Environmental Information Provision as a Public Policy Instrument

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Abstract

The present paper examines the use of information provision financed by the revenues of existing environmental taxation in the case of products that generate damages to the consumers of these products as well as an environmental externality. We show that when information provision is used alone it is welfare dominated by taxation, except if information can be provided costesly. The zero cost case, although not realistic, indicates the potential of information provision and leads us to examine the combined use of the two policies. We find that a policy regime that combines information provision and taxation dominates taxation in terms of welfare. This is because a uniform taxation levies a heavier than the optimal burden on the informed consumers and allows the uninformed consumer to partially free ride on the informed consumers voluntary actions. The combination of policies regime reduces this problem, allocating the effort of reducing the consumption of the environmentally damaging good more efficiently among consumers. Therefore, recycling of environmental tax revenues to finance information provision improves welfare.

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1 Introduction

Although incentive-based approaches, when implemented, have increased cost efficiency of pollution control, they have not been able to fully explore all potential gains. There is a number of reasons for this, including among other things the high administrative costs resulting from the number and the complexity of the substances to be controlled and the indirect costs that charges generate, such as those resulting from price increases and the interaction with the existing set of taxes. Continuing the research for improving the design of pollution control policies, economists have proposed alternative methods, such as voluntary compliance programs (see for example, Glachant (1994) and Boyd et al. (1998)), recycling of environmental tax revenues to finance the reduction of distortionary taxation (see for example Bovenberg and Goulder (1996) and Goulder (1995)) and information provision (Tietenberg and Wheeler (1998)), which could either broaden the scope or complement incentive-based instruments. The present paper contributes to this discussion by examining the use of information provision financed by the revenues of existing environmental taxation in the case of products that, in addition to environmental externalities, impose damages directly to the consumers of these goods.

Throughout their life cycle, products generate environmental impacts from extracting and processing raw materials; during production, assembly, and distribution; due to their packaging, use, and maintenance; and at the end of their life. Some types of products are also responsible for damages inflicted directly on the consumers of these goods. Examples of such products could include fruits and vegetables grown with the use of pesticides, which generate environmental externalities during production and also health problems to consumers from residues of pesticides; and household products such as paints that, apart from the environmental damages associated with their production and disposal, they release toxic chemicals that create health problems during their use.[1] In such cases, individuals have incentives to take actions to reduce the use of these types of products, by choosing, if available, other less harmful types of the product and by doing so they can also contribute to pollution control. To take action, individuals have to be aware of the damages associated with the particu-

[1] Tietenberg and Wheeler (1998) offer a number of examples of products and processes that generate damages to individual consumers and they suggest information provision as the appropriate policy.

lar type of the product, which is not always the case. Thus, when the government is called to intervene, the question arises as to whether it should use economic incentives or provide information or a combination of these two policies and if possible finance the information campaign with the tax revenues. The paper addresses this particular question.

In particular we examine the case of a differentiated product offered in two types produced by oligopolists competing in prices. During its lifetime this product generates an environmental externality, hereafter called external damages, and during its use imposes damages on the individual consumer of the product, hereafter called individual damages. The magnitude of both types of damages depends on the product type. We normalize assuming that one type of the product causes no damage at all, hereafter called the clean good, while the other type of the product, hereafter called the dirty good, generates both types of damages. We assume that consumers take into account the individual damages if they have information about these damages. However, consumers' knowledge (perception) of individual damages is imperfect. For simplicity, we assume that there are two groups of consumers, those that have perfect knowledge of the individual damages, and those that have no knowledge at all. The informed consumers substitute away from the dirty and towards the clean good. As a result, the relative price of the two goods changes, leading the uninformed consumers to purchase more of the dirty and less of the clean good. Since the dirty good generates also external damages, the uninformed consumers free ride on the informed consumers' efforts.

We consider three types of policy intervention. First, we assume that the regulator levies a tax on the dirty good. We find that the optimal tax consists of a Pigouvian tax, a part equal to the marginal individual damage, and a subsidy that corrects for the market distortion. We assume that the regulator does not have information on the true type of each consumer and thus, it levies a uniform tax on all consumers despite their type. Since the informed consumers purchase already less of the dirty good, the tax rate is higher for them and lower for the uninformed consumers relative to its optimal value. Since the dirty good generates external damages, the inability of the regulator to distinguish between the two groups of consumers exacerbates the free riding problem.

Second, we assume that the regulator decides to correct the information asymmetry problem by informing consumers about the negative effects of consuming the dirty good. Upon receiving the information, the previously uninformed consumers change their behavior. Since information provision is costly, we solve for the optimal fraction of consumers to be informed by the government. We find that the optimal fraction of consumers to which the government provides information is positively related to both individual and external damages and negatively to the market distortion. Information provision, although it avoids exacerbation of the free riding effect, it is often welfare dominated by taxation which is a more direct and thus powerful instrument to internalize external damages and to correct for the market distortion. This is not the case if information can be provided at low marginal cost and the fraction of consumers that are initially informed is low enough. In such case, the government informs all consumers and as a result the information asymmetry distortion is eliminated entirely, while the environmental externality is also reduced. In this case, information provision improves welfare over taxation.

