

# Participation in and Compliance with Public Voluntary Environmental Programs: An Evolutionary Approach

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Version: February 29, 2004

The joint evolution of participating and complying firms in a public VA, along with the evolution of the pollution stock is examined. Replicator dynamics modeling participation and compliance are combined with pollution stock dynamics. Fast-slow selection dynamics are used to capture the fact that decisions to participate in and further comply with the public VA evolve in different time scales. Evolutionary stable (ES) equilibria depend on the structure of the legislation and auditing probability. Partial participation and partial compliance can be ES equilibria, with possible multiplicities, in addition to the monomorphic equilibria of full (non) compliance. Convergence to these equilibria could be monotonic or oscillating. Full participation and compliance can be attained if the regulator is pre-committed to certain legislation and inspection probabilities, or by appropriate choices of the legislatively set emission level and the non-compliance fine.

*Key Words:* Voluntary agreements, participation, compliance, evolutionary stability, replicator dynamics.

*JEL Classification:* Q2, L5.

## 1. INTRODUCTION

Recent experience in environmental regulation indicates that traditional regulatory instruments, such as emission taxes, subsidies, or tradeable permit systems etc., might not be entirely successful in reversing the environmental degradation process, creating the need for an evolution of environmental policy towards new instruments. Voluntary approaches appear to be an alternative to traditional instruments to pollution control, since they offer certain advantages over mandatory regulations that impose technology restrictions or penalties on firms. They are expected to increase economic and environmental effectiveness, as well as social welfare, since they allow firms greater flexibility in their pollution control strategies and also have the potential to reduce transaction and compliance costs.<sup>1</sup> Voluntary

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<sup>1</sup>The theoretical analysis of voluntary approaches to environmental regulation has been mainly developed in the recent decade. See for example the work of Carraro and Siniscalco (1996),

approaches can be placed into three basic categories, based mainly on the degree of public intervention.<sup>2</sup> *Negotiated agreements*, which are the most common cases of voluntary approaches, imply a bargaining process between the regulatory body and a firm or an industry group, in order to jointly set the environmental goal and the means of achieving it. *Unilateral agreements* are environmental improvement programs prepared and voluntarily adopted by firms themselves.<sup>3</sup> *Public voluntary agreements* are environmental programs developed by a regulatory body with which firms can only agree with and in return they may receive technical aid, supplementary funds and/or favorable publicity<sup>4</sup>.

The potentially most serious drawback of voluntary approaches (VAs) is that they leave room for "free-riding" behavior. Particularly, in an industry-wide public voluntary agreement where the attainment of an environmental target requires collective action, individual firms may have incentives not to reduce their emissions but to rely upon other firms to carry out the actions necessary to attain the target. These firms can decide not to participate in the achievement of the established goal either ex-ante (non-participation), or ex-post after signing the agreement (non-compliance). Therefore, it is possible that the incentive to free-ride may impede the establishment of a public VA, or may result in a failure of the agreement because signatory firms do not comply with the rules of the VA.<sup>5</sup> This suggests some limitations in the ability of VAs to attain desired targets. In fact there are some reservations regarding the ability of public VAs to improve environmental quality as an independent policy tool, based on empirical observations. According to a report by Environment Canada, *"the industrial sectors that relied solely on self monitoring or voluntary compliance<sup>6</sup> had a compliance rating of 60% versus the 94% average compliance rating of those industries which were subject to federal regulations combined with a consistent inspection program"*.<sup>7</sup> These findings imply that both participation in and compliance with the agreement's provisions and goals are important in successful VAs. Indeed without appropriate threats of sanctions or enforcement schemes, there may be a problem of compliance or uneven application. Thus it seems that a successful VA scheme may need to include a mix of voluntary and mandatory features, to ensure that polluting agents will not only sign the public VA but also comply with its provisions and reduce their emissions to a desirable level. Examples of successful public voluntary programs include the EPA's "33-50" program that seeks to encourage firms in the US Chemical industry to voluntarily reduce the discharges of 17 high-priority toxic chemicals under the background threat of legislation (Khanna and Damon, 1998), the environmental management system certification standards "EMAS" and "ISO 14000" (Šauer et al.,

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Segerson and Miceli (1998), Segerson and Dawson (2000), Brau et al. (2001), Lyon and Maxwell (2003).

<sup>2</sup>This refers primarily to the degree the of authority's impact on a certain hierarchical level of public administration (Šauer et al., 2001).

<sup>3</sup>Such voluntary approaches are also known as "self-regulation".

<sup>4</sup>Polluting firms may receive rights to use an ecological logo or certification label.

<sup>5</sup>Nevertheless, despite the presence of apparent incentives to free-ride it is possible to have an equilibrium in which the environmental target is achieved and only a subset of firms in the industry comply with the agreement's provisions, while the remaining free-ride (Dawson and Segerson, 2001).

<sup>6</sup>The term compliance refers to the state of conformity with the existing environmental provisions.

<sup>7</sup>"Enforcement vs Voluntary Compliance: An Examination of the Strategic Enforcement Initiatives Implemented by the Pacific and Yukon Regional Office of Environment Canada, Report No. DOEFRAP 19983.

2001), the "US. Conservation Reserve Program" (CRP) that used cost-sharing and other financial inducements to achieve reduction of agricultural pollution through voluntarily participation in soil conservation and other erosion control programs (Segerson and Miceli, 1998) and its successor "Environmental Quality Incentives Program" (EQIP) (Dawson and Segerson, 2001). Examples of public VA schemes<sup>8</sup> include the "Canadian Industry Program for Energy Conservation" (CIPEC), the "US Green Lights" , the "Motor Challenge" programs for industry, as well as the "Golden Carrot" program for manufactures of highly energy-efficient refrigerators which have been recently consolidated with the "Motor Challenge" (OECD 1998). The Canadian "Climate Change Voluntary Challenge and Registry Program" permits least-cost actions from industry, business, government and public institutions to limit or reduce net greenhouse gas emissions, and participants are free from government regulatory requirements.

In the present paper we study the long-run structure of a public VA where the regulator makes an offer to a large number of homogeneous firms to reduce emissions in order to voluntarily attain, by using flexible cost saving methods, a desired target ambient pollution level.<sup>9</sup> The type of VA programme we study has many similarities with voluntary climate change programs or the various Energy Star programs.<sup>10</sup> If the offer attains full participation, a target ambient pollution stock is attained. If there is no full participation there is a deviation from the target and a positive probability of legislation that will regulate the sector through conventional instruments such as taxes or emission limits. Thus free riding, in the sense of not participating and expecting to avoid regulation because others are participating, may trigger regulation. Participating firms are not directly observed by the regulator so there could be incentives not to comply. The regulator tries to deter non-compliance by random auditing and fines to those found not in compliance with the VA programme. The probability of auditing may increase with deviations from the target ambient pollution level.

In modelling the process where firms decide whether to participate in the agreement under a probabilistic regulation threat, we adopt an evolutionary framework. The basic characteristic of this framework is that, although firms are profit maximizers in the output choice, when it comes to choosing a strategy regarding participating in the VA programme, or whether to comply or not, they adopt a more passive decision making and not an explicit optimizing behavior.<sup>11</sup> This more passive decision making is modelled by an evolutionary process where decisions are taken by comparing the profits of a strategy to participate and comply with the corresponding expected profits of a nonparticipating, non-complying firm. Successful strategies, in the sense of those attaining higher expected profits, are imitated by other firms with a probability proportional to the difference between the corresponding profits. Thus profit differentials exercise evolutionary pressures on the composition of the population so that more successful strategies increase their share

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<sup>8</sup>It should be noted that while "ProjectXL" and "Common Sense Initiative" involve negotiation, they also resemble public voluntary programs.

<sup>9</sup>The flexible methods of reducing emissions through the VA program have a weak cost advantage relative to regulation like, for example, the XL Project or the EPA's 33-50.

<sup>10</sup>See, for example, OECD (1998).

<sup>11</sup>This evolutionary approach might be interpreted as encompassing ideas of bounded rationality since it can be associated with firms' bounded ability to fully perceive either advantages associated with flexibilities and cost superiority of the VA programme, or costs associated with probabilistic fines. For general presentations of these approaches see for example Nelson (1995) and Conlisk (1996).

in the total population of firms. A simple way to model the movements in the composition of the population of firms regarding participation in and compliance with the VA is the use of *replicator dynamics*.<sup>12</sup> We use replicator dynamics as our selection dynamics to model in two stages the evolution of: (i) the decision to sign or not the agreement, and (ii) the decision to comply or not with the agreement's provisions after signing it. The use of replicator dynamics allows us to determine strategies regarding participation and compliance which are evolutionarily stable (ES).<sup>13</sup> We further elaborate on the selection dynamics by considering the situation where decisions to participate or not evolve fast, since when the offer is made there is usually a time framework determined by legal considerations,<sup>14</sup> while decisions regarding compliance after participation are unconstrained and we expect them to evolve much more slowly. This suggests that the ES equilibrium composition of firms regarding participation in the VA is reached faster than the ES equilibrium composition regarding compliance, which suggests that selection dynamics operate in a *fast-slow dynamics* framework.

