# Asset allocation in the Athens Stock Exchange:
# A variance sensitivity analysis

**by**

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Abstract

This paper provides an analysis of asset allocation using univariate portfolio GARCH models from the Athens Stock Exchange. We use daily data for the period January 1997 to February 2005. Our analysis adopts the methodology due to Manganelli (2004) and we are able to recover from the univariate approach the multivariate dimension of the portfolio allocation problem. Manganelli (2004) suggests that such a dual problem can be solved with the application of a variance sensitivity analysis which considers the change in the portfolio variance induced by an infinitesimal change in the portfolio allocation. Our main findings are based on the estimation of the variance sensitivity for a portfolio of two assets and the way sensitivity has been changing over time and this has implications for risk management. In addition we compute the second derivative of the estimated variance with respect to portfolio weights and this gives an indication of the benefits arising from diversification at any given point of time.

**Keywords:** asset allocation, GARCH models, risk management, sensitivity analysis

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## 1. Introduction

The recent development of risk management tools is the result of the need for comprehensive measures in order portfolio managers and financial institutions to be able to calculate market risk. This need has become more evident during the 1990s which has witnessed the bankruptcy of major financial institutions like the BCCI and Barrings international banks as well as the increased volatility in equity markets as a result of the dramatic rise of investment in emerging markets.

The Basel Committee of Banking Supervision through the 1988 Basel Accord and the 1996 Amendment of the Basel Accord (or Basel II) which will be in force in 2007 has set the regulation framework for the world financial system. There are three main tools available to regulators for the measurement and control of financial risk, namely minimum risk capital requirements; inspections and reporting requirements and public disclosure and market discipline. Risk management is mainly linked with the minimum risk capital requirements which are imposed by the regulatory body. The Basel Committee currently recommends two types of models for measuring market risk on a daily basis, with VaR being the most popular one.

Value-at-Risk has become the standard tool used by financial analysts to measure market risk. VaR is defined as a certain amount lost on a portfolio of financial assets with a given probability over a fixed number of days. The confidence level represents 'extreme market conditions' with a probability that is usually taken to be 99% or 95%. This implies that in only 1% (5%) of the cases will lose more than the reported VaR of a specific portfolio. VaR has become a very popular tool among financial analysts which is widely used because of its simplicity. Essentially the VaR provides a single number that represents market risk and therefore it is easily understood.[1]

During the last decade several approaches in estimating the profit and losses distribution of portfolio returns have been developed and a substantial literature of empirical applications have emerged. The first approach is based on parametric models such as Riskmetrics (1996) and GARCH models.[2] The second approach for the estimation of the distribution of profits and losses is the non-parametric historical simulation. The group for the estimation of the VaR are the semiparametric models including the extreme value theory and the conditional autoregressive value-at-risk with regression quantile method.[3]

The estimation of multivariate GARCH models is an extensively used approach for risk management. However, it is recognized that such estimation is very demanding because we are required to estimate large number of parameters whose number increases exponentially as the number of variables increases. Recently, Manganelli *et al*. (2002) and Manganelli (2004) have developed an approach which provides a solution to the multivariate problem with the GARCH estimation. This approach is based on the estimation of univariate portfolio models and then with the use of certain statistical tools we are able to recover the multivariate dimension which is lost in the estimation of the univariate models. The key idea to this new methodology is to consider that the estimated univariate portfolio variance is a function of the weights of the assets that form the portfolio. Then next step of this approach is to take the first and second derivatives of the variance subject to these

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[1] See also Bank for International Settlements (1988, 1999a,b,c, 2001).
[2] Since their introduction by Engle (1982), ARCH models have been used extensively to estimate the volatility of financial assets.
[3] Jorion (2000) and Alexander (2002, 2005) provide a comprehensive analysis of the VaR models.

weights. This will assist us to deduce important information with respect to the local behaviour (i.e. around the portfolio weights) of the estimated variance.

The idea of employing measures of sensitivity to the weights of the portfolio allocation has previously used in a number of alternative models of estimating VaR. Thus, Garman (1996) argues that the computation of the derivative of the VaR with respect to the individual elements of a specific portfolio in order to evaluate the potential influence of trading on the VaR of a financial institution or a company. Moreover, Gourieroux *et al*. (2000) have adopted the proposal set forth by Garman (1996) to provide an analysis of its theoretical implications to alternative VaR specifications. Manganelli (2004) argues that the same type of analysis can be done for the case of the variance of a portfolio given that the corresponding variance-covariance matrix.

Manganelli (2004) makes a significant contribution to the issue of using the sensitivity analysis for achieving optimal asset allocation within the context of univariate GARCH models as we already explained above. Their approach has a number of significant implications. First, the GARCH sensitivity analysis can be used by portfolio managers and investors to test whether their actual portfolio has minimum variance. According to Manganelli (2004) this test amounts to a value of zero for all derivatives with respect to portfolio weights. Second, this methodology can also be used if we want to examine the effect that any asset has on the variance of the portfolio. This will provide valuable information to portfolio managers in order to identify the main sources of market risk and/or to investigate the influence that a specific transaction exercises on the portfolio variance. Finally, Manganelli (2004) develop a simple method for the estimation of the full variance–covariance matrices of portfolio assets.

VaR models as well as sensitivity analysis has mostly applied on mature markets. However, during the 1990s there has been a growing significance of emerging markets for investors. At the same time these markets exhibit substantial volatility as opposed to that of the mature markets. Such observed volatile behaviour od stock prices in these new markets have made investors to become more cautious in their investment decisions while it has also led for the increased need for a more careful study of price volatility in stock markets. Indeed, recently we observe an intensive research from academics, financial institutions and regulators of the banking and financial sectors to better understanding the operation of emerging markets.

