k is non-radial and has an input-conserving orientation, which however cannot be converted into a cost-saving measure.

According to Reinhard, Lovell and Thijssen (1999), the minimum feasible use of the K input for the ith firm can be computed through the fitted frontier function assuming $ln\,TE_i^O = 0$, i.e.,

$$ln\, y_i = \hat{\beta}_0 + \sum_{k=1}^{K-1} \hat{\beta}_k \, ln\, x_{\iota\kappa} + \hat{\beta}_{\mathrm{K}} \, ln\, x_{iK}^I \tag{8}$$

Then, there are two alternatives: either we can solve (8) for $ln\, x_{iK}^I$ and then compute ITE_i^k using the observed x_{iK}, or we can combine (8), (3) and (5) to shown that

$$ln\, ITE_i^K = ln\, x_{iK}^I - ln\, x_{iK} = \frac{ln\, TE_i^O}{\hat{\beta}_K} \tag{9}$$

or equivalently, $ITE_i^K = \left(TE_i^O\right)^{1/\hat{\beta}_K}$. Then, by using again Schmidt and Sickles (1984) normalization, we can compute input-specific technical efficiencies as:

$$ln\, \hat{ITE}_i^K = \max_i \left\{ \frac{\hat{v}_{i0} + \sum_{i=1}^{K} \hat{v}_{ik} \, ln\, x_{ik}}{\hat{\beta}_K} \right\} - \left(\frac{\hat{v}_{i0} + \sum_{i=1}^{K} \hat{v}_{ik} \, ln\, x_{ik}}{\hat{\beta}_K} \right) \tag{10}$$

which ensures that they lie in the interval $(0,1]$.

3. Data

The data used in the empirical application were extracted from a survey undertaken by the Institute of Agricultural Economics and Rural Sociology of Greece. The analysis focuses on a sample of 190 olive-growing farms, located in the three most productive regions of Greece (Peloponissos, Crete and Sterea Ellada). Observations were obtained for the 1992-93 crop year. The sample was selected with respect to production area, the total number of farms within the area, the number of olive trees on the farm, the area of cultivated land and the share of olive oil production in farm output.

The dependent variable is the olive-oil production measured in kilograms. The inputs included as explanatory variables are: (a) *labor*, comprising hired (permanent and casual), family and contract labor, measured in working hours. It covers farm activities such as plowing, fertilization, chemical spraying, harvesting, irrigation, pruning, transportation, administration and other services; (b) *fertilizers*, including nitrogenous, phosphate, potash, complex and others, measured in kilograms; (c) *other intermediate inputs* expenses, consisting of pesticides, fuel and electric power, irrigation taxes, depreciation, interest payments, fixed assets interest, taxes and other miscellaneous expenses, measured in drachmas (constant 1990 prices); (d) *land*, including only the share of farm's land devoted to olive-tree cultivation measured in stremmas (one stremma equals 0.1 ha).

Aggregation over the various components of the above input categories (except of land input) was conducted using *Divisia* indices with cost shares serving as weights. To avoid problems associated with units of measurement, all variables were converted into indices, with the basis for normalization being the representative olive-growing farm. The choice of the representative farm was based on the smallest deviation of the variables (i.e. output and input levels) from the sample means. The econometric estimation of the model was carried out using Gauss (ver 3.2.26).

4. Empirical Results

Parameter estimates of the Cobb-Douglas SVCF model for olive-growing farms in Greece are presented in Table 1. The hypothesis of random coefficient variation cannot be rejected by the Breusch-Pagan LM-test lending support to SVCF model. Indeed, individual response coefficients vary considerably among sample farms (see Table 1) implying that farmers are using quite different farming practices. Mean response coefficients along with the corresponding t-ratios are presented in the first two columns of Table 1. These estimates indicate that land exhibited the highest output elasticity (0.516) followed by labor (0.399), other intermediate inputs (0.106), and fertilizers (0.036). For land input the relevant range of variation is between 0.539

and 0.441, for labor between 0.586 and 0.309, for other intermediate inputs between 0.134 and 0.098 and for fertilizers between 0.153 and 0.011. Mean estimate of returns to scale is found to be close to unity (1.057) varying from a maximum of 1.285 to a minimum of 1.012.[9]

