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Abstract

The literature so far has analyzed the effects of Minimum Quality Standards in oligopoly, using models of pure vertical differentiation, with only two firms, and perfect information. We analyze products that are differentiated horizontally and vertically, with imperfect consumers information, and more than two firms. We show that a MQS changes the consumers' perception of produced qualities. This increases the firms' returns from quality enhancing investments, notwithstanding contrary strategic effects. As a consequence, MQS policies may be desirable as both, firms and consumers, can gain. This contrasts with previous results in the literature and provides a justification for the use of MQS to improve social welfare.

Keywords: Minimum Quality Standards, Imperfect Consumer Information, Oligopoly, Horizontal and Vertical Product Differentiation

JEL Classification: L0, L5

1 Introduction

The quality of goods is at the center of much policy debate. In Europe, quality certification and quality standards have become a matter of increasing concern for firms and policy makers over the years, along with the completion of the common market and the implementation of uniform trading rules. In many industries, ranging from passenger transport, to food, or children toys, quality and safety regulation are also intertwined and almost indistinguishable. Also, it is often claimed that consumers have to make their purchases without having the full knowledge to assess the quality of the goods; and in many cases qualities are difficult to ascertain even by experts equipped with the necessary tools. To capture these features, the present paper deals with mandatory Minimum Quality Standards (MQS) and studies their effects on product qualities and on welfare in an oligopolistic industry where buyers have less than full information about the quality of goods.

Unlike the present paper, the existing literature on MQS is largely and almost uniquely based upon models of oligopolistic competition with *pure vertical product differentiation*. These models, derived from the seminal paper by Gabszewicz and Thisse (1979), share some well-known typical features. First, the firms' products differ only in *one* aspect, the vertical quality dimension, which is observable and which, if increased, leads to higher utility for each consumer. Second, in a duopoly, the rival firms try to differentiate their products, in order to relax price competition (Shaked and Sutton, 1981). This implies that one of the two producers assigns to its product a low level of quality, even if it costs nothing to increase it. Third, once a MQS is introduced at a level that lies between the low and the high unregulated quality levels (Ronnen, 1991), the equilibrium price of goods and the equilibrium profits of firms decrease. Indeed, since the standard reduces the distance between the quality levels of the two firms, more intense price competition brings forth lower prices. This, by the way, also explains why aggregate profits fall. In this vein of the literature, other papers (Crampes and Hollander, 1995; Scarpa, 1998; Valletti, 2000; Jinji and Toshimitsu, 2004), explore the issues of consumers' and firms' gains and losses, and of quality changes, always based on models of pure vertical differentiation.

Another noteworthy common trait of the literature on MQS is that perfect information is generally assumed. This is at odds with the very beginning of explanations of MQS-like practices (Leland, 1979), where, absent strategic interplays between producers, these are interpreted as corrections to a "lemon" phenomenon. In a way, a MQS in Leland (1979) has the effect of cutting away the lowest tail of a distribution of qualities, thereby increasing the average marketed quality in a competitive setting. Roughly speaking, the present paper is in the spirit of this interpretation of MQS, although the effects on consumers' expected quality here has a feedback on the firms' behavior in a strategic setting.

As to the results obtained in the literature on MQS in duopoly and under perfect information, some remarks are in order. The predicted decrease in firms' profits may be consistent with some real world cases, but it appears to contrast with other cases in which firms, or association of producers, voluntarily agree to impose Minimum Quality Standards or, in a similar way, quality certification standards. Documented instances can be found, e.g., in agriculture; for instance, Ferguson and Carman (1999) report how the California kiwifruit producers agreement to a standard allowed them to withstand competition from New Zealand. Furthermore, the assumption that consumers are perfectly informed prevents the applicability of results on consumer welfare to situations where MQS should be introduced because of concerns about imperfect consumers' information.

As to the differentiation feature, it has already been noted (Garella, 2003),[1] that in a model with perfect information the effects of MQS on quality levels, are ambiguous if goods are differentiated also horizontally, as in the Hotelling (1929) linear city. The *average* quality in the market is found to be possibly decreasing because quality competition between firms is characterized by downward sloping reaction functions: in a duopoly when a MQS forces the low quality firm to increase quality, the high quality firm reacts by lowering the quality of its own product. This feature of reaction functions is preserved in the model presented below, although the horizontal aspect of differentiation is represented here according to the so-called "Dixit-Spence-Bowley" model (Spence, 1976; Dixit, 1979; Bowley, 1924) without spatial competition.

Our model is also characterized by imperfect information. Each consumer is assumed to receive some evidence about the utility that he will derive by consuming the good. This piece of information can be alternatively interpreted as relating to the goods true quality level, which may differ by what is written on the product "label"; or as relating to the carefulness with which the good is manufactured, which may lead to failures of the good or of some of its components. This piece of information is in effect amenable to an idiosyncratic disturbance on the quality observation of the

[1] Perverse quality reactions to the standard had also been found by Scarpa (1995) under pure vertical differentiation.

single consumer. The consumers' willingness to pay for the good is affected by the information received, and we exclude from the analysis any sort of signaling or certification activity by firms that may cancel or counter the effect of this information.[2]

In line with the bulk of the literature, we model firms choosing quality levels at the first stage of a game and at the second and final stage they choose prices. Qualities are costly, so that higher quality implies higher production costs which are sunk in the price competition stage.