Finally, we consider the case in which the regulator uses a combination of the two policy instruments, financing the cost of information provision with the tax revenues. We derive the optimal tax rate and the fraction of consumers targeted with information and we find that each component of this policy is lower relative to the case in which it is used alone. We find that individual consumers purchase less of the clean good under the combination of policy instruments relative to taxation, but since more consumers are informed, the aggregate demand for the clean good increases. Therefore, the combination of policies unambiguously shifts aggregate demand towards the clean good in the same time that it reduces, relative to the case of taxation, the free riding effect, allocating thus, more efficiently the effort to internalize the externality among consumers. Within a combined policy instruments regime, using the tax revenue to finance information provision could improve welfare.

Our work relates to the recent proposal by Tietenberg and Wheeler (1998) of expanding environmental policy into the provision of information. They review the different settings (household, consumption, employment and community) under which provision of information is important. They further assess the role of government in detecting environmental risks, assuring reliable information, and providing new channels –when the existing are not enough– through which information can be used. This study provides substantial support to our argument in favour of informative advertisement. An earlier study by Kennedy, Laplante and Maxwell (1994) examines the provision of information in the presence of environmental externalities. They show that when conventional environmental policies are not available, publicly provided information may improve welfare. However, when conventional policies are available, the situations that call for public provision of information are very limited. More recently Bansala and Gangopadhyay (2003) compare a tax on the dirty good with a subsidy on the clean good assuming environmentally aware consumers and imperfect market structure. They find that a subsidy reduces total pollution and enhances aggregate welfare, while a tax increases total pollution and may reduce aggregate welfare.

Section 2 presents the model establishing the need for regulatory intervention. Section 3 considers the case of taxation while Section 4 the case of information provision. Section 5 examines the case of combined policy instruments regime and Section 6 concludes the paper. The proofs of all the results presented in the paper are delineated in the Appendix.

2 The model

Assume a differentiated product, which generates individual and environmental damages. This product is offered in two types and the magnitude of both individual and external damages differs between them. For simplicity we assume that one type of the good causes no damage at all, hereafter called the clean good, while the other, called the dirty good, generates positive individual and external damages. We further assume that if consumers are informed, they take into account the individual damage. The utility that the informed consumer derives from the consumption of the dirty good decreases, while that from the clean good increases. One interpretation is that the latter effect captures the increased appreciation of the clean good as consumers learn about the bad characteristics of the dirty good.

Within this framework, the utility of the informed consumer is,

$$U(q_c, q_d, \theta_d, \gamma) = (a + \theta_c)\, q_c + (a - \theta_d) q_d - \frac{1}{2}\left(q_c^2 + q_d^2 + 2\gamma q_c q_d\right) \ ,$$

where q_j, $j = c, d$ are the quantities consumed of the clean and the dirty good respectively, θ_d, is the perception of the negative effect associated with the consumption of the dirty good, and θ_c represents the increase in the evaluation of the clean good. The consumer's utility maximization problem yields the direct demands for each good,

$$q_c = \frac{A_c + \Theta_c}{1-\gamma^2},\ q_d = \frac{A_d - \Theta_d}{1-\gamma^2}, \tag{1}$$

where, $A_j = (1-\gamma)a + \gamma p_k - p_j$, and $\Theta_j = \theta_j + \gamma\theta_k$, with $j,k = c,d$ and $j \neq k$.

However, only a fraction of the consumers are informed about the individual damage that the dirty good generates. For simplicity, we assume that there are two groups of consumers, those that have perfect knowledge of the negative effect associated with the dirty good, and those that have no knowledge at all. The informed consumers, which form μ fraction of the population, use the correct values of the parameters, that is, θ_c and θ_d, while the uniformed consumers set, $\theta_c = \theta_d = 0$ and thus, cannot distinguish between the two goods. The total demand for the clean and the dirty good is, $Q_d = \mu q_{di} + (1-\mu)q_{dn}$ and $Q_c = \mu q_{ci} + (1-\mu)q_{cn}$ respectively, where q_{di} and q_{dn} (q_{ci} and q_{cn}) are the quantities of the dirty (clean) good consumed by the informed and the uniformed consumer respectively. That is, $q_{ji} \equiv q_j(\theta_d, \theta_c)$ and $q_{jn} \equiv q_j(\theta_d = \theta_c = 0)$, where $j = c, d$.

The two types of the good are offered by two oligopolists competing in prices. For simplicity we assume that they both produce with the same constant marginal cost c. In the absence of government intervention, each firm j solves the following profit maximization problem,

$$\max_{p_j} \pi_j = (p_j - c)\left[\mu q_{ji} + (1-\mu) q_{jn}\right] \ ,$$

which yields the price reaction function,

$$p_j = \frac{1}{2}\left[(1-\gamma)a + c + \mu\left(\theta_j + \gamma\theta_k\right)\right] + \frac{1}{2}\gamma p_k \ ,$$

where $j,k = c,d$ and $j \neq k$.

