

Design of Public Voluntary Environmental Programs for Nitrate Pollution in Agriculture: An Evolutionary Approach

by

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Abstract

The joint evolution of participation and compliance of farmers in a public VA, along with the evolution of the pollution stock is examined. Replicator dynamics, modeling participation and compliance, are combined with pollution stock dynamics. Fast-slow selection dynamics are used to capture the fact that distinct decisions to participate in and comply with the public VA evolve in different time scales. Conditions for evolutionary equilibria and evolutionary stable strategies regarding participation and compliance are derived. Depending on the structure of the legislation and auditing probability, polymorphic equilibria indicating partial participation and partial compliance or monomorphic equilibria of full (or non) compliance could be the outcome of the evolutionary processes. Multiple equilibria and irreversibilities are possible, while convergence to evolutionary equilibria could be monotonic or oscillating. Full participation and compliance can be attained if the regulator is pre-committed to certain legislation and inspection probabilities, or by appropriate choices of the legislatively set emission level and the non-compliance fine. Budget constraints associated with monitoring costs seem to produce polymorphic equilibria.

Keywords: Voluntary agreements, participation, compliance, evolutionary stability, replicator dynamics, budget constraint.

JEL Classification: Q2, L5.

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1 Introduction

Voluntary approaches (VAs) to environmental regulation have been regarded as alternative instrument to pollution control. They are expected to increase economic and environmental effectiveness as well as social welfare, relative to traditional policy instruments¹, since they allow farmers greater flexibility in their pollution control strategies and have also the potential to reduce transaction and compliance costs.² VAs can be classified into three basic categories, based mainly on the degree of public intervention.³ *Negotiated agreements* that imply a bargaining process between the regulatory body and a farmer or an industry group to jointly set the environmental goal and the means of achieving it. *Unilateral agreements* are environmental improvement programs prepared and voluntarily adopted by farmers themselves. *Public voluntary agreements* are environmental programs developed by a regulatory body and farmers can only agree to adopt them or not.

The potentially most serious drawback of VAs is that they leave room for free-riding. Particularly, in public VA where the attainment of an environmental target requires collective action, individual farmers may have incentives not to reduce their emissions but to rely upon other farmers to carry out the actions necessary to attain the target. Farmers can decide not to participate in the achievement of the established goal either ex-ante (non-participation), or ex-post after signing the agreement (non-compliance). It is possible that free-riding may impede the establishment of a public VA, or may result in a failure of the agreement because signatory farmers do not comply with the rules of the VA.⁴ This suggests some limitations in

¹Such as emission taxes, subsidies, or tradeable permit systems etc.

²The theoretical analysis of VAs to environmental regulation has been mainly developed in the recent decade. See for example the work of Carraro and Siniscalco (1996), Segerson and Miceli (1998), Segerson and Dawson (2000), Brau et al. (2001), Lyon and Maxwell (2003).

³Examples of successful public VAs include the EPA's "33-50" program that seeks to encourage firms in the US Chemical industry to voluntarily reduce the discharges of 17 high-priority toxic chemicals under the background threat of legislation, the "US. Conservation Reserve Program" that used cost-sharing and other financial inducements to achieve reduction of agricultural pollution through voluntarily participation in soil conservation and other erosion control programs and its successor "Environmental Quality Incentives Program", "Canadian Industry Program for Energy Conservation", the "US Green Lights" , the "Motor Challenge" programs for industry, as well as the "Golden Carrot" program for manufactures of highly energy-efficient refrigerators which have been recently consolidated with the "Motor Challenge" (OECD 1998). While "ProjectXL" and "Common Sense Initiative" involve negotiation, they also resemble public VAs.

⁴Despite the presence of apparent incentives to free-ride it is possible to have an equilib-

the ability of VAs to attain desired targets. In fact there are some reservations, based on empirical observations, regarding the ability of public VAs to improve environmental quality as an independent policy tool. According to a report by Environment Canada, industrial sectors that relied solely on self monitoring or voluntary compliance had a sufficiently lower average compliance rating (60% vs 94%) of those industries which were subject to federal regulations combined with a consistent inspection program.⁵ Indeed without appropriate threats of sanctions or enforcement schemes, there may be a problem of compliance or uneven application. Both participation in and compliance with the agreement are important and thus a successful VA scheme may need to include a mix of voluntary and mandatory features, to ensure that polluting agents will not only sign the public VA but also comply with its provisions and established goals.

The present paper studies the long-run structure of a public VA where the regulator makes an offer to a large number of homogeneous farmers to reduce nitrate emissions in order to voluntarily attain, by using flexible cost saving methods, a desired ambient pollution level.⁶ The type of VA we study has many similarities with voluntary climate change programs or the various Energy Star programs.⁷ If the offer attains full participation, a target ambient pollution stock is attained. If there is no full participation then there is a deviation from the target and a positive probability of legislation through conventional instruments such as direct regulation. Thus limited participation may trigger regulation. Participating farmers are not directly observed by the regulator so there could be incentives not to comply. The regulator tries to deter non-compliance by random auditing and fines to those found not in compliance with the VA.

The general set up of compliance and auditing developed in this paper here can be used as a basis in order to gain some insights regarding nitrate pollution regulation. In particular, a similar type of regulatory framework can be regarded for the EU Nitrate Directive (91/676/EEC) that aims to re-

rium in which the target is achieved and only a subset of firms in the industry comply with the agreement's provisions, while the remaining free-ride (Dawson and Segerson, 2001).

⁵Enforcement vs Voluntary Compliance: An Examination of the Strategic Enforcement Initiatives Implemented by the Pacific and Yukon Regional Office of Environment Canada, Report No. DOEFRAP 19983.

⁶The flexible methods of reducing emissions through the VA program have a weak cost advantage relative to regulation like, for example, the XL Project or the EPA's 33-50.

⁷See, for example, OECD (1998).

duce water pollution caused by nitrates generated from agricultural sources. The importance of the particular issue lays in the fact that only a minority of Member States have fully applied the directive and the Commission has opened a number of infringement proceedings against Member States for non-implementation. In this context the directive entails two regulatory frameworks (Article 4): (i) codes of good agricultural practice to be implemented on voluntary basis supplemented where necessary by the provision of training and information⁸ and, (ii) a mandatory framework involving obligatory measures to be implemented in action programs for nitrate vulnerable zones⁹. Thus the developed general conceptual model of VA and nitrate pollution generation in this paper can be associated to some extent with the regulatory framework of the Nitrate Directive.

In modelling the process where farmers decide whether to participate in the agreement under a probabilistic regulation threat, we adopt an evolutionary framework. The basic characteristic of this framework is that, although farmers are profit maximizers in the output choice, when it comes to choosing a strategy regarding participating in the VA, or whether to comply or not, they adopt a more passive decision making and not an explicit optimizing behavior.¹⁰ This more passive decision making is modelled by an evolutionary process where decisions are taken by comparing the profits of a strategy to participate in and comply with the VA, with corresponding expected profits of a nonparticipating, non-complying farmer. Successful strategies are those attaining higher expected profits and are imitated by other farmers with a probability proportional to the difference between corresponding profits. Profit differentials exercise evolutionary pressures on the

⁸These codes contain provisions covering issues such as: (a) periods when application of fertilizers is inappropriate, (b) application of fertilizer to steeply sloping ground, (c) fertilizer application to water-saturated, flooded, frozen or snow-covered ground, (d) the conditions application of fertilizer near water courses, (e) the capacity and construction of storage vessels for livestock manure and (f) procedures for fertilizer applications.

⁹Action programs consist of the following mandatory measures: (a) measures prescribed in the code(s) of good agricultural practice, (b) rules determining time periods where application of certain types of fertilizers is prohibited, (c) measures concerning the minimum acceptable capacity of storage vessels for manure, and (d) limitation in the application of fertilizers on particular grounds.

¹⁰This evolutionary approach might be encompassing ideas of bounded rationality since it can be associated with firms' bounded ability to fully perceive either advantages associated with flexibilities and cost superiority of the VA, or costs associated with probabilistic fines. For general presentations of these approaches see for example Nelson (1995) and Conlisk (1996).

composition of population of farmers so that more successful strategies increase their share in the total population of farmers. A simple way to model the movements in the composition of population of farmers regarding participation in and compliance with the VA is the use of *replicator dynamics*.¹¹ We use replicator dynamics as selection dynamics to model in two stages the evolution of: (i) the decision to sign or not the agreement, and (ii) the decision to comply or not with the agreement's provisions after signing it.

The use of replicator dynamics allows us to determine evolutionary equilibria (EE), which can be related to evolutionary stable (ES) strategies regarding participation and compliance.¹² We further elaborate on the selection dynamics by considering the situation where decisions to participate or not evolve fast, since when the offer is made there is usually a legal time framework,¹³ while decisions regarding compliance after participation are unconstrained and expected to evolve much more slowly. This suggests that the evolutionary equilibrium composition of farmers regarding participation is reached faster than the ES equilibrium composition regarding compliance, implying that selection dynamics operate in a *fast-slow dynamics* framework.

Our contribution lies in using, for the first time to our knowledge, an evolutionary approach with fast-slow selection dynamics to jointly determine the steady-state equilibrium fraction of signatory and complying farmers, as well as the corresponding steady-state equilibrium emission stock. Using this approach we are able to determine "*which strategies survive in the long-run*", in the sense of evolutionary stability, define the structure that a long-term VA would have, in terms of participation and compliance, and identify policy rules that might produce desirable VAs.¹⁴ Our analysis indicates that the value and characteristics of the legislation and auditing probability are of crucial importance for the resulting long-term equilibrium outcome. Under different assumptions about the legislation probability, the fast time dynamic system can alternatively converge to a polymorphic or monomorphic steady state, implying either partial or full (or non) participation in the public VA. Similarly by choosing the structure of the auditing probab-

¹¹For definitions, see, for example, Weibull (1995). For applications of this methodology to common property resources see, Sethi and Somanathan (1998).

¹²A strategy is ES if it can not be invaded by a mutant strategy. (See for example Weibull (1995) page 36)

¹³EPA's National Environmental Performance Track accepts applications twice a year.

¹⁴For a similar approach regarding the regulation of a renewable resource, see Xepapadeas (2005).

ity, the regulator can achieve partial or full (or non) compliance. There is a possibility of unique or multiple EE with potential irreversibilities, while the convergence to these equilibria could be monotonic or oscillating. If full participation and full compliance, are regarded as the desired outcome for the regulator, they can be attained if the regulator is pre-committed to certain legislation and inspection probabilities, or by appropriate choices of the legislation mandate and the non-compliance fine. Finally we show that under a limiting budget for financing auditing inspections, which is partly financed exogenously and partly through collected fines, a polymorphic compliance equilibrium is the most likely outcome.