The results on Table 1 indicate that the maximum response coefficients are not coming from the same farm in the sample. Hence, the estimates of farm-specific output-oriented technical efficiency using Kalirajan and Obwona's (1994a) procedure may not lead to meaningful results. However, for comparison purposes we present also the corresponding estimates using Kalirajan and Obwona's (1994a) procedure. The results are presented in Table 2 in the form of frequency distribution within a decile range.

According to the proposed measure, mean output-oriented technical efficiency was found to be 88.60% (upper panel of Table 2). As it is shown there is a great concentration of farms in the upper tail of the distribution as almost the 81% of the farms exhibit mean output-oriented technical efficiency above 80%. Estimates of input-specific technical efficiencies, obtained from (10), indicate that land is utilized more efficiently in the production process (80.68%) followed by labor (76.79%), other intermediate inputs (52.22%) and fertilizers (32.90%). Further, all individual measures of input-specific technical efficiencies indicate a considerable variation among farms in the sample, which is more intense in other intermediate inputs and fertilizers.

On the other hand, farm-specific estimates of output-oriented and input-specific technical efficiencies based on the Kalirajan and Obwona (1994a) procedure are presented in the lower panel of Table 2. As it is shown, there are considerable differences in all four alternative measures both in their mean estimates as well as in their frequency distribution. Mean output-oriented technical efficiency was found to be 66.51%, almost 14% lower than the estimates obtained from the proposed measure. This is rather expected as we have shown that, whenever the maximum response coefficients are coming from different farms, the frontier as defined by Kalirajan and Obwona (1994a) is at the end infeasible for all of them. Mean input-specific technical efficiencies were also found to have significant differences: for land is 93.53%, for intermediate inputs is 84.09%, for labor is 75.42%, and for fertilizers is 35.81%. The variation among farms is not so large except for fertilizers where the minimum value is 11.49%. It is noteworthy the fact, that for land and intermediate inputs, all the sample participants have technical efficiency scores above 80 and 70%, respectively.

Besides the differences in average estimates, the correlation between the two sets of technical efficiency estimates is very low suggesting that the ranking would also be different. Table 3 presents the ranking of the ten most and least efficient farms according to alternative measures discussed previously. Regarding the output-oriented measure, it can be seen that farms that are found to be technically efficient according to the proposed procedure are not also technically efficient using Kalirajan and Obwona's (1994a) procedure. Nevertheless, there exists a relative concordance between the two indices concerning the least efficient farms. The Spearman correlation coefficients reported in Table 2 confirm this finding. They also show an insignificant correlation between the estimated input-specific technical efficiencies of land. On the other hand, there is a significantly negative correlation between the input-specific estimates for labor, fertilizers, and other intermediate inputs as the relevant Spearman correlation coefficients were found to be statistically significant at the 1% level. This means that farms that are found to be technically inefficient according to Kalirajan and Obwona's (1994a) input-specific technical efficiency measures for these inputs are also among the most efficient according to the proposed input-specific measure.

Moreover, as it is shown in Table 3, the concordance between output-oriented and all input-specific efficiency measures, estimated by using the proposed procedure, is very high although there are some exceptions (first 6 columns of table 3). This indicates that a farm which is technical efficient (inefficient) according to output-oriented measure is also among the most (least) efficient farms according to all input-specific measures. However, the same is not true when the Kalirajan and Obwona's (1994a) procedure is used. As it can be seem from the last 5 columns of Table 3, farms that are technically efficient in an output-oriented perspective they are not efficient in the use of labor, fertilizers and other intermediate inputs. There is, however, a concordance between output-oriented efficiency and the input-specific efficiency for land.