This paper focuses on the issue of asset allocation to one European emerging market the Athens Stock Exchange (ASE). We use daily data for 30 companies listed in ASE for the period 14 January 1997 to 10 February 2005. We apply the sensitivity analysis proposed by Manganelli (2004) and we provide an evaluation of this approach with the results obtained from the estimation of three alternative models, namely the Dynamic Conditional Correlation model (DCC), the Orthogonal GARCH model (OGARCH) and the Exponentially Weighted Moving Average (EWMA) model. For this application we first estimate the variance sensitivity for a portfolio with two assets traded at ASE and then we estimate minimum variance portfolios for any given point of time. The main finding of our analysis is the use of sensitivity analysis for asset allocation at this emerging market provides a suitable measure for the diversification opportunities at any given point of time and it leads to substantial efficiency gains.

The rest of the paper is organized as follows. Section 2 presents the theoretical aspects of sensitivity analysis. In section 3 we discuss the alternative GARCH specification and the estimation of minimum variance portfolios. Section 4 reports the empirical results with the summary and the concluding remarks given in a final section.

## 2. GARCH models and variance sensitivity analysis

In this and the next section we draw on Manganelli (2004) and we provide a description of the theoretical consideration of the variance sensitivity analysis within the univariate portfolio GARCH models and show how the multivariate dimension of the portfolio allocation problem may be recovered from the univariate approach. The starting point of the analysis is the argument made by Nijman and Sentana (1996) that a linear combination of variables generated by a multivariate GARCH process will only be a weak GARCH process. An implication of this result is that an attempt to fit GARCH processes directly to portfolio returns will generally lead to a model misspecification.

Manganelli (2004) developed an alternative methodology based on the notion of “quasi-maximum-likelihood” introduced by White (1994), assuming that any GARCH model is only a rough approximation of the true relationship among the observed data. The basic proposition put forward by Manganelli (2004) is that when we change the weights of a portfolio this will lead to a change of the time series of portfolio returns. This change will then lead to an alteration of the information available for the estimation of the univariate GARCH process. A result of this procedure will be that the derived variance can be considered as function of the portfolio returns via two channels, the first being the vector of portfolio returns and the second being the estimated parameters.[4] Drawing on Maganelli (2004, p373-374) we briefly sketch the basics theoretical derivations.[5]

Let $y_t$ denotes the return of the portfolio consisted by $n+1$ assets and let $y_{t.i}$ be the $i$th stock return, for $t=1,.....,T$ and $i=1,.......,n+1$. Denoting the weight of asset $i$ by $a_i$, the portfolio return at time $t$ is $y_t = \sum_{i=1}^{n+1} a_i y_{t,i}$. Given that the weights $a_i$ must sum to one, we can express one weight as a function of the others, $a_{n+1} = 1 - \sum_{i=1}^{n} a_i$ .

Then, let us assume that $y_t$ follows a zero-mean process with a GARCH$(p,q)$ conditional variance $h_t$:

$$y_t = \sqrt{h_t}\varepsilon_t \qquad \varepsilon_t \mid \Omega_t \sim (0,1) \tag{1}$$

$$h_t = z_t'\theta \tag{2}$$

where

$\underset{mx1}{z_t} = (1, y_{t-1}^2,......, y_{t-q}^2, h_{t-1},......., h_{t-p})'$, $\underset{mx1}{\theta} = (\alpha_0, \alpha_1,.....,\alpha_q, \beta_1,......,\beta_p)'$, and

$m = p+q+1$

$\Omega_t = \{a, [y_{r,1}]_{t=1}^{t-1},....,[y_{r,n+1}]_{r=1}^{t-1}\}$, is defined as the information set of the model, where $a$ denotes the $n$-vector of portfolios weights.[6] It is clear from the definition of the

[4] As Manganelli (2004, p. 373) points out that the estimated parameters depend on the time series of portfolio returns used in estimation).
[5] For a full analysis of the mathematical and statistical derivations, (Manganelli, 2004).

information set that a change in the vector of portfolio weights implies a change in the information set. This is due to the fact that the actual series of the stock returns are included in the information. Since, the problem to evaluate the potential influence of a transaction on the estimated variance can become extremely complicated as a result of the required re-estimation of the complete model, Manganelli (2004) suggest an alternative simple method.

This method calls for the calculation of the first derivative of the variance with respect to the weights. Thus, a positive derivative would indicate that the change in weights due a trade on a particular asset will result to an increase of the variance of the portfolio while a negative derivative will lead to a reduction in the portfolio's variance. To analyze this point let us define $\hat{h} = \hat{z}_t' \hat{\theta}$ to be the estimated variance. The computation of the first derivative of $\hat{h}_t$ is based on the recognition that both the vectors $\hat{z}_t$ and $\hat{\theta}$ (the vector of the estimated coefficients) are functions of the weight $a$. Then the first derivative is derived as follows:

$$\underset{xx1}{\frac{\partial \hat{h}}{\partial a}} = \underset{nxm}{\frac{\partial \hat{z}_t'}{\partial a}} \underset{nx1}{\hat{\theta}} + \underset{nxm}{\frac{\partial \hat{\theta}'}{\partial a}} \underset{mx1}{\hat{z}}_t \tag{3}$$

In order to analyse carefully the local behaviour of the estimated variance with respect to the portfolio allocation we derive the second derivative, which will allows us to examine the concavity its concavity. This is given by:

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[6] As Manganelli (2004, p. 373) notes, that the $(n+1)$ weight equals one minus the sum of the other weights. The respective $(n+1)$ asset is considered to be the benchmark asset against which the sensitivity is conducted.