2 A Model of Agricultural nitrate pollution and Regulation

Assume an industrial sector consisting of $i = 1, 2, \dots, n$ small and identical farmers, which operate under competitive conditions and emit into the ambient environment. Emissions accumulate in the environment and cause external damages, which exceed the socially-desirable levels without regulation. The regulator proposes formally a "take-it-or-leave-it" environmental protection scheme and gives each farmer in the industrial sector a chance to voluntarily meet an exogenously determined emission level e_v . This type of public VA offers full flexibility to choose the profit-maximizing and legislative preemptive means of achieving the target and could provide cost advantages over legislative regulation.

In particular the regulator proposes a long-term "preemptive" public VA¹⁵ to which farmers can only agree or not. If all farmers follow the agreement then total emissions in the ambient environment will be $E_v = ne_v$, where we assume that the nitrate pollution stock S accumulates according to:

$$\dot{S}(t) = E(t) - \varphi(S(t)), \quad E(t) = \sum_{i=1}^n e_i(t) \quad (1)$$

where $E(t)$ denotes total nitrate emissions at time t due to agricultural activities, and $\varphi(S(t))$ denotes emissions outflows due to natural environmental self cleaning process and environmental feedbacks.

¹⁵Such VAs indirectly reduce expected production costs because they reduce the probability of facing a (more costly) direct regulatory regime (Brau et al., 2001).

Let, $\bar{S}(t)$ be the path of nitrate pollution stock under full participation and compliance to the agreement. If there is no full participation, a deviation at time t is expected between the observed and desired nitrate pollution stock, denoted by $\Delta S(t) = S(t) - \bar{S}(t)$.¹⁶ Participation in the VA does not imply that a farmer will also comply with its provisions. Thus although the regulator has full observability of participating farmers, we assume that simultaneous control of all signatory farmers is prohibitively costly. The mechanism usually applied to verify compliance and identify compliance problems, is inspection of randomly chosen signatory farmers. Therefore a positive $\Delta S(t)$ might be the result of either partial participation and non-compliance by some of participating farmers, or under full participation, the result of non-compliance by some signatory farmers. It would be intuitive to assume that from a farmer's point of view the subjective probability of having legislation introduced at time t depends on the deviation $\Delta S(t)$ and the proportion of participating farmers $x(t)$, or¹⁷

$$p(t) = p(\Delta S(t), x(t), \omega_v(t)), \text{ with } \frac{\partial p(\cdot)}{\partial \Delta S} > 0, \frac{\partial p(\cdot)}{\partial x} < 0 \quad x \in [0, 1] \quad (2)$$

where $\omega_v(t)$ is a vector of other parameters affecting the probability of regulation.¹⁸ The probability of regulation would increase due to either an increase in the deviation $\Delta S(t)$ or a decrease in the number of participating farmers.

We further specify the probability structure, by assuming that the probability of introducing legislation is common to all farmers and that: $p(0, 1) = 0$;¹⁹ $p(\Delta S, x | \Delta S > 0, x < 1) > 0$; $p(\Delta S, 1 | \Delta S > 0) = 0$ hold respectively. That is, if everybody participates, then the deviation is due to non-compliance. We assume that $(\Delta S(t), x(t))$ are observable by the regulator and become public information, while there is uncertainty regarding the vector ω_v . farmers can use announced $(\Delta S(t), x(t))$ to calculate subjective probabilities, but there is uncertainty regarding the probability law $p(\Delta S(t), x(t), \omega_v(t))$, thus farmers use model (2) as a benchmark for some fixed value of the vector ω_v .

¹⁶We do not consider uncertainty issues.

¹⁷Segerson and Miceli (1998) assume a fixed legislation probability.

¹⁸It may include legislative procedures, transaction costs, etc.

¹⁹The possibility of $p(0, x | x < 1) = 0$, which allows for overcompliance by some firms so that the target is achieved even if some firms are not participating, is not considered. The possibility of overcompliance implies the introduction of another strategy, $e_{OC} < e_v$.

If farmers believe that the only factor that affects the probability of legislation is the nitrate pollution deviation ΔS , then the probability can be simplified to²⁰

$$p(t) = p(\Delta S(t), \omega_v(t)), \text{ with } \frac{\partial p(\cdot)}{\partial \Delta S} > 0 \quad (3)$$

The decision to participate and then comply or not depends on the structure of profits. In our model, each farmer produces an output Q and emissions e . The cost function $C(Q, e)$ is a continuous function where $C_Q > 0$, $C_e < 0$, $C_{QQ} > 0$ and $C_{ee} > 0$. We assume that the VA offers only a cost advantage to participating and complying farmers since it deters the introduction of relatively more costly mandatory regulation and allows greater flexibility in the processes of emissions reduction.²¹ The profit function is defined as $\Pi(e) = \max_Q \{PQ - C(Q, e)\}$.

At the unregulated equilibrium a farmer chooses emissions $e_o = \arg \max_e \Pi(e)$. Therefore when a farmer decides not to participate in the VA, and continues producing at the profit-maximizing emission level without facing a legislative mandate, then profits are defined as $\Pi_N(e_o)$. If a farmer decides to sign the VA and voluntarily cut emissions at the agreed level e_v , then profits are $\Pi_v(e_v) = \max_Q \{PQ - C_v(Q, e_v)\}$.

If a farmer decides not to participate in the VA and mandatory legislation is used to introduce either an emission tax τ , or an emission limit (performance standard) \bar{e} , then its profit function could be defined as:

$$\Pi_L(e, \tau) = \max_Q \{PQ - C_L(Q, e) - \tau e\} \quad (4)$$

$$\Pi_L(e, \bar{e}) = \max_Q \{PQ - C_L(Q, e) : e \leq \bar{e}\} \quad (5)$$

In both cases $C_v(Q, e) < C_L(Q, e)$ under the cost advantage assumption of the VA. So under legislation profits can be defined as $\Pi_L(e_L)$, where $e_L(\tau) = \arg \max_e \Pi_L(e, \tau)$ under taxation, or $e_L = \arg \max_e \Pi_L(e)$ subject to $e \leq \bar{e}$, under a performance standard. Under standard assumptions $e_L = \bar{e}$.²²

²⁰It seems that ΔS shall always be part of the subjective probability in every case. If the subjective probability is a function of participation proportion x alone, then the incentive to participate is not linked with the achievement of the environmental target e_v .

²¹We assume that the VA does not improve a firm's public image and increase consumers' goodwill.

²²The target $\bar{e} = e_L$ can be achieved either through taxation, if the tax rate is chosen

If a participating farmer decides not to comply with the VA and emit at the unregulated level e_o , then there is a possibility that the farmer is caught after a random inspection. If the non-complying farmer is not inspected then profits are $\Pi_N(e_o)$. If caught the farmer is subjected to individual legislation and a non-compliance fine F . The profits of a non-complying farmer which is caught after a random inspection is $\Pi_C(e_L, F) = \Pi_L(e_L) - F$.

Since $e_o > e_L \geq e_v$ the structure of costs and profits imply:

$$\Pi_o(e_o) > \Pi_v(e_v) > \Pi_L(e_L) > \Pi_C(e_L, F)$$

In the case of non-participation in the agreement the imposition of legislation is probabilistic, therefore the expected profits of non-participating farmers are:

$$\mathcal{E}\Pi_N = p\Pi_L(e_L) + (1 - p)\Pi_N(e_o), \quad p = p(\Delta S, x, \omega_v) \quad (6)$$

Thus the sufficient condition for participation in the VA is

$$\Pi_v(e_v) \geq p\Pi_L(e_L) + (1 - p)\Pi_N(e_o) \quad (7)$$

Let q be the subjective probability that a participating farmer will be inspected and let z be the proportion of participating farmers that comply with the terms of the VA. A farmer's subjective probability of being audited can generally be defined by $q(\omega_c)$, where ω_c is a vector of parameters. It is assumed that this function is common for all farmers and can be further specified in the following cases.

In the first case the regulator exercises fixed monitoring effort and makes a fixed number of inspections, say \bar{n} per period. The regulator announces this policy and precommits to a certain auditing probability, which is known by the polluters. In this case the audit probability is fixed, or²³

$$q(\omega_c) \equiv \bar{q} \quad (8)$$

An alternative assumption would be that the regulator exercises variable

such that $e_L(\tau) = \bar{e}$ is a solution to $\max_e \Pi_L(e, \tau)$, or through a performance standard \bar{e} .

²³This is a common assumption in the enforcement literature in environmental economics (e.g. Malik, 1993; Garvie and Keeler, 1994; Segerson and Miceli, 1998; Stranlund and Dhana, 1999).

monitoring effort, dependent on state variables of the problem observed by the regulator.²⁴ One such variable is the deviations from the desired nitrate pollution stock ΔS ; and/ or the share of violators $1 - u$, $u \in [0, 1]$ detected during an audit. The regulator increases the monitoring effort if the stock or the share of violators is increasing. This policy can be regarded as a type of no full commitment - or partial commitment - auditing policy on the regulator's part. The regulator might, for example, not audit individual farmers if the deviation ΔS is sufficiently low, but start inspections if the deviation increases beyond a certain level.²⁵ The farmers are made aware of the results of the inspections, through public announcements and/or private communications, and perceive that if the deviation or the share of violators increases, more monitoring effort will be exercised and thus the subjective probability of being audited increases. In this case the probability q can be specified as stock dependent auditing probability:

$$q = q(\Delta S, \omega_c), \quad q'(\Delta S, \omega_c) > 0, \quad q(0, \omega_c) = 0 \quad (9)$$

where ω_c is a vector of parameters similar to ω_v .

If farmers use the observed u as an estimate for their perceived z , that is they set $u = z$, a compliance dependent auditing probability is defined as:

$$q = q(z, \omega_c), \quad q'(z, \omega_c) < 0, \quad q(1, \omega_c) = 0, \quad q(0, \omega_c) > 0 \quad (10)$$

It is expected that the value of $q(0)$ will be large but not unity since not every farmer is audited²⁶ even if nobody complies, while if everybody is complying the subjective probability is $q(1) = 0$.

If (9) and (10) are taken together, a general formulation of the subjective audit probability with joint dependence on compliance and stocks would be:

$$q = q(z, \Delta S, \omega_c) \quad (11)$$

In this context the expected profits of a participating, non-complying farmer

²⁴In the enforcement literature, variable monitoring effort is usually related to firm specific variables (e.g. Malik, 1990; VanEgteren and Weber, 1996).

²⁵Grieson and Singh (1990), Khalil (1997), and Franckx (2002) analyze no commitment frameworks. An environmental regulator chooses which firm to inspect without observing firms' actions but after observing ambient pollution.

