

Preserving Biodiversity: Ambiguity and Safety Rules

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Abstract

Safety rules are developed, for biodiversity preservation. These rules are designed to take into account the impact of uncertainty and worst case scenarios, which when combined with unregulated ecosystem management decisions, might produce extinction of species. The safety rules take the form of fixed land allocation and fixed harvesting rules under uncertainty. We explore how model uncertainty affects these safety rules relative to the classic risk aversion case and how a measure of precaution against worst case scenarios can be formulated.

Key words Biodiversity Preservation, Model Uncertainty, Safety Rules

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1 Introduction

In the recent years the term biodiversity, which refers to the variety of life in all its manifestations, has become widespread. Importance is attached to it by environmental groups, political decision makers, the press and the scientific community. Biodiversity is a very complex phenomenon which requires

proper measurement and quantification so the term could to be more operational. The ideal Measurement of biodiversity is very difficult since no single objective measure is possible, but only measures related to particular purposes or applications. Species richness which refers to the number of species present, and which we adapt for measuring biodiversity in our approach, has been extensively employed in the literature and in some way consists a common currency of the study of biodiversity.¹

Biodiversity is the source of all biological wealth, and provides the basis for life on earth, including that of humans. Biological resources supply all of our food and water, support the primary production process, and maintain the balance of the earth's atmosphere. Biodiversity also consists the basic material for a good life, health, harmonically social relations and security. The fundamental ethnical, social and cultural values of the biological resources, as well as their contribution to the economic development measured in monetary terms, have been broadly recognized as well.² Therefore the preservation of biodiversity is essential for the future of the planet, since the lost of biodiversity affects negatively the operation of terrestrial ecosystems. Species extinction and biodiversity loss is however a real phenomenon. As pointed out in the recent Millennium Ecosystem Assessment (MEA) report, ([8]) over the past few hundred years humans, through their management actions and impacts on ecosystems, have increased the species extinction rate by as much as 1000 times over background rates typical over the planets history. Furthermore genetic diversity has declined globally particularly among cultivated species.³

Species extinction could be intensified by high levels of uncertainty about the ecological, social, and economic dynamics of the changing earth which make the emergence of unexpected events, and worst case scenarios likely. These unexpected events when combined with human management decisions might accelerate or cause the extinction of species. Uncertainty about the forces driving the social, biological and economics changes and the aversion to ambiguity about the true dynamics of our system create a need of precaution

¹See for example Gaston [5],

²See for example Perrings et. al. [10], for the value of resource conservation to society.

³It has been estimated that 50% of the biotic diversity will be lost in the next century (Soule [12])

against surprising consequences that might occur and which may result in species extinction. The above creates the need to control the dynamics of our system, under uncertainty and worst case scenarios.

Although economists try to manage ecosystems and biodiversity in an optimal way most of the times, the complexity of ecosystems might make the optimization exercises difficult even at a theoretical level. On the other hand if we are interested in preserving diversity it might be useful to think about managing ecosystems using safety rules, which when applied make sure that a species or a set of species will not go extinct.⁴ Safety rules in biodiversity preservation acquire greater importance when the ecosystem manager faces model uncertainty, or ambiguity, in the Knightian sense, where the state space of outcomes is known but information is too imprecise to be summarized by probabilities. This case is very relevant in ecosystem management where the extreme complexity of the system makes difficult to derive exact probability distributions in order to deal with the systems' uncertainties and to manage risks. In this situation worst case events might cause surprises and extinction of species. Since these extinctions have occurred in reality, dealing with worst case scenarios implies that ecosystem management and biodiversity preservation are associated with a precautionary principle, where the management rules are such that species will not go extinct under worst case scenarios.

Since species extinction through human actions is related to decisions regarding the management of ecosystems and cultivated species⁵ the purpose of this paper is to develop rules which preserve biodiversity, in the sense of preventing certain species from going extinct, in an environment where species are cultivated and harvested, and a certain amount of land is devoted to them so they can grow and harvested. In this context our rules are land allocation rules and harvesting rules⁶ which are *not* optimal in the sense of optimizing an objective, but can be regarded as *safety rules*, since they provide condi-

⁴Safety regulation is a more general issue in economics. For a general discussion of the role of economic analysis in the development of environmental health and safety regulation see Arrow et al. [1]. For a discussion of safety standards in species protection see for example Holt and Tisdell [13].