5. Concluding Remarks

The SVCF model developed by Kalirajan and Obwona (1994a) have all interesting features of a non-neutral frontier model, but their procedure for estimating both the output-oriented and the input-specific technical efficiency measures is not free of theoretical and methodological problems. Specifically, it has been shown that the frontier as defined by Kalirajan and Obwona (1994a) is in practice infeasible for any sample participant and theoretically improper whenever the maximum response coefficients are not coming from the same production unit. Consequently, by using it to compute the maximum attainable output yields misleading results regarding both the magnitude of technical efficiency and the ranking of firms according to their efficiency scores. The main reason behind these problems is Kalirajan and Obwona's (1994a) assumption that the best practice methods refer to each input separately instead of the whole set of inputs used by a firm.

In order to overcome these problems, we suggest an alternative procedure for measuring output-oriented and input-specific technical efficiencies within the SVCF model. The proposed measures are respectively analogous to those used by Huang and Liu (1994) in the maximum likelihood formulation of the non-neutral frontier model and Kopp's (1981) definition of single-factor efficiency measure. After these adjustments, the SVCF model may be seen as a promising alternative to Huang and Liu (1994) non-neutral frontier model.

The empirical results of the present study indicate that there are significant differences in efficiency score estimates (in terms of their means as well as of their frequency distribution) between the proposed measures and those used by Kalirajan and Obwona (1994a), as the maximum response coefficients are not coming from the same farm in the sample. This is true for both the output-oriented and the input-specific technical efficiency scores. In addition, ranking of farms according to their efficiency scores is also found to be different. Thus, different conclusions are drawn which may mislead policy-makers in designing appropriate measures for improving individual performance.

References

Breusch, T.S. and Pagan, A.R. (1979). “A simple test for heteroscedasticity and random coefficient variation”, *Econometrica*, Vol. 47, pp. 1287-1294.

Forsund, F.R. (1996). “On the calculation of the scale elasticity in DEA models”, *Journal of Productivity Analysis*, Vol. 7, pp. 283-302.

Griffiths, W.E. (1972). “Estimation of actual response coefficients in the Hildreth-Houck random coefficient model”, *Journal of the American Statistical Association*, Vol. 67, pp. 633-635.

Hildreth, C. and Houck, J.P. (1968). “Some estimators for linear model with random coefficients”, *Journal of American Statistical Association*, Vol. 63, pp. 584-595.

Huang, C.J. and Liu, J.T. (1994). “Estimation of a non-neutral stochastic frontier production function”, *Journal of Productivity Analysis* Vol. 5, pp. 171-180.

Huang, Y. and Kalirajan, K.P. (1997). “Potential of China’s grain production: Evidence from the household data”, *Agricultural Economics* vol. 17, pp. 191-199.

Kaliajan, K.P. and Huang, Y. (2001). “Does China have a grain problem? An empirical analysis”, *Oxford Development Studies* vol. 29, pp. 45-55.

Kalirajan, K.P. and Obwona, M.B. (1994a). “Frontier production function: The stochastic coefficients approach”, *Oxford Bulletin of Economic and Statistics* Vol. 56, pp. 87-96.

Kalirajan, K.P. and Obwona, M.B. (1994b). “A measurement of firm- and input-specific technical and allocative efficiencies”, *Applied Economics* vol. 26, pp. 393-39.

Kalirajan, K.P., Obwona, M.B. and Zhao, S. (1996). “A Decomposition of total factor productivity growth: The case of Chinese agricultural growth before and after reforms”. *American Journal of Agricultural Economics*, vol. 78, pp. 331-338.

Kopp, R.J. (1981). “The measurement of productive efficiency: A reconsideration”, *Quarterly Journal of Economics*, Vol. 96, pp. 477-503.

Reinhard, S., Lovell, C.A.K. and Thijssen, G.J. (1999). “Econometric estimation of technical and environmental efficiency: An application to Dutch dairy farms”, *American Journal of Agricultural Economics*, Vol. 81, pp. 44-60.

Salim, R.A. and Kalirajan, K.P. (1999). “Sources of output growth in Bangladesh food processing industries: A decomposition analysis”, *Developing Economies* vol. 37, pp. 355-374.