²⁶This can be associated with a binding budget constrained for inspection costs.

are:

$$\mathcal{E}\Pi_N = q\Pi_C(e_L, F) + (1 - q)\Pi_N(e_o) \quad (12)$$

and the sufficient condition for complying with the agreement's provisions is:

$$\Pi_v(e_v) \geq q\Pi_C(e_L, F) + (1 - q)\Pi_N(e_o) \quad (13)$$

Given the above framework we explore how imitation and adaptation of behavior, resulting in higher profits, determine which strategies (participate or not/comply or not) will survive in the long run. We model the selection dynamics that can be used to determine the ES strategies by replicator dynamics.

3 Replicator Dynamics as Selection Dynamics

Assume that at a given time t the industrial sector consists of two groups of farmers, each group following different strategy concerning participation in the public VA. Let $x(t)$ denote the proportion of farmers participating in the agreement, while $x_N(t)$ the remaining proportion of non-signatory agents at time t , with $x(t) + x_N(t) = 1$.

In every time period dt there is a positive probability adt that a farmer i , following a certain strategy, will compare its profits and consequently its strategy, with the corresponding profits and strategy of another randomly chosen farmer j .²⁷ If farmer i perceives that the farmer j 's profits are sufficiently higher, then it switches its strategy. There is imperfect information concerning the difference in the expected profits of the two strategies, since there is uncertainty in the law determining the probability of legislation and possible uncertainty regarding the true cost functions. In this context the higher the profits difference is, the higher the probability is that farmer i will change strategy. Particularly, farmer i that did not participate in the public VA at time t , might decide to switch strategy and sign the agreement if its expected profits $\mathcal{E}\Pi_N$, defined by (6), are less than the profits $\Pi_v(e_v)$ of the participating farmer. Therefore, the probability that a non-participating farmer will change its strategy and ultimately sign the public

²⁷In motivating the replicator dynamics we follow Gindis (2000).

VA, after comparing profits, is given as:

$$P_{NV}^t = \begin{cases} \beta [\Pi_v(e_v) - p\Pi_L(e_L) - (1-p)\Pi_N(e_o)] & \text{for } \Pi_v(e_v) > \mathcal{E}\Pi_N \\ 0 & \text{for } \Pi_v(e_v) \leq \mathcal{E}\Pi_N \end{cases}$$

The expected proportion of farmers that decide to participate in the public VA at time $t + dt$ is given as:

$$\begin{aligned} \mathcal{E}x^{t+dt} &= x^t + \alpha dt x^t \sum_{j=1}^n x_N \beta (\Pi_N(e_o) - \Pi_v(e_v)) \\ \mathcal{E}x^{t+dt} &= x^t + \alpha dt x^t \beta (\Pi_v(e_v) - \bar{\Pi}(e)) \end{aligned}$$

where $\bar{\Pi}(e)$ denotes average profits for the whole population, defined as:

$$\bar{\Pi}(e) = x\Pi_v(e_v) + (1-x)\mathcal{E}\Pi_N = x\Pi_v(e_v) + (1-x) [\Pi_N(e_o) - p\Delta\Pi_L^N] \quad (14)$$

where $\Delta\Pi_L^N = (\Pi_N(e_o) - \Pi_L(e_L))$ are the profit losses under the non-participating strategy when legislation is imposed.

The population of farmers in the industrial sector is assumed to be large and thus we can replace $\mathcal{E}x^{t+dt}$ by x^{t+dt} . Moreover, if we subtract from both sides the term x^t , divide by dt and finally take the limit as $dt \rightarrow 0$, we derive an equation that describes the behavior of the fraction x over time:

$$\dot{x} = \alpha\beta x^t [\Pi_v(e_v) - \bar{\Pi}(e)]$$

This is the replicator dynamics equation, which indicates that the frequency of the signatory strategy increases when its profits $\Pi_v(e_v)$ are above the average profits $\bar{\Pi}(e)$. If we substitute from (14) then the replicator dynamics equation is rewritten as:

$$\dot{x} = \alpha\beta x(1-x)[p\Delta\Pi_L^N - \Delta\Pi_v^N] \quad (15)$$

where $\Delta\Pi_v^N = (\Pi_N(e_o) - \Pi_v(e_v))$ are the profit losses under the participating, complying strategy.

It has already been mentioned that participation in the VA does not imply that a farmer will also comply with its provisions. We assume that in choosing between compliance or not farmers imitate successful strategies, as in the choice of participation strategy, by collecting (incomplete) information

regarding expected profits of non-complying farmers. Let $z(t)$ denote the proportion of farmers complying with the agreement, while $z_N(t)$ the remaining proportion of non-complying farmers at time t , with $z(t) + z_N(t) = 1$.

After following the same conceptual framework, the replicator dynamics equation for the compliance strategy is defined as:

$$\dot{z} = \gamma \delta z^t [\Pi_v(e_v) - \bar{\Pi}_{VN}(e)]$$

where γ and δ correspond to α and β above, and $\bar{\Pi}_{VN}(e)$ are the average profits for the whole population of signatory farmers.²⁸

Then the replicator dynamics equation for the complying strategy is:

$$\dot{z} = \gamma \delta z(1 - z) [q \Delta \Pi_C^N - \Delta \Pi_v^N] \quad (16)$$

where $\Delta \Pi_C^N = (\Pi_N(e_o) - \Pi_C(e_L, F))$ are the profit losses under the non-complying, participating strategy when both legislation and fine are imposed.

Steady states (S-S) (or stationary points or rest points, or critical points) of the replicator dynamics equations (15) or (16) can be used to define evolutionary equilibria. Following standard stability classification an (S-S) is *stable* (or *Lyapunov stable*) if no small perturbation from the (S-S) induces a movement away from (S-S), it is *asymptotically stable* (AS) if it is stable and small perturbations induce a movement back towards (S-S), or to put it differently if the solution of the replicator dynamic equation tends to the (S-S) from initial conditions in the neighborhood of (S-S) as $t \rightarrow \infty$. We will define as *evolutionary equilibrium* (EE) an AS steady state under the replicator dynamics (Gindis (2000)). The (S-S) is *globally asymptotically stable* (GAS) if it converges to the (S-S) independent of initial conditions for any initial state in the open interval $(0, 1)$ ²⁹. A stable (S-S) is a Nash equilibrium of the game defined in terms of two farmers following strategies of participation or non-participation, or compliance or non-compliance. Furthermore, if a strategy \hat{x} is an evolutionary stable (ES) strategy then it corresponds to an EE under the replicator dynamics. Conversely, a strategy \hat{x} is an ES strategy if it is a *strongly stable* equilibrium point of the replicator dynamics equation, where strong stability means that if \hat{x} is contained in a *convex hull*

²⁸ Where $\bar{\Pi}_{VN}(e) = z \Pi_v(e_v) + (1 - z)[q \Pi_C(e_L, F) + (1 - q) \Pi_N(e_o)]$.

²⁹ We can not request convergence from the boundaries 0 and 1 since they are invariant.

of the strategy simplex, all strategies in the neighborhood of \hat{x} converge to \hat{x} (see, for example, Hofbauer and Sigmund 2003).³⁰ Since GAS equilibria can be associated with the notion of strong stability, GAS steady states under the replicator dynamics can be regarded as reflecting ES strategies.

The evolution of the emission stock is affected by the decisions to participate in the agreement and further comply with its provisions and established goals. Therefore the nitrate pollution stock dynamic equation (1) can be further specified as:

$$\dot{S} = n\{x[ze_v + (1 - z)\mathcal{E}e_L(q)] + (1 - x)e_o\} - \varphi(S) \quad (17)$$

where $\mathcal{E}e_L(q) = qe_L + (1 - q)e_o$ are the expected emissions of a non-complying but participating farmer. Finally, (17) can be further specified by assuming that the emissions outflows term is linear, implying that $\varphi(S) = bS$ with $b > 0$.

The combination of the replicator dynamics equations (15) or (16) with an emission dynamic equation (17) can be used to develop a unified dynamical system which characterize, participation, compliance and the associated movement of the nitrate pollution stock.

4 Fast - Slow Selection Dynamics in the Evolution of Public Voluntary Agreements

The purpose of introducing different time scales in the replicator dynamics framework is to capture the fact, observed in real situations, that when a VA of that type is offered, the composition regarding participation is finalized relatively fast. Since farmers have to decide whether to accept the offer within a relatively small time interval, determined by legal procedures, we expect evolutionary pressures to work relatively fast. On the other hand compliance behavior is not constrained by a time framework so we expect evolutionary pressures to operate more slowly relative to the participation case. This implies that the rate of change of x with respect to time is “large” in absolute value, while the rate of change of z is relatively slower. That is, $\left|\frac{dx}{dt}\right| \equiv |\dot{x}| \gg \left|\frac{dz}{dt}\right| \equiv |\dot{z}|$.

³⁰A convex hull of a set A is the smallest convex set containing A . For example, the convex hull of three noncollinear points is the triangle with these points as vertices.

The above argument implies that in (15) and (16) we can set $\alpha\beta = 1$ and $\gamma\delta = \varepsilon$ where ε is a small positive parameter. Assuming that the natural system evolves in a time scale which is comparable to the slow compliance variable, then our dynamic system can be written in a fast time scale as:

$$\frac{dx}{d\tau} = f_1(x, S) \quad (18)$$

$$\frac{dz}{d\tau} = \varepsilon f_2(z, S) \quad (19)$$

$$\frac{dS}{d\tau} = \varepsilon f_3(x, z, S) \quad (20)$$

System (18)-(20) is the fast time system (FTS).³¹ If fast time is scaled such that $\tau = t/\varepsilon$, so that $d\tau = dt/\varepsilon$ then the dynamics system characterizing participation, compliance and nitrate pollution accumulation in slow time can be written:

$$\varepsilon \dot{x} = f_1(x, S) \quad (21)$$

$$\dot{z} = f_2(z, S) \quad (22)$$

$$\dot{S} = f_3(x, z, S) \quad (23)$$

The problem defined in the dynamical system (21)-(23) is a *singular perturbation* problem.³² The general method for analyzing it, is to consider the systems at the limit $\varepsilon \rightarrow 0$. If the solutions satisfy certain regularity conditions for $\varepsilon = 0$, then solutions for small ε can be approximated by the solutions for $\varepsilon = 0$. By taking $\varepsilon = 0$ in system (21)-(23) we obtain the *reduced* system, or else known as slow-time system (STS), where the equation $0 = f_1(x, S)$ provides, if it can be solved for x , the equilibrium participation rate for fixed level of S , as:

$$x = h(S) \quad (24)$$

The solutions of (24) are equilibria of the FTS (18)-(20), defined for $\varepsilon \rightarrow 0$ and denoted by $h_j(S)$, $j = 1, \dots, J$.³³ For the stable equilibria from the set

³¹Where f_i , $i = 1, 2, 3$ represent the right hand sides of (15), (16) and (17) respectively.