Our contribution lies therefore in using, for the first time to our knowledge, an evolutionary approach with fast-slow selection dynamics to jointly determine the steady-state equilibrium fraction of signatory and complying firms, as well as the corresponding steady-state equilibrium emission stock. Using this approach we are able to determine "*which strategies survive in the long-run*", in the sense of evolutionary stability, define the structure that a voluntary agreement would have in the long run and identify policy rules that might produce desirable ES VAs.<sup>15</sup> The analysis indicates that the value and characteristics of the legislation probability and the auditing probability are of crucial importance for the resulting long-term equilibrium outcome. Under different assumptions about the legislation probability, the fast time dynamic system can alternatively converge to a polymorphic or monomorphic steady state, implying either partial or full (non) participation in the public VA. Similarly by choosing the structure of the auditing probability, the regulator can achieve partial or full (non) compliance. There is a possibility of unique or multiple ES equilibria with potential irreversibilities, while the convergence to these equilibria could be monotonic or oscillating. Full participation and full compliance, which can be regarded as the desired outcome for the regulator, can be attained if the regulator is pre-committed to certain legislation and inspection probabilities, or by appropriate choices of the legislatively set emission level and the non-compliance fine.

## 2. THE INDUSTRIAL MODEL

Assume an industrial sector consisting of  $i = 1, 2, \dots, n$  small and identical firms. Firms operate under competitive conditions and emit into the ambient environment. Emissions accumulate in the environment and cause external damages. Due to the externality emissions exceed the socially-desirable levels without regulation. The regulator proposes formally a "take-it-or-leave-it" environmental protection scheme

<sup>12</sup>For definitions, see, for example, Weibull (1995). For applications of this methodology to common property resources see, Sethi and Somanathan (1998).

<sup>13</sup>A strategy is evolutionary stable if it can not be invaded by a mutant strategy. (See for example Weibull (1995) page 36)

<sup>14</sup>For example EPA's National Environmental Performance Track accepts applications twice a year.

<sup>15</sup>For a similar approach regarding the regulation of a renewable resource, see Xepapadeas (2003)

and gives each firm in the industrial sector a chance to voluntarily meet an exogenously determined emission level  $e_v$ . This type of public VA offers full flexibility to choose the profit-maximizing and legislative preemptive means of achieving the target and could provide cost advantages over legislative regulation (Brau et al., 2001, Segerson 2001).

In particular the regulator proposes a long-term "preemptive" public voluntary environmental contract<sup>16</sup> to which firms can only agree or not. If all firms follow the agreement then total emissions in the ambient environment will be  $E_v = ne_v$ , assuming that the pollution stock  $S$  accumulates according to:

$$\dot{S}(t) = E(t) - \varphi(S(t)), \quad E(t) = \sum_{i=1}^n e_i(t) \quad (1)$$

where  $E(t)$  denotes total emissions at time  $t$  due to industrial activities, and the term  $\varphi(S(t))$  denotes emissions outflows reflecting natural environmental self cleaning process and environmental feedbacks.

Let,  $\bar{S}(t)$  be the path of the pollution stock under full participation and compliance to the agreement. If there is no full participation, a deviation is expected between the observed stock of pollution and the desired stock  $\bar{S}(t)$ . We denote this deviation at time  $t$  by  $\Delta S(t) = S(t) - \bar{S}(t)$ . Participation in the agreement does not imply that a firm will also comply with the agreement. Thus we assume that although the regulator has full observability of the participating firms, simultaneous control of all signatory firms is prohibitively costly. In this case inspection of randomly chosen signatory firms is the mechanism usually applied to verify compliance and identify compliance problems. Therefore a positive  $\Delta S(t)$  might be the result of either partial participation and non-compliance by some of the participating firms, or under full participation, the result of non-compliance by some the firms that have already signed the agreement. Let  $x(t)$  denote the proportion of participating firms at time  $t$ . It would be intuitive to assume that from a firm's point of view the subjective probability of having legislation introduced at time  $t$  depends on the deviation  $\Delta S(t)$  and the proportion of participating firms  $x(t)$ , or<sup>17</sup>

$$p(t) = p(\Delta S(t), x(t), \omega_v(t)), \quad \text{with } \frac{\partial p(\cdot)}{\partial \Delta S} > 0, \quad \frac{\partial p(\cdot)}{\partial x} < 0 \quad x \in [0, 1] \quad (2)$$

where  $\omega_v(t)$  is a vector of other parameters affecting the probability of regulation which may include legislative procedures, transaction costs, etc.

An increase in the deviation  $\Delta S(t)$  increases the probability of regulation, while an increase in the number of participating firms reduces the probability of regulation. To provide further structure to the probability of introducing legislation we assume that this probability is common to all firms and that:  $p(0, 1) = 0$ ;<sup>18</sup>  $p(\Delta S, x | \Delta S > 0, x < 1) > 0$ ;  $p(\Delta S, 1 | \Delta S > 0) = 0$ . That is, if everybody participates, then the deviation is due to non-compliance and the regulator has to resort to other methods such as random inspections and fines which are discussed below.

<sup>16</sup>Such voluntary approaches indirectly reduce expected production costs because they reduce the probability of facing a (more costly) direct regulatory regime (Brau et al., 2001).

<sup>17</sup>Segerson and Miceli (1998) assume a fixed legislation probability.

<sup>18</sup>The possibility of  $p(0, x | x < 1) = 0$ , which allows for overcompliance by some firms so that the target is achieved even if some firms are not participating, is not considered. The possibility of overcompliance implies the introduction of another strategy,  $e_{OC} < e_v$ . This case is left as an area for further research.

We assume that  $(\Delta S(t), x(t))$  are observable by the regulator and become public information, while there is uncertainty regarding the vector  $\omega_v$ . Firms can use announced  $(\Delta S(t), x(t))$  to calculate subjective probabilities, but there is uncertainty regarding the probability law  $p(\Delta S(t), x(t), \omega_v(t))$ , thus firms use model (2) as a benchmark for some fixed value of the vector  $\omega_v$ .

If the firms believe that the crucial factor that affects the probability of legislation is not the proportion of participating firms but only the deviations from the desired pollution path, then this probability can be further simplified to<sup>19</sup>

$$p(t) = p(\Delta S(t), \omega_v(t)), \text{ with } \frac{\partial p(\cdot)}{\partial \Delta S} > 0 \quad (3)$$

The decision to participate and then to comply or not depends on the structure of profits. In our model, each firm produces an output  $Q$  and emissions  $e$ . The cost function  $C(Q, e)$  is a continuous function where  $C_Q > 0$ ,  $C_e < 0$ ,  $C_{QQ} > 0$  and  $C_{ee} > 0$ . We assume that the VA offers only a cost advantage to participating and complying firms since participation and compliance deter the introduction of relatively more costly mandatory regulation. Moreover participation in the public VA allows for greater flexibility in the processes of emissions reduction, and offers lower compliance and transaction costs.<sup>20</sup>

The firm's profit function is defined as  $\Pi(e) = \max_Q \{PQ - C(Q, e)\}$ . At the unregulated equilibrium a firm chooses emissions  $e_o = \arg \max_e \Pi(e)$ . Therefore when a firm decides not to participate in the VA, either ex-ante or ex-post, and continues producing at the profit-maximizing emission level without facing a legislative mandate or paying a fine, then profits are defined as  $\Pi_N(e_o)$ .

If a firm decides to sign the VA in the first stage and voluntarily comply with its provisions to emit at the agreed level  $e_v$  then profits are  $\Pi_v(e_v) = \max_Q \{PQ - C_v(Q, e_v)\}$ , where  $C_v(Q, e)$  is the cost function under the flexibility provided by the VA.

If a firm decides not to participate in the VA and mandatory legislation is used to introduce regulation, then its profit function could be defined as:

$$\Pi_L(e, \tau) = \max_Q \{PQ - C_L(Q, e) - \tau_L e\} \quad (4)$$

$$\Pi_L(e) = \max_Q \{PQ - C_L(Q, e)\} \text{ with } e \leq \bar{e} \quad (5)$$

if the legislation introduces an emission tax  $\tau$ , or an emission limit (performance standard)  $\bar{e}$ . In both cases  $C_v(Q, e) < C_L(Q, e)$  under the cost advantage assumption of the VA. So under legislation profits can be defined as  $\Pi_L(e_L)$ , where  $e_L(\tau) = \arg \max_e \Pi_L(e, \tau)$  under taxation, or  $e_L = \arg \max_e \Pi_L(e)$  subject to  $e \leq \bar{e}$ , under a performance standard. Under standard assumptions  $e_L = \bar{e}$ .<sup>21</sup>

If a firm that has already signed the agreement decides not to comply and emit at the unregulated level  $e_o$ , then there is a possibility that the firm could be caught after a random inspection. If caught the firm is subjected to individual legislation

<sup>19</sup>It seems that  $\Delta S$  shall always be part of the subjective probability in every case. If the subjective probability is a function of participation proportion  $x$  alone, then the incentive to participate is not linked with the achievement of the environmental target  $e_v$ .

<sup>20</sup>We assume that the VA does not improve a firm's public image and increase consumers' goodwill. Therefore total revenues remain unchanged whether firms participate in the agreement or not and whether they comply with it or not.