⁵The MEA states that genetic diversity has declined globally particularly among cultivated species.

⁶Deriving safety rules which are optimal in the sense of maximizing an objective determined in terms of harvesting values and possible existence values is an area for further research

tions under which species biomasses will not fall below a prespecified level, so that a species or a set of species will not go extinct. The safety - harvesting and land allocation rules - are time invariant for the chosen lower bounds for biomasses and are determined under certainty and uncertainty.⁷ The basic contribution of this paper is that safety rules are derived and compared under conditions of classical risk aversion, and under conditions of model uncertainty or ambiguity in the Knightian sense. The analysis under model uncertainty aversion is important since it can incorporate worst case scenarios and an associated precautionary principle. In particular by adopting a special case of recursive multiple priors models [4], which incorporates worst case scenarios, we derive safety rules that can preserve diversity even when a worst case scenario occurs.⁸ Therefore we incorporate precaution into harvesting and land allocation rules and explore the role of precaution against the impact of model uncertainty in the development of the above rules.

Using a model of population dynamics with species interactions, where an ecosystem manager can choose harvesting rules and land allocation rules, our results suggest that under uncertainty we can define combinations of harvesting and land allocation rules such that the biomass of a given species is kept above a predetermined level with a given probability. This probability is bounded both above and below. We also show the deviations between safety rules when a probability distribution for the uncertain events can be specified, which is the classic risk case, and safety rules under model uncertainty or ambiguity and worst case scenarios which is the case of Knightian uncertainty. These deviation provide a quantification of the precautionary principle in biodiversity preservation.

The rest of the paper is organized as follows. In the next section we present initially a deterministic model with two species, allowing interactions between them. In the sequence we extend it by introducing uncertainty, and we determine upper and lower bounds for the probabilities of the two biomasses to above a prespecified proportion of their initial values. and afterwards extend it in the n biomasses case. The upper and lower bounds depend on the harvesting rules and land allocation rules, and constitute our safety rules. In Section 3 we consider that the decision maker faces model uncertainty in the Knightian sense, we adopt the k -Ignorance framework which

⁷See Regan et. al. [11], for treatment of uncertainty in populations models.

⁸Recursive multiple priors models along with with robust control methods [7], comprise the two mainly approaches in desicion making under ambiguity aversion

is consistent with the axiom of uncertainty or ambiguity aversion of Gilboa and Schmeidler [6], and show how model uncertainty affects the assessment of the above bounds, which reflects the precautionary principle. The last section concludes.

2 Preservation of Biodiversity

2.1 A Deterministic Model of Population Dynamics with Species Interactions

We consider a landscape normalized to unity where two species could potential coexist. Let B_{i0} for $i = 1, 2$ be the initial biomasses of the two species. In this case the evolution of the initial biomasses B_{1t}, B_{2t} , through time can be described by the following system of deterministic differential equations:⁹

$$\begin{aligned} dB_1 &= B_1[f_1(\mathbf{w}) - d_1]d\tau - a_{12}B_2d\tau - h_1d\tau \\ dB_2 &= B_2[f_2(\mathbf{w}) - d_2]d\tau - a_{21}B_1d\tau - h_2d\tau, \tau \geq t \end{aligned} \quad (1)$$

which can be rewritten as:

$$dB/d\tau = AB - h,$$

$$A = \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} \\ -a_{21} & f_2(\mathbf{w}) - d_2 \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

In the above equation $f_i(\mathbf{w}) - d_i$, $i = 1, 2$, are the instantaneous, net of death, growth rates, $\mathbf{w} = (w_1, w_2)$ denote the the land allocation rule, $\mathbf{h} = (h_1, h_2)$ h_i are the harvesting rates, and finally a_{12} and a_{21} refer to the interaction coefficient as in Vandermeer [14]. Initially by considering the equation:

$$\begin{aligned} |A - \lambda I| = 0 &\Leftrightarrow [f_1(\mathbf{w}) - d_1 - \lambda][f_2(\mathbf{w}) - d_2 - \lambda] - a_{12}a_{21} = 0 \Leftrightarrow \\ \lambda_{12} &= \frac{+[(f_1(\mathbf{w}) - d_1) + (f_2(\mathbf{w}) - d_2)] \pm \sqrt{\Delta}}{2} \\ \Delta &= [(f_1(\mathbf{w}) - d_1) + (f_2(\mathbf{w}) - d_2)]^2 + 4(f_1(\mathbf{w}) - d_1)(f_2(\mathbf{w}) - d_2)a_{12}a_{21} \end{aligned}$$

we can calculate the characteristics roots λ_1, λ_2 of the matrix A . By suitable choice of the values of (\mathbf{w}, \mathbf{h}) we can obtain:

⁹See for example Brock and Xepapadeas [3], or Benyes [2]

- negative characteristics roots $\lambda_i, i = 1, 2,$
- attain a solution of the above 2×2 system of ODEs which satisfy:

$$B_{1t} \geq \overline{B}_1, B_{2t} \geq \overline{B}_2$$

where $(\overline{B}_1, \overline{B}_2) > 0$ are some prespecified desirable levels. This solution takes the form: $(B_{1t}, B_{2t}) = \Omega e^{\Lambda t} c - A^{-1}h$, with Ω the matrix of characteristics vectors x ($Ax = \lambda x$), and $e^{\Lambda t}$ a diagonal matrix with diagonal elements $e^{\lambda_i t}$, with $c = \Omega^{-1}B_0 + \Omega^{-1}A^{-1}h$ a vector of constants which we calculate using the initial conditions (B_0 is a given matrix of the initial values of the two biomasses),

- as t increases the limit values are higher than the above prespecified levels.

If the above conditions are satisfied, then we have that in the simple deterministic case, we achieve the positive biomasses for both species for all times. Therefore by suitable choosing the values of the controls of our system (w_1, w_2, h_1, h_2) we can have safety rules for the preservation of biodiversity in the landscape.

The above model is a relative simple model, which we use as a vehicle for the introduction of a more suitable set-up for the study of biodiversity. A more interesting and more realistic model is the one examined in the next section where the landscape manager takes into account uncertainty relatively to the evolution of biomasses through time.

2.2 Preservation of Biodiversity under Uncertainty

Therefore in this section we extend our model by introducing uncertainty, which is a more suitable set up for studying biodiversity preservation. We prove safety rules for preserving biodiversity initially in case of two species and afterwards by extending to the n -species case.

In this case the evolution of the initial biomasses B_{10}, B_{20} , through time we assume that are given by the following system of stochastic differential equations:

$$\begin{aligned} dB_1 &= B_1[f_1(\mathbf{w}) - d_1]dt - a_{12}B_2dt - h_1dt - \sigma_1(\mathbf{w})dz_1 \\ dB_2 &= B_2[f_2(\mathbf{w}) - d_2]dt - a_{21}B_1dt - h_2dt - \sigma_2(\mathbf{w})dz_2 \end{aligned} \quad (2)$$

where the parameters are defined as above, where with $\sigma_i^2(\mathbf{w}) > 0$ denote the variance per unit time, and with dz_1, dz_2 two correlated Brownian motions and let ρ the correlation coefficient between them.

Using matrix notation the equation (2), is able to written as:

$$\begin{aligned}
dB &= ABdt - hdt + \Sigma dZ \quad \text{where} & (3) \\
dB &= \begin{bmatrix} dB_1 \\ dB_2 \end{bmatrix}, \\
A &= \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} \\ -a_{21} & f_2(\mathbf{w}) - d_2 \end{bmatrix}, \\
\Sigma &= \begin{bmatrix} -\sigma_1(\mathbf{w}) & 0 \\ 0 & -\sigma_2(\mathbf{w}) \end{bmatrix}, h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \\
dZ &= \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix},
\end{aligned}$$

The above equation (3), multiplied from the left by a suitable matrix using the two dimensional Ito formula (see Oksedal [9], theorem 4.2.1 page 48), becomes:¹⁰

$$d(e^{-At} B_t) = e^{-At} dB - e^{-At} ABdt = -e^{-At} hdt + e^{-At} \Sigma dZ, \quad (4)$$

$$\text{where } e^F = \sum_{n=1}^{\infty} \frac{1}{n!} F^n = F + \frac{1}{2!} F^2 + \frac{1}{3!} F^3 + \dots \quad (5)$$

$$\text{here } F = -At \quad (6)$$

Equivalently:

¹⁰In our case F is the matrix $-At$, where of course the elements of this matrix converge in a real number. This holds because of each element of this matrix is upper bounded of the sum $a_k = \sum_{k=1}^{\infty} \frac{2^{k-1}}{k!} (-tx)^k$, with x the maximum of the four elements of the matrix A in equation (3). For the above general term holds a known convergence criterion : $\limsup \left| \frac{a_{k+1}}{a_k} \right| < 1$ and therefore the series converge.

$$\begin{aligned}
e^{-At}B_t - B_0 &= - \int_0^t e^{-As}h_s ds + \int_0^t e^{-As}\Sigma dZ_s \\
B_t &= e^{At}B_0 - \int_0^t e^{A(t-s)}h_s ds + \int_0^t e^{A(t-s)}\Sigma dZ_s \quad (7) \\
\text{with } B_0 &= \begin{bmatrix} B_{10} \\ B_{20} \end{bmatrix}.
\end{aligned}$$

where let:

$$e^{At} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \quad (8)$$

with A_i for $i = 1, ..4$, depending on the values of the interaction coefficients a_{ij} and on f_i , h_i and which we can calculate using the relationship (5) . So using the relationships (7), (8) we are able to derive that:

$$B_{1t} = A_1B_{10} + A_2B_{20} + g_1(h_1, h_2) + \int_0^t G_1 dZ_1 + \int_0^t G_2 dZ_2, \quad (9)$$

$$B_{2t} = A_3B_{10} + A_4B_{20} + g_2(h_1, h_2) + \int_0^t G_3 dZ_1 + \int_0^t G_4 dZ_2. \quad (10)$$

with G_i functions of f_i , h_i , and σ_i , with the property to belong in the class $V = V(0, T)$.^{11 12} The four integrals in the above two equations are stochastic integrals with the property, for all the possible combinations of i, j

$$E \int_0^t G_i dZ_j = 0. \quad (11)$$

In the sequence suppose that the landscape manager concerns about the preservation of biodiversity or better the planner is interested on the sustainability on the landscape and wants the preservation of biomasses in higher levels than $\frac{1}{n}$ of the initial values. Using equations, (9) and (10), because of (11) and the fact that the biomasses are bounded from their initial values,

¹¹The quantities $g_1(h_1, h_2)$, $g_2(h_1, h_2)$ are due to the integral $\int_0^t e^{A(t-s)}h_s ds$.

¹² V is the set of measurable and adapted functions f with the property $E \int_0^T f(t, \omega)^2 dt < \infty$. Then for the corresponding stochastic integral holds that: $E \int_0^T f(t, \omega) dZ_t = 0$.

using standard operations from the probability theory, we are able to obtain upper and lower bounds for the probabilities of biomasses to be higher than the desirable levels. Then we have that :

$$\begin{aligned} \left(A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}\right) - \frac{1}{n} &\leq \Pr(B_{1t} > \frac{1}{n}B_{10}) \leq \frac{n}{B_{10}}(A_1B_{10} + A_2B_{20} + g_1) \\ &= n\left(A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}\right) \end{aligned} \quad (12)$$

$$\begin{aligned} \left(lA_3 + A_4 + \frac{g_2}{B_{20}}\right) - \frac{1}{n} &\leq \Pr(B_{2t} > \frac{1}{n}B_{20}) \leq \frac{n}{B_{20}}(A_3B_{10} + A_4B_{20} + g_2) \\ &= n\left(lA_3 + A_4 + \frac{g_2}{B_{20}}\right) \end{aligned} \quad (13)$$

$$\text{where } l = \frac{B_{10}}{B_{20}}$$

In the above expressions each one of the A_i are defined as $A_i = A_i(w_1, w_2, h_1, h_2)$ thus the associated probability bounds depend on the land allocation weights (w_1, w_2) and on the harvesting rules (h_1, h_2) . Therefore the landscape manager can suitable specify a tome invariant land allocation and harvesting rule, (w_1, w_2, h_1, h_2) , so that, the probability that the biomass of species i at any instant of time exceeds a prespecified level, which is proportional to the initial species biomass does not fall below a lower bound and does not exceed an upper bound.¹³ So with the parameters defined as above we have proven that:

Proposition 1 *Given a certain land allocation rule and harvesting rule (w_1, w_2, h_1, h_2) , the upper and lower bounds so that the probabilities of biomasses of species $i = 1, 2$, are higher than $\frac{1}{n}$ of the initial biomasses values are given by equations (12) and (13) respectively.*

The land allocation and harvesting rule (w_1, w_2, h_1, h_2) that satisfies proposition 1, is therefore a safety rule since it bounds the probability of having the biomasses at any point in time above a certain level, which is $\frac{1}{n}B_{i0}$, $i = 1, 2$. By choosing this level, that is by choosing $1/n$, the safety rules are determined and could ensure the preservation of biodiversity, in the sense of not having species going extinct with a probability that is between bounds defined by land allocation and harvesting rules.