³²For the analysis of problems in a fast-slow time framework see, for example, Wasow (1965, Chapter X) or Sastry (1999, Chapter 6).

³³Where J is the number of these equilibria.

of equilibria of (24), the slow variables evolve as:

$$\dot{z} = f_2(z, S) \quad (25)$$

$$\dot{S} = f_3(h(S), z, S) \quad (26)$$

The analysis of the dynamic system (25) and (26) can be used to determine the long-run ES compliance and nitrate pollution stock (z^*, S^*) . Then the long-run ES participation in the VA will be determined as $h(z^*, S^*)$.³⁴

5 Long-Run Structure for a Public VA

The conceptual framework developed above is used to determine the long-run structure regarding participation in and compliance with a public VA. Since the long-run structure is determined as a stable equilibrium of the replicator dynamics equation, it has the property of a stable EE. To illustrate the importance of the structure of legislation and auditing probabilities in determining the long-run structure for the public VA, we classify the following analysis according to the characteristics of these probabilities.

5.1 Participation Decision and Evolutionary Participation Equilibria

The decision regarding participation in a public VA is reached faster and it is affected by the structure of the subjective probability of introducing legislation. The legislation probability can either depend on the nitrate pollution stock solely or depend jointly on nitrate pollution stock and proportion of participating farmers.

5.1.1 nitrate pollution Stock Dependence of the Legislation Probability

Assume that the subjective probability of introducing legislation depends only on the nitrate pollution deviation ΔS . In the fast time participation system (FTPS) the observed emission stock S and the deviation ΔS , are

³⁴In more technical terminology the dynamic system (25) and (26) is defined on the stable two-dimensional manifold (or union of) $M = \left\{ (z, S, x) : g(x, z, S) = 0 : x_j^F(z, S) \right\}$. Solutions of the slow system (21)-(23) at least locally are attracted to this manifold.

both regarded as fixed and parameters. As a consequence the legislation probability is fixed, implying that $p = p(\overline{\Delta S})$.

Under this definition the slow time compliance nitrate pollution system (STCPS), is defined as:

$$0 = x(1-x) [p(\overline{\Delta S})\Delta\Pi_L^N - \Delta\Pi_v^N] \quad (27)$$

$$\dot{z} = z(1-z) [q(z, \Delta S, \omega_c) \Delta\Pi_C^N - \Delta\Pi_v^N] \quad (28)$$

$$\dot{S} = n\{x[ze_v + (1-z)\mathcal{E}e_L(q(z, \Delta S, \omega_c))] + (1-x)e_o\} - bS \quad (29)$$

The solution of the replicator dynamics equation (27) provides the S-S participation rates x^* , for fixed level of S , which correspond to an EE. Two S-S exist, $x_1^* = 1$ and $x_2^* = 0$, implying either full or non-participation in the public VA.

The derivative of (27) with respect to x defines the stability condition:

$$\frac{d\dot{x}}{dx} = (1-2x)\Omega \quad (30)$$

where $\Omega = [p(\overline{\Delta S})\Delta\Pi_L^N - \Delta\Pi_v^N]$. There is a critical probability value, defined as $\hat{p}(\overline{\Delta S})$, that sets $\Omega = 0$ and behaves as a bifurcation parameter.³⁵ The sign of the expression Ω , and therefore the stability of the steady states, depend on the magnitude of the fixed legislative probability $p(\overline{\Delta S})$ relative to the critical value $\hat{p}(\overline{\Delta S})$. Specifically, if the regulator announces and commits to a legislative probability higher than the critical value, then $\Omega > 0$. On the other hand, if $p(\overline{\Delta S}) < \hat{p}(\overline{\Delta S})$, then $\Omega < 0$.

Under this definition it follows that:

$$\begin{aligned} \text{If } p(\overline{\Delta S}) > \hat{p}(\overline{\Delta S}) \text{ then } \frac{d\dot{x}}{dx}\Big|_{x_1^*=1} < 0 \text{ and } \frac{d\dot{x}}{dx}\Big|_{x_2^*=0} > 0 \\ \text{If } p(\overline{\Delta S}) < \hat{p}(\overline{\Delta S}) \text{ then } \frac{d\dot{x}}{dx}\Big|_{x_1^*=1} > 0 \text{ and } \frac{d\dot{x}}{dx}\Big|_{x_2^*=0} < 0 \end{aligned}$$

In the first case, farmers perceive that the introduction of the legislation is highly likely. Therefore farmers prefer the profit loss $\Delta\Pi_v^N$ under the public VA to the higher profit losses $\Delta\Pi_L^N$, realized if legislation is imposed. Consequently, all farmers participate in the public VA and $x_1^* = 1$ is GAS. Furthermore the ambient nitrate pollution stock is equal to the industrial emission target E_v . In the second case, the legislation mandate appears less likely and farmers can maintain the unregulated profits $\Pi_N(e_o)$. Therefore

³⁵ Where $\hat{p}(\overline{\Delta S}) = \frac{\Pi_N(e_o) - \Pi_v(e_v)}{\Pi_N(e_o) - \Pi_L(e_L)} < 1$, since $\Pi_N(e_o) - \Pi_v(e_v) < \Pi_N(e_o) - \Pi_L(e_L)$.

no farmer has the incentive to participate in the public VA and receive reduced profits by $\Delta\Pi_v^N$, so $x_2^* = 0$ is GAS.

These findings can be summarized in the following proposition:

Proposition 1 *Under an emission stock dependent legislative probability the FTPS converges to a monomorphic equilibrium. If $p(\overline{\Delta S}) \in (\hat{p}(\overline{\Delta S}), 1]$, then there is full participation in the public VA and $x_1^* = 1$ is the GAS evolutionary equilibrium. If $p(\overline{\Delta S}) \in [0, \hat{p}(\overline{\Delta S}))$, then there is non-participation in the public VA and $x_2^* = 0$ is the GAS evolutionary equilibrium. By the strong stability property of the GAS steady states, participation or non participation are ES strategies, for the appropriate value of the subjective legislation probability.*

>From the total differential of $\Omega = 0$ we obtain,

$$\frac{d\hat{p}(\overline{\Delta S})}{de_L} = \frac{\hat{p}(\Delta S)\Pi'_L(e_L)}{\Delta\Pi_L^N} < \mathbf{0} \quad (31)$$

Thus, the higher the legislative emission e_L ³⁶ is set by the regulator, the lower is the critical probability value $\hat{p}(\overline{\Delta S})$. There is a trade-off between the announced legislative set emission level and the commitment to a given legislation probability value. Through a stricter legislation the range of legislation probability values that induce participation becomes wider, allowing the regulator to achieve the stable EE outcome by committing to a lower legislation probability.

5.1.2 nitrate pollution Stock and Participation Dependence of the Legislation Probability

Assume that the subjective probability of introducing legislation depends jointly on nitrate pollution deviation ΔS and the participation proportion x . Under $p = p(\Delta S, x)$ and (11) the STCPS is defined as:

$$0 = x(1-x) [p(\Delta S, x)\Delta\Pi_L^N - \Delta\Pi_v^N] \quad (32)$$

$$\dot{z} = z(1-z) [q(\cdot)\Delta\Pi_C^N - \Delta\Pi_v^N] \quad (33)$$

$$\dot{S} = n\{x(\Delta S)[ze_v + (1-z)\mathcal{E}e_L(q(\cdot))] + (1-x(\Delta S))e_o\} - bS \quad (34)$$

³⁶As noted above, a target e_L can be attained either through emissions taxes or emission limits. From our assumptions it follows that $\Pi'_L(e_L) < 0$.

The fast time dynamic equation (32) defines two monomorphic S-S: $x_1^* = 1$ and $x_2^* = 0$, as well a polymorphic critical point $x_3^*(\Delta S) \in (0, 1)$ that is defined by $\Omega = [p(\Delta S, x_3^*)\Delta\Pi_L^N - \Delta\Pi_v^N] = 0$. The stability condition for these S-S is given by:

$$\frac{d\dot{x}}{dx} = (1 - 2x)\Omega + x(1 - x)p'(\Delta S, x)\Delta\Pi_L^N \quad (35)$$

There is a critical probability value $\hat{p}(\Delta S, x_3^*)$ that sets Ω equal to zero and corresponds to the critical point $x_3^*(\Delta S)$. Furthermore if $x < x_3^*$ then $p(\Delta S, x) > \hat{p}(\Delta S, x_3^*)$ and $\Omega > 0$, while if $x > x_3^*$ then $p(\Delta S, x) < \hat{p}(\Delta S, x_3^*)$ and $\Omega < 0$. Then,

$$\left. \frac{d\dot{x}}{dx} \right|_{x_1^*=1}, \left. \frac{d\dot{x}}{dx} \right|_{x_2^*=0} > 0 \text{ and } \left. \frac{d\dot{x}}{dx} \right|_{x_3^*} < 0$$

The FTPS converges to the polymorphic EE x_3^* , implying that only a sub-group of polluting farmers participate in the public VA in the long-run. This happens because, in the case of full participation $p(\Delta S, 1 | \Delta S > 0) = 0$ holds, giving farmers an incentive not to participate in the VA, when participation is already high. On the other hand, $p(\Delta S > 0, 0) \rightarrow 1$, giving farmers an incentive to participate in the agreement when participation is very low.

These findings can be summarized in the following proposition:

Proposition 2 *Under a legislative probability that depends jointly on participation proportion and nitrate pollution stock the participation system converges to a GAS polymorphic EE, implying partial participation to the public VA. Partial participation is an ES strategy.*

Furthermore,

$$\frac{dx}{de_L} = \frac{p(\Delta S, x)\Pi_L'(e_L)}{p'(\Delta S, x)(\Delta\Pi_L^N)} > 0 \quad (36)$$

Under the threat of a stricter legislative regulation, the participating proportion increases, shifting the polymorphic x_3^* steady state upwards, closer to the full participation critical point, $x_1^* = 1$. Therefore through proper design of the legislation mandate the regulator can induce the majority of farmers to participate in the VA.