The two reaction functions are solved for the equilibrium prices,

$$p_c^* = \frac{B + \mu\Theta_c^p}{4-\gamma^2} \ , \text{and} \ \ p_d^* = \frac{B - \mu\Theta_d^p}{4-\gamma^2} \ , \tag{2}$$

where, $B = (2+\gamma)\left[(1-\gamma)\,a + c\right]$ and $\Theta_j^p = (2-\gamma^2)\,\theta_j + \gamma\theta_k$, with $j,k = c,d$ and $j \neq k$. The fact that informed consumers have higher demand for the clean good allows the firm producing it to raise its price relative to the case that all consumers were uninformed, while the firm producing the dirty good decreases its price. The effect of informed consumers' presence on price is proportional to the fraction of the informed consumers and is positively related to both θs.

Substituting the values of p_c and p_d from equation (2) into the direct demand functions given in equation (1), yields the equilibrium demand of both groups of individuals for both types of the differentiated good, that is, q^*_{ci}, q^*_{cn}, q^*_{di} and q^*_{dn}, and the aggregate demand, Q^*_c and Q^*_d. Substituting the values of q_{ji} and q_{jn}, $j = c, d$ in the utility function, we get the inverse utility of the informed, $V_i(q_{ci}, q_{di}, \theta_c, \theta_d, \gamma)$, and the uniformed consumer, $V_n(q_{cn}, q_{dn}, \theta_c, \theta_d, \gamma)$, both evaluated at the true values of θs. We evaluate the uninformed consumer's utility at $\theta_j > 0$, in order to get its true and not the perceived value. Proposition 1 compares the case in which there are no informed consumers to the case in which a fraction μ of the consumers are informed. We denote with a superscript $*$ the equilibrium values of the variables when a fraction μ of consumers are informed and with a hat the equilibrium values when all consumers are uninformed.

Proposition 1 *In the absence of policy intervention, the response of the informed consumers to individual damages yields the following results:*
(i) $q^*_{ci} > \widehat{q}_c > q^*_{cn}$ *and* $q^*_{di} < \widehat{q}_d < q^*_{dn}$.
(ii) $Q^*_c > \widehat{Q}_c$ *and* $Q^*_d < \widehat{Q}_d$.
(iii) The informed consumer's utility exceeds that of the uninformed, $V^*_i - V^*_n$, *if the dirty good's damages on individuals are significant relative to the existing market distortion.*

The informed (uninformed) consumers' consumption of the clean good increases (decreases) and that of the dirty good decreases (increases) relative to the case that no consumer was informed. The direct effect of informed consumer's action to take into account individual damages dominates the indirect effect of the resulting price changes and thus, she consumes more of the clean and less of the dirty good relative to the case that all consumers were uninformed. The uninformed consumer's choices are influenced only by the indirect price effect and thus, she consumes more of the dirty and less of the clean good.

The aggregate output of the clean good increases and that of the dirty decreases. Although the uninformed individuals' consumption of the dirty (clean) good increases (decreases) responding to price changes, total consumption of the clean (dirty) good increases (decreases). The increase in the aggregate consumption of the clean good is positively related to both μ and θs, that is $\frac{\partial\left(Q_c^* - \widehat{Q}_c\right)}{\partial\mu} > 0$, and $\frac{\partial\left(Q_c^* - \widehat{Q}_c\right)}{\partial\theta} > 0$.

The shift in the informed consumer's demand away from the dirty and towards the clean good decreases the level of her individual damage, but in the same time has a negative effect on her utility since she consumes higher quantity of the more expensive good.[2] The more prominent the market imperfection is, for given values of θ_d and θ_c, the higher is the cost imposed on the informed consumer. In such case, the informed consumer's utility will be lower than the uninformed one, especially if the informed consumers represent a small fraction of the population.

Assuming that the government does not intervene directly to correct the market imperfection, there are two reasons calling for government's intervention: first, there is an information asymmetry problem, since only a fraction of the consumers take into account the individual damage and second an externality problem which cannot be eliminated even when all consumers are informed. In what follows we examine two types of regulatory response, namely a tax on the dirty good and information provision regarding the damages that consumption of the dirty good causes on consumers.

3 Optimal taxation

Assume that the regulator imposes a tax t on the dirty good. The firm that offers the dirty good solves the following profit maximization problem,

$$\max_{p_d} \pi_d = (p_d - t - c_d)\left[\mu q_{di} + (1-\mu) q_{dn}\right] .$$

The two firms' price reaction functions yield the prices as functions of the parameters and the tax rate,

$$p_c = \frac{B + \mu\Theta_c^p + \gamma t}{4-\gamma^2} \text{ , and } p_d = \frac{B - \mu\Theta_d^p + 2t}{4-\gamma^2} . \tag{3}$$

[2] If $\theta_d > \theta_c$, the uninformed consumers' total consumption of both goods exceeds that of the informed consumers. From equations (6) and (7) in the Appendix we derive, $(q_{ci}^* + q_{di}^*) - (q_{cn}^* + q_{dn}^*) = \frac{\Theta_c - \Theta_d}{1-\gamma^2} < 0$ if $\theta_d > \theta_c$.