The results that we obtain may help explain why MQS policies are adopted in many real-life situations in which consumers' information is often imperfect. In a model where firms are symmetric, the equilibrium qualities always improve after the introduction of the standard. This is an important result. Indeed, in spite of the firms' reaction functions in quality being downward sloping, so that strategic effects on quality are negative, the effect of a MQS is that of reducing consumers' uncertainty about the quality of the two goods. This positive countervail effect leads to an increased willingness to pay for quality by consumers and to an increased effort in producing high quality by the firms. We also find that firms' profits increase, a result that would explain the use of quality certification by (far-sighted) producers' associations and also the producers' willingness to accept MQS policies.

The effects on consumers' welfare are the most difficult to assess. Consumers that receive a correct information about goods coexist in the model with consumers that receive information that the goods are of lower quality than it is true. This second type of consumer is more willing to pay for the goods when MQS are introduced, because the range of quality uncertainty is reduced. However, this implies that firms are confronted with higher average demand and therefore will increase their equilibrium prices. This price effect hurts the perfectly informed consumers. Consumer's expected surplus can be increased, unless the signals that consumers receive about qualities are good with very large probability. Furthermore, too strong strategic effects in the quality game (namely high substitutability between the goods) also reduce the positive effects on consumers' welfare.

Our model has the advantage of allowing an analysis for a number of firms larger than two. The signs of the impact on qualities, profits, and welfare of MQS are confirmed for the case with n firms. The unregulated

[2] Note that in an *oligopoly* of the sort here analyzed consumers could interpret signals coming through *prices* only if they knew all the details of the model. In particular, consumers should be able to assess by how much firms deviate from their mutual best replies, and therefore they should know the exact specification of the firms' cost functions, as well the probability distribution of quality signals.

quality level decreases with the number of firms. Moreover, the quality improvement obtained by a MQS decreases, as the strategic effects fade away with an increase in the number of competitors.

The remainder of the paper is organized as follows. In Section 2 the model with two firms is presented and the main results of our analysis are stated. In Section 3 welfare analysis is conducted. Section 4 extends our results to the $n-$firm case. Finally, Section 5 concludes.

2 The Model

In a market for differentiated goods, two firms compete by choosing the quality level of their products and their prices. These choices are modelled as a two stage game where at each stage moves are simultaneous.

The population of consumers is composed by individuals with identical tastes, summarized by the (gross) utility function

$$U(x_1, x_2) = (\alpha + s_1)x_1 + (\alpha + s_2)x_2 - (x_1^2 + x_2^2 + 2\gamma x_1 x_2)/2 + m \qquad (1)$$

where x_i represents the quantity of good i bought by the representative consumer, for $i = 1, 2$, and m is the respective quantity of the "composite good". Further, s_i represents a quality parameter for good i, while the parameter γ is related to the degree of substitutability between the two goods and is restricted to lie in the interval $[0, 1]$. When $\gamma = 0$ the goods are independent and when $\gamma = 1$ they are perfect substitutes. Let p_1 and p_2 denote the unit prices for the respective goods, while the price of the composite good is normalized to be equal to 1.

Under *perfect* information, maximization of utility with respect to x_1 and x_2 gives the (inverse) demand functions for the representative consumer, $p_i = (a + s_i) - x_i - \gamma x_j$, for $i, j = 1, 2$, and inverting we obtain the demand functions,

$$x_i(p_i, p_j, s_i, s_j) = \frac{\alpha(1-\gamma) + (s_i - \gamma s_j) - p_i + \gamma p_j}{1-\gamma^2}, \text{ for } j \neq i. \qquad (2)$$

For the sake of simplicity we normalize henceforth the mass of consumers to $N = 1$.

Under *imperfect* information, each consumer receives a private signal about the quality of the good (as after imperfect inspection). The consumer's signal about good i is denoted σ_i. Consumers in proportion μ

receive a correct signal about good i, $\sigma_i = s_i$. While consumers in proportion $1-\mu$ receive the wrong signal $\sigma_i = s_0 < s_i$. Messages are independent draws. Therefore the expected proportion of consumers that receive the correct information about the two qualities is μ^2, while $(1-\mu)^2$ are expected to receive a wrong information about both goods, and $2\mu(1-\mu)$ are expected to receive a wrong information about at least one of the two goods. This set up is a simplified, and therefore more manageable, account of what could be a more general environment.[3]

Demand for good 1 by $N = 1$ *ex-ante* identical consumers is:

$$\begin{aligned} q_1 &= \mu^2 x_1(p_1, p_2, s_1, s_2) + \mu(1-\mu)\, x_1(p_1, p_2, s_1, s_0) + \\ &\quad (1-\mu)\mu x_1(p_1, p_2, s_0, s_2) + (1-\mu)^2 x_1(p_1, p_2, s_0, s_0). \end{aligned} \tag{3}$$