5.2 Compliance Decisions and Evolutionary Compliance Equilibria

Assume that the regulator has set $p(\overline{\Delta S}) > \hat{p}(\overline{\Delta S})$ and thus the full participation S-S $x_1^* = 1$ is an EE and an ES strategy in the fast time.³⁷ We examine now the second level of decision, which is to comply or not with the VA. Substituting the GAS steady state, $x_1^* = 1$, the slow-time system is defined in general terms as:

$$\dot{z} = z(1-z) [q(z, \Delta S, \omega_c) \Delta \Pi_C^N - \Delta \Pi_v^N] \quad (37)$$

$$\dot{S} = n\{ze_v + (1-z)\mathcal{E}e_L(q(z, \Delta S, \omega_c))\} - bS \quad (38)$$

The system has a hierarchical structure, if the audit probability q is independent of the nitrate pollution stock S , implying that the S-S and the stability properties of the replicator dynamics (37) can be determined first and then used to determine the nitrate pollution stock S-S of equation (38). In the following we examine how alternative assumptions about the structure of the auditing probability affect the compliance EE and nitrate pollution stock, given the full participation decisions.

5.2.1 Fixed Auditing Probability

Assume that the regulator is committed to a fixed auditing probability. Participating farmers know exactly the probability \bar{q} under which they may experience profit losses $\Delta \Pi_C^N$, if caught violating the agreement. Based on this knowledge they choose their evolutionary strategy of whether or not to comply.

Under this assumption there are two monomorphic S-S of the replicator dynamic satisfying the equilibrium condition $\dot{z} = 0$ of the (37), implying either full compliance $z_1^* = 1$ or non-compliance $z_2^* = 0$ with the agreement. The stability condition is determined by:

$$\frac{dz}{dz} = (1-2z)\Phi \quad (39)$$

where $\Phi = [\bar{q}\Delta \Pi_C^N - \Delta \Pi_v^N]$. There is a critical probability value $\hat{\bar{q}}$ that sets

³⁷It makes no sense to examine the S-S when $x_2^* = 0$, is the ES strategy since non-participating firms are not expected to do "self-regulation".

$\Phi = 0$. In particular, if $\bar{q} > \hat{\hat{q}}$ then $\Phi > 0$ and if $\bar{q} < \hat{\hat{q}}$ then $\Phi < 0$. Thus the stability conditions of the replicator dynamic becomes:

$$\begin{aligned} \text{If } \bar{q} > \hat{\hat{q}} \text{ then } \frac{d\dot{z}}{dz} \Big|_{z_1^*=1} < 0 \text{ and } \frac{d\dot{z}}{dz} \Big|_{z_2^*=0} > 0 \\ \text{If } \bar{q} < \hat{\hat{q}} \text{ then } \frac{d\dot{z}}{dz} \Big|_{z_1^*=1} > 0 \text{ and } \frac{d\dot{z}}{dz} \Big|_{z_2^*=0} < 0 \end{aligned}$$

The $\dot{S} = 0$ isocline defines the corresponding nitrate pollution stock equilibrium. If $\bar{q} > \hat{\hat{q}}$ then full compliance, $z_1^* = 1$, is the EE with $S_1^* = \frac{ne_v}{b}$, which is the desired nitrate pollution stock level. If $\bar{q} < \hat{\hat{q}}$ then non compliance, $z_2^* = 0$, is EE with $S_2^* = \frac{n\mathcal{E}e_L(\bar{q})}{b} > S_1^*$. In this case the nitrate pollution stock dynamic isocline $\dot{S} = 0$ is a linear curve with negative slope defined as:

$$z(S) = \frac{bS}{n\{e_v - \mathcal{E}e_L(\bar{q})\}} - \frac{\mathcal{E}e_L(\bar{q})}{\{e_v - \mathcal{E}e_L(\bar{q})\}} = AS - B$$

where $A < 0$ and $B < 0$. The STCPS converges to a GAS monomorphic S-S.³⁸

The above conclusions can be summarized in the following proposition:

Proposition 3 *Under a fixed auditing probability the compliance-nitrate pollution system converges to a GAS monomorphic S-S. If $\bar{q} \in (\hat{\hat{q}}, 1)$ then there is full compliance with the public VA and the S-S $z_1^* = 1$ is the ES strategy. If $\bar{q} \in [0, \hat{\hat{q}})$ then there is non-compliance with the public VA and the S-S $z_2^* = 0$ is the ES strategy.*

Furthermore,

$$\frac{d\hat{\hat{q}}}{dF} = \frac{\hat{\hat{q}}\Pi'_C(e_L, F)}{\Delta\Pi_C^N} < 0 \quad (40)$$

The higher the non-compliance fine is, the lower the critical probability value $\hat{\hat{q}}$ is.³⁹ Thus, by an appropriate choice of the fine, and provided that this choice is politically feasible, the regulator can lower the number of random inspections and achieve full compliance, as well as the desired goal E_v , with less monitoring expenses.

³⁸For details see the Appendix.

³⁹This is the evolutionary analogue to Franckx (2002) result, which indicates that the only role the fine plays is that when it increases the equilibrium inspection probability is reduced.

5.2.2 Compliance Dependent Auditing Probability

Under an auditing probability which is dependent on the fraction of the complying farmers, defined as $q(z)$, $z_1^* = 1$ and $z_2^* = 0$ are EE for (37). Furthermore an additional EE $z_3^* \in (0, 1)$ may exist, which also satisfies the equilibrium condition $\dot{z} = 0$. This S-S defines a critical probability value $\hat{q}(z_3^*)$ such that $\Phi = [\hat{q}(z_3^*)\Delta\Pi_C^N - \Delta\Pi_v^N] = 0$.

In this case the stability condition is defined as:

$$\frac{d\dot{z}}{dz} = (1 - 2z)\Phi + z(1 - z)q'(z) (\Delta\Pi_C^N) \quad (41)$$

Due to (10), it holds that $\Phi > 0$ for $z < z_3^*$ since $q(z) > \hat{q}(z_3^*)$ and $\Phi < 0$ for $z > z_3^*$ since $q(z) < \hat{q}(z_3^*)$. It can be easily seen that the monomorphic EE of the replicator dynamic are not asymptotically stable since:

$$\left. \frac{d\dot{z}}{dz} \right|_{z_1^*=1} = -\Phi > 0 \quad \text{and} \quad \left. \frac{d\dot{z}}{dz} \right|_{z_2^*=0} = \Phi > 0$$

Under full compliance the regulator may respond with a reduced or even zero number of random inspections, due to condition (10). This gives participating farmers an incentive to violate the agreement. On the other hand, under non compliance the value of $q(0)$ is sufficiently high, this gives participating farmers an incentive to comply with the agreement's provisions. In this case since,

$$\left. \frac{d\dot{z}}{dz} \right|_{z_3^*} = z_3^*(1 - z_3^*) q'(z_3^*) \Delta\Pi_C^N < 0$$

the replicator dynamic converges to a polymorphic EE, implying that only a sub-group of participating farmers complies with the public VA.

For the $\dot{S} = 0$ isocline we have that $S_1^* = \frac{ne_v}{b} < S_2^* = \frac{n\mathcal{E}e_L(q(0))}{b}$, evaluated at $z_1^* = 1$ and $z_2^* = 0$ respectively, with

$$\frac{dz}{dS} = b / (n\{e_v - \mathcal{E}e_L(q(z)) + (1 - z) (\partial q(z)/\partial z) (e_L - e_o)\})$$

Thus in general the $\dot{S} = 0$ isocline defines a non linear relationship on the (z, S) space, which could be monotonically decreasing, or having decreasing and increasing parts (See Figure 1)⁴⁰. As shown in the Appendix the S-S

⁴⁰Moreover, it is clear that $\left. \frac{dz}{dS} \right|_{z_1^*=1} = \frac{b}{n\{e_v - e_o\}} < 0$, while $\left. \frac{dz}{dS} \right|_{z_2^*=0} =$

defined by the intersection of $z = z_3^*$ and the $\dot{S} = 0$ isocline, with $S_3^* = n[z_3^*e_v + (1 - z_3^*)\mathcal{E}e_L(q(z_3^*))]/b$ is unique and GAS in the interval $(0, 1)$, with monotonic or oscillating approach dynamics. Therefore, in this case the following proposition holds:

Proposition 4 *Under a compliance dependent auditing probability, partial compliance to the public VA is the ES strategy.*

Furthermore,

$$\frac{dz_3^*}{dF} = -\frac{q(z_3^*)}{q'(z_3^*)\Delta\Pi_C^N} > 0 \quad (42)$$

Therefore increasing the fine, increases the equilibrium compliance proportion and shifts the polymorphic steady state upwards, closer to the full compliance critical point. So under the appropriate adjustments of the fine, compliance in the left side neighborhood of $z_1^* = 1$ is a GAS evolutionary equilibrium.

[Figure 1]

5.2.3 Emission Stock Dependent Auditing Probability

Assume that the auditing probability depends on the deviation from the established environmental goal. In the slow time scale the observed emission stock and thus the auditing probability $q(\Delta S)$ are no longer fixed. In this case the equilibrium condition $\dot{z} = 0$ for the replicator dynamic equation (37) defines the two monomorphic S-S $z_1^* = 1$ and $z_2^* = 0$, as well as a potential third one $z_3^* \in (0, 1)$, determined by a critical emission stock level \hat{S} , such that $\Phi = [q(\Delta\hat{S})\Delta\Pi_C^N - \Delta\Pi_v^N] = 0$.

The type of the evolutionary equilibrium for the STCPS depends on the relationship between the critical emission stock level \hat{S} , the full compliance emission stock level S_1^* and the non-compliance stock level S_2^* . In this case the nitrate pollution stock isocline $\dot{S} = 0$ is a non-linear, monotonic curve with negative slope,⁴¹ while the critical emission stock level \hat{S} corresponding to $\dot{z} = 0$ is a vertical line in the (z, S) space. If $S_1^* > \hat{S} > S_2^*$ then the

$\frac{b}{n\{e_L - e_o + \frac{\partial q(z)}{\partial z}(e_L - e_o)\}} \leq 0$, supporting the potential existence of decreasing and increasing parts for the $\dot{S} = 0$ isocline.

⁴¹The slope of the pollution stock isocline $\dot{S} = 0$ is clearly negatively defined and equal to $\frac{dz}{dS} = \frac{b-(1-z)\frac{\partial q(\Delta S)}{\partial S}(e_L - e_o)}{n\{e_v - \mathcal{E}e_L(q(\Delta S))\}} < 0$.

intersection of \hat{S} with $\dot{z} = 0$ corresponds to $z_3^* \in (0, 1)$ and the STCPS has three isolated S-S, while if $\hat{S} < S_1^* < S_2^*$ or $\hat{S} > S_2^* > S_1^*$ the STCPS has two isolated S-S (see Figure 2a). The properties of these EE are summarized in the following proposition:

Proposition 5 *Under an emission stock dependent auditing probability the CPSTS could converge to either a polymorphic or monomorphic compliance evolutionary equilibrium. If $\hat{S} < S_1^* < S_2^*$ then there is a GAS full compliance EE, and $z_1^* = 1$ is the ES strategy. If $S_1^* > \hat{S} > S_2^*$ then there is a GAS partial compliance EE and $z_3^* \in (0, 1)$ is the ES strategy with oscillating approach dynamics. If $\hat{S} > S_1^* > S_2^*$ then there is a GAS no compliance EE and $z_2^* = 0$ is the ES strategy.*

For proof see Appendix

Furthermore since

$$\frac{d\hat{S}}{dF} = -\frac{q(\Delta S)}{q'(\Delta S)(\Delta \Pi_C^N)} < 0 \quad (43)$$

the critical emission stock level declines with the level of the fine and the vertical isocline moves closer to the full compliance emission stock level in figure 2b. Moreover the polymorphic equilibrium point moves closer to the monomorphic steady state point A, implying that with the proper design of the non-compliance fine the regulator can induce a larger share of participating farmers to comply.