<sup>21</sup>Furthermore, under standard assumptions the target  $\bar{e} = e_L$  can be achieved either through taxation, if the tax rate is chosen such that  $e_L(\tau) = \bar{e}$  is a solution to  $\max_e \Pi_L(e, \tau)$ , or through a performance standard  $\bar{e}$ .

with a performance standard  $\bar{e} = e_L$  and a non-compliance fine  $F$ . Assuming that the individual performance standard is set at  $e_L$  defined above, the profits of a non-complying firm which is caught after a random inspection is  $\Pi_C(e_L, F) = \Pi_L(e_L) - F$ . If the non-complying firm is not inspected then profits are simply  $\Pi_o(e_o)$ .

It holds that  $e_o > e_L \geq e_v$ , then our assumptions regarding the structure of costs and profits imply:

$$\Pi_o(e_o) > \Pi_v(e_v) > \Pi_L(e_L) > \Pi_C(e_L, F)$$

Since, in the case of non-participation in the agreement, the imposition of legislation is probabilistic, the expected profits of non-participating firms are :

$$\mathcal{E}\Pi_N = p\Pi_L(e_L) + (1 - p)\Pi_N(e_o), \quad p = p(\Delta S, x, \boldsymbol{\omega}_v) \quad (6)$$

Therefore a sufficient condition for participation in the VA is

$$\Pi_v(e_v) \geq p\Pi_L(e_L) + (1 - p)\Pi_N(e_o) \quad (7)$$

Let  $q$  be the subjective probability that a participating firm will be inspected and let  $z$  be the proportion of participating firms that comply with the terms of the agreement. A firm's subjective probability of being audited can be defined in a general form by  $q(\boldsymbol{\omega})$ , where  $\boldsymbol{\omega}$  is a vector of parameters. It is assumed that this function is common for all firms and can be further specified in the following cases.

In the first case the regulator exercises fixed monitoring effort and makes a fixed number of inspections, say  $\bar{n}$  per period. In doing so the regulator announces this policy and thus precommits to a certain auditing probability which is known by the polluters. In this case the audit probability is fixed, or<sup>22</sup>

$$q(\boldsymbol{\omega}) \equiv \bar{q} \quad (8)$$

An alternative assumption would be that the regulator exercises variable monitoring effort, which depends on state variables of the problem that the regulator can observe.<sup>23</sup> One such variable is the deviations from the desired pollution stock  $\Delta S$ ; another variable is the share of violators  $u$  detected during an audit. The regulator increases the monitoring effort if the stock is declining or the share of violators is increasing. This policy can be regarded as a type of no full commitment - or partial commitment - auditing policy on the regulator's part. The regulator might, for example, not audit individual firms if the deviation  $\Delta S$  is sufficiently low, but the regulator might start inspecting if the deviation increases beyond a certain level.<sup>24</sup> The firms are made aware of the results of the inspections, say through public announcements and/or private communications, and perceive that if the deviation increases or the share of violators increases, more effort will be exercised and thus the subjective probability of being audited increases. In this case the probability  $q$  can be specified as stock dependent auditing probability:

<sup>22</sup>This is a common assumption in the enforcement literature in environmental economics (e.g. Malik, 1993; Garvie and Keeler, 1994; Segerson and Miceli, 1998; Stranlund and Dhana, 1999).

<sup>23</sup>In the enforcement literature, variable monitoring effort is usually related to firm specific variables (e.g. Malik, 1990; VanEgteren and Weber, 1996).

<sup>24</sup>Grieson and Singh (1990), Khalil (1997), and Franckx (2002) analyze no commitment frameworks. Franckx relates individual auditing to the level of ambient pollution which is a global state variable. An environmental regulator chooses which firm to inspect without observing firms' actions but after observing ambient pollution.

$$q = q(\Delta S, \omega_c), \quad q'(S, \omega_c) > 0, \quad q(0, \omega_c) = 0 \quad (9)$$

where  $\omega_c$  is a vector of parameters similar to  $\omega_v$ .

If the firms use the observed  $u$  as an estimate for their perceived  $z$ , that is they set  $u = z$ , a compliance dependent auditing probability is defined as:

$$q = q(z, \omega_c), \quad q'(z, \omega_c) < 0, \quad q(1, \omega_c) = 0, \quad q(0, \omega_c) > 0 \quad (10)$$

It is expected that the value of  $q(0)$  will be large but not unity since not every firm is audited even if nobody complies, while  $q(1) = 0$ , since if everybody is complying the subjective probability of paying the noncompliance fine (and facing the legislation) is zero.

If (9) and (10) are taken together, a more general formulation for the subjective auditing probability with joint dependence on compliance and stocks would be:

$$q = q(z, \Delta S, \omega_c) \quad (11)$$

In this context the expected profits of a participating but non-complying firm are :

$$\mathcal{E}\Pi_N = q\Pi_C(e_L, F) + (1 - q)\Pi_N(e_o) \quad (12)$$

and the sufficient condition for complying with the agreement's provisions is:

$$\Pi_v(e_v) \geq q\Pi_C(e_L, F) + (1 - q)\Pi_N(e_o) \quad (13)$$

Given the above framework we explore how imitation and enforcement of behavior resulting in higher profits will determine which strategies (participate or not/comply or not) will survive in the long run. We model the selection dynamics that can be used to determine the ES strategies by replicator dynamics.

### 3. REPLICATOR DYNAMICS

Assuming that at a given time  $t$  the industrial sector consists of two groups of firms, each group could follow two possible strategies concerning the offered public VA, participate or not. Let  $x(t)$  denote the proportion of firms participating in the agreement, while  $x_N(t)$  denotes the remaining proportion of non-signatory agents at time  $t$ , with  $x(t) + x_N(t) = 1$ .

In every time period  $dt$  there is a positive probability  $adt$  that a firm  $i$ , following a certain strategy, will compare its profits and consequently its strategy, with the corresponding profits and strategy of another randomly chosen firm  $j$ .<sup>25</sup> If firm  $i$  perceives that the other's profits are higher, then it switches its strategy to firm  $j$ 's strategy. There is imperfect information concerning the difference in the expected profits of the two strategies, since there is uncertainty in the subjective probability law determining the probability of legislation, and possible uncertainty regarding the true cost functions under the voluntary agreement or under regulation. In this context the higher the difference between profits is, the higher the probability is that firm  $i$  will perceive it and change strategy. Particularly, firm  $i$  that did not participate in the public VA in time  $t$ , might decide to switch strategy and finally sign the agreement if its expected profits  $\mathcal{E}\Pi_N$  without participation, defined by (6),

<sup>25</sup>In motivating the replicator dynamics we follow Gindis (2000).



are less than the corresponding profits  $\Pi_v(e_v)$  of the participating firm. Therefore, the probability that a non-participating firm will change its strategy and ultimately sign the public VA, after comparing profits, is given as:

$$P_{NV}^t = \begin{cases} \beta [\Pi_v(e_v) - p\Pi_L(e_L) - (1-p)\Pi_N(e_o)] & \text{for } \Pi_v(e_v) > \mathcal{E}\Pi_N \\ 0 & \text{for } \Pi_v(e_v) \leq \mathcal{E}\Pi_N \end{cases}$$

The expected proportion of firms that decides to participate in the voluntary public VA in time  $t + dt$  is given as:

$$\mathcal{E}x^{t+dt} = x^t + \alpha dt x^t \sum_{j=1}^n x_N \beta (\Pi_N(e_o) - \Pi_v(e_v))$$

or alternatively if we use the definition of the average profits:

$$\mathcal{E}x^{t+dt} = x^t + \alpha dt x^t \beta (\Pi_v(e_v) - \bar{\Pi}(e))$$

where  $\bar{\Pi}(e)$  is defined as the average profit for the whole population at time  $t$ :

$$\bar{\Pi}(e) = x\Pi_v(e_v) + (1-x)\mathcal{E}\Pi_N = x\Pi_v(e_v) + (1-x) [\Pi_N(e_o) - p(\Pi_N(e_o) - \Pi_L(e_L))]$$

It has been assumed that the population of firms in the industrial sector is large, thus we can replace  $\mathcal{E}x^{t+dt}$  by  $x^{t+dt}$ . Moreover, if we subtract from both sides the term  $x^t$ , divide by  $dt$  and finally take the limit as  $dt \rightarrow 0$ , we derive an equation that describes the behavior of the fraction  $x$  over time. This is the replicator dynamic equation:

$$\dot{x} = \alpha \beta x^t [\Pi_v(e_v) - \bar{\Pi}(e)]$$

Replicator dynamics indicates that the frequency of the signatory strategy increases exactly when its profits  $\Pi_v(e_v)$  are above the average profits  $\bar{\Pi}(e)$ . If we substitute the profit definitions and drop  $t$ , then the replicator dynamics equation is rewritten as follows:

$$\dot{x} = \alpha \beta x(1-x) [p(\Pi_N(e_o) - \Pi_L(e_L)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (14)$$

However, it has already been mentioned that participation in the agreement does not imply that a firm will also comply with the agreement. We assume that in choosing between compliance or not, firms imitate successful strategies as in the choice of the participation strategy defined above by collecting (incomplete) information regarding expected profits of non-complying firms. Let  $z(t)$  denote the proportion of firms complying with the agreement, while  $z_N(t)$  denotes the remaining proportion of non-complying firms at time  $t$ , with  $z(t) + z_N(t) = 1$ .