Prices of both goods are higher relative to the unregulated case for all $t > 0$, and they are increasing in the tax rate. The price of the clean good increases because of the strategic complementarity in the choice variables. The overall effect of the informed consumers' action and the government's policy on p_c is positive but on p_d is ambiguous. The price of the dirty good could increase or decrease depending on the values of the parameters.

Substituting the values of p_c and p_d from equation (3) into the direct demand functions given in equation (1), yields the individual's demand as a function of the tax rate, that is, $q_{ji}(t)$ and $q_{jn}(t)$, from which we obtain, $Q_j(t)$, $j = c, d$. Substituting the values of $q_{ji}(t)$ and $q_{jn}(t)$ into the individual's utility function, yields the indirect utility of the informed, $V_i(t)$, and the uniformed $V_n(t)$, consumer both evaluated at the true values of θs. The regulator derives the optimal tax rate from the social welfare maximization problem, $\max_t W = \mu V_i(t) + (1-\mu)V_n(t) - cQ_c(t) - (c+d)Q_d(t)$, taking into account the externality problem, where d is the marginal external damage. Lemma 1 gives the optimal tax rate.

Lemma 1 *The optimal tax rate is $t^* = t^d - s$, consisting of a tax $t^d = d + (1-\mu)\theta_d$ to correct for the externality and the individual damage, and a subsidy, s, to correct for the market imperfection. Both parts of the optimal tax take into account the information asymmetry among consumers.*

Since the government has only one policy instrument, the tax consists of three components each addressing one of the existing distortions. If there was no market imperfection, that is, $\gamma = 1$, and all consumers were informed, t^* collapses to the Pigouvian tax, that is, if $\gamma = 1$ and $\mu = 1$ then $t^* = d$. In the absence of external damages and market imperfections, the tax corrects for the individual damages, by imposing a cost $t^* = (1-\mu)\theta_d$ to all consumers regardless of whether they are informed or not. In the absence of all damages, the government corrects for the market imperfection by subsidizing the production of the dirty good, that is, $t^* = -s$. Whether at the equilibrium a tax or a subsidy is needed depends on the relative strength of the distortions.

In what follows we concentrate in the case where the market distortion is not prominent, i.e. γ is high enough, and thus, we have a positive tax. Proposition 2 summarizes the results in this case. We denote with a superscript t the equilibrium values of the variables in the case of taxation.

Proposition 2 *A positive tax on the dirty good has the following effects on the equilibrium values:*
(i) $q_{ci}^t - q_{ci}^* = q_{cn}^t - q_{cn}^* > 0$.
(ii) $Q_c^t > Q_c^*$, *and* $Q_d^t < Q_d^*$.
(iii) A uniform tax on the dirty good ignores the asymmetric behavior of consumers, allowing the uninformed consumers to partially free ride on the informed consumers' voluntary actions. As a consequence, the informed consumer's utility decreases as a result of taxation, that is, $V_i^t < V_i^*$.

Taxation increases the clean good's consumption of both groups of individuals and thus the aggregate consumption relative to the benchmark case of no policy intervention. Viewed as a percentage increase over the unregulated equilibrium, the uninformed consumer's demand for the clean good increases by a higher percentage relative to the demand of the informed consumer, that is, $\frac{q_{ci}^t - q_{ci}^*}{q_{ci}^*} < \frac{q_{cn}^t - q_{cn}^*}{q_{cn}^*}$. This is because informed individuals consume more clean good even before taxation.

Apart from the usual problems related to the fact that one instrument addresses three distortions, there is another problem specific to this case raising from the fact that the regulator is unable to distinguish between the two groups of individuals. Since the tax is uniform, the informed consumers pay, as part of the total charge, $(1-\mu)\theta_d$ intended to provide a signal to decrease their consumption of the dirty good because of individual damages, although they have already taken into account individual damages. Similarly, the uninformed consumers pay only $(1-\mu)\theta_d$, instead of θ_d, which would be the optimal price signal (equal to their marginal damage). Therefore, at the equilibrium, the informed consumers purchase less than the full information optimal quantity of the dirty good while the uninformed consume more than the optimal quantity. Since the dirty good imposes external damages in addition to individual damages, the inability of the regulator to distinguish between the two groups of individuals allows uninformed consumers to partially free ride. A uniform tax is not the right instrument to address the individual damage problem. For this reason, the next Section explores information provision as an alternative policy instrument.