[Figure 2]

5.2.4 Joint Dependence of Auditing Probability on Compliance and nitrate pollution Stock

Assume that the auditing probability depends jointly on nitrate pollution stock and the proportion of complying farmers. Under $q = q(\Delta S, z)$ the condition $\dot{z} = 0$ for (37) defines two equilibria, $z_1^* = 1$ and $z_2^* = 0$, and a possible third one which is implicitly defined by an isocline $l(S)$ with the property:

$$z = l(S) : \Phi = [q(\Delta S, z)(\Delta \Pi_C^N) - (\Delta \Pi_v^N)] = 0$$

and slope $\frac{dz}{dS} = -\frac{\partial q(\Delta S, z)/\partial S}{\partial q(\Delta S, z)/\partial z} > 0$, that reflect the farmers' beliefs about the variability of the auditing probability value due to changes in the levels of

the state variables S and z . If participating farmers perceive that changes in the nitrate pollution stock can not affect the auditing probability value, then $\partial q(\Delta S, z)/\partial S = 0$ and the auditing probability depends only on the compliance proportion and the isocline is parallel to the horizontal axis as in case 5.2.2. If participating farmers perceive that $\partial q(\Delta S, z)/\partial z = 0$, then the auditing probability depends only on the nitrate pollution stock and the isocline is vertical to the horizontal axis as in case 5.2.3. Thus the case of joint dependency of the auditing probability on S and z is a generalization of the two previous cases. It can be shown that the results are similar to the more specific cases above and can be summarized in the following proposition:

Proposition 6 *Under an auditing probability with joint dependence on compliance levels and nitrate pollution stock, the evolutionary equilibrium of the CPSTS regarding compliance could be monomorphic, or polymorphic with possible multiple steady states. The type of the prevailing EE depends mainly on the slope and position of the $z = l(S)$ isocline. The flatter the isocline is, the more likely it is that the EE equilibrium implies partial compliance which is the ES strategy in the case of a unique GAS evolutionary equilibrium. The more steeper the isocline is the more likely it is that EE equilibrium implies full compliance as the ES strategy*

For details see Appendix and Figures 3 and 4.

The EE outcome can be further affected through the non-compliance fine, since it determines the position of the isocline $z(S)$. The higher the fine is, the more participating farmers tend to comply with the VA. Consequently the regulator can shift the isocline upwards, bringing the polymorphic equilibrium point closer to the monomorphic steady state, through the announcement of a sufficiently higher fine F .

[Figures 3 and 4]

5.2.5 Compliance Equilibria with a general legislation probability

Under a legislation probability jointly dependent on nitrate pollution deviation ΔS and participation proportion x the EE steady state $x_3^*(\Delta S)$ in the fast time implies partial participation. To analyze the evolution of compliance

and nitrate pollution stock we define the slow time system by substituting $x_3^*(\Delta S)$. Therefore under (11) the CPSTS is:

$$\dot{z} = z(1-z) [q(z, S)(\Delta \Pi_C^N) - (\Delta \Pi_v^N)] \quad (44)$$

$$\dot{S} = n\{x_3^*(\Delta S)[ze_v + (1-z)\mathcal{E}e_L(q(z, S))] + (1-x_3^*(\Delta S))e_o\} - bS \quad (45)$$

Under $p = p(\Delta S, x)$, the nitrate pollution stock isocline $z = k(S) : \dot{S} = 0$ could be monotonic curve with negative slope or have increasing and decreasing parts, depending on the type of the audit probability. In this case the $\dot{S} = 0$ isocline takes the following general form:

$$\psi(z, S) = z - \left(\frac{bS}{nx_3^*(\Delta S)\Delta e_L^v} - \frac{x_3^*(\Delta S)\mathcal{E}e_L(q(\cdot)) + (1-x_3^*(\Delta S))e_o}{x_3^*(\Delta S)\Delta e_L^v} \right) = 0$$

$$e_v - \mathcal{E}e_L(q(z, S)) = \Delta e_L^v$$

As previously the EE (z^*, S^*) of the CPSTS is highly dependent on the structure of the auditing probability q . Based on the conceptual framework developed in the previous section we conclude that:

Proposition 7 *Under a participation and nitrate pollution stock dependent legislation probability and a fixed or state variable dependent auditing probability, the EE equilibrium implies partial participation in the public VA and full, non or partial compliance of the participating subgroup of farmers.*

The CPSTS could be either characterized by a unique equilibrium or multiple equilibria and irreversibilities, with the final outcome crucially depending on initial conditions.⁴²

6 The Impact of Auditing Costs and Budget Constraint on Evolutionary Equilibria

In this section we explicitly introduce a budget constraint that determines a maximum number of inspections. We assume that the available budget for auditing is exhausted in each period and that it consists of two components. An amount K exogenously determined by the regulator and the

⁴²Only in the case of a fixed audit probability the $\dot{S} = 0$ isocline is clearly a monotonic curve with negative slope and the CPSTS has a unique EE.

sum of noncompliance fines F collected from participating farmers found in non-compliance after a random inspection.⁴³ Thus the budget is partially fine financed and is determined endogenously. Particularly under (11) the flexible budget the period t can be defined as:

$$B_t = K_t + q(z, \Delta S, \omega_c)(1 - z)F$$

The number of realized audits and therefore the auditing probability are dependent on the available budget of the regulatory body, implying that the auditing probability is endogenous to the budget. An increase in the budget allows the a higher number of inspections and increases the auditing probability. Thus a more general formulation for the subjective audit probability (11) can be written as:

$$q = q(z, \Delta S, B, \omega_c) \quad (46)$$

where $\frac{\partial q}{\partial B} > 0$ with $\frac{\partial^2 q}{\partial B^2} < 0$ and $0 < \frac{\partial q}{\partial B} < 1$. Moreover, we assume that $q = q(z, \Delta S, 0, \omega_c) = 0$, implying that no inspection can be conducted if monitoring expenses cannot be covered.⁴⁴

Under this definition the available budget can take the general form:

$$B_t = K_t + q(z, \Delta S, B, \omega_c)(1 - z)F \quad (47)$$

$$\text{or } B_t = B(K, S, z, F) \quad (48)$$

In this case even if the auditing probability is regarded as independent of z ,⁴⁵ it is eventually dependent on the compliance proportion, through the sum of collected fines, since it is defined as $\bar{q} = q(B)$ with $B_t = B(K, z, F)$.⁴⁶ After taking the total derivative of (47) the relationship between the budget and the following variables: K , S , z and F is determined. It holds that $\frac{\partial B}{\partial K}$, $\frac{\partial B}{\partial F}$ and $\frac{\partial B}{\partial S} > 0$ while $\frac{\partial B}{\partial z} < 0$, denoting that the available budget increases either as K , F or ΔS increases and decreases as z increases.

Under the budget constraint and the general definition of the legislation

⁴³The noncompliance fine F is assumed to be fixed even though it could depend on compliance proportion and/or the pollution stock.

⁴⁴No assumption is made about covering the monitoring costs in the following period.

⁴⁵This corresponds to the case \bar{q} , as developed in the previous sections.

⁴⁶The same holds under $q = q(\Delta S, B(K, S, z, F))$. In this case the CPSDS behaves as in the $q(z, \Delta S)$ case.

and audit probability, (2) and (??), the CPSTS is defined as:

$$0 = x(1-x) [p(\Delta S, x)(\Delta S, x)\Delta\Pi_L^N - \Delta\Pi_v^N] \quad (49)$$

$$\dot{z} = z(1-z) [q(\cdot)\Delta\Pi_C^N - \Delta\Pi_v^N] \quad (50)$$

$$\dot{S} = n\{x(\Delta S)[ze_v + (1-z)\mathcal{E}e_L(q(\cdot))] + (1-x(\Delta S))e_o\} - bS \quad (51)$$

Thus, the CPSTS behaves as in the case where $q = q(z)$, or $q = q(z, \Delta S)$. Based on the conceptual framework developed in the previous sections we conclude that under a flexible partially fine financed budget constraint, the behavior of the system is similar to the behavior under state dependent auditing probabilities.

The most notable difference between the present and previous CPSTS is that under the flexible budget described above, there can be no commitment to a fixed auditing probability and polymorphic EE are expected instead of monomorphic. Of course one component of the budget is exogenous then commitment to a certain fixed amount K is equivalent with commitment to a certain fixed auditing probability

To explore the impact of the exogenously amount K on the critical auditing probability $\hat{q}(B(K, z_3^*, F))$ and compliance fraction z of the GAS polymorphic EE described in section 5.2.1 and 5.2.2 respectively, we consider the following derivatives.

$$\frac{d\hat{q}(B(K, z_3^*, F))}{dK} = -\frac{\partial\hat{q}(B(K, z_3^*, F))}{\partial B} \frac{\partial B}{\partial K} < 0 \quad \text{and} \quad \frac{dz}{dK} = -\frac{\partial B/\partial K}{\partial B/\partial z} > 0$$

It follows that as the amount K increases, the critical audit probability value decreases and the compliance proportion increases. Specifically, the second derivative implies that the as the amount K increases, the number of financial feasible inspections increases, inducing more participating farmers to comply with the VA at the equilibrium. Under these circumstances the polymorphic steady state z_3^* shifts upwards, closer to the full compliance critical point. However, full compliance can not be achieved given the repelling property of the full compliance S-S.

7 Concluding Remarks

The purpose of this paper is to analyze the long-run structure of a public VA regarding participation and compliance of farmers and to specify certain characteristics that a VA should possess in order to induce the majority of or even all polluting farmers to participate in and comply with the VA. In this context we examine the evolution of participation in and compliance with the public VA, along with the evolution of nitrate pollution stock. Individual polluting farmers' decisions about whether or not to participate in and comply with the VA were based on the evolutionary processes of comparing expected profits associated with the different decisions, and were modelled by replicator dynamics operating in fast and slow time scales.