After following the same conceptual framework as above, the replicator dynamics equation for the compliance strategy is defined as:

$$\dot{z} = \gamma \delta z^t [\Pi_v(e_v) - \bar{\Pi}_{VN}(e)]$$

where  $\gamma$  and  $\delta$  correspond to  $\alpha$  and  $\beta$  above, and  $\bar{\Pi}_{VN}(e)$  is the average profits for the whole population of signatory firms defined as:

$$\bar{\Pi}_{VN}(e) = z\Pi_v(e_v) + (1-z)[q\Pi_C(e_L, F) + (1-q)\Pi_N(e_o)]$$

Then the specific form of the replicator dynamics equation for the complying strategy is defined as:

$$\dot{z} = \gamma\delta z(1-z) [q(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (15)$$

The evolution of the emission stock is affected by the decisions to participate in the agreement and further comply with its provisions and established goals. Therefore the pollution stock dynamic equation (1) can be further specified as:

$$\dot{S} = n\{x[ze_v + (1-z)(qe_L + (1-q)e_o)] + (1-x)e_o\} - \varphi(S) \quad (16)$$

#### 4. FAST - SLOW SELECTION DYNAMICS IN THE EVOLUTION OF PUBLIC VOLUNTARY AGREEMENTS

The purpose of introducing different time scales in the replicator dynamics framework characterizing the evolution of voluntary agreements is to capture the fact, observed in real situations, that when a VA of the type analyzed here is offered, the composition regarding participation is finalized relatively fast. Since firms have to decide whether to accept the offer within a relatively small time interval determined by legislative procedures, we expect evolutionary pressures to work relatively fast. On the other hand compliance behavior is not constrained by a time framework so we expect evolutionary pressures to operate more slowly relative to the participation case. This implies that the rate of change of  $x$  with respect to time is “large” in absolute value, while the rate of change of  $z$  is relatively slower. That is,  $|\frac{dx}{dt}| \equiv |\dot{x}| \gg |\frac{dz}{dt}| \equiv |\dot{z}|$ .

The above argument implies that in (14) and (15) we can set  $\alpha\beta = 1$  and  $\gamma\delta = \varepsilon$  where  $\varepsilon$  is a small positive parameter. Assuming that the natural system evolves in a time scale which is comparable to the slow compliance variable, then our dynamic system can be written in a fast time scale as:

$$\frac{dx}{d\tau} = f_1(x, S) \quad (17)$$

$$\frac{dz}{d\tau} = \varepsilon f_2(z, S) \quad (18)$$

$$\frac{dS}{d\tau} = \varepsilon f_3(x, z, S) \quad (19)$$

where  $f_i$ ,  $i = 1, 2, 3$  represent the right hand sides of (14), (15) and (16) respectively. System (17)-(19) is the fast time system (FTS). If fast time is scaled such that  $\tau = t/\varepsilon$ , so that  $d\tau = dt/\varepsilon$  then the replicator dynamics system characterizing participation, compliance and pollution accumulation can be written in slow time as:

$$\varepsilon\dot{x} = f_1(x, S) \quad (20)$$

$$\dot{z} = f_2(z, S) \quad (21)$$

$$\dot{S} = f_3(x, z, S) \quad (22)$$

The problem defined in the dynamical system (20)-(22) is a *singular perturbation* problem.<sup>26</sup> The general method for analyzing it, is to consider the systems at the

<sup>26</sup>For the analysis of problems in a fast-slow time framework see, for example, Wasow (1965, Chapter X) or Sastry (1999, Chapter 6).

limit  $\varepsilon \rightarrow 0$ . If the solutions satisfy certain regularity conditions for  $\varepsilon = 0$ , then solutions for small  $\varepsilon$  can be approximated by the solutions for  $\varepsilon = 0$ . By taking  $\varepsilon = 0$  in system (20)-(22) we obtain the *reduced* system

$$0 = f_1(x, S) \quad (23)$$

$$\dot{z} = f_2(z, S) \quad (24)$$

$$\dot{S} = f_3(x, z, S) \quad (25)$$

In the reduced system equation (23) provides, if it can be solved for  $x$ , the equilibrium participation rate for fixed level of  $S$ , as

$$x = h(S) \quad (26)$$

The solutions of (26) are equilibria of the FTS (17)-(19) defined for  $\varepsilon \rightarrow 0$  as:

$$\begin{aligned} \frac{dx}{d\tau} &= f_1(x, S) \\ \frac{dz}{d\tau} &= 0 \\ \frac{dS}{d\tau} &= 0 \end{aligned}$$

The equilibria of the FTS are denoted by  $h_j(S)$ ,  $j = 1, \dots, J$ , where  $J$  is the number of these equilibria. For the stable equilibria from the set of equilibria of (26), the slow variables evolve as:

$$\dot{z} = f_2(z, S) \quad (27)$$

$$\dot{S} = f_3(h(S), z, S) \quad (28)$$

The analysis of the dynamic system (27) and (28) can be used to determine the long-run ES compliance and pollution stock  $(z^*, S^*)$ . Then the long-run ES participation in the VA will be determined as  $h(z^*, S^*)$ .<sup>27</sup>

## 5. LONG-RUN STRUCTURES OF A PUBLIC VA

The conceptual framework developed above is used to determine the long-run structure regarding participation in and compliance with a public VA. Since the long-run structure is determined as a stable equilibrium of the replicator dynamics equation, it has the property of evolutionary stability.<sup>28</sup> To illustrate the importance of the legislation and auditing probabilities in determining these long-run structures, we classify the following analysis according to the characteristics of these probabilities.

<sup>27</sup>In more technical terminology the dynamic system (27) and (28) is defined on the stable two-dimensional manifold (or union of)  $M = \left\{ (z, S, x) : g(x, z, S) = 0 : x_j^F(z, S) \right. \\ \left. \text{is stable in FTS} \right\}$ . Solutions of the slow system (20)-(22) at least locally are attracted to this manifold.

<sup>28</sup>Formally a strategy  $\hat{x}$  is an evolutionary stable strategy if it is a strongly stable equilibrium point of the replicator dynamics equation. Strong stability means that if  $\hat{x}$  is contained in a convex hull of the strategy simplex, all strategies in the neighborhood of  $\hat{x}$  converge to  $\hat{x}$ . (See, for example, Hofbauer and Sigmund (2003).)

### 5.1. Pollution Stock Dependent Legislation Probability

Assume that the subjective probability of introducing legislation depends only on the deviation from the targeted emission stock level, defined as  $\Delta S$ . Then in the fast time system the observed emission stock  $S$  and the deviation from the target  $\Delta S$ , are both fixed and anticipated as parameters. As a consequence the legislation probability is fixed, implying that  $p = p(\overline{\Delta S})$ . Under this definition the slow time dynamic system (23)-(25) is defined as:

$$0 = x(1-x) [p(\overline{\Delta S})(\Pi_N(e_o) - \Pi_L(e_L)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (29)$$

$$\dot{z} = z(1-z) [q(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (30)$$

$$\dot{S} = n\{x[ze_v + (1-z)(qe_L + (1-q)e_o)] + (1-x)e_o\} - \varphi(S) \quad (31)$$

The solution of the replicator dynamics equation (29) provides the long-term equilibrium participation rates  $x^*$ , for fixed level of observed emission stock  $S$ . Two steady states exist,  $x_1^* = 1$  and  $x_2^* = 0$ , implying either full or non-participation in the public VA.

The stability condition is obtained by taking the derivative of (29) with respect to  $x$ , or:

$$\frac{d\dot{x}}{dx} = (1-2x)\Omega \quad (32)$$

where  $\Omega = [p(\overline{\Delta S})(\Pi_N(e_o) - \Pi_L(e_L)) - (\Pi_N(e_o) - \Pi_v(e_v))]$ . There is a critical probability value, defined as  $\hat{p}(\Delta S)$ , that sets  $\Omega$  equal to zero and behaves as a bifurcation parameter.<sup>29</sup> It can easily be seen that the sign of the expression  $\Omega$ , and therefore the stability of the steady states, depend on the magnitude of the fixed legislative probability  $p(\overline{\Delta S})$  relative to the critical probability value  $\hat{p}(\Delta S)$ . Specifically, if the regulator can announce and commit to a legislative probability higher than the critical value, then  $\Omega > 0$ . On the other hand, if the probability  $p(\overline{\Delta S})$  is lower than the critical value, then  $\Omega < 0$ .

Under this definition it follows that:

$$\begin{aligned} \text{If } p(\overline{\Delta S}) > \hat{p}(\Delta S) \text{ then } \frac{d\dot{x}}{dx} \Big|_{x_1^*=1} < 0 \text{ and } \frac{d\dot{x}}{dx} \Big|_{x_2^*=0} > 0 \\ \text{If } p(\overline{\Delta S}) < \hat{p}(\Delta S) \text{ then } \frac{d\dot{x}}{dx} \Big|_{x_1^*=1} > 0 \text{ and } \frac{d\dot{x}}{dx} \Big|_{x_2^*=0} < 0 \end{aligned}$$

In the first case, firms perceive that the implementation of the legislation mandate is highly likely. Therefore firms prefer the profit loss  $\Pi_N(e_o) - \Pi_v(e_v)$  under the public VA, to the higher profit losses  $\Pi_N(e_o) - \Pi_L(e_L)$  which will be realized if legislation is finally imposed. Consequently, all firms participate in the public VA and  $x_1^* = 1$  is stable, while  $x_2^* = 0$  is unstable. Furthermore the ambient pollution stock is equal to the industrial emission target  $E_v$ . In the second case, the legislation mandate appears less likely and firms can maintain the unregulated profits  $\Pi_N(e_o)$ . Therefore no firm has the incentive to participate in the public VA and receive reduced profits by  $\Pi_N(e_o) - \Pi_v(e_v)$  so  $x_2^* = 0$  is stable.