$$\underset{nxn}{\frac{\partial^2 \hat{h}_t}{\partial a \partial a'}} = \underset{nxm}{\frac{\partial \hat{z}'_t}{\partial a}} \underset{nxm}{\frac{\partial \hat{\theta}}{\partial a}} + \underset{nxm}{\frac{\partial \hat{\theta}'}{\partial a}} \underset{mxn}{\frac{\partial \hat{z}_t}{\partial a'}} + \left( \underset{1xm}{\theta'} \otimes \underset{nxn}{I} \right) \underset{mnxn}{\frac{\partial}{\partial a'}} vec\left( \frac{\partial \hat{z}'_t}{\partial a} \right) + \left( \underset{1xm}{\hat{z}'_t} \otimes \underset{nxn}{I_n} \right) \frac{\partial}{\partial a'} vec\left( \frac{\partial \hat{\theta}'}{\partial a} \right) \qquad (4)$$

where $\otimes$ indicates the Kronecker product and $I_n$ is an ($nxn$) identity matrix.

Manganelli (2004, p. 374-375 and Appendix B) provide a full mathematical analysis of the evaluation and properties of the first and second derivatives.

## 3. Asset allocation and variance sensitivity

The sensitivity analysis approach discussed in section 2 can now be employed in order to estimate large variance-covariance matrices as well as to analyze the optimal conditional portfolio allocation in a mean-variance framework. Again, we draw on Manganelli (2004).

Using the standard mean-variance model due to Markowitz we consider the following portfolio allocation problem:

$$\max_a E_t[u(y_t)] = E_t[y_t] - k(\mathrm{var}_t(y_t) + E_t[y_t]^2) \qquad (5)$$

where $y_t \sum_{i=1}^{n+1} a_i y_{t,i}$ , is the portfolio return at time $t$, $a$ is the $n$-vector of weights, and $E_t$ and $\mathrm{var}_t$ denote, respectively, the conditional expectation and conditional variance at time $t$, given the information set $\Omega_t$. We maximize a function of the conditional mean and the conditional variance with respect to portfolio weights.[7]

The maximization problem given by (9) can be solved by maximizing a function $n$ variables, in this case the portfolio weights, of which the first and second derivatives. These derivatives can be derived from the corresponding likelihood using

[7] As before, we assume that the weights sum to one.

the same procedure described in section 2. As Manganelli (2004) points out the solution of this maximization problem is straightforward under the assumption that the function is sufficiently well behaved and we are not required to estimate a variance covariance matrix. Furthermore, it is important to know a priori the potential misspecification of the univariate GARCH models, although this is not feasible on theoretical grounds.

Following Manganelli (2004) we describe how the sensitivity analysis can be used to estimate large variance-covariance matrices as well. This is a three-step procedure:

The first step involves the minimization of portfolio variance with respect to the weights. Essentially this is considered as a special case of equation (5), in which we set the conditional mean equal to zero. This may be a good approximation in applications with daily data. In the second step we calculate the second derivatives of the portfolio variance with respect to the weights. Theoretically, these derivatives should be constant and independent of the values of the weights. However, in practice this does not hold as a result of the misspecification of the univariate GARCH(1,1) model. A possible correction of this problem is offered by the computation of the second derivative that corresponds to the minimum-variance portfolio derived in step 1. The final step involves the definition and derivation of the variance-covariance matrix at time $t$ (see Manganelli, 2004, p. 376-377; and Manganelli *et al*. 2002; for a complete analysis of the derivation of the full system). Thus, we define $\omega = [a', 1 - \iota' a]'$ to be the $(n+1)$-vector of weights corresponding to each asset included in the portfolio, where $i$ is an $n$-vector of weights of ones. Then, we derive the variance-covariance matrix ($\Sigma_t$) which is the solution to the following system:

(a) $\omega^{*\prime}\Sigma_t\omega^* = h_t(a^*)$

(b) $\frac{\partial\omega'\Sigma_t\omega}{\partial a}|_{a=a^*} = 0$

(c) $\left(\frac{\partial^2\omega'\Sigma_t\omega}{\partial a\partial a'}\right)|_{a=a^*} = K_t$

where $K_t$ is an $(nxn)$ matrix consisting of the estimated second derivatives whereas $a^*,\omega^*$ are the optimal weights associated to the minimum-variance portfolio computed in step 1.

We also note that the variance-covariance matrix $\Sigma_t$ has $(n+2)(n+1)/2$ parameters to be estimated. Specifically, from condition (a) we obtain one parameter; from condition (b) we obtain $n$ parameters; and from condition (c) we obtain $n(n+1)/2$ parameters. Moreover, Manganelli (2004) argues that the solution to the system described by (a)-(c) results to coefficients of a paraboloid with vertex that corresponds to the minimum-variance portfolio and curvature $K_t$. Given that the minimum variance is strictly positive then the estimated variance-covariance matrix must be positive definite.[8]

The analytic solution to the system given by conditions (a)-(c) takes the following steps. We partition the variance-covariance matrix as follows:

$$\Sigma_t = \begin{bmatrix} A_t & b_t \\ b_t' & c_t \end{bmatrix}$$

where $A_t$ is an $(nxn)$ matrix, $b_t$ is an $(nx1)$ vector and $c_t$ is a scalar. Then we obtain:

$$\omega'\Sigma_t\omega = [a', 1-\iota'a]\begin{bmatrix} A_t & b_t \\ b_t' & c_t \end{bmatrix}\begin{bmatrix} a & \\ 1 & \iota'a \end{bmatrix} = a'(A_t - \iota b_t' - b_t\iota' + c_t\iota\iota')a + 2(b_t' - c_t\iota')a + c_t \quad (6)$$

[8] Manganelli (2004, p.374) provides a proof to this result.