A similar expression holds for good 2. From the notation it should be clear that, for instance, $x_1(p_1, p_2, s_1, s_0) = \dfrac{\alpha(1-\gamma) + (s_1 - \gamma s_0) - p_1 + \gamma p_2}{1-\gamma^2}$. Therefore, after some manipulations, the (expected) demand function under imperfect information, q_i, is given by:

$$q_i(p_i, p_j, s_i, s_j, s_0, \mu) = \frac{(1-\gamma)[\alpha + s_0(1-\mu)] + \mu(s_i - \gamma s_j) - p_i + \gamma p_j}{1-\gamma^2}. \tag{4}$$

In order to simplify notation, let $w = (1-\gamma)[\alpha + s_0(1-\mu)]$, while $z_i = \mu(s_i - \gamma s_j) + w$, for $i \neq j$. Then, substituting into (4), we obtain:

$$q_i = \frac{z_i - p_i + \gamma p_j}{1-\gamma^2}, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{5}$$

This is a linear demand function, that depends upon the qualities, s_1 and s_2, upon the information precision, μ and s_0, and upon prices.

We assume that each one of the two firms produces only one of the two goods; further, that its cost of producing quantity q_i of good i whose quality is s_i is given by:

$$C_i(q_i, s_i) = cq_i + s_i^2 \tag{6}$$

[3] For instance, true quality of each good results from (i) producer's investment in quality, determining its average quality, s, and (ii) a disturbance, ε, generated by a random process. Firms could observe but not credibly reveal ε. Consumers would know its distribution. Through "inspection" some consumers would get to perfectly know ε. Some other would only learn that ε is in some range (e.g. below a threshold level).

As can be seen, we maintain the hypothesis that variable costs are independent of quality, while fixed costs are increasing, at an increasing rate, in quality. To simplify the exposition we assume the quality-independent marginal cost to be $c = 0$. Then in the last stage, i.e. in the price competition stage, firm i solves the maximization problem,

$$\max_{p_i} \pi_i = p_i \left(\frac{z_i - p_i + \gamma p_j}{1 - \gamma^2} \right) - s_i^2 \tag{7}$$

where s_1 and s_2 are given as first stage choices. The best reply function for firm i as a function of the rival's price p_j is $p_i(p_j) = \frac{z_i}{2} + \frac{\gamma}{2} p_j$.Therefore, by solving the system of first order conditions, we obtain the Nash equilibrium prices, in terms of the original parameters,

$$p_i^* = \frac{(2+\gamma)(1-\gamma)[\alpha + s_0(1-\mu)] + \mu[s_i(2-\gamma^2) - \gamma s_j]}{4-\gamma^2} \tag{8}$$

Obviously, $p_1^* = p_2^*$ holds only if $s_1 = s_2$. Note that the equilibrium prices are increasing in s_0, the value of the lower signal for the wrongly informed consumers. The same is true for the equilibrium quantities that, from the first order conditions of (7), are given by $q_i^* = p_i^*/(1-\gamma^2)$. Moreover, the maximization problem for firm i at the first stage, is:

$$\max_{s_i} \pi_i^* = \frac{1}{1-\gamma^2} \left[\frac{(2+\gamma)(1-\gamma)[\alpha + s_0(1-\mu)] + \mu[s_i(2-\gamma^2) - \gamma s_j]}{(4-\gamma^2)} \right]^2 - s_i^2. \tag{9}$$

This problem admits a maximum for all $\gamma < 0.9325$. [4] The best reply functions in qualities are downward sloping when the second order conditions are satisfied. This can be easily checked, since from the first order condition we get the best reply function for firm i as:

$$s_i(s_j) = \mu(2-\gamma^2) \frac{(2+\gamma)(1-\gamma)[\alpha + s_0(1-\mu)] - \mu\gamma s_j}{(4-\gamma^2)^2(1-\gamma^2) - \mu^2(2-\gamma^2)^2}, \tag{10}$$

where the denominator is positive if the second order condition is satisfied. Therefore, qualities are strategic substitutes.

Exploiting symmetry, the Nash equilibrium qualities turn out to be:

$$s_i = s^* = \frac{\mu(2-\gamma^2)[\alpha + s_0(1-\mu)]}{(2-\gamma)^2(1+\gamma)(2+\gamma) - \mu^2(2-\gamma^2)}, \ i = 1, 2. \tag{11}$$

[4] While for each $\gamma \geq 0.9325$, μ should be smaller than $\mu_{cr}(\gamma) \equiv (4-\gamma^2)\sqrt{(1-\gamma^2)}/(2-\gamma^2)$, with $d\mu_c/d\gamma < 0$ and $\mu_c(1) = 0$.