4 Information provision

Assume that the regulator decides to improve the situation by informing consumers about the negative effects of consuming the dirty good. The regulator targets a fraction ϕ of consumers who, upon receiving the information, take into account the individual damage that the consumption of the dirty good imposes on them. Since the government does not know the true type of each individual (informed or uninformed) it targets all consumers. We assume that information reaches consumers in each group with equal probability. Therefore, the provision of information affects the aggregate demand for both goods. More precisely, upon information reaching a fraction ϕ of consumers, the aggregate demand of the dirty and the clean good becomes, $Q_d = mq_{di} + (1-m)q_{dn}$ and $Q_c = mq_{ci} + (1-m)q_{cn}$ respectively, where $m = \mu + (1-\mu)\phi$ and $1 - m = (1-\mu)(1-\phi)$. The cost of reaching a fraction ϕ of consumers is, $A(\phi) = \omega\frac{\phi^2}{2}$, where ω is the slope of the marginal cost of advertisement.

The two reaction functions resulting from the duopolists' profit maximization problems are solved for the prices,

$$p_c = \frac{B + m\Theta_c^p}{4-\gamma^2}\ \text{, and}\ \ p_d = \frac{B - m\Theta_d^p}{4-\gamma^2}\ . \qquad (4)$$

Contrary to the case of taxation, information provision does not have any direct effect on prices. The policy intervention has only an indirect effect on prices, through the change in both goods' demand. As a result of information provision, the demand of an additional $\phi(1-\mu)$ fraction of consumers, who become informed, changes. Comparison of equation (4) to (2) reveals that the firm producing the dirty good responds to this development by decreasing its price, while the firm producing the clean good increases its price relative to the unregulated equilibrium.[3] Thus, while the tax has a positive effect on both prices, in the case of information provision the prices adjust to the change in consumers' behavior yielding a higher price for the clean good but a lower price for the dirty good.

Substituting the values of p_c and p_d from equation (4) into the direct demand functions given in equation (1), yields the individual's demand as a function of ϕ, that is, $q_{ji}(\phi)$ and $q_{jn}(\phi)$, from which we obtain, $Q_j(\phi)$, $j = c, d$. Substituting the values of $q_{ji}(\phi)$ and $q_{jn}(\phi)$ into the individual's utility function, yields the indirect

[3] From the definition of m we have that $m > \mu$, $\forall\ \mu < 1$ and $\phi > 0$.

utility of the informed, $V_i(\phi)$, and the uniformed $V_n(\phi)$, consumer both evaluated at the true values of θs. The optimal fraction of consumers to which the regulator provides information is derived from the social welfare maximization problem, $\max_{\phi} W = mV_i(\phi) + (1-m)V_n(\phi) - cQ_c(\phi) - (c+d)Q_d(\phi) - \omega\frac{\phi^2}{2}$. Lemma 2 gives the optimal ϕ.

Lemma 2 *The optimal fraction of consumers to which the regulator provides information ϕ^*, is*
(i) inversely related to the initial fraction of the informed consumers, and $\phi^* = 0$ for $\mu = 1$,
(ii) increasing in the marginal external damage,
(iii) decreasing as the market imperfection gets stronger for $\theta_d > \theta_c$. The effect of market imperfection on ϕ^* diminishes as θ_c gets closer to θ_d and vanishes when $\theta_c = \theta_d \neq 0$, and
(iv) is increasing in both θs..

Although ϕ^* depends on the marginal external damage and the market imperfection, information provision concerns primarily individual damages and for that reason, the regulator does not provide any information, that is, $\phi^* = 0$ when either all consumers are informed ($\mu = 1$), or the marginal individual damage is zero ($\theta_c = \theta_d = 0$). Contrary to taxation, information provision is a policy instrument that only indirectly deals with the external damage and the market imperfection. It spreads existing "good behavior" among the population instead of providing economic incentives by changing the relative prices directly. In this framework, information provision could be called a "focused" or "soft" policy instrument.

Proposition 3 summarizes the effect of information provision on the choice variables and welfare. We denote with a superscript ϕ the equilibrium values of the variables in the case of information provision.

Proposition 3 *When the market imperfection is not prominent, such that both ϕ^* and t^* are positive, information provision has the following effects on the equilibrium values of output and welfare:*
(i) $q_{c\nu}^{\phi} < q_{c\nu}^{*} < q_{c\nu}^{t}$ and $q_{d\nu}^{\phi} > q_{d\nu}^{*} > q_{d\nu}^{t}$, $\nu = i, n$,

(ii) $Q_c^{\phi} > Q_c^*$ *and* $Q_d^{\phi} < Q_d^*$*,*
(iii) $Q_c^t > Q_c^{\phi}$ *and* $Q_d^t < Q_d^{\phi}$*, if* $t > (1-\mu)\frac{\Theta_c^p}{\gamma}\phi$*, and*
(iv) $W^t > W^{\phi}$ *if the cost of information is sufficiently high, while* $W^{\phi} > W^t$ *if the cost of information is low and information is provided to the entire population.*

Since the regulation does not directly affect prices, the only effect on the quantities demanded is the enhancement of the indirect price effect discussed in Section 2. Thus, information provision, through the increase in the number of informed consumers, yields a decrease (increase) of individual consumer's demand for the clean (dirty) good relative to the unregulated case, that is, $q_{c\nu}^{\phi} < q_{c\nu}^*$ and $q_{d\nu}^{\phi} > q_{d\nu}^*$. The increase in the number of informed consumers -who consume a higher quantity of the clean good relative to the uninformed consumers- more than compensates the decrease in the quantities of the clean good demanded by both types of individuals. Aggregate demand for the clean (dirty) good increases (decreases) relative to the unregulated equilibrium, that is, $Q_c^{\phi} > Q_c^*$ and $Q_d^{\phi} < Q_d^*$. Compared to the case of taxation, aggregate quantity demanded of the clean good could be either higher or lower. When external and individual damages are more prominent relative to the market imperfection, which is the case examined here, $Q_c^t > Q_c^{\phi}$ and $Q_d^t < Q_d^{\phi}$.