Therefore, we have

$$\frac{\partial \omega' \Sigma_t \omega}{\partial a} = 2(A_t - \iota b_t' - b_t \iota' + c_t \iota\iota')a + 2(b_t - c_t \iota) \tag{7}$$

$$\frac{\partial^2 \omega' \Sigma_t \omega}{\partial a \partial a'} = 2(A_t - \iota b_t' - b_t \iota' + c_t \iota\iota') \tag{8}$$

If we combine the relationships given by (6)-(8) with conditions (a)-(c) we get the following values for the elements of the variance-covariance matrix $\Sigma_t$ (Manganelli, p. 374):

$$A_t = 0.5K_t + h(a^*)\iota\iota' + 0.5a^{*'} K_t a^* \iota\iota' - 0.5\iota a^{*'} K_t - 0.5K_t a^{*'} \iota \tag{9}$$

$$b_t = h_t(a^*)\iota + 0.5a^{*'} K_t a^* \iota - 0.5K_t a^* \tag{10}$$

$$c_t = h_t(a^*) + 0.5a^{*'} K_t a^* \tag{11}$$

**4. Empirical results**

We implement the sensitivity analysis described in the previous sections using a sample of selected stocks traded at the Athens Stock Exchange (ASE). The empirical application of the methodology developed by Manganelli (2004) is twofold. First, we estimate the sensitivity of GARCH variances within a two asset portfolio framework. Second, we calculate the minimum-variance portfolios for five different portfolios (two assets, five assets, ten assets, twenty assets and thirty assets) and we then compare its performance with that of the Dynamic Conditional Correlation model, the Orthogonal GARCH model and the Exponentially Weighted Moving Average model.

We begin our empirical analysis with the estimation of the first and second derivatives of GARCH variances using a two-asset portfolio composed of the stocks of IATRIKO (medical services) and CHIPITA (food services) traded at the ASE. Daily data is used obtained from the Athens Stock Exchange database and the sample covers the period 14 January 1997 to 10 February 2005.[9]

We estimate univariate GARCH (1,1) models for 31 portfolios constructed from these two assets. The weight $(a)$ for IATRIKO ranges from -1 to 2, with a step increase of 0.1. For each estimated GARCH model, the first and second derivatives of the estimated variance with respect to the weight (a) have been calculated. The estimated variances on 10 February 2005 for the 31 portfolios with respect to the weight (a) along with the first and second derivatives are illustrated in Figure 1. There are several points to be made regarding these plots. The variance corresponding to $a = 0$ is the variance of CHIPITA whereas the variance corresponding to $a = 1$ is that of IATRIKO. Furthermore, those portfolios that have a weight greater than 1 are short of CHIPITA and those which have a weight less than zero are short of IATRIKO. The shape of the estimated variance shown in Figure 1 as we have already explained in section 2 is tied with the potential gains from portfolio diversification. Thus given that the estimated variance is considered to be a parabolic and convex function of portfolio weights $a$ this implies that there are substantial gains from portfolio diversification measured in terms of risk reduction.

Assuming that the true variance-covariance matrix is known then the corresponding estimated variance would be exactly a parabola. Our univariate

[9] All estimations have been run with the Matlab codes developed by Manganelli.

GARCH estimates produce results close to the theoretical considerations and therefore we may argue that they are good approximations of the unknown true model. This argument is further strengthened by examining the shape of the first and second derivatives (see Manganelli, 2004). In the function was an exact parabola then the first derivative would be a straight line with a positive slope and the second derivative would be a straight line with a slope equal to zero. From Figure 1 we observe that both derivatives are near to the values implied by theory.

Figure 2 shows the plots of the first derivatives of the estimated variance, $\frac{\partial \hat{h}_t(a)}{\partial a}$, for the two degenerate portfolios, i.e. CHIPITA ($a = 0$) and IATRIKO $(\alpha = 1)$ .These plots show the magnitude by which the variance would decrease or increase over time, in the case that an investor moves away from the corner solution of holding either stock. Similar patterns can be derived for any portfolio weight. Therefore, the investor or the portfolio manager has a complete set of information with respect to the effects in terms of risk when a change in the composition of the current portfolio is considered.

Figure 2 also shows that the first derivative of CHIPITA is always positive whereas the corresponding one for IATRIKO is mostly negative. This finding implies that during the period under investigation the minimum variance portfolio was formed by a convex combination of these two assets (Manganelli, p. 379). Moreover, we observe that towards the end of the sample both first derivatives were positive and this is an indication that during those days the portfolio manager is required to take a short position on CHIPITA in order to attain the minimum variance portfolio.

Finally, Figure 2 provides useful information with respect to the sources of risk of a particular portfolio. Thus, Manganelli (2004) shows that the greater in absolute value the first derivative is, the greater the risk reduction following a portfolio reallocation will be. The first derivative of the portfolio which is only composed by the IATRIKO stock is much higher on average (in absolute value) than the first derivative of the portfolio including only the CHIPITA stock. Thus, we may conclude that during the 1990s an investor could gain more in terms of risk reduction if he/she diversified away from the portfolio with only IATRIKO stock than from the CHIPITA portfolio.