Note that for $s_0 < s^*$ to hold, the following condition should be satisfied:

$$\frac{s_0}{a} < \frac{\mu(2-\gamma^2)}{(2-\gamma)^2(1+\gamma)(2+\gamma) - \mu(2-\gamma^2)}, \tag{12}$$

i.e. the quality perceived by the uninformed consumers should not be too high. In the sequel, we shall assume that (12) holds. As stated in the following Lemma, the equilibrium quality is inversely related to the degree of product substitutability. Market competition is stronger when γ is high and this fades away the firms' incentives to invest in quality.

Lemma 1 *The equilibrium quality level in an unregulated duopoly s^* is decreasing in γ and is therefore highest when the goods are independent.*

We can now state our main result.

Proposition 2 *The introduction of a MQS, denoted by Q, such that $Q > s_0$, leads to an increase in the equilibrium quality level. Moreover, the elasticity $\eta = \frac{\partial s^*}{\partial s_0}\frac{s_0}{s^*}$ is positive and decreasing in μ.*

Clearly, if $Q > s_0$, consumers who receive the wrong quality signal will update it to $s_0 = Q$, and this will increase their willingness to pay for the particular good. This in turn will increase the firms' incentives to invest in quality. Note that the increase in quality is obtained also for values of the standard that lie below the unregulated quality level s^*. Indeed, this is the situation that one may have in mind. Finally, the elasticity of equilibrium quality with respect to the MQS is positive and increasing in the proportion of uninformed consumers, $1-\mu$. This would indicate that a market with more severe information problem is a more fertile ground for the use of MQS policies.

What about the effect of γ on the effectiveness of a MQS policy? From (11), we get,

Lemma 3 *The MQS effectiveness on quality is decreasing in the degree of substitutability between the two goods.*

Indeed, one has that $\frac{\partial^2 s^*}{\partial s_0 \partial \gamma} < 0$. The policy instrument is most effective for independent goods, where there are no strategic effects. This confirms that strategic effects decrease the marginal rentability of investments in quality while direct effects tend to increase it.

3 Welfare Analysis

The welfare analysis of the effects of a MQS depend upon its impact on qualities, prices, quantities and firms' profits. We can start from the following result:

Proposition 4 *Equilibrium prices, quantities, and profits are increased if a standard Q, such that $s_0 < Q < s^*$, is introduced.*

The firms' equilibrium profits always increase due to the introduction of the standard. Consumers that are receiving $\sigma = s_0$, i.e. wrong information, about one or both products revise their expectations if a standard prevails. Their willingness to pay is increased with respect to the no-standard case, and their demand for both goods shifts up. Since firms offer goods of higher quality (Proposition 1), they can charge higher prices too. The increase in prices though is moderate and consumers buy more of the higher quality goods.

Note however from (25) and (11) that the hedonic price, namely the price-quality ratio, is given by,

$$\frac{p^*}{s^*} = \frac{(4-\gamma^2)(1-\gamma^2)}{(2-\gamma^2)\mu} \tag{13}$$

Thus, the hedonic price does not depend on s_0 and is inversely related to μ and γ.An increase in the proportion of informed consumers, or the degree of product substitutability, intensifies market competition and as a result, consumers enjoy lower prices per unit of quality offered.

The expected utility of consumers varies according to their information regarding the qualities of the two goods. We can distinguish an *ex-ante* and an *interim* stage. At the first stage one can define the expected utility of a consumer who does not know which signals he will receive (*ex-ante* expected utility). The second is the stage at which each consumer has received the individual signals σ_1 and σ_2. At the interim stage, the four groups of consumers obtain different (*interim*) expected utility. Obviously, a consumer does not know to which group he will be assigned and therefore *ex-ante* utility is the expectation of the *interim* utilities, weighted by the appropriate probabilities, $(1-\mu)^2, (1-\mu)\mu, \mu(1-\mu)$, and μ^2.

It is easy to show that the interim utility of consumers that get the wrong information for both goods, and therefore would buy on the quality perception s_0 for both goods, is increased by the standard. Consumers who receive a wrong signal about both goods, call it "group 1", have net utility,

$$U(x_{1u},x_{2u}) = (\alpha + s_0)(x_{1u}+x_{2u}) - (x_{1u}^2+x_{2u}^2+2\gamma x_{1u}x_{2u})/2 + m - p_1 x_{1u} - p_2 x_{2u}, \tag{14}$$

where $p_1 = p_2 = p^*$ and $x_{1u} = x_{2u} = x_i(p^*, p^*, s_0, s_0)$ (see (2)). Substituting from (25) and after some manipulations, we get Lemma 3:

Lemma 5 *The interim expected utility of consumers receiving the wrong signal that both products are of low quality is increased after the introduction of a MQS.*

On the other hand, the utility of those consumers that at the interim stage are perfectly informed is increased by the standard only in a region of the parameters, because of the price increase effect they pay for enjoying the equilibrium quality increase. The net utility of consumers who receive the true signals, call it "group 4", is given by:

$$U(x_{1f},x_{2f}) = (\alpha+s^*)(x_{1f}+x_{2f}) - (x_{1f}^2+x_{2f}^2+2\gamma x_{1f}x_{2f})/2 + m - p_1 x_{1f} - p_2 x_{2f}, \tag{15}$$

where $p_1 = p_2 = p^*$ and $x_{1f} = x_{2f} = x_i(p^*, p^*, s^*, s^*)$ (see (2)). Substituting from (25) and (11) into (15) and after some manipulations, we get:

Lemma 6 *The interim expected utility of consumers receiving the true signal about both products increases always after the introduction of a MQS only if $\gamma > .765$ and $\mu > \mu^{\#}(\gamma)$, where $\mu^{\#}(\gamma) \equiv \left(4-\gamma^2\right)(1-\gamma^2)/(2-\gamma^2)$.[5]. For the rest of the parameter values, it increases (decreases) with a MQS provided that s_0 is sufficiently high (low).*

Finally, we have no clear-cut results about those consumers (group 2 and group 3) who get a signal that only one of the qualities is s_0. Notice only that the relative magnitude of this group decreases monotonically as the product $\mu(1-\mu)$ is increased and tends to 0 for μ tending to 1 or 0.

Nevertheless, by evaluating the weighted sum of the interim utilities of the four groups, we obtain the expected consumer surplus. We then get the following result for consumer surplus as well as total welfare.

Proposition 7 *(a) Total welfare in the industry is increased by the introduction of a standard over a compact set in the (γ, μ) space with measure larger than 92% of the region of admissible values of (γ, μ) pairs; in particular, such that either $\gamma < 0.823$ or $\gamma > 0.823$ and $\mu < \mu^w(\gamma, s_0)$ where*

[5] Note that $\mu^{\#}(\gamma) < \mu_c(\gamma)$ for all γ; thus there is a (small) permissible region of the parameters where a standard increases the informed consumers surplus.

$\partial\mu^w/\partial\gamma < 0$, $\partial\mu^w/\partial s_0 > 0$ and $\mu^w(0.823, 0) = 1$. (b) Expected consumer surplus is increased by the introduction of a standard for a region in the (γ, μ) space larger than 90% of all values of (γ, μ) pairs; in particular, such that either $\gamma < 0.765$ or $\gamma > 0.765$ and $\mu < \mu^c(\gamma, s_0)$ where $\partial\mu^c/\partial\gamma < 0$, $\partial\mu^c/\partial s_0 > 0$ and $\mu^c(0.765, 0) = 1$.

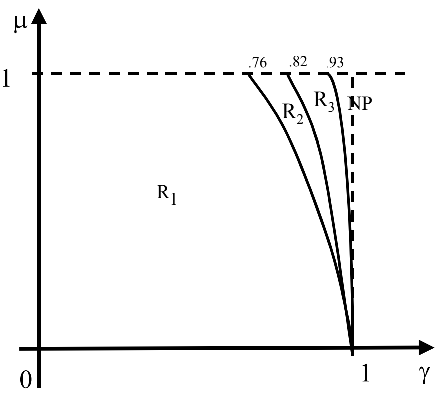

Figure 1: In R_1 and R_2 TW increases always with MQS; in R_1 CS increases always with MQS; NP is the non-admissible region

Proposition 3 is illustrated in Figure 1, where the region denoted R_1 represents the region where consumer surplus is *always* increasing in the MQS, while the region formed by the sum of R_1 and R_2 is the region over which total welfare in the industry is *always* increasing in the standard. In the rest of the (admissible) region, total welfare and consumer surplus are also increasing in the standard, but only if s_0 is sufficiently large.

As for *ex-post* utility one has two possibilities to consider. The first is that a wrongly informed consumer never gets to know the true qualities and remains with the information given at the interim stage. Then the sum over all consumers of the ex-post utility would be equal to the expected utility, if the population of consumers is large enough. In that case the results in Proposition 3 would apply.

The second possibility for ex-post utility is that a consumer ex-post gets to know the quality of the good he consumes. This implies that a consumer belonging, say, to the group that receives the wrong information about both products buys a quantity of each good given by $x_u(p^*p^*, Q, Q) = \dfrac{(\alpha + Q - p)}{1+\gamma}$, where Q is the level of the standard. Ex post they get to know the true quality and re-evaluate their utility according to the quantities they have bought. We have no clear-cut results over the behavior of consumer surplus in the whole parameter space, but we can assert that total welfare in the industry is *always* increasing in the MQS in this case.

4 Extension to more than 2 firms

Let $x = (x_1, \cdots, x_n)$, $s = (s_1, \cdots, s_n)$ and $p = (p_1, \cdots, p_n)$ be the vectors of the quantities, qualities and prices of the n differentiated goods produced in the industry. The net utility function of the representative consumer, setting $m = 0$ to shorten it, is given by

$$U(x,s,p) = \sum_{i=1}^{n}(a+s_i)x_i - \frac{x_1^2 + \cdots x_n^2 + 2\gamma x_1 x_2 + \cdots + 2\gamma x_{n-1}x_n}{2} - \sum_{i=1}^{n} p_i x_i \tag{16}$$

or else,

$$U(x,s,p) = \sum_{i=1}^{n}(a+s_i)x_i - \frac{(\sum_{i=1}^{n} x_i^2 + 2\gamma \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} x_i x_j)}{2} - \sum_{i=1}^{n} p_i x_i \tag{17}$$

As a result, under perfect information the inverse demand function for firm i is,

$$p_i = (a+s_i) - x_i - \gamma x_{-i}, \text{ where } x_{-i} = \sum_{j\neq i} x_j, \tag{18}$$

and its demand function is given by,

$$x_i = \frac{a(1-\gamma) - [1+\gamma(n-2)](p_i - s_i) + \gamma(p_{-i} - s_{-i})}{(1-\gamma)[1+\gamma(n-1)]},$$

where $p_{-i} = \sum_{j\neq i} p_j$ and $s_{-i} = \sum_{j\neq i} s_j$.