Schmidt, P. and Sickles, R. (1984). “Production frontiers and panel data”, *Journal of Business and Economic Statistics*, Vol. 2, pp. 367-374.

Tsionas, E.G. (2002). “Stochastic frontier models with random coefficients”, *Journal of Applied Econometrics* vol. 17, pp. 127-147.

TABLE 1

Parameter Estimates of the Cobb-Douglas Stochastic Varying Coefficient Frontier Model

Parameter	Estimate	t-stat	Max Coefficients[1]		Min Coefficients[1]		Std Dev
Constant	0.164	(2.114)	0.197	(70)	0.144	(94)	0.006
β_A	0.516	(2.592)	0.539	(109)	0.441	(94)	0.017
β_L	0.399	(1.798)	0.568	(94)	0.309	(30)	0.039
β_C	0.036	(1.241)	0.153	(94)	0.011	(109)	0.025
β_O	0.106	(1.277)	0.134	(23)	0.098	(77)	0.006
RTS	1.057		1.285	(94)	1.012	(30)	0.051
LM-test:	514.02	$\chi^2_{14,0.05} = 23.69$					

where, **A** stands for land, **L** for labour, **C** for fertilizers, **O** for other intermediate inputs and **RTS** for returns to scale.

[1] The numbers in parentheses are the corresponding farms with the maximum or minimum coefficient.

TABLE 2

Frequency Distribution of Output- and Input-Oriented Multiple- and Single-Factor Technical Efficiency Estimates

(%)	$MFTE_i^O$	$SFTE_{Ai}^I$	$SFTE_{Li}^I$	$SFTE_{Ci}^I$	$SFTE_{Oi}^I$
Present Study					
<10	0 (0.0)	0 (0.0)	0 (0.0)	78 (41.1)	35 (18.4)
10-20	0 (0.0)	1 (0.5)	3 (1.6)	16 (8.4)	18 (9.5)
20-30	0 (0.0)	3 (1.6)	10 (5.3)	12 (6.3)	13 (6.8)
30-40	0 (0.0)	10 (5.3)	9 (4.7)	16 (8.4)	7 (3.7)
40-50	3 (1.6)	9 (4.7)	9 (4.7)	13 (6.8)	11 (5.8)
50-60	8 (4.2)	10 (5.3)	10 (5.3)	6 (3.2)	14 (7.4)
60-70	12 (6.3)	14 (7.4)	22 (11.6)	8 (4.2)	17 (8.9)
70-80	15 (7.9)	21 (11.1)	13 (6.8)	6 (3.2)	23 (12.1)
80-90	32 (16.8)	29 (15.3)	32 (16.8)	12 (6.3)	13 (6.8)
90-100	120 (63.2)	93 (48.9)	82 (43.2)	23 (12.1)	39 (20.5)
Mean	88.60	80.68	76.79	32.90	52.22
Max	100	100	100	100	100
Min	40.71	17.53	10.53	0.05	0.08
Kalirajan and Obwona (1994a)					
<10	0 (0.0)	0 (0.0)	0 (0.0)	1 (0.5)	0 (0.0)
10-20	0 (0.0)	0 (0.0)	0 (0.0)	16 (8.4)	0 (0.0)
20-30	0 (0.0)	0 (0.0)	0 (0.0)	82 (43.2)	0 (0.0)
30-40	35 (18.4)	0 (0.0)	0 (0.0)	34 (17.9)	0 (0.0)
40-50	56 (29.5)	0 (0.0)	0 (0.0)	25 (13.2)	0 (0.0)
50-60	45 (23.7)	0 (0.0)	3 (1.6)	11 (5.8)	0 (0.0)
60-70	4 (2.1)	0 (0.0)	19 (10.0)	10 (5.3)	0 (0.0)
70-80	5 (2.6)	0 (0.0)	131 (68.9)	8 (4.2)	39 (20.5)
80-90	7 (3.7)	27 (14.2)	28 (14.7)	1 (0.5)	138 (72.6)
90-100	38 (20.0)	163 (85.8)	9 (4.7)	2 (1.1)	13 (6.8)
Mean	66.51	93.53	75.42	35.81	84.09
Max	92.57	100	100	100	100
Min	30.46	81.81	54.37	11.49	72.94
Rho[1]	0.388	-0.388*	-0.898*	-0.667*	-0.916*
N	190	190	190	190	190