Next we consider the information obtained from Figure 3. We report the plots of the second derivative of CHIPITA as well as its difference from the average second derivatives computed over all 31 portfolios. Theory suggests that in the case of a model which is correctly specified the second derivative should be a flat line since it should not depend on the portfolio composition. In this case if we take the difference between the average second derivatives and that of CHIPITA should be zero. Indeed, in Figure 3 we observe that the derived difference is quite smooth around zero and this evidence tied with our analysis of Figure 1 provides further support in favour of our univariate GARCH model as being a good approximation of the true variance.[10]

As with the first derivative, the second derivative provides important information to the risk manager with respect to the size of change of the variance sensitivity when a change in the portfolio allocation takes place. This implies that the greater is the size of the second derivative, the greater the change in the variance sensitivity will be and this will lead to the need for a smaller portfolio reallocation in order to attain a given size of variance reduction. In Figure 3 we observe that in the last five years the impact on variance due to portfolio reallocations is much greater compared to that effect during the 1990s. Specifically, the average value of the second

[10] The reason that the difference is not exactly equal to zero is certainly due to the misspecification of the univariate GARCH model.

derivative was 12.36 between 1990 and 1999, whereas in the period 2000 to 2005 a substantial increase has been documented. These findings suggest that there has been a significant increase in the concavity of the portfolio variance (as a function of weight $a$) for the stocks of CHIPITA and IATRIKO over the last five years and risk managers active in the Athens Stock Exchange should take this information into consideration.

The next stage of the present analysis deals with the implementation of the methodology developed by Manganelli (2004) and discussed in section 3. The purpose is to estimate full variance-covariance matrices and to find the allocation minimizing the portfolio variance. We test this approach with the use of different subsamples of the Athens Stock Exchange general index for the period 14 January 1997 to 10 February 2005.

The performance of the variance sensitivity methodology is then evaluated against three alternative specifications of multivariate models. These specification include the Dynamic Conditional Correlation (DCC), the Orthogonal GARCH (OGARCH) and the Exponentially Weighted Moving Average (EWMA). We briefly discuss these models.

The Dynamic Conditional Correlation was proposed by Engle (2000) and Engle and Sheppard (2002) and is considered a generalization of the Constant Conditional Correlation developed by Bollerslev (1990). In this model instead of assuming that the conditional correlations are constant, these are directly parameterized. The estimation of this multivariate specification is done with the use of a two-step procedure as suggested by Engle (2000). The first step involves the estimation of the univariate GARCH models for each asset while in the second step we fit the conditional correlation specification to the standardized residuals calculated in the first step.

The Orthogonal GARCH model was developed by Alexander and Chibumba (1995) and Alexander (2000) and is based on a principal component GARCH methodology. This approach also involves a two-step procedure. In the first step we construct unconditionally uncorrelated factors which are linear combinations of the original returns series. The second step involves fitting the univariate GARCH models to the constructed principal components. Assuming that the conditional variance-covariance matrix of the principal components series is diagonal (i.e. the conditional correlations are set to zero) it would be possible to recover, with the use of a fixed mapping matrix, the variance-covariance matrix of the original stocks.[11]

Finally, the Exponentially Weighted Moving Average method has been extensively used during the 1990s as a tool for risk management. This method involves the computation of the variance-covariance matrix at time $t$ as a convex function of the lagged one period variance-covariance matrix and the matrix of squared and cross-product lagged returns. For daily data the weight (decay coefficient), $\lambda$, is usually set equal to 0.94.

The estimation of the three alternative specifications starts with the estimation of the variance-covariance matrix on 10 February 2005. Next we compute the weights that lead to the derivation of the minimum-variance portfolio. Manganelli (2004) shows that if we partition the variance-covariance matrix following eq. (6), then we can derive the optimal weights if we set eq. (7) to zero.

[11] The DCC approach has two shortcomings. The first is the inability to allow for homogeneous distributions across correlations while the second refers to the estimation of the identical pair of parameters for all equations under consideration. The OGARCH approach has also a limitation since it requires a very large sample in order to obtain a significant degree of variability of the variance-covariance matrix.

$$a^* = (\hat{A}_t - \iota \hat{b}'_t - \hat{b}_t \iota' + \hat{c}_t \iota\iota')^{-1}(\hat{c}_t \iota - \hat{b}_t)$$

We then estimate the univariate GARCH variance associated to this portfolio and present the annualized estimated in Table 1. We conduct this exercise for the DCC, OGARCH and EWMA models and for five alternative portfolios with 2, 5, 10, 20 and 30 portfolios.[12]

The estimation of the variance sensitivity analysis (VSA) model is conducted with the direct minimization of the univariate GARCH variance with respect to portfolio weights.[13] We observe that convergence is achieved very quickly and is very robust to the choice of the initial conditions and this implies that the objective function is behaves appropriately even when we consider the case of problems with high dimensions.[14] Following Manganelli (2004) we choose as initial conditions of the variance sensitivity analysis model the optimal weights of exponentially weighted moving average model. Table 1 presents the complete results.

The picture emerging from Table 1 is that the VSA model outperforms the three alternative models in comparison. This result is the outcome of the fact that the VSA model is constructed as to estimate the minimum-variance portfolio based on the univariate GARCH model. Furthermore, we observe the performance of the VSA model relative to the other competing models increases as the number of stocks increase.[15] Thus, we see that while in the case of the two-asset portfolio the differences in the minimum variances are almost zero, as we move towards larger

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[12] A full list of the companies used in the analysis is given in Appendix A. Assets are progressively aggregated in the order reported in this Appendix.

[13] We use the function *fminunc* in Matlab and we insert as inputs the first and second analytical derivatives calculated in section 2.

[14] Convergence for a 30-asset portfolio occurs in less than 20 iterations for randomly chosen initial conditions.

[15] The outperformance of the VSA model is measured by the percentage difference in annualized volatility.

portfolios these differences get larger for both the DCC and OGARCH models. With five stocks, DCC and OGARCH overestimate the minimum-variance portfolio by about 4% and 3%. When we look into the case with ten stocks then the difference rises to 13% and 14%, respectively, while for the case of twenty and third stocks it ranges from 52% to approximately 143%. These results lead to the conclusion that as the number of stocks rises, the number of restrictions which the typical multivariate GARCH models rises and its solution becomes very complicated.