Under imperfect information, i.e. when the probability of observing the true quality s_i is μ (with $1-\mu$ the probability of believing that the quality is s_0) and the quality signals are independent draws, the demand function of firm i (after manipulations) turns out to be surprisingly simple. Letting $[a+(1-\mu)s_0](1-\gamma)=V$, it is given by:

$$q_i(p,s,n)=\frac{V-[1+\gamma(n-2)](p_i-\mu s_i)+\gamma(p_{-i}-\mu s_{-i})}{(1-\gamma)[1+\gamma(n-1)]}. \tag{19}$$

Accordingly, the firm i's profit function can be written as $\pi_i(p,s,n)=p_i q_i(p,s,n)-s_i^2$, and standard calculations show that the best price reply function is given by,

$$p_i(p_{-i},s,n)=\frac{V+\gamma p_{-i}+\mu s_i[1+\gamma(n-2)]-\mu\gamma s_{-i}}{2[1+\gamma(n-2)]} \tag{20}$$

Adding the first order conditions of the profit maximization problem, we obtain:

$$\sum_{i=1}^{n}p_i=p_i+p_{-i}=\frac{nV+\mu(1-\gamma)(s_i+s_{-i})}{2+\gamma(n-3)} \tag{21}$$

Substituting p_{-i} into (20) and solving, we obtain the Nash equilibrium prices for given s:

$$p_i(s_i,s_{-i},n)=\frac{[2+\gamma(2n-3)]V+\mu\Psi(\gamma,n)s_i-\mu\gamma[1+\gamma(n-2)]s_{-i}}{[2+\gamma(n-3)][2+\gamma(2n-3)]} \tag{22}$$

where $\Psi(\gamma,n)=[2+3\gamma(n-2)+\gamma^2(n^2-5n+5)]$. Further, the first order conditions of the profit maximization imply that $q_i=p_i(-\partial q_i/\partial p_i)$. Accordingly, from (19) the firm's profits can be written as:

$$\pi_i(s_i,s_{-i},n)=(p_i)^2\frac{[1+\gamma(n-2)]}{(1-\gamma)[1+\gamma(n-1)]}-s_i^2 \tag{23}$$

Finally, maximizing $\pi_i(s_i,s_{-i},n)$ w.r.t s_i and exploiting symmetry, we get the equilibrium quality:

$$s_n^*=\frac{\mu\Psi(\gamma,n)[1+\gamma(n-2)][a+s_0(1-\mu)]}{[2+\gamma(n-3)]^2[1+\gamma(n-1)][2+\gamma(2n-3)]-\mu^2[1+\gamma(n-2)]\Psi(\gamma,n)} \tag{24}$$

Clearly, the effect of a MQS on equilibrium quality is positive for all n. This is so, because the cost of investing in quality, for a single firm, is independent of a MQS. The marginal benefit of investing, instead, depends upon a MQS as both, the equilibrium price and the equilibrium quantity do so. As $\partial V/\partial s_0>0$, it is obvious from (22) that the marginal benefit increases with the MQS.

Now from the focs of (23) we know that

$$p_n^*(\partial p_i/\partial s_i)\frac{[1+\gamma(n-2)]}{(1-\gamma)[1+\gamma(n-1)]} = 2s_n^*$$

where from (22) $\partial p_i/\partial s_i$ does not depend on s_0. Accordingly, equilibrium prices increase with the MQS. Also, equilibrium quantities increase with the MQS, as $q_n^* = p_n^*(-\partial q_i/\partial p_i)$ and the last term is independent of s_0. Further, it is easy to see from (23) that the equilibrium profits π_n^* are proportional to $(s_n^*)^2$, with the coefficient of proportionality being independent of s_0.In fact, this coefficient turns out to be equal to

$$-1 + \frac{4(1-\gamma)[1+\gamma(n-1)]}{[1+\gamma(n-2)]}(\partial p_i/\partial s_i)^{-2}$$

Therefore, the firms' profits increase with the MQS whatever the number of firms in the industry is. Proposition 4 summarizes our findings:

Proposition 8 *In an oligopoly with $n > 2$ firms, equilibrium qualities, prices, quantities and profits are increased if a MQS higher than s_0 is introduced.*

The reasoning is as follows. Since quality is higher with the MQS, prices will be higher too. More important, the effect on consumers' expectations about quality is the same as for the case with two firms: consumers' willingness to pay is increased for each true quality level. This is equivalent to an outward shift in demand function for each firm, and cannot be detrimental to profits. Finally, the analysis for consumers' welfare is more complex than for the case with two firms, but analogous results could be obtained.