Information provision improves welfare relative to the unregulated case, $W^{\phi} > W^*$. However, this improvement is less than the welfare improvement obtained by the tax policy when market imperfections are not too prominent relative to the external and individual damages. The dominance of taxation over the "soft" policy instrument relates to the fact that taxation deals with all three imperfections in a more direct way. Therefore, when the market imperfection is not too prominent, the tax rate is high and thus, taxation is more effective than information provision in shifting consumption away from the dirty and towards the clean type of the differentiated good, yielding thus a higher level of welfare. Information provision avoids the problem of distributing the burden in favor of uninformed consumers, associated with taxation, and for this reason improves welfare. However, since it does not provide any direct price signal is not as effective as taxation, at least when information cost is sufficiently high.

The impact of information provision changes drastically if information can be provided almost costlesly. In such case, the government could provide information to the entire population, and thus, all consumers take into account individual damages.

The information asymmetry distortion is eliminated entirely, while the environmental externality is to a large extent internalized indirectly. The welfare difference $W^{\phi}_{|\omega=0,\ \phi=1} - W^{t}$, is higher the lower is the fraction of the informed consumers before the regulatory intervention, and the higher is θ_c. Although the zero cost case is an extreme case, it indicates the potential importance of information provision when information cost is low enough. In reality the cost of information provision could be covered by the revenues of an environmental tax, if the two policy instruments were used jointly, a case we examine in the next Section.

5 Combination of taxation and information provision

The discussion in the previous Sections indicates the inability of either taxation or information provision alone to deal efficiently and effectively with the existing distortions. The present Section examines the case in which the regulator uses a combination of the two policy instruments, namely a tax on the dirty good, t_σ, and information provision, ϕ_σ. The cost of providing information could be partially or totally financed by the tax revenues.[4] As in the case of information provision, considered in the previous Section, the aggregate demand of the dirty and the clean good is, $Q_d = m_\sigma q_{di} + (1 - m_\sigma)q_{dn}$ and $Q_c = m_\sigma q_{ci} + (1 - m_\sigma)q_{cn}$ respectively, where $m_\sigma = \mu + (1 - \mu)\phi_\sigma$ and $1 - m_\sigma = (1 - \mu)(1 - \phi_\sigma)$.

The reaction functions resulting from the duopolists' profit maximization problems are solved for the prices,

$$p_c = \frac{B + m_\sigma \Theta^p_c + \gamma t_\sigma}{4 - \gamma^2} \text{ , and } p_d = \frac{B - m_\sigma \Theta^p_d + 2t_\sigma}{4 - \gamma^2} \ . \tag{5}$$

Despite the strong similarities of the above expressions to those in (3) and (4), the prices under the combined policies scenario are different from those in the case of taxation and information provision. Although the effect of government's intervention is clearly positive on the price of the clean good, the price of the dirty good could be lower or higher than the price in the unregulated case, depending on the values of m_σ and t_σ. In fact, p_d increases the higher is the tax rate but decreases the higher is m_σ.

[4] We choose not to impose a balanced budget constraint, in order to avoid further complication of the analysis.

Substituting the values of p_c and p_d from equation (5) into the direct demand functions given in equation (1), yields $q_{ji}(t_\sigma, \phi_\sigma)$ and $q_{jn}(t_\sigma, \phi_\sigma)$, from which we obtain, $Q_j(t_\sigma, \phi_\sigma)$, $j = c, d$. Following the same steps as in both previous cases, we define social welfare as a function of the government's choice variables. The solution of the social welfare maximization problem yields the optimal t_σ and ϕ_σ which are presented in Lemma 3.

Lemma 3 *In the case that the government implements a policy that combines a tax on the dirty good with information provision, we have* $t^*_\sigma = t^* - G\phi^*_\sigma$*, where* G *is defined in the Appendix. The value of the tax is lower relative to the case that taxation is used alone,* $t^*_\sigma < t^*$*, and the fraction of consumers to whom the government provides information is lower relative to the case that information provision is used alone,* $\phi^*_\sigma < \phi^*$*.*

The optimal tax rate under the tax policy equals the weighted sum of the two policy instruments under the combined policy instruments case, and $t^*_\sigma = t^*$ only if $\phi^*_\sigma = 0$. Since information provision increases the number of informed individuals, the required tax rate is lower. The increase in the fraction of informed individuals diminishes the problem of misallocating the burden of reducing the externality, a problem we have associated with taxation in Section 3. Proposition 4 summarizes the effect of the combined policy on the choice variables and welfare. We denote with a superscript σ the equilibrium values of the variables in this case.