In contrast the results for the EWMA model provide a rather different outcome since its performance does not deteriorate as much as well as quick as the DCC and OGARCH models. Manganelli (2004) explains this behaviour of the EWMA model on the grounds of its construction. Since we use the same weight $\lambda$ for all variance and covariance terms this amounts to the estimation of this model's portfolio variance directly, with coefficient $\lambda$. Certainly, this does necessarily imply that the EWMA model provides reasonable estimates of the variance-covariance matrix, (see Manganelli, 2004, p. 384).[16]

**5. Summary and concluding remarks**

In this paper we provided a variance sensitivity analysis using daily data from the Athena Stock Exchange, a closely monitored emerging market. Variance sensitivity analysis has been recently proposed by Manganelli (2004) in order to resolve the problem that arises when we are trying to model asset volatility using multivariate GARCH models. These models become cumbersome since they require

[16] It is worthwhile to note that the computation time of the VSA model for a thirty-asset portfolio it takes less than minutes to attain optimization .

strong assumptions to make estimations feasible while their dimension increase exponentially as the number of variables increases.

A common procedure to avoid the problems raised by the estimation is to fit univariate GARCH models to the time series data of portfolio returns, but this approach has as a shortcoming the loss of the multivariate dimension of the portfolio allocation. Manganelli (2004) develops an approach which utilizes the GARCH environment giving at the same time tractable computations and clear-cut conclusions. He suggests to evaluate the impact of a portfolio reallocation on the estimated variance by calculating the sensitivity of the estimated variance with respect to the weight of the stock involved in the transaction. This task can be accomplished by using as a sensitivity measure the derivative of the estimated variance with respect to portfolio weights. Furthermore, this approach allows us to estimate full variance-covariance matrix.

Our sample consists of daily returns of thirty assets traded at the Athens Stock Exchange for the period 14 January 1997 to 10 February 2005. This is an emerging market which has been closely monitored by portfolio managers as a result of its high returns during the last decade. We conduct our analysis by constructing different portfolios with two, five, ten, twenty and thirty assets. First, we considered a two-asset portfolio consisting of the stocks of two major firms, CHIPITA and IATRIKO, which are traded in the ASE. After the estimation of the variance sensitivity we examined how this sensitivity has been changing over time and emphasize its implications for risk management in this emerging stock market. Furthermore, we calculate the second derivative of the estimated variance for this portfolio with respect to the portfolio weights. The second derivative is a measure that provides an indication of the benefits measured in terms of risks that arise from portfolio diversification between the two assets under examination.

Second, following Manganelli (2004) we also compute the minimum variance portfolio at any given point of time for alternative portfolios constructed from the general index of ASE. The performance of this methodology was assessed against three popular multivariate GARCH specifications, namely DCC, OGARCH and EWMA models. The overall results of the present analysis leads to the conclusion that the adopted methodology provides more efficient results than the competing models. An important point to be made is that the degree of misspecification of the estimated univariate GARCH is insignificant. Finally, our results are in line with those reported by Manganelli (2004) for the NYSE suggesting that this methodology performs well on daily data derived from mature as well as emerging markets and thus it can be considered a useful tool for risk managers.

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# APPENDIX

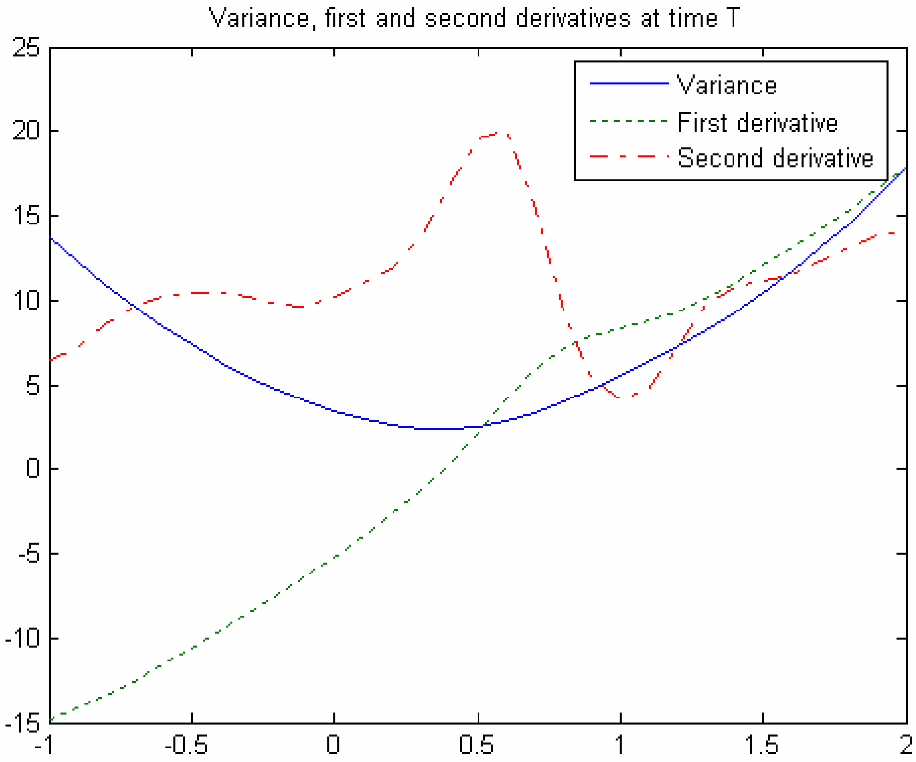
## Table A.1 List of stocks used in the analysis