These findings can be summarized in the following proposition:

**PROPOSITION 1.** *Under an emission stock dependent legislative probability the fast time dynamic system converges to a monomorphic equilibrium. If  $p(\overline{\Delta S}) \in (\hat{p}(\Delta S), 1]$ , then there is full participation in the public VA and  $x_1^* = 1$  is the ES*

<sup>29</sup>  $\hat{p}(\Delta S)$  is defined as  $\hat{p}(\Delta S) = \frac{\Pi_N(e_o) - \Pi_v(e_v)}{\Pi_N(e_o) - \Pi_L(e_L)} < 1$ , since  $\Pi_N(e_o) - \Pi_v(e_v) < \Pi_N(e_o) - \Pi_L(e_L)$ .

equilibrium. If  $p(\overline{\Delta S}) \in [0, \hat{p}(\Delta S))$ , then there is non-participation in the public VA and  $x_2^* = 0$  is the ES equilibrium.

Furthermore the total derivative of  $\Omega = 0$  defines the relationship between the critical legislation probability value  $\hat{p}(\Delta S)$  and the legislative emissions  $e_L$ <sup>30</sup> and we obtain:

$$\frac{d\hat{p}(\Delta S)}{de_L} = \frac{\hat{p}(\Delta S)\Pi'_L(e_L)}{\Pi_N(e_o) - \Pi_L(e_L)} < \mathbf{0} \quad (33)$$

It is obvious that the higher the  $e_L$  is, the lower the critical probability value  $\hat{p}(\Delta S)$  is. Therefore the regulator can achieve full participation in the environmental agreement by commitment to a given  $e_L$ , instead of commitment to a legislation probability.

At this point we assume for simplicity that the regulator has set  $p(\overline{\Delta S}) > \hat{p}(\Delta S)$  and therefore the full participation steady state  $x_1^* = 1$  is ES in the fast time.<sup>31</sup> This steady state of the fast time dynamic system (29) is substituted in the slow time dynamic system, from which the long-term compliance and emission stock critical points are determined.

$$\dot{z} = z(1-z)[q(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (34)$$

$$\dot{S} = n\{ze_v + (1-z)(qe_L + (1-q)e_o)\} - \varphi(S) \quad (35)$$

The system has a hierarchical structure, implying that the stable equilibria of the replicator dynamics (34) can be determined first and then used to determine the pollution stock equilibria of equation (35). In order to more clearly determine the steady state pollution stock, (35) can be further specified by assuming that the emissions outflows term is linear, implying that  $\varphi(S) = bS$  with  $b > 0$ .

Under this definition the emissions stock dynamic isocline  $\dot{S} = 0$  is a linear equation defined as:

$$\begin{aligned} z(S) &= \frac{bS}{n\{e_v - [qe_L + (1-q)e_o]\}} - \frac{\{qe_L + (1-q)e_o\}}{\{e_v - [qe_L + (1-q)e_o]\}} \\ &= AS - B, \quad A < 0, \quad B < 0 \end{aligned}$$

However the auditing probability  $q$  can either be fixed  $\bar{q}$  or dependent on the state variables of the problem, that is, the deviations from the desired pollution stock  $\Delta S$  and the share of non-complying firms. Therefore it is interesting to examine how these alternative assumptions about the auditing probability affect the resulting equilibrium compliance and pollution stock.

#### 5.1.1. Case 1: Fixed Auditing Probability

Assume that the regulator is committed to a fixed auditing probability. Participating firms know exactly the probability  $\bar{q}$  under which they may experience profit losses  $\Pi_N(e_o) - \Pi_C(e_L, F)$ , if caught violating the agreement's provisions by

<sup>30</sup>As noted above, a target  $e_L$  can be attained either through emissions taxes, tradable emission permits or emission limits. From our assumptions about the industrial model it follows that  $\Pi'_L(e_L) < 0$ .

<sup>31</sup>It makes no sense to examine the slow time dynamic system when  $x_2^* = 0$ , since non-participating firms are not expected to do "self-regulation".

the regulator. Based on this knowledge they define their evolutionary strategy of whether or not to comply with the agreement.

Under this assumption the slow time dynamic system (34) and (35) becomes:

$$\dot{z} = z(1-z)[\bar{q}(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (36)$$

$$\dot{S} = n\{ze_v + (1-z)(\bar{q}e_L + (1-\bar{q})e_o)\} - bS \quad (37)$$

In this case there are two steady-states, satisfying the equilibrium condition  $\dot{z} = 0$ . The equilibrium outcome is always monomorphic, implying either full compliance  $z_1^* = 1$  or non-compliance  $z_2^* = 0$  with the agreement. These steady states corresponds to two parallel isocline. The stability condition that defines the prevailing evolutionary sustainable equilibrium is:

$$\frac{d\dot{z}}{dz} = (1-2z)\Phi \quad (38)$$

where  $\Phi = [\bar{q}(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))]$ . There is a critical probability value  $\hat{\bar{q}}$  that sets  $\Phi = 0$ . In particular, if  $\bar{q} > \hat{\bar{q}}$  then  $\Phi > 0$  and if  $\bar{q} < \hat{\bar{q}}$  then  $\Phi < 0$ . Thus the stability conditions becomes:

$$\begin{aligned} \text{If } \bar{q} > \hat{\bar{q}} \text{ then } \frac{d\dot{z}}{dz} |_{z_1^*=1} < 0 \text{ and } \frac{d\dot{z}}{dz} |_{z_2^*=0} > 0 \\ \text{If } \bar{q} < \hat{\bar{q}} \text{ then } \frac{d\dot{z}}{dz} |_{z_1^*=1} > 0 \text{ and } \frac{d\dot{z}}{dz} |_{z_2^*=0} < 0 \end{aligned}$$

If  $\bar{q} > \hat{\bar{q}}$  then full compliance is the ES outcome, while if  $\bar{q} < \hat{\bar{q}}$  then no compliance is the ES strategy. The  $\dot{S} = 0$  isocline defines the corresponding pollution stock equilibrium. Thus, if  $z_1^* = 1$  then  $S_1^* = \frac{ne_v}{b}$ , which is a target pollution stock level, while if  $z_2^* = 0$  is then  $S_2^* = \frac{n\{\bar{q}e_L + (1-\bar{q})e_o\}}{b} > S_1^*$

The above conclusions can be summarized in the following proposition:

**PROPOSITION 2.** *Under a fixed auditing probability the slow time dynamic system converges to a monomorphic equilibrium. If  $\bar{q} \in (\hat{\bar{q}}, 1)$  then there is full compliance with the public VA and  $z_1^* = 1$  is the ES equilibrium. If  $\bar{q} \in [0, \hat{\bar{q}})$  then there is non-compliance in the public VA and  $z_2^* = 0$  is the ES equilibrium.*

By taking the total derivative of  $\Phi = 0$  we obtain the relationship that connects the critical auditing probability value  $\hat{\bar{q}}$  and the non-compliance fine  $F$ . It follows that:

$$\frac{d\hat{\bar{q}}}{dF} = \frac{\hat{\bar{q}}\Pi'_C(e_L, F)}{\Pi_N(e_o) - \Pi_C(e_L, F)} < 0 \quad (39)$$

It is evident that the higher the non-compliance fine is, the lower the critical probability value  $\hat{\bar{q}}$  is. This shoes that a lower number of random inspections may induce participating firms to comply with the agreement when fines are high. Therefore the regulator can achieve full compliance and the established industrial environmental goal  $E_o$  with less monitoring effort.

### 5.1.2. Case 2: Compliance Dependent Auditing Probability

Under the assumption that the auditing probability is dependent on the fraction of complying participating firms, the slow time dynamic system (34) and (35) becomes:

$$\dot{z} = z(1-z)[q(z)(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (40)$$

$$\dot{S} = n\{ze_v + (1-z)(q(z)e_L + (1-q(z))e_o)\} - bS \quad (41)$$

It can easily be seen that the equilibria of compliance replicator dynamics (40) are  $z_1^* = 1$  and  $z_2^* = 0$ . Nevertheless, there could be an additional critical point, defined as  $z_3^* : q(z_3^*)(\Pi_N - \Pi_C) - (\Pi_N - \Pi_v)$ ,  $z_3^* \in (0, 1)$ , that also satisfies the equilibrium condition  $\dot{z} = 0$ .