The effect of raising the number of firms n on the equilibrium outcome is worth investigating too. The following proposition summarizes the results

Proposition 9 *Unregulated quality is decreasing in n. The absolute effectiveness of a MQS policy is also decreasing with n, while the elasticity of quality to the MQS is independent of n.*

The conclusion that the absolute effectiveness of a MQS is decreasing with n relates to the sign of the cross second derivative $\frac{\partial^2 s}{\partial n \partial s_0}$, and it conforms with the idea that the intensity of the strategic effects fades away as the number of competitors increases. The result about the elasticity says that in percentage terms, two industries with a different number of competitors would show similar reactions in quality to the introduction of a MQS. One may also note, that the elasticity $\eta = \frac{\partial s^*}{\partial s_0}\frac{s_0}{s^*}$ is increasing in $(1-\mu)$ for all n, so that the percentage increase in quality is higher the higher the proportion of uninformed consumers in the population.

5 Conclusions

We have reconsidered the effects of Minimum Quality Standards policies in oligopoly, introducing imperfect information, and brand differentiation. The analysis above has shown that both, firms and consumers, may gain from the introduction of mandatory quality standards. This is a novel result. The key to the result is in the updating by consumers of their expectations about the true quality of the good when a MQS is introduced. This updating may be obtained, more generally, in models where the information acquisition process is less specific. For instance, one may assume that consumers observe a stochastic signal that is centered around the true quality. The MQS would be effective then as far as it cuts from below the support of the distribution of the signal.

This shows that the policy implications and the rationale for the observed use of MQS and of mandatory certification policies markedly differ in the two cases of perfect and imperfect information, because in the first case the firms' reaction to the standard does not depend upon the change in consumers' expectations. Our analysis thus sheds new light in the real world where quality concerns are strong because consumers have imperfect information, for instance, they have difficulties in assessing the way in which goods are manufactured.

We have excluded from our analysis any sort of signaling or certification activity by firms that may cancel or counter the effect of the information received by consumers. The interaction between MQS policies and certification-signaling activities by producers and by their associations is an unexplored field that is left for future research.

6 Appendix

Proof of Lemmata 1 and 2: Taking the derivative of (11) w.r.t γ, we get $\frac{\partial s^*}{\partial \gamma} = \frac{\mu[\alpha+s_0(1-\mu)]\left(2\gamma^5-\gamma^4-8\gamma^3+2\gamma^2+8\gamma-8\right)}{[(2-\gamma)^2(1+\gamma)(2+\gamma)-\mu^2(2-\gamma^2)]^2}$. Therefore, the effect of γ on quality is signed as negative if the term in round parentheses is negative, which is true for all values of γ in the $[0,1]$ interval.

Further, $\frac{\partial^2 s^*}{\partial \gamma \partial s_0} = \frac{\mu(1-\mu)\left(2\gamma^5-\gamma^4-8\gamma^3+2\gamma^2+8\gamma-8\right)}{[(2-\gamma)^2(1+\gamma)(2+\gamma)-\mu^2(2-\gamma^2)]^2}$, which is negative for the same reason. ■

Proof of Proposition 1: If $Q > s_0$, consumers who receive the wrong quality signal will update it to $s_0 = Q$. Notice from (11) that $\dfrac{ds^*}{ds_0} > 0$, so that the effect of an increase in the standard on equilibrium qualities is positive. Finally, the elasticity η is given by $\eta = \frac{(1-\mu)s_0}{\alpha + s_0(1-\mu)}$.■

Proof of Proposition 2: Substituting (11) into (8) and simplifying, we obtain

$$p_i^* = p^* = \frac{(4-\gamma^2)(1-\gamma^2)[\alpha + s_0(1-\mu)]}{(2-\gamma)^2(1+\gamma)(2+\gamma) - \mu^2(2-\gamma^2)}, \ i = 1,2 \tag{25}$$

Moreover, from the first order conditions (7), we know that $q_i^* = q^* = \frac{p^*}{1-\gamma^2}$. It is then clear that $\frac{dp^*}{ds_0} > 0$ and $\frac{dq^*}{ds_0} > 0$, so that both equilibrium prices and quantities are increasing in s_0 and therefore in the standard. Turning to profits, by the first order conditions of (7), equilibrium profits are equal to $\frac{(p^*)^2}{1-\gamma^2} - (s^*)^2$. Substituting p^* from (25) and s^* from (11), we get after some manipulations:

$$\pi^* = \frac{[(4-\gamma^2)^2(1-\gamma^2) - \mu^2(2-\gamma^2)^2][\alpha + s_0(1-\mu)]^2}{[(2-\gamma)^2(1+\gamma)(2+\gamma) - \mu^2(2-\gamma^2)]^2}, \ i = 1,2 \tag{26}$$

which are positive if the second order condition is satisfied. Clearly, equilibrium profits are increasing in s_0, and thus increase due to the introduction of the standard. ■

Proof of Lemma 3: After the substitution we obtain $CS_u = U(x_{1u}, x_{2u})$ as a function of a, γ, μ and s_0. In order to sign $\partial CS_u/\partial s_0$, we first take its derivative w.r.t. s_0. It can be checked that $\partial^2 CS_u/\partial s_0^2 > 0$; hence, $\partial CS_u/\partial s_0$ takes its lowest value at $s_0 = 0$. Moreover, it can be checked that $\partial CS_u/\partial s_0 \mid_{s_0=0} > 0$; hence $\partial CS_u/\partial s_0$ is positive for all parameter values.