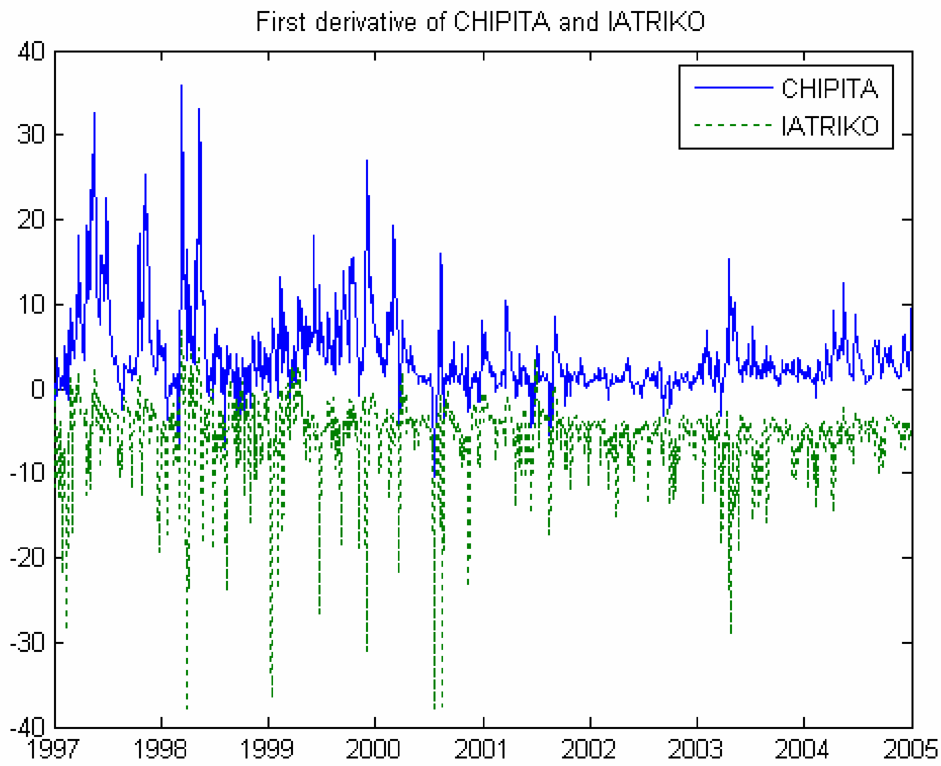
| A/A | FIRMS | NAME | TICKER |
|---|---|---|---|
| 1 | ALPHA | ALPHA ΤΡΑΠΕΖΑ Α.Ε. | **ALPHA** |
| 2 | ΑΤΤΙΚΑ | ATTICA Α.Ε. ΣΥΜΜΕΤΟΧΩΝ | **ATTICA** |
| 3 | ΤΣΙΠ | CHIPITA INTERNATIONAL | **CHIP** |
| 4 | ΕΕΕΚ | COCA-COLA Α.Ε. | **EEEK** |
| 5 | ΑΡΒΑ | S&B ΒΙΟΜΗΧΑΝΙΚΑ ΟΡΥΚΤΑ Α.Ε. | **ARVA** |
| 6 | ΑΒΑΞ | J&P ΑΒΑΞ Α.Ε. | **ABAX** |
| 7 | ΑΕΓΕΚ | ΑΕΓΕΚ | **AEGEK** |
| 8 | ΑΚΤΩΡ | ΑΚΤΩΡ | **AKTOR** |
| 9 | ΑΛΕΚ | ΑΛΟΥΜΙΝΙΟ ΤΗΣ ΕΛΛΑΔΟΣ | **ALEK** |
| 10 | ΒΙΟΧΚ | ΒΙΟΧΑΛΚΟ Ε.Β. ΧΑΛΚΟΥ & ΑΛΟΥΜΙΝΙΟΥ | **BIOXK** |
| 11 | ΣΑΡ | ΓΡ.ΣΑΡΑΝΤΗΣ Α.Β.Ε.Ε. | **SAR** |
| 12 | ΜΥΤΙL | ΜΥΤΙΛΗΝΑΙΟΣ Α.Ε. | **MYTIL** |
| 13 | ΤΙΤΚ | Α.Ε. ΤΣΙΜΕΝΤΩΝ ΤΙΤΑΝ | **TITK** |
| 14 | ΓΕΝΑΚ | ΕΘΝΙΚΗ ΑΞΙΟΠ.ΑΚΙΝ.& ΕΚΜ/ΕΩΣ ΓΕΝ.ΑΠΟΘ. Α.Ε. | **GENAK** |
| 15 | ΕΛΑΙΣ | ΕΛΑΙΣ Α.Ε. | **ELAIS** |
| 16 | ΕΛΒΑ | ΕΛΒΑΛ Α.Ε. ΕΠΕΞ.ΑΛΟΥΜΙΝΙΟΥ | **ELVA** |
| 17 | ΕΛΤΕΧ | ΕΛΛΗΝΙΚΗ ΓΕΩΔΥΝΑΜΙΚΗ | **ELTEX** |
| 18 | ΕΤΕ | ΕΘΝΙΚΗ ΤΡΑΠΕΖΑ | **ETE** |
| 19 | ΕΜΠ | ΕΜΠΟΡΙΚΗ ΤΡΑΠΕΖΑ | **EMP** |
| 20 | ΕΕΓΑ | "Η ΕΘΝΙΚΗ" ΑΣΦΑΛΕΙΩΝ | **EEGA** |
| 21 | ΗΡΑΚ | ΑΓΕΤ ΗΡΑΚΛΗΣ | **HRAK** |
| 22 | ΙΑΤΡ | ΙΑΤΡΙΚΟ ΚΕΝΤΡΟ | **IATR** |
| 23 | ΙΝΤΚΑ | ΙΝΤΡΑΚΟΜ Α.Ε. | **INTKA** |
| 24 | ΛΑΜΨΑ | ΛΑΜΨΑ Α.Ε. ΕΛΛ.ΞΕΝΟΔΟΧΕΙΩΝ | **LAMPSA** |
| 25 | ΜΑΙΚ | Μ.Ι.ΜΑΙΛΛΗΣ Α.Ε.Β.Ε. | **MAIK** |
| 26 | ΡΟΚΚΑ | ΜΕΤΑΛ. ΑΡΚΑΔΙΑΣ Χ.ΡΟΚΑΣ | **ROKKA** |
| 27 | ΜΕΤΚ | ΜΕΤΚΑ Α.Ε. | **METK** |
| 28 | ΜΗΧΚ | ΜΗΧΑΝΙΚΗ | **MHXK** |
| 29 | ΟΤΕ | ΟΤΕ Α.Ε. | **OTE** |
| 30 | ΝΙΚΑΣ | Π.Γ.ΝΙΚΑΣ Α.Ε. | **NIKAS** |