In this case the stability condition that determines the equilibrium type of each steady state, is defined as:

$$\frac{d\dot{z}}{dz} = (1-2z)\Phi + z(1-z)q'(z)(\Pi_N(e_o) - \Pi_C(e_L, F)) \quad (42)$$

where  $\Phi = [q(z)(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))]$ . There is a critical probability value  $\hat{q}(z_3^*)$ , which corresponds to the steady state  $z_3^*$  and sets  $\Phi$  equal to zero. It is evident that if the initial compliance fraction  $z$  is lower than the critical proportion  $z_3^*$ , then due to condition (10) the existing auditing probability is higher than  $\hat{q}(z_3^*)$  and  $\Phi > 0$ . On the other hand, if  $z > z_3^*$  then  $q(z) < \hat{q}(z_3^*)$  and  $\Phi < 0$ . The sign of the expression  $\Phi$  affects the stability of the steady states and in particular, it can be seen that the slow time dynamic system can not converge to a monomorphic equilibrium since:

$$\frac{d\dot{z}}{dz} \Big|_{z_1^*=1} = -\Phi > 0 \text{ and } \frac{d\dot{z}}{dz} \Big|_{z_2^*=0} = \Phi > 0$$

Therefore it is clear that the ES equilibrium does not correspond to full compliance or non-compliance with the public VA. If there is full compliance then condition (10) holds and the regulator may respond to this with a reduced or even zero number of random inspections. This gives participating firms a financial incentive to violate the agreement. On the other hand, in the case of no compliance the value of  $q(0)$  is sufficiently high, making the profit losses at the level  $\Pi_N(e_o) - \Pi_C(e_L, F)$  highly likely. This gives non-participating firms a financial incentive to comply with the agreement's provisions, since the  $\Pi_N(e_o) - \Pi_v(e_v)$  profit losses are preferable to the profit losses under the legislation and the non-compliance fine. Thus, the slow time dynamic system converges to a polymorphic stable equilibrium, implying that only a sub-group of participating firms complies with the public VA, since:

$$\frac{d\dot{z}}{dz} \Big|_{z_3^*} = z_3^*(1-z_3^*)q'(z_3^*)(\Pi_N(e_o) - \Pi_C(e_L, F)) < 0$$

It is noticeable that the stability of the particular steady state  $z_3^*$  is independent of the existing auditing probability value. This implies that the initial distribution of complying firms does not affect the equilibrium outcome and thus partial compliance is the ES outcome, with an equilibrium pollution stock level  $S_3^* = \frac{n\{z_3^*e_v + (1-z_3^*)[q(z_3^*)e_L + (1-q(z_3^*))e_o]\}}{b}$ .

Therefore, in this case the following proposition holds:

PROPOSITION 3. *Under a compliance dependent auditing probability, partial compliance to the public VA is the ES outcome.*

Finally, after taking the total derivative of  $\Phi$ , the relationship between the compliance fraction  $z$  and the noncompliance fine  $F$  is obtained as:

$$\frac{dz}{dF} = -\frac{q(z)}{q'(z)(\Pi_N(e_o) - \Pi_C(e_L, F))} > 0 \quad (43)$$

The higher the fine for non-compliance is, the higher the  $\Pi_N(e_o) - \Pi_C(e_L, F)$  profit losses are for violators. Therefore under the threat of a higher fine, participating firms have a financial incentive to comply with the regulator and fulfill the individual environmental target  $e_v$ . Consequently an increased non compliance fine, increases the equilibrium compliance proportion and shifts the polymorphic steady state upwards, closer to the full compliance critical point. It is evident that the higher the fine is, the more participating firms comply with the public VA. So under the appropriate adjustments of the fine, compliance in the left side neighborhood of  $z_1^* = 1$  is an ES outcome.

### 5.1.3. Case 3: Emission Stock Dependent Auditing Probability

Assume that the auditing probability depends on the deviation from the established environmental goal. It is important to mention that the observed emission stock is no longer fixed in the slow time. In this case the slow time dynamic system is defined as:

$$\dot{z} = z(1-z)[q(\Delta S)(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (44)$$

$$\dot{S} = n\{ze_v + (1-z)(q(\Delta S)e_L + (1-q(\Delta S))e_o)\} - bS \quad (45)$$

The compliance replicator dynamics equation (44) has two equilibria, defined as  $z_1^* = 1$  and  $z_2^* = 0$ . Moreover, the equilibrium condition  $\dot{z} = 0$  is further satisfied by a critical emission stock level  $\hat{S}$ , with corresponding probability value  $\hat{q}(\Delta S)$  which sets  $\Omega = [q(\Delta S)(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))]$  equal to zero. From condition (9), it is evident that  $\Phi > 0$  if  $q(\Delta S) > \hat{q}(\Delta S)$  and  $\Phi < 0$  if  $q(\Delta S) < \hat{q}(\Delta S)$ . Thus the stability condition is:

$$\frac{d\dot{z}}{dz} = (1-2z)\Phi \quad (46)$$

However, in slow time the deviation  $\Delta S$  is no longer fixed. It can be easily seen that the type of the prevailing equilibrium depends on the existing relationship between the critical emission stock level  $\hat{S}$ , the full compliance emission stock level  $S_1^*$  and/or the non-compliance stock level  $S_2^*$ .

Assume that the critical emission stock level  $\hat{S}$  lies beneath both the full compliance emission stock level  $S_1^*$  and non-compliance stock level  $S_2^*$ . This implies that  $\hat{q}(\Delta S) < q_1(\Delta S) < q_2(\Delta S)$  and thus  $\Phi > 0$ . In this case (46) implies that:

$$\text{if } \hat{S} < S_1^* < S_2^* \text{ then } \left. \frac{d\dot{z}}{dz} \right|_{z_1^*=1} < 0 \text{ and } \left. \frac{d\dot{z}}{dz} \right|_{z_2^*=0} > 0$$

Thus, the ES equilibrium is monomorphic with full compliance  $z_1^* = 1$ , and equilibrium pollution stock  $S_1^* = \frac{ne_v}{b}$  (See figure 1a).



Assume that the critical emission level lies between the full and non-compliance emission stock level. This implies that  $q_1(\Delta S) < \hat{q}(\Delta S)$  with  $\Phi < 0$  and  $\hat{q}(\Delta S) < q_2(\Delta S)$  with  $\Phi > 0$ . In this case (46) implies that:

$$\text{if } S_1^* > \hat{S} > S_2^* \text{ then } \frac{d\dot{z}}{dz} \Big|_{z_1^*=1} > 0 \text{ and } \frac{d\dot{z}}{dz} \Big|_{z_2^*=0} > 0$$

Thus, the full compliance and non-compliance critical points are both unstable in the long run, implying that the ES equilibrium can not be monomorphic. Nevertheless the slow time dynamic system has an additional steady state, defined as  $z_3^*$ , indicating a potential polymorphic equilibrium. To characterize the equilibrium type of this steady state we define the Jacobian (linearization) matrix  $J$  around this point:

$$J = \begin{bmatrix} 0 & z(1-z)q'(\Delta S)(\Pi_N(e_o) - \Pi_C(e_L, F)) \\ n\{e_v - (q(\Delta S)e_L + (1-q(\Delta S))e_o)\} & n(1-z)q'(\Delta S)(e_L - e_o) \end{bmatrix}$$

For  $J$  the trace  $Tr(J) < 0$  while the determinant  $Det(J) > 0$  is positively defined, since we have  $\frac{d\hat{S}}{dz}, \frac{d\hat{S}}{dS} < 0$  and  $\frac{d\dot{z}}{dS} > 0$ . However, the discriminant  $\Delta = [Tr(J)]^2 - 4Det(J)$  can be positive, negative or even zero. Consequently the equilibrium in the partial compliance critical point  $z_3^*$  can be a stable focus, a stable proper node or even a stable improper node. Qualitative analysis of the phase diagram in Figure 1b suggests that  $z_3^*$  is a stable focus.

Therefore in this case compliance and the pollution stock fluctuate with the pollution stock converging to  $S_3^* = \frac{n\{z_3^*e_v + (1-z_3^*)[q(\Delta S)e_L + (1-q(\Delta S))e_o]\}}{b}$ .

Assume that the full compliance emission stock level  $S_1^*$  and non-compliance stock level  $S_2^*$  lie beneath the critical emission level. This implies that  $\hat{q}(\Delta S) > q_1(\Delta S) > q_2(\Delta S)$  and thus  $\Phi < 0$ . In this case (46) implies that:

$$\text{If } \hat{S} > S_1^* > S_2^* \text{ then } \frac{d\dot{z}}{dz} \Big|_{z_1^*=1} > 0 \text{ and } \frac{d\dot{z}}{dz} \Big|_{z_2^*=0} < 0$$

Thus, the ES equilibrium is monomorphic with no compliance  $z_2^* = 0$  and equilibrium pollution stock  $S_2^* = \frac{n\{q(\Delta S)e_L + (1-q(\Delta S))e_o\}}{b}$  (See figure 1a).

[Figure 1]

We summarize in the following proposition.

**PROPOSITION 4.** *Under an emission stock dependent auditing probability the slow time dynamic system can converge to either a polymorphic or monomorphic compliance equilibrium. If  $\hat{S} < S_1^* < S_2^*$  then there is full compliance and  $z_1^* = 1$  is the ES equilibrium. If  $S_1^* > \hat{S} > S_2^*$  then there is partial compliance and  $z_3^* \in (0, 1)$  is the ES equilibrium with fluctuation approach dynamics. If  $\hat{S} > S_1^* > S_2^*$  then there is no compliance and  $z_2^* = 0$  is the ES equilibrium.*

Furthermore since

$$\frac{dS}{dF} = -\frac{q(\Delta S)}{q'(\Delta S)(\Pi_N(e_o) - \Pi_C(e_L, F))} < 0 \quad (47)$$

the critical emission stock level declines with the level of the fine and the vertical isocline moves closer to the full compliance emission stock level in figure 1b. Moreover the polymorphic equilibrium point moves closer to the monomorphic steady state point A, implying that with the proper design of the non-compliance fine the regulator can induce a larger share of participating firms to comply.