The main finding is that the structure of the legislation and auditing probability, the levels of legislative emissions and non-compliance fines are the main factors characterizing the evolutionary equilibria and evolutionary stable strategies. If the legislation probability is fixed in fast time, and is set higher than a critical value, then the equilibrium outcome is monomorphic implying that all the farmers participate in the agreement. On the other hand, if the legislation probability depends jointly on emission stock and participation proportion, the evolutionary equilibrium is polymorphic, implying partial participation. In this case the regulator can lead the equilibrium outcome sufficiently to full participation through the proper design of the legislation mandate and particularly through the magnitude of the legislative emissions e_L .

By committing to a fixed auditing probability, higher than a critical value, the regulator can achieve full compliance of participating farmers. The same outcome can be achieved under certain initial conditions when the auditing probability depends on the nitrate pollution stock and the complying proportion. In this case however the dynamic system describing the evolution of compliance and the nitrate pollution stock can alternatively converge to a partial compliance steady state, either monotonically or oscillating. Under certain conditions the compliance-nitrate pollution stock system is characterized by multiple equilibria and irreversibilities. The introduction of a budget constraint in covering monitoring costs, with partial financing through the collection of fines, leads the compliance-nitrate pollution stock system to a polymorphic evolutionary equilibrium, implying

partial compliance of participating farmers to the agreement's provisions.

In conclusion, the more complex the structure of the legislation and audit probability is, the more likely is that the evolutionary equilibrium is polymorphic, and depends largely on the initial conditions. With no binding budget constraint regarding monitoring costs, commitment to legislation and auditing probabilities along with properly chosen legislative mandate and non compliance fines can induce full participation and compliance with the public VA. If these conditions are not fulfilled or the available budget is limited then partial participation, partial compliance with multiple equilibria and irreversibilities and even fluctuation in the nitrate pollution stock are possible evolutionary outcomes.

The present paper developed a general framework for analyzing participation in and compliance with voluntary environmental agreements. In this generalized context, the results obtained in this paper might provide some insights related to the expected efficiency and the long run outcome regarding participation in and compliance with the EU Nitrate Directive (91/676/EEC). It seems that the voluntary attainment of the Directive's target depends on the existence of a credible threat of mandatory regulation which would imply implementation of action programmes and extension of vulnerable zones. Crucial to the attainment of compliance is also, according to our results, the existence of some non-compliance penalty, or some mechanism that will effectively decrease the profits of noncomplying farmers. More precise analysis and prediction regarding the long-run impacts of the EU Nitrate Directive requires a model that is more specific to the Directive's structure and incorporates the principle of the cross-compliance of aid. This is the following step of ongoing research which would entail a more specific tailoring to the Directive's structure, of the general concepts and mechanism developed in this paper

Appendix

In order to characterize the way the STCPS converges to the equilibrium the linearization matrixes J around the S-Ss are defined along with their traces $Tr(J) = \frac{dz}{dz} + \frac{d\dot{S}}{dS}$, determinants $Det(J) = \frac{dz}{dz} \frac{d\dot{S}}{dS} - \frac{dz}{dS} \frac{d\dot{S}}{dz}$ and discriminants $\Delta = [Tr(J)]^2 - 4Det(J)$.

Fixed Auditing Probability

The STCPS is defined as:

$$\begin{aligned}\dot{z} &= z(1-z) [\bar{q}\Delta\Pi_C^N - \Delta\Pi_v^N] \\ \dot{S} &= n\{ze_v + (1-z)\mathcal{E}e_L(\bar{q})\} - bS\end{aligned}$$

where $\Phi = \bar{q}\Delta\Pi_C^N - \Delta\Pi_v^N$ and $\mathcal{E}e_L(\bar{q}) = \bar{q}e_L + (1-\bar{q})e_o$, and the linearization matrix J is:

$$J = \begin{bmatrix} (1-2z)\Phi & 0 \\ n\{e_v - \mathcal{E}e_L(\bar{q})\} & -b \end{bmatrix}$$

with $\frac{d\dot{S}}{dS} < 0$ and $\frac{dz}{dz} < 0$. If $\bar{q} > \hat{\bar{q}}$ then $\frac{dz}{dz} < 0$ for the full compliance critical point z_1^* . Therefore $Tr(J) < 0$, while $Det(J) > 0$. The discriminant $\Delta \gtrless 0$, thus the critical point z_1^* can be a stable focus or node. The same conclusion holds for the non-compliance critical point z_2^* if $\bar{q} < \hat{\bar{q}}$. \square

Compliance Dependent Auditing Probability

The STCPS is defined as:

$$\begin{aligned}\dot{z} &= z(1-z) [q(z)\Delta\Pi_C^N - \Delta\Pi_v^N] \\ \dot{S} &= n\{ze_v + (1-z)\mathcal{E}e_L(q(z))\} - bS\end{aligned}$$

where $\Phi = q(z)\Delta\Pi_C^N - \Delta\Pi_v^N$ and $\mathcal{E}e_L(q(z)) = q(z)e_L + (1-q(z))e_o$ and the linearization matrix J is:

$$J = \begin{bmatrix} (1-2z)\Phi + z(1-z)\frac{\partial q(z)}{\partial z}\Delta\Pi_C^N & 0 \\ n\{e_v - \mathcal{E}e_L(q(z)) + (1-z)\frac{\partial q(z)}{\partial z}(e_L - e_o)\} & -b \end{bmatrix}$$

For $z_1^* = 1$, $Tr(J) \leq 0$, $Det(J) < 0$, for $z_2^* = 0$, $Tr(J) \leq 0$, $Det(J) < 0$, for $z^* = z_3^* \in (0,1)$, $Tr(J) < 0$, $Det(J) > 0$. Thus z_1^* , z_2^* correspond to unstable S-S since the matrix J has at least one positive eigenvalue. Then the S-S corresponding to z_3^* is GAS. Since the determinant of the

linearization matrix does change sign in the interval $(0, 1)$ the S-S (z_3^*, S_3^*) is unique by the index theorem. Since the discriminant $\Delta \stackrel{\geq}{=} 0$ the partial compliance S-S (z_3^*, S_3^*) can either be a stable focus, a stable node. \square

Emission Stock Dependent Auditing Probability

The CPSTS is defined as:

$$\begin{aligned} \dot{z} &= z(1-z) [q(\Delta S)\Delta\Pi_C^N - \Delta\Pi_v^N] \\ \dot{S} &= n\{ze_v + (1-z)\mathcal{E}e_L(q(\Delta S))\} - bS \end{aligned}$$

where $\Phi = q(\Delta S)\Delta\Pi_C^N - \Delta\Pi_v^N$ and $\mathcal{E}e_L(q(\Delta S)) = q(z)e_L + (1-q(z))e_o$, and the linearization matrix J is:

$$J = \begin{bmatrix} (1-2z)\Phi & z(1-z)\frac{dq(\cdot)}{dS}\Delta\Pi_C^N \\ n\{e_v - \mathcal{E}e_L(q(\Delta S))\} & n(1-z)\frac{dq(\cdot)}{dS}(e_L - e_o) - b \end{bmatrix}$$

with $\frac{d\hat{S}}{dS}, \frac{d\hat{z}}{dz} < 0$ and $\frac{d\hat{z}}{dS} > 0$ for $z \in (0, 1)$, $q(\Delta\hat{S})\Delta\Pi_C^N - \Delta\Pi_v^N = 0$. We examine the linearization matrix at the following cases:

1. $z_1^* = 1, \hat{S} < S_1^* < S_2^*$, the CPSTS has two isolated S-S. Furthermore $q(\Delta\hat{S}) < q(\Delta S_1^*) \implies \Phi > 0$. then $Tr(J) < 0$ and $Det(J) > 0$. The S-S $z_1^* = 1$ with $S_1^* = \frac{ne_v}{b}$ is GAS. The S-S with $z_2^* = 0$ is not stable since $Det(J) < 0$.
2. $z_3^* \in (0, 1), S_1^* > \hat{S} > S_2^*$ the CPSTS has three isolated S-S. The S-Ss with $z_1^* = 1, z_2^* = 0$ are not stable since for $q(\Delta\hat{S}) > q(\Delta S_1^*) \implies \Phi < 0$, and for $q(\Delta\hat{S}) < q(\Delta S_2^*) \implies \Phi > 0$, therefore $Det(J) < 0$, at both $z_1^* = 1, z_2^* = 0$. For the point $z_3^* \in (0, 1)$ we have $\Phi = 0$ and $Tr(J) < 0, Det(J) > 0$. Therefore the S-S at z_3^* is GAS in for $z \in (0, 1)$. Qualitative analysis of the phase diagram in Figure 2b suggests that z_3^* is a stable focus, implying that compliance and the nitrate pollution stock fluctuate with the nitrate pollution stock converging to $S_3^* = \frac{n\{z_3^*e_v + (1-z_3^*)\mathcal{E}e_L(q_3(\Delta S))\}}{b}$.
3. $z_2^* = 0, \hat{S} > S_2^* > S_1^*$, the CPSTS has two isolated S-S. Furthermore $q(\Delta\hat{S}) > q(\Delta S_2^*) \implies \Phi < 0$. then $Tr(J) < 0$ and $Det(J) > 0$. The S-S $z_2^* = 0$ with $S_2^* = \frac{n\mathcal{E}e_L(q_2(\Delta S))}{b}$ is GAS. The S-S with $z_1^* = 1$ is not stable since $Det(J) < 0$. \square

Joint Dependence of Auditing Probability on Compliance and nitrate pollution Stock

The CPSTS is defined as:

$$\begin{aligned}\dot{z} &= z(1-z) [q(\Delta S, z)\Delta\Pi_C^N - \Delta\Pi_v^N] \\ \dot{S} &= n\{ze_v + (1-z)\mathcal{E}e_L(q(z))\} - bS\end{aligned}$$

where $\Phi = q(\Delta S, z)\Delta\Pi_C^N - \Delta\Pi_v^N$, $\mathcal{E}e_L(q(z)) = q(\Delta S, z)e_L + (1-q(\Delta S, z))e_o$, $\Delta e_L^o = (e_L - e_o)$, and the linearization matrix J is:

$$J = \begin{bmatrix} (1-2z)\Phi + z(1-z)\frac{\partial q(\cdot)}{\partial z}\Delta\Pi_C^N & z(1-z)\frac{dq(\cdot)}{dS}\Delta\Pi_C^N \\ n\{e_v - \mathcal{E}e_L(q(z)) + (1-z)\frac{\partial q(\cdot)}{\partial z}\Delta e_L^o\} & n\{e_v - \mathcal{E}e_L(q(z)) + (1-z)\frac{dq(\cdot)}{dS}\Delta e_L^o\} - b \end{bmatrix}$$

with $\frac{dz}{dS} > 0$ and $\frac{d\dot{S}}{dS} < 0$, $\frac{d\dot{S}}{dz} \geq 0$ depending on the assumption made about the slope of the isocline $\dot{S} = 0$. Along the isocline $z = l(S)$ that determines the potential z_3^* S-S, the probabilities $\hat{q}(\Delta S, z)$ satisfy the equality $\Phi = 0$. Every other combination outside the isocline switches the sign of Φ . In particular, since $\frac{\partial q}{\partial z} < 0$ and $\frac{\partial q}{\partial S} > 0$, for combinations located on the right of the isocline we have $q(\Delta S, z) > \hat{q}(\Delta S, z)$ and $\Phi > 0$, while on the left of $l(S)$ we have $\Phi < 0$ since $q(\Delta S, z) < \hat{q}(\Delta S, z)$.