where, **A** stands for land, **L** for labour, **C** for fertilisers and pesticides and **O** for other capital inputs. In parentheses are the corresponding percentage values. **Rho** stands for Spearman correlation coefficient. (*) indicate statistical significance at the 1% level.

TABLE 3

Ranking of Farms According to the Suggested Output-Oriented MFTE Index

Present Study					Kalirajan and Obwona (1994a)				
TE_i^O	ITE_i^A	ITE_i^L	ITE_i^C	ITE_i^O	TE_i^O	ITE_i^A	ITE_i^L	ITE_i^C	ITE_i^O
1	1	1	1	1	38	40	167	151	170
2	4	2	2	6	39	35	169	153	162
3	3	3	5	3	61	31	159	162	174
4	5	8	4	4	31	34	156	157	180
5	2	5	8	8	48	23	158	164	176
6	6	6	6	11	37	24	171	155	166
7	7	11	7	7	27	37	150	166	172
8	8	7	13	5	36	32	157	168	157
9	11	9	10	9	44	56	151	149	178
10	14	13	9	10	78	25	146	170	181
181	181	177	181	181	171	10	10	11	11
182	182	182	176	177	175	9	9	10	10
183	183	185	183	183	186	8	8	9	9
184	184	184	182	187	184	7	7	8	8
185	185	183	185	185	181	6	6	7	7
186	186	186	186	186	183	5	5	6	6
187	184	187	189	184	179	4	4	5	5
188	188	188	188	189	188	3	3	4	4
189	189	190	187	188	187	2	2	3	3
190	190	189	190	190	190	1	1	2	2

Endnotes

[1] This should not be confused with that, in most efficiency measurement approaches with the exception of Free Disposable Hull, inefficiencies are calculated with respect to unobservable input-output combinations, because of the convexity assumption.

[2] This implies that all inefficient firms suffer a uniform reduction in input productivity without altering their input bundle (Huang and Liu, 1994).

[3] The hypothesis of firm-specific coefficients could be tested by using the standard Breusch-Pagan LaGrange multiplier test (Breusch and Pagan, 1979).

[4] It is obvious that when all best response coefficients are coming from the same firm, there is at least one firm in the sample that operates efficiently. If however the best response coefficients are coming from different firms, none of the firms in the sample operates with full efficiency. This is a rather simply way to check the origin of the best response coefficients.

[5] In a stochastic frontier model it is quite likely that none of the firms in the sample operates with full efficiency, but this is due to stochastic disturbances and not because the frontier is not feasible to sample participants.

[6] Since the Cobb-Douglas is self-dual, the same argument can be applied to (1). Let suppose that after having estimated (1) we would like to use the dual representation of the best practice technology (i.e., cost frontier) as a benchmark. Then we have first to compute the corresponding individual cost functions by relying on (1) and then to use these individual cost functions to construct the underlying cost frontier. This cost frontier will not satisfy the property of linear homogeneity if the minimum response coefficients are coming from the different firms.

[7] For the Kalirajan and Obwona (1994a) study in particular, which reports estimates of the potential and actual output for each firm separately, it can be seen that the maximum response coefficients are from the firm with identification number 43.

[8] Notice that this formulation is observationally equivalent to a (fixed coefficient) neutral frontier model with heteroscedastic statistical noise (Salim and Kalirajan, 1999; Tsionas, 2002), but it is estimated with a completely different method. Here we follow Kalirajan and Obwona (1994a) and regard (4) as a non-neutral frontier model.

[9] As noted by Kalirajan and Obwona (1994a) the hypothesis of constant returns to scale cannot easily be tested since the imposition of the corresponding restrictions in the model complicates its estimation.