5.1.4. *Case 4: Joint Dependence of Auditing Probability on Compliance Level and Pollution Stock*

Assume that the auditing probability depends jointly on the observed emission stock level and the proportion of complying firms. Under this assumption the slow time dynamic system (34) and (35) becomes:

$$\dot{z} = z(1-z)[q(\Delta S, z)(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (48)$$

$$\dot{S} = n\{ze_v + (1-z)(q(\Delta S, z)e_L + (1-q(\Delta S, z))e_o)\} - bS \quad (49)$$

As before the compliance replicator dynamics equation (44) has two equilibria, defined as  $z_1^* = 1$  and  $z_2^* = 0$ . There could exist however a third  $\dot{z} = 0$  isocline defined as:

$$z = l(S) : \Phi = [q(\Delta S, z)(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] = 0$$

The corresponding probabilities  $\hat{q}(\Delta S, z)$  of the  $(z, S)$  combinations lying along this isocline satisfy the equality  $\Phi = 0$ . Therefore every other combination outside the isocline switches the sign of the expression  $\Phi$ . Particularly, each combination located on the right of the isocline gives  $\Phi > 0$  since  $q(\Delta S, z) > \hat{q}(\Delta S, z)$ , while combinations located on the left give  $\Phi < 0$  since  $q(\Delta S, z) < \hat{q}(\Delta S, z)$ .

In this case the stability condition is defined as:

$$\frac{d\dot{z}}{dz} = (1-2z)\Phi + z(1-z)q'(\Delta S, z)(\Pi_N(e_o) - \Pi_C(e_L, F)) \quad (50)$$

While the replicator dynamics equilibria  $z_1^* = 1$  and  $z_2^* = 0$  correspond to two parallel isoclines respectively, the isocline of the additional critical point  $z_3^*$  corresponding to  $z = l(S)$  has a positive slope (see figure 2) since:

$$\frac{dz}{dS} = -\frac{\partial q(\Delta S, z)/\partial S}{\partial q(\Delta S, z)/\partial z} > 0$$

The partial derivatives reflect the firms' beliefs about the variability of the auditing probability's value due to changes in the levels of the state variables. For example, if polluting firms give high significance to increases in the observed emission stock level, then both the probability value and the relative partial derivative are expected to be high and vice versa.

Assume that the firms perceive that the auditing probability's value is more sensitive to compliance proportion changes, or alternatively that it is less sensitive to emission stock changes. In this case the isocline is flatter and cuts the emission stock dynamic isocline at a critical point located on the right of the full compliance steady state in figure 2b.

[Figure 2]

On the left of the isocline  $\Phi < 0$ , giving  $\frac{d\dot{z}}{dz} > 0$  for  $z_1^* = 1$ , while on the right of the isocline  $\Phi > 0$ , giving  $\frac{d\dot{z}}{dz} > 0$  for  $z_2^* = 0$ . As a consequence neither the full nor the non-compliance critical points are stable. In this case the ES equilibrium is the polymorphic critical point, equilibrium pollution stock level  $S_3^* = \frac{n\{z_3^*e_v + (1-z_3^*)[q(\Delta S, z_3^*)e_L + (1-q(\Delta S, z_3^*))e_o]\}}{b}$  and partial compliance  $z_3^* \in (0, 1)$ .

Assume that participating firms perceive that the auditing probability's value is more sensitive to emission stock changes, or alternatively that it is less sensitive to compliance proportion changes. In this case the isocline  $z(S)$  is steeper and cuts the emission stock dynamic isocline at a critical point located on the left of the full compliance steady state (figure 2a), which is an unfeasible area of combinations since it lies above the  $z_1^* = 1$  isocline. Consequently this critical point  $C$ , is not feasible. It is evident that on the right of the  $z(S)$  isocline the corresponding probability values are higher than the critical values  $\hat{q}(\Delta S, z)$ , giving  $\Phi < 0$ , thus full compliance or  $z_1^* = 1$  is the ES equilibrium.<sup>32</sup>

We summarize in following proposition:

**PROPOSITION 5.** *Under an auditing probability with joint dependence on compliance levels and pollution stock, the slow time dynamic system can converge either to a polymorphic or monomorphic equilibrium. The type of the prevailing ES equilibrium depends mainly on the slope and position of the  $z = \phi(S)$  isocline. The flatter the isocline is, the more likely it is that the ES equilibrium implies partial compliance. The more steeper the isocline is the more likely it is that ES equilibrium implies full compliance.*

Finally, the equilibrium outcome can be further affected through the value of the non-compliance fine, since it determines the position of the isocline  $z(S)$ . It has already been mentioned that the higher the non-compliance fine is, the more participating firms tend to comply with the agreement and control their emission production. Consequently the regulator can shift the isocline upwards, bringing the polymorphic equilibrium point closer to the monomorphic steady state, through the announcement of a sufficiently higher fine  $F$ . Therefore, it is logical to expect that under the proper design the slow time dynamic system can eventually converge to the full compliance critical point.

## 6. JOINT DEPENDENCE OF LEGISLATION PROBABILITY ON POLLUTION STOCK AND PARTICIPATION IN THE PUBLIC VA

Assume that the subjective probability of introducing legislation depends jointly on the deviation from the targeted emission stock level  $\Delta S$  and the participation proportion  $x$ . This implies that  $p = p(\Delta S, x)$ . Under this assumption the slow time dynamic system (23)-(25) is defined as:

$$0 = x(1-x)[p(\Delta S, x)(\Pi_N(e_o) - \Pi_L(e_L)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (51)$$

$$\dot{z} = z(1-z)[q(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (52)$$

$$\dot{S} = n\{x(\Delta S)[ze_v + (1-z)(qe_L + (1-q)e_o)] + (1-x(\Delta S))e_o\} - bS \quad (53)$$

The fast time dynamic equation (51) has three equilibria: the two critical points,  $x_1^* = 1$  and  $x_2^* = 0$ , and the third critical point  $x_3^*(\Delta S) \in (0, 1)$ , which sets

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<sup>32</sup>It should be noted that if participating firms perceive that changes in the observed emission stock level can not affect the auditing probability value, then the partial derivative  $\partial q(\Delta S, z)/\partial S$  is zero. Thus the auditing probability depends only on the proportion of complying firms and the isocline is parallel to the horizontal axis as in case 2. If participating firms perceive that the auditing probability value is not affected by changes in the compliance fraction, then the partial derivative  $\partial q(\Delta S, z)/\partial z$  is zero. The auditing probability depends only on the observed emission stock and the isocline is vertical to the horizontal axis as in case 3.

$\Omega = [p(\Delta S, x)(\Pi_N(e_o) - \Pi_L(e_L)) - (\Pi_N(e_o) - \Pi_v(e_v))]$  equal to zero, implies partial participation and depends on the deviation  $\Delta S$ . The stability condition for these equilibria is:

$$\frac{d\dot{x}}{dx} = (1 - 2x)\Omega + x(1 - x)p'(\Delta S, x)(\Pi_N(e_o) - \Pi_L(e_L)) \quad (54)$$

It can easily be seen that if the probability value is equal to the critical value  $\hat{p}(\Delta S, x)$  then  $\Omega$  is set equal to zero. Therefore the magnitude of the legislative probability  $p(\Delta S, x)$  compared to the critical probability value  $\hat{p}(\Delta S, x)$  affects the sign of the  $\Omega$  and the stability of the steady states. Obviously, if the legislative probability is higher than the critical value, then  $\Omega > 0$ . On the other hand, if the probability  $p(\Delta S, x)$  is lower than the critical value, then  $\Omega < 0$ . Under this definition it follows that:

$$\left. \frac{d\dot{x}}{dx} \right|_{x_1^*=1}, \left. \frac{d\dot{x}}{dx} \right|_{x_2^*=0} > 0 \text{ and } \left. \frac{d\dot{x}}{dx} \right|_{x_3^*} < 0$$

In this case the ES equilibrium indicates that there is partial participation in the environmental public VA. This happens because, in the case of full participation the condition  $p(\Delta S, 1 | \Delta S > 0) = 0$  holds, giving polluting firms a financial incentive not to participate in the agreement. On the other hand, in the case of non-participation, we have  $p(\Delta S > 0, 0) = 1$ , thus profit losses equal to  $\Pi_N(e_o) - \Pi_L(e_L)$  are certain, thus giving polluting firms a financial incentive to participate in the agreement's provisions since the  $\Pi_N(e_o) - \Pi_v(e_v)$  profit losses are preferable. Obviously, the particular fast time dynamic system converges to a polymorphic stable equilibrium  $x_3^*$ , implying that only a sub-group of individual polluting firms participate in the public VA in the long-run.

These findings can be summarized in the following proposition:

**PROPOSITION 6.** *Under a legislative probability that depends jointly on participation proportion and pollution stock the fast time dynamic system converges to a polymorphic equilibrium, implying partial participation  $x_3^*$  to the public VA.*

After taking the total derivative of  $\Omega$  with respect to the participation fraction  $x$  and the legislative emissions  $e_L$ , we obtain:

$$\frac{dx}{de_L} = \frac{p(\Delta S, x)\Pi_L'(e_L)}{p'(\Delta S, x)(\Pi_N(e_o) - \Pi_L(e_L))} > 0 \quad (55)$$

Obviously, the higher the legislative emissions  $e_L$  are, the higher the  $\Pi_N(e_o) - \Pi_L(e_L)$  profit losses are for polluting firms that decide not to participate. Therefore under the threat of a higher tax rate  $\tau$  or stricter emissions limit  $\bar{e}$ , individual firms have a financial incentive to participate in the public VA and the participation fraction increases. It can be seen that an increase in the legislative emissions  $e_L$ , shifts the polymorphic  $x_3^*$  steady state upwards, closer to the full participation critical point,  $x_1^* = 1$ . Therefore through proper design of the legislation mandate the regulator can induce the majority of polluting firms to participate in the agreement.