Under a sufficiently vertical isocline $z = l(S)$, so that the intersection of $z = l(S)$ with $\dot{S} = 0$ provides a z_3^* in the non feasible region of $z > 1$ to the left of the $(z_1^* = 1, S_1^*)$ point, the linearization matrix J around the S-S $z_1^* = 1$ gives $Tr(J) < 0$, and $Det(J) > 0$ since $\frac{d\dot{S}}{dz}, \frac{d\dot{S}}{dS} < 0$, and $\frac{dz}{dz} < 0$. The full compliance EE is stable. Since the discriminant $\Delta \leq 0$ the full compliance EE z_1^* can either be a stable focus or a stable node.

Under a flat enough isocline $z = l(S)$ so that the intersection of $z = l(S)$ with $\dot{S} = 0$ provides a $z_3^* \in (0, 1)$ the linearization matrix J around z_3^* has $\frac{dz}{dz} < 0$. In this case $Tr(J) < 0$, while $Det(J) > 0$, if $\frac{d\dot{S}}{dz} < 0$, which means that the $\dot{S} = 0$ isocline has a negative slope in the neighborhood z_3^* . Then the (z_3^*, S_3^*) S-S is an asymptotically stable EE with monotonic or fluctuating approach dynamics. If the S-S is unique in $(0, 1)$ then it is GAS and partial compliance is the ES strategy. If there are more than one S-S in $(0, 1)$, resulting from an $\dot{S} = 0$ isocline with decreasing and increasing parts, we expect locally asymptotically stable and unstable EE. Furthermore if $z_3^* \in (0, 1)$ then the $z_1^* = 1$ and $z_2^* = 0$ S-Ss are not stable. \square

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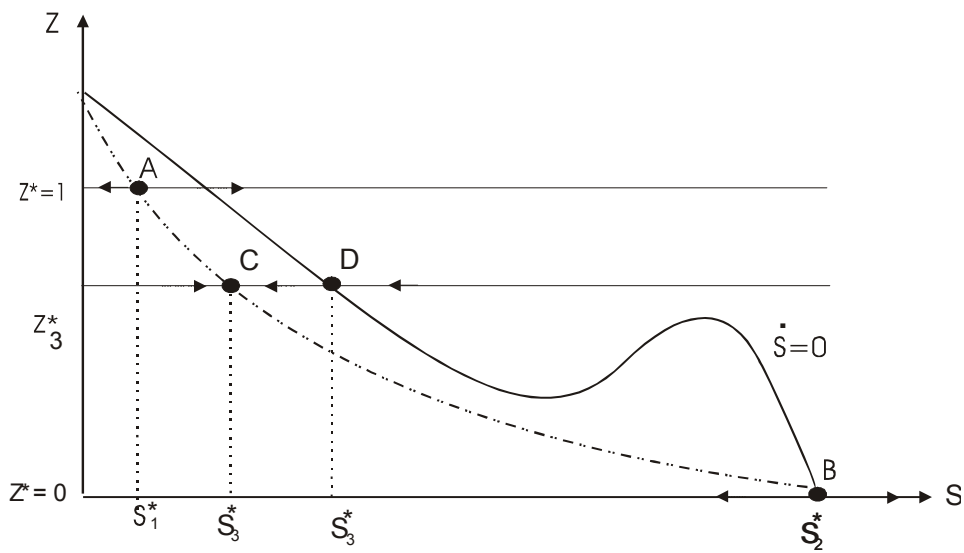
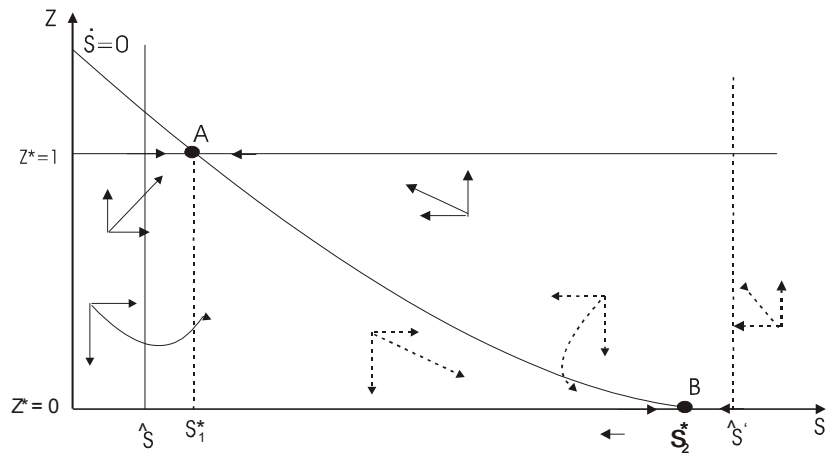
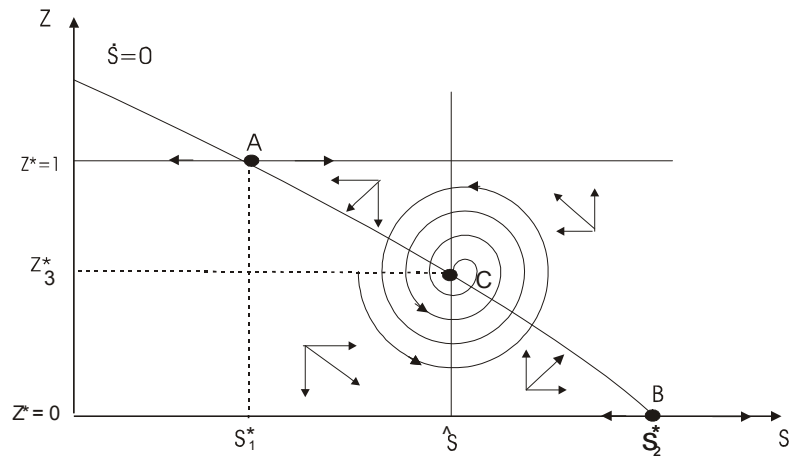


Figure 1: Evolutionary equilibrium with compliance dependent auditing probability



(a)



(b)

Figure 2: Evolutionary equilibria under pollution stock dependent auditing probability

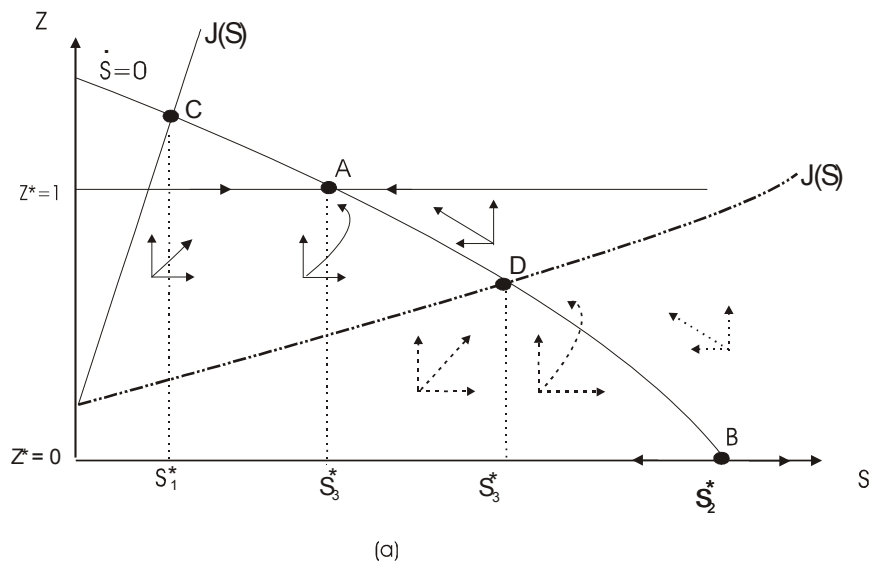


Figure 3: Evolutionary equilibria under a general auditing probability

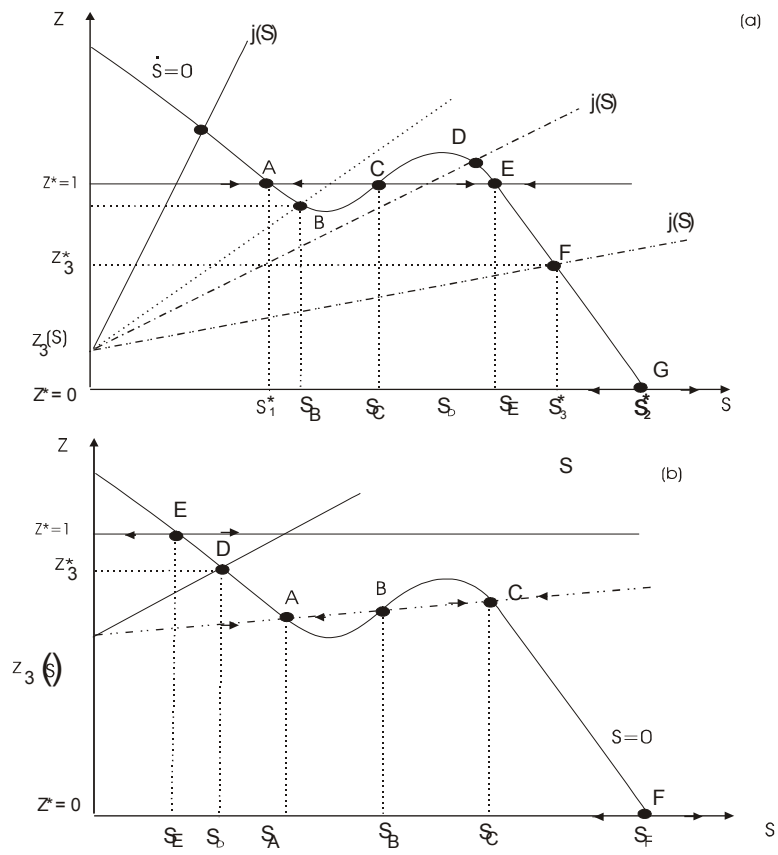


Figure 4: Multiple equilibria