¹³The elements A_i , can be calculated by using numerical methods, but this goes beyond the scope of the present work.

2.3 The multi species case

Our model can be extended to the multi species case. Particularly then the evolution of the biomass of the k^{th} species is given by the following equation:

$$dB_k = B_k[f_k(\mathbf{w}) - d_k]dt - h_k dt - \sum_{\substack{j \neq k \\ j=1, \dots, n}} a_{kj} B_j dt - \sigma_k(\mathbf{w}) dz_k \quad k = 1, \dots, n \quad (14)$$

with $\mathbf{w} = (w_1, \dots, w_n)$ been the land allocation rule

Using matrix notation the system of equations (3) now takes the following form:

$$dB = ABdt - hdt + \Sigma dZ \text{ where} \quad (15)$$

$$dB = \begin{bmatrix} dB_1 \\ \dots \\ dB_n \end{bmatrix},$$

$$A = \begin{bmatrix} f_1(\mathbf{w}) - d_1 & -a_{12} & \dots & \dots & -a_{1n} \\ -a_{21} & f_2(\mathbf{w}) - d_2 & \dots & \dots & -a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ -a_{n1} & -a_{n2} & \dots & \dots & f_n(\mathbf{w}) - d_n \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} -\sigma_1(\mathbf{w}) & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & -\sigma_n(\mathbf{w}) \end{bmatrix}, \quad h = \begin{bmatrix} h_1 \\ \dots \\ h_n \end{bmatrix}$$

$$dZ = \begin{bmatrix} dZ_1 \\ \dots \\ dZ_n \end{bmatrix}$$

Applying the same methodology as above obtain again that:

$$B(t) = e^{At} B_0 - \int_0^t e^{A(t-s)} h_s ds + \int_0^t e^{A(t-s)} \Sigma dZ_s$$

$$\text{where now } B_0 = \begin{bmatrix} B_{10} \\ \dots \\ B_{n0} \end{bmatrix}$$

with the exponential matrix now be the $n \times n$:

$$e^{At} = \begin{bmatrix} A_{11} & \dots & A_{1n} \\ \dots & A_{kj} & \dots \\ A_{n1} & \dots & A_{nn} \end{bmatrix}$$

Therefore the k^{th} biomass is given by:

$$B_{kt} = \sum_{i=1}^n A_{ki} B_{i0} + g_k(h_1, \dots, h_n) + \sum_{i=1}^n \int_0^t G_i dZ_i \quad k = 1, \dots, n$$

and the upper and lower bounds now are given by the following equation:

$$\left(\frac{A_1}{l_{1k}} + A_k + \frac{A_n}{l_{nk}} + \frac{g_k}{B_{k0}} \right) - \frac{1}{\gamma} \leq \Pr(B_{kt} > \frac{1}{\gamma} B_{k0}) \leq n \left(\frac{A_1}{l_{1k}} + A_k + \frac{A_n}{l_{nk}} + \frac{g_k}{B_{k0}} \right)$$

$$\text{with } l_{jk} = \frac{B_{k0}}{B_{j0}} \quad k = 1, \dots, n \quad j \neq k \quad (16)$$

Therefore we have proven that in the multi species n - biomasses case with the parameters defined as above that:

Proposition 2 *Safety rules for land allocation and harvesting* $(w_1, \dots, w_n; h_1, \dots, h_n)$, can determine upper and lower bounds for the probabilities that the biomasses of species $i = 1, 2, \dots, n$, are higher than $\frac{1}{\gamma}$ of the initial biomasses values. The safety rules and the corresponding bounds are characterized by (16).