To analyze compliance and evolution of the pollution stock we analyze the slow time reduced system by substituting the stable critical point  $x_3^*(\Delta S)$  of the fast time system. In this case the slow time dynamic system is defined as:

$$\dot{z} = z(1-z)[q(\Pi_N(e_o) - \Pi_C(e_L, F)) - (\Pi_N(e_o) - \Pi_v(e_v))] \quad (56)$$

$$\dot{S} = n\{x_3^*(\Delta S)[ze_v + (1-z)(qe_L + (1-q)e_o)] + (1-x_3^*(\Delta S))e_o\} - bS \quad (57)$$

Under the assumption of a legislation probability jointly dependent on emission stock and participation proportion, the emissions stock dynamic isocline  $z = k(S)$  :  $\dot{S} = 0$  corresponds to a non-linear curve (see Figure 3) defined as:

$$k(S) = A(S)S - B(S) = \frac{bS}{nx_3^*(\Delta S)\{e_v - [qe_L + (1-q)e_o]\}} - \frac{x_3^*(\Delta S)\{qe_L + (1-q)e_o\} + (1-x_3^*(\Delta S))e_o}{x_3^*(\Delta S)\{e_v - [qe_L + (1-q)e_o]\}}$$

As previously the equilibrium solution  $(z^*, S^*)$  of the slow time dynamic system is highly dependent on the structure of the auditing probability  $q$ , which can either be fixed  $\bar{q}$  or dependent on the state variables of the problem. Based on the same conceptual framework developed in the previous section we conclude that:

*PROPOSITION 7. Under a participation dependent legislation probability and a fixed or state variables dependent auditing probability, the ES equilibrium implies partial participation in the public VA and full, non or partial compliance of the participating subgroup of firms.*

It should be noted that because of non-linearities the compliance - pollution system could be characterized by multiple equilibria and irreversibilities as shown in figure 3b with the final outcome crucially depending on initial conditions.

[Figure 3]

## 7. CONCLUDING REMARKS

The purpose of this paper is to analyze the long-run structure of a public VA and to specify certain characteristics that a public VA should possess in order to induce the majority of or even all polluting firms to participate in the VA and comply with its provisions. In this context we examine the evolution of participation in and compliance with the public VA, along with the evolution of the industrial pollution stock. Individual polluting firms' decisions about whether or not to participate in the agreement and furthermore whether or not to comply with it, were based on evolutionary processes of comparing expected profits associated with the different decisions, and were modelled by replicator dynamics operating in fast and slow time scales.

The main finding is that the structure of the legislation and auditing probability and the levels of legislative emissions and non-compliance fines are the main factors characterizing the ES outcomes. If the legislation probability is emission stock dependent, consequently fixed in fast time, then the equilibrium outcome is monomorphic implying either full or non-participation. Specifically, if the regulator announces a legislative probability higher than the critical value and commits to it, then all the firms participate in the agreement. On the other hand, if the legislation probability depends jointly on emission stock and participation proportion, the evolutionary sustainable equilibrium is polymorphic, implying partial participation. In this case the regulator can lead the equilibrium outcome closer to or even

achieve full participation through the proper design of the legislation mandate and particularly through the magnitude of the legislative emissions  $e_L$ . By committing to a fixed auditing probability, higher than a certain critical value, the regulator can achieve full compliance of the participating firms. The same outcome can be achieved under certain initial conditions when the auditing probability depends on specific state variables. The slow time dynamic system can alternatively converge to a partial compliance steady state, either monotonically or oscillating, and under certain conditions the compliance-pollution stock system is characterized by multiple equilibria and irreversibilities, which can be eliminated by commitment to specific auditing probabilities and/or fines. Finally, it is evident that the more complex the design of the public VA is, the less likely is the final achievement of the desired environmental target through full participation and compliance. Specifically, the more complex the structure of the legislation probability is, the more dependent the equilibrium outcome is on the initial conditions and the more likely it is that multiple equilibria and irreversibilities exist.<sup>33</sup> In conclusion it seems that commitment to legislation and auditing probabilities along with properly chosen legislative emission levels and non compliance fines are the main factors in order to induce the entire industrial sector to participate and comply with the public VA. If these conditions are not fulfilled partial participation, partial compliance and even fluctuation in the pollution stock are possible equilibrium outcomes of the public VA.

In this paper, although legislation and compliance probabilities have been endogenized, their cost to the regulator has not been taken into account. An interesting area of further research would be to include the cost of legislation and the cost of auditing in order to derive optimal auditing along with optimal thresholds for introducing legislation. In this context the replicator dynamics equations will be dynamic constraints to the regulators's optimization problem, combining bounded rationality and optimization notions.

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<sup>33</sup>Experience in Belgium and Denmark indicates that firms refused to accept the defined framework of VAs by pointing out its complexity and the reality that, with its application, it will approach traditional regulation (Sauer et al., 2001).

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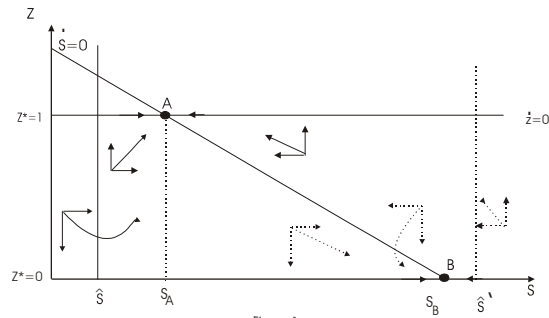


Figure 1a

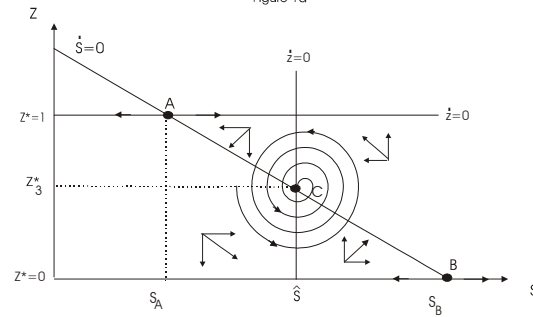


Figure 1b

**FIG. 1** Equilibrium with emission stock dependent auditing probability

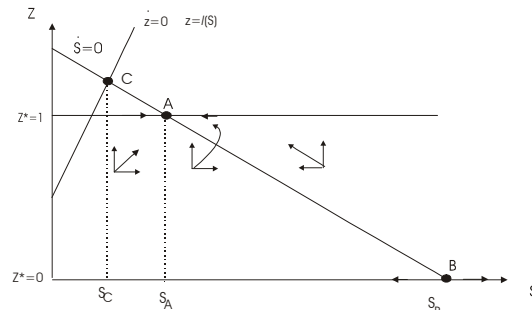


Figure 2a

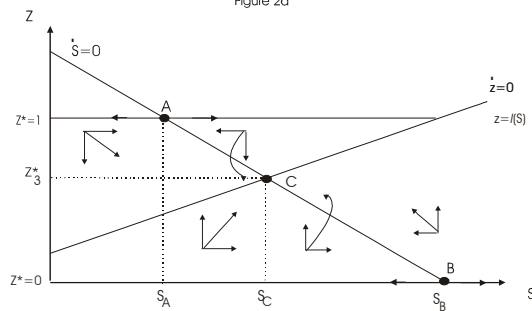
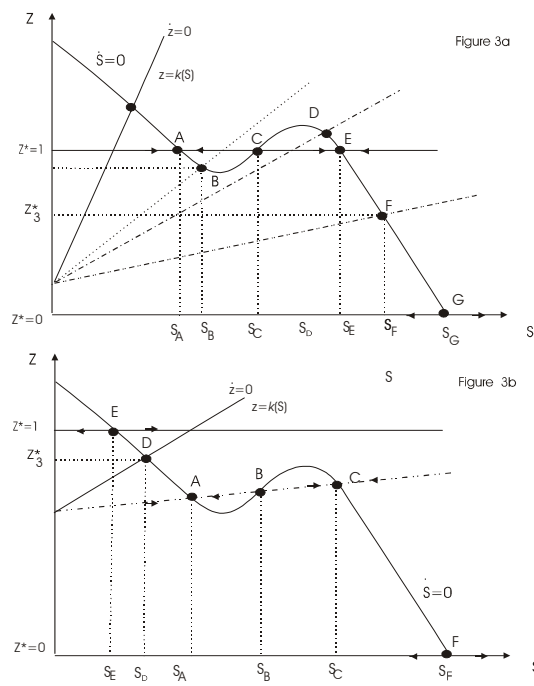


Figure 2b

**FIG. 2** Equilibrium with auditing probability jointly dependent on emission stock and compliance



**FIG. 3** Equilibrium with participation dependent legislation probability and state variables dependent auditing probability