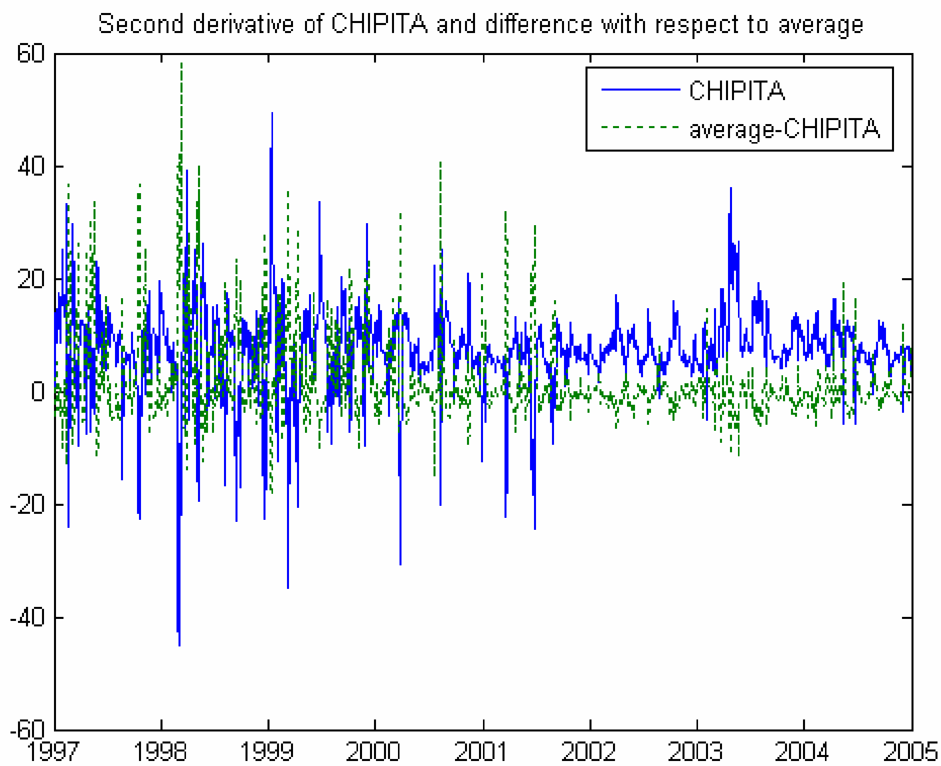
Table 1. Comparison between the VSA methodology and alternative multivariate GARCH models

| | Portfolio with 2 assets | | | Portfolio with 5 assets | | | Portfolio with 10 assets | | | Portfolio with 20 assets | | | Portfolio with 30 assets | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Min vol | % VSA | Seconds | Min vol | % VSA | Seconds | Min vol | % VSA | Seconds | Min vol | % VSA | Seconds | Min vol | % VSA | Seconds |
| DCC | 27.57% | 0.81 | 6 | 19.64% | 4.08 | 18 | 19.27% | 13.18 | 24 | 13.07 | 52.64 | 43 | 10 | 112.23 | 79 |
| OGARCH | 27.41% | 0.25 | 3 | 19.39% | 2.77 | 4 | 19.49% | 14.47 | 8 | 16.51 | 92.77 | 15 | 13 | 143.19 | 23 |
| EWMA | 27.62% | 1.00 | 1 | 18.92% | 0.25 | 1 | 17.96% | 5.52 | 1 | 10.21 | 19.25 | 1 | 3 | 34.12 | 1 |
| VSA | 27.34% | 0 | 22 | 18.87% | 0 | 21 | 17.02% | 0 | 76 | 8.56 | 0 | 103 | 0 | 0 | 222 |

*Notes: DCC is the dynamic conditional correlation; OGARCH is the orthogonal GARCH; EWMA is the exponentially weighted moving average. For each portfolio we report the univariate GARCH annualized volatility associated with the minimum-variance weights implied by the estimated variance-covariance matrix, the percentage difference with respect to VSA and the computation time to estimate the model.*


Variance, first and second derivatives at time T
Variance
First derivative
Second derivative
25
20
15
10
5
0
-5
-10
-15
-1
-0.5
0
0.5
1
1.5
2


First derivative of CHIPITA and IATRIKO
CHIPITA
IATRIKO
40
30
20
10
0
-10
-20
-30
-40
1997
1998
1999
2000
2001
2002
2003
2004
2005


Second derivative of CHIPITA and difference with respect to average
CHIPITA
average-CHIPITA
60
40
20
0
-20
-40
-60
1997
1998
1999
2000
2001
2002
2003
2004
2005