Proof of Lemma 4: Similarly, we obtain $CS_f = U(x_{1f}, x_{2f})$ as a function of a, γ, μ and s_0. We again take $\partial^2 CS_u/\partial s_0^2$ and check that is positive. Hence, $\partial CS_u/\partial s_0$ takes its lowest value at $s_0 = 0$. It can then be checked that $\partial CS_f/\partial s_0 \mid_{s_0=0} > 0$ only if $\gamma > .765$ and $\mu > \mu^{\#}(\gamma)$, where $\mu^{\#}(\gamma) \equiv \left(4-\gamma^2\right)(1-\gamma^2)/(2-\gamma^2)$. Clearly, for this range of parameters $\partial CS_f/\partial s_0$ is always positive.

For the rest of the parameters, where $\partial CS_f/\partial s_0 \mid_{s_0=0} < 0$, one can check that $\partial CS_f/\partial s_0 \mid_{s_0^{\max}} > 0$, where $s_0^{\max}$ is the maximum permissible s_0 and is given by the L.H.S. of (12) multiplied by a. We then conclude by continuity, that there exists a $\widetilde{s}_0$ s.t. for all $s > \widetilde{s}_0$, CS_f is increasing in s_0; otherwise it is decreasing in s_0.

Summarizing all the above, one can say that CS_f is increasing in s_0 provided that s_0 is not too low.

Proof of Proposition 3: We first calculate the interim utility of group 2 and 3 that consist of consumers who receive the wrong signal about good 1 and 2, respectively. Given the symmetric form of the net utility function, we need only find the interim utility of any one of these two groups. For instance, the interim utility of group 3 is given by,

$$U(x_{1i}, x_{2u}) = (\alpha+s^*)(x_{1i}+x_{2u})-(x_{1i}^2+x_{2u}^2+2\gamma x_{1i}x_{2u})/2+m-p_1x_{1i}-p_2x_{2u},$$

where $p_1 = p_2 = p^*$, $x_{1i} = x_i(p^*, p^*, s^*, s_0)$ and $x_{2u} = x_i(p^*, p^*, s_0, s^*)$ (see (2)). Let $CS_p = U(x_{1i}, x_{2u}) = U(x_{1u}, x_{2i})$. Then $CS = \mu^2 CS_f + 2\mu(1-\mu)CS_p + (1-\mu)^2 CS_u$ and $TW = CS + 2\pi^*$, and both are functions of the parameters a, γ, μ and s_0.

We then follow the same steps as in Lemma 4. It can be checked that both $\partial^2 CS/\partial s_0^2$ and $\partial^2 TW/\partial s_0^2$ are always positive. Evaluating then $\partial CS/\partial s_0$ at $s_0 = 0$, we can assert that CS is always increasing in s_0 if either $\gamma < 0.765$, or $\gamma > 0.765$ and $\mu < \mu^c(\gamma, s_0)$, with $\partial\mu^c/\partial\gamma < 0$, $\partial\mu^c/\partial s_0 > 0$ and $\mu^c(0.765, 0) = 1$. Similarly, evaluating $\partial TW/\partial s_0$ at $s_0 = 0$, we can assert that TW is always increasing in s_0 if either $\gamma < 0.823$, or $\gamma > 0.823$ and $\mu < \mu^w(\gamma, s_0)$, with $\partial\mu^w/\partial\gamma < 0$, $\partial\mu^w/\partial s_0 > 0$ and $\mu^c(0.823, 0) = 1$. Further, it can be checked that $\mu^c(\gamma, s_0) < \mu^w(\gamma, s_0) < \mu_{cr}(\gamma)$ for all permissible values of s_0. ■

Proof of Proposition 5: Taking the derivative of (24) w.r.t. n and manipulating, we can assert that it is always negative. Further, observe that $\partial s_n^*/\partial s_0$ is proportional to s_n^*, with the coefficient of proportionality independent of n. Therefore, $\partial^2 s_n^*/\partial s_0 \partial n$ has the same sign as $\partial s_n^*/\partial n$, i.e. the effectiveness of the MQS decreases with n. As for the elasticity, starting from (24), one has that the elasticity $\eta \equiv \frac{\partial s^*}{\partial s_0}\frac{s_0}{s^*}$ is given by $\eta = \frac{s_0(1-\mu)}{a+s_0(1-\mu)}$, independent of n and decreasing in μ.

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