3 Safety Rules under Knightian Uncertainty

Suppose now that the ecosystem manager is facing model uncertainty in the sense that he/she is not sure about the benchmark model, (2) or (14). In this case the manager faces model uncertainty in the Knightian sense or ambiguity. Basically there are two main approaches in the modern literature analyzing a situation where the agent is not sure about the initially estimated model, which are consistent with uncertainty or ambiguity aversion [6]. The robust dynamic control approach and the multiple priors models. In the first approach the decision maker is unsure about his/her model, in the sense that there is a group of approximate models that are also considered as possibly true given a set of finite data. These approximate models are obtained by disturbing a benchmark model, and the admissible disturbances reflect the

set of possible probability measures that the decision maker is willing to consider. The objective of the resulting robust dynamic control problem, is to choose a rule that will work under a range of different model specifications. [7] The other approach is the recursive multiple priors models and has been introduced by Epstein and Wang [4]. A special case of this approach is the k -ignorance framework which we adopt in order to explore how model uncertainty affects the probability bounds which have developed in the previous section.

In particularly in the two biomasses case, the system of equations given by (3), now can be written as:

$$\begin{aligned}
dB &= ABdt - hdt + \Sigma RdZ \text{ where} & (17) \\
dB &= \begin{bmatrix} dB_1 \\ dB_2 \end{bmatrix}, \\
A &= \begin{bmatrix} f_1(\mathbf{w}_1) - d_1 & -a_{12} \\ -a_{21} & f_2(\mathbf{w}_2) - d_2 \end{bmatrix}, \\
\Sigma &= \begin{bmatrix} -\sigma_1 & 0 \\ 0 & -\sigma_2 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1 - \rho^2} \end{bmatrix} \\
h &= \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, dZ = \begin{bmatrix} dZ_1 \\ dZ_2 \end{bmatrix}
\end{aligned}$$

where ρ is the correlation coefficient between the two Brownian motions in the initial system (3), and dZ_1, dZ_2 are now two independent Brownian motions.

Adopting the terminology of robust control we consider measurable drift distortions. More specifically the initial Brownian motions dZ_i $i = 1, 2$, are replaced by :

$$Z_i(t) = \widehat{Z}_i(t) + \int_0^t \varepsilon_i(s)ds \quad , \quad i = 1, 2 \quad (18)$$

where \widehat{Z}_i are Brownian motions and ε_i are measurable functions. By doing this, the system (17) takes the form:

$$\begin{aligned}
dB &= ABdt - hdt + \Sigma RE dt + \Sigma Rd\widehat{Z} \text{ where} \\
E &= \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} & (19)
\end{aligned}$$

Applying the same methodology as above the above system becomes:

$$\begin{aligned}
d(e^{-At}B_t) &= e^{-At}dB - e^{-At}ABdt = -e^{-At}hdt + e^{-At}\Sigma R d\widehat{Z}, \quad (20) \\
e^{-At}B_t - B_0 &= -\int_0^t e^{-As}h_s ds + \int_0^t e^{-As}\Sigma R E ds + \int_0^t e^{-As}\Sigma R d\widehat{Z}_s \\
B_t &= e^{At}B_0 - \int_0^t e^{A(t-s)}h_s ds + \int_0^t e^{A(t-s)}\Sigma R E ds + \int_0^t e^{A(t-s)}\Sigma R d\widehat{Z}_s
\end{aligned}$$

If we compare the above equation with (7) we can see that there is an extra term $\int_0^t e^{A(t-s)}\Sigma R E dt$ which acts as a measure of precaution and reflects the impact of model uncertainty. This has as a result the introduction of two extra terms in each one of the equations (9), (10) respectively. Therefore the upper and lower bounds change, depending on the structure of the problems' parameters.

In particular, by considering distortions in the above model, the initial measure P is replaced by another probability measure Q . The discrepancy between the two measures is measured by the relative entropy, $R(Q//P) = \int_0^{+\infty} e^{-\delta t} E_Q[\frac{1}{2}\varepsilon_t^2] dt$. According to the k -Ignorance framework we consider the following instantaneous relative entropy constraint¹⁴ : $Q(\tau) = \{Q : E_Q[\frac{1}{2}\varepsilon_t^2] \leq \tau, \text{ for all } t\}$, which restricts the set of models the decision maker considers at each instant of time. In this case the worst case perturbation is:

$$\varepsilon_{it}^* = -\sqrt{2\tau_i}$$

By adopting this approach the distortions now are constants negative numbers and therefore we can calculate the new integral and afterwards the adjusted bounds. Particularly by examining the possible cases of the signs of the matrix in the integral $\int_0^t e^{A(t-s)}\Sigma R E dt$, we derive for these matrices:

¹⁴This is in contrast to robust control approach where we consider a lifetime constraint.

$$\begin{aligned}
e^{A(t-s)} &= \begin{bmatrix} + & - \\ - & + \end{bmatrix}, \Sigma = \begin{bmatrix} - & 0 \\ 0 & - \end{bmatrix}, e^{A(t-s)}\Sigma = \begin{bmatrix} - & + \\ + & - \end{bmatrix} \\
R &= \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix}, E^* = \begin{bmatrix} -\sqrt{2\tau_1} \\ -\sqrt{2\tau_2} \end{bmatrix} \\
RE^* &= \begin{bmatrix} -\sqrt{2\tau_1} \\ -\sqrt{2\tau_1}\rho - \sqrt{2\tau_2}\sqrt{1-\rho^2} \end{bmatrix} \\
e^{A(t-s)}\Sigma RE^* &= \begin{bmatrix} \text{positive} - (\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2}) & \text{positive} \\ \text{negative} + (\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2}) & \text{positive} \end{bmatrix}
\end{aligned}$$

Therefore from the final equation we obtain that: In case where the term $(\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2})$ is negative the first element of the matrix is positive and the second is negative, where when $\tau_1 = \tau_2$ the above condition is satisfied if $\rho + \sqrt{1-\rho^2} < 0 \iff \rho < -\frac{\sqrt{2}}{2}$.¹⁵ Then (12) and (13), now become:

$$\begin{aligned}
pos + (A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}) - \frac{1}{n} &\leq \Pr(B_{1t} > \frac{1}{n}B_{10}) \leq n(A_1 + \frac{A_2}{l} + \frac{g_1}{B_{10}}) + pos \\
neg + (lA_3 + A_4 + \frac{g_2}{B_{20}}) - \frac{1}{n} &\leq \Pr(B_{2t} > \frac{1}{n}B_{20}) \leq n(lA_3 + A_4 + \frac{g_2}{B_{20}}) + neg \\
\text{where } l &= \frac{B_{10}}{B_{20}}
\end{aligned}$$

It can be seen from the above equations that when there is model uncertainty, and the ecosystem manager is uncertainty averse, then for given land allocation and harvesting safety rule the probability bounds corresponding to sustaining the species biomasses above a certain level change relative to the risk aversion case. In particular when $\sqrt{2\tau_1}\rho + \sqrt{2\tau_2}\sqrt{1-\rho^2} < 0$, the bounds that correspond to the first biomass are higher and that to the second one are lower as compared to the bounds proposed under the risk aversion case. The changes in the safety rules relative to the risk averse case, which correspond to the changes in the probability bounds reflects precaution against model uncertainty.

¹⁵In case where this condition is not satisfied we are not able to derive a similar result.

4 Conclusions

This paper introduces initially a simple deterministic model which we use as a vehicle for the development of a more complex models for the study of rules for biodiversity preservation. In the sequence by modeling the evolution of the biomasses of species using Brownian motions we control the dynamics of our system under uncertainty and worst case scenarios. Thus the main contribution of this work is the development of safety rules for preserving biodiversity. Under certainty, safety rules take the form of fixed land allocation and harvesting rules that make the species biomasses converge in the long run above some predetermined levels. Under uncertainty, initially in the case of two biomasses and afterwards by extending in the multi species case, we show that safety rules can determine upper and lower bound for the probability that a species biomass will exceed a predetermined level at each point in time. Furthermore, by taking into account model uncertainty, using the k -ignorance framework we recalculate the above bounds. The difference, in the values of the bounds and the associated differences in the safety rules, between the classic risk aversion case and the k -ignorance case can be interpreted as a measure of precaution against uncertainty and worst case scenarios.¹⁶

The model can be further analyzed by considering numerical methods for determining combinations of safety rules and probability bounds so that more insights can be gained regarding the impact of precaution. Further extension could include the optimal choice of the safety rules by maximizing some objective related to the value of ecosystem services.

¹⁶Similar type differences between risk and uncertainty aversion in portfolio selection problems are analyzed in [15].

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