Proposition 4 *When the market imperfection is not prominent, the combined policy instruments regime has the following effects on the equilibrium values of output and welfare:*
(i) $q^\phi_{c\nu} < q^*_{c\nu} < q^\sigma_{c\nu} < q^t_{c\nu}$, $\nu = i, n$, *if* $t^*_\sigma > \frac{(1-\mu)\Theta^q_c}{\gamma}\phi^*_\sigma$,
(ii) $Q^\sigma_c > Q^t_c$ *and* $Q^\sigma_d < Q^t_d$, $\forall\ \phi > 0$,
(iii) The combined policy instruments regime improves welfare.

The combined policy instruments regime shifts the individuals' demand more towards the clean good relative to both the unregulated and the information provision cases, but is less aggressive than the taxation case. Although individual consumers purchase less of the clean good under the combined policy instruments regime than under taxation, the fact that more consumers are informed results in an increase of the aggregate demand for the clean good. Therefore, the combination of policies unambiguously shifts aggregate demand towards the clean good in the same time that it reduces, relative to the case of taxation, the burden of informed consumers in reducing the externality. Since the combined policy instruments regime allocates the effort to internalize the externality among individuals more efficiently, it is also welfare superior. Therefore, the government can improve welfare if it uses part of the tax revenues to finance the provision of information.

6 Conclusions

In this paper we examine information provision as an alternative policy instrument to complement existing environmental taxation in the case that consumption of goods generates damages to individual consumers as well as an environmental externality. We show that when information provision is used alone, although it constitutes a welfare improvement over the case of no intervention, it is welfare dominated by taxation, except if information cost is low enough. However, a policy regime that combines information provision and taxation dominates taxation in terms of welfare. This is because a uniform taxation levies a heavier than the optimal burden on the informed consumers and allows the uninformed consumer to partially free ride on the informed consumers voluntary actions. The combination of policy instruments regime allocates the effort of reducing the consumption of the environmentally damaging good more efficiently among consumers. Therefore, recycling of environmental tax revenues to finance information provision can improve welfare.

Since the area of information provision has not yet received appropriate attention in the literature, there are many directions in which the current analysis can be extended. Since it is clear that the firm producing the clean good has incentives to provide information to consumers, the private actions should also be examined in order to decide whether private incentives are compatible with the efficient outcome or they lead to lower or excessive action. Another direction in which the analysis can be extended is to consider the problem in a dynamic framework, where the fraction ϕ of consumers acquiring information evolves over time as a function of the accumulated information provided by the government.

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8 Appendix

8.1 Proof of Proposition 1.

Proof. (i) Substituting the values of p_c and p_d from equation (2) into the direct demand functions given in equation (1), yields the equilibrium level of output,

$$q_{ci}^* = \frac{C + (4-\gamma^2)\,\Theta_c - \mu\Theta_c^q}{(1-\gamma^2)(4-\gamma^2)}, \; q_{cn}^* = \frac{C - \mu\Theta_c^q}{(1-\gamma^2)(4-\gamma^2)}. \tag{6}$$

$$q_{di}^* = \frac{C - (4-\gamma^2)\,\Theta_d + \mu\Theta_d^q}{(1-\gamma^2)(4-\gamma^2)}, \; q_{dn}^* = \frac{C + \mu\Theta_d^q}{(1-\gamma^2)(4-\gamma^2)}, \tag{7}$$

where, $C = (2+\gamma)(1-\gamma)(a-c)$ and $\Theta_j^q = 2\theta_j + \gamma(3-\gamma^2)\theta_k$, with $j,k = c,d$ and $j \neq k$. There are two effects on the equilibrium outputs. A direct effect, representing the attempt of the informed consumers to take into account the individual damage, due to which the informed consumers' demand for the clean (dirty) good increases (decreases) (second term in q_{ci} and q_{di}). An indirect effect due to the change in both prices (last term in q_{ci}, q_{di}, q_{cn} and q_{dn}). The direct dominates the indirect effect on the informed consumer's demand, and thus, at the equilibrium, the informed consumer purchases more of the clean and less of the dirty good relative to the case that all

consumers were uninformed, that is, $q_{ci}^* > \widehat{q}_c$ and $q_{di}^* < \widehat{q}_d$.[5] On the contrary, the uninformed consumer, responding to price changes only, purchases more of the dirty and less of the clean good, that is, $\widehat{q}_d < q_{dn}^*$ and $\widehat{q}_c > q_{cn}^*$. The difference between the consumption of the informed and uninformed consumers depends on the values of θs, that is, $q_{ci}^* - q_{cn}^* = \frac{\Theta_c}{1-\gamma^2} > 0$, and $q_{di}^* - q_{dn}^* = -\frac{\Theta_d}{1-\gamma^2} < 0$.
(ii) The total output for each good is,

$$Q_c^* = \frac{C + \mu\Theta_c^p}{(1-\gamma^2)(4-\gamma^2)}, \; Q_d^* = \frac{C - \mu\Theta_d^p}{(1-\gamma^2)(4-\gamma^2)} \; . \tag{8}$$

The aggregate equilibrium quantity of the clean good increases and that of the dirty good decreases relative to the case that all consumers were uninformed, that is, $Q_c^* > \widehat{Q}_c$ and $Q_d^* < \widehat{Q}_d$. The change is proportional to the fraction of the informed consumers and is positively related to both θ_c and θ_d. In the symmetric case, that is, $\theta_c = \theta_d$, we have $\Theta_c^q = \Theta_d^q$, and thus, the increase in Q_c^* equals the decrease in Q_d^*.
(iii) Substituting the values of q_{ji} and q_{jn}, $j = c, d$ from equations (6) and (7) in the utility function, we get the inverse utility of the informed, $V_i(q_{ci}, q_{di}, \theta_c, \theta_d, \gamma)$, and the uniformed consumer, $V_n(q_{cn}, q_{dn}, \theta_c, \theta_d, \gamma)$, both evaluated at the true values of θs. The difference between the indirect utility of the informed and the uninformed consumer is,

$$V_i^* - V_n^* = \frac{(4-\gamma^2+4\mu)\left(\theta_c^2+\theta_d^2\right) + 2\gamma\left[(4-\gamma^2)+2(3-\gamma^2)\mu\right]\theta_c\theta_d - 2(1-\gamma)(\theta_d-\theta_c)B}{2(4-5\gamma^2+\gamma^4)}. \tag{9}$$

The nominator is positive when the market structure approaches perfect competition, that is, as γ approaches 1, and for small differences between the values of θ_d and θ_c. In either case the value of the last, negative term on the nominator diminishes. The difference between the indirect utility of the informed and the uninformed consumer increases in both θ_d and θ_c when γ approaches 1. ■

8.2 Proof of Lemma 1

Proof. Substituting the values of p_c and p_d from equation (3) into the direct demand functions given in equation (1), yields the individual's demand as a function of the

[5] Note that, $\left(4-\gamma^2\right)\Theta_j - \mu\Theta_j^q = \left[\left(2-\gamma^2\right)+2(1-\mu)\right]\theta_j + \gamma\left[1+\left(3-\gamma^2\right)(1-\mu)\right]\theta_k > 0$, with $j, k = c, d$ and $j \neq k$.

tax rate,

$$q_{ci} = \frac{C + (4-\gamma^2)\Theta_c - \mu\Theta_c^q + \gamma t}{(1-\gamma^2)(4-\gamma^2)}, \; q_{cn} = \frac{C - \mu\Theta_c^q + \gamma t}{(1-\gamma^2)(4-\gamma^2)}, \tag{10}$$

$$q_{di} = \frac{C - (4-\gamma^2)\Theta_d + \mu\Theta_d^q - (2-\gamma^2)t}{(1-\gamma^2)(4-\gamma^2)}, \; q_{dn} = \frac{C + \mu\Theta_d^q - (2-\gamma^2)t}{(1-\gamma^2)(4-\gamma^2)}. \tag{11}$$

The higher the tax rate is, the more clean and the less dirty good is consumed by both groups of consumers (last term in the expressions in equation (10) and (11)). The difference between the consumption of the informed and uninformed consumers remains the same as the unregulated case. The total output for each good is,

$$Q_c = \frac{C + \mu\Theta_c^p + \gamma t}{(1-\gamma^2)(4-\gamma^2)}, \; Q_d = \frac{C - \mu\Theta_d^p - (2-\gamma^2)t}{(1-\gamma^2)(4-\gamma^2)}. \tag{12}$$

On the aggregate, the higher the tax rate is, the more of the clean and the less of the dirty good is consumed.

Substituting the values of q_{ji} and q_{jn}, $j = c, d$ from equation (10) and (11) into the consumer's utility function, we get the indirect utility of the informed consumer, $V_i(q_{ci}, q_{di}, \theta_c, \theta_d, \gamma, t)$, and the uniformed consumer, $V_n(q_{cn}, q_{dn}, \theta_c, \theta_d, \gamma, t)$, both evaluated at the true values of θs.[6] That is, the regulator evaluates both the informed and the uninformed consumers' utility taking into account the true value of the damage that the consumption of the dirty good imposes on them and the increased appreciation of the clean good. The regulator also accounts for the externality generated by the consumption of the dirty good by subtracting dQ_d from the social welfare, where d is the marginal external damage. The regulator derives the optimal tax rate from the social welfare maximization problem,

$$\max_t W = \mu V_i(t) + (1-\mu)V_n(t) - cQ_c(t) - (c+d)Q_d(t),$$

The solution of the above maximization problem yields,

$$t^* = t^d - \frac{C - (4 - 3\gamma^2 + \gamma^4)(d + \theta_d) - \gamma\left[4 - (1+\mu)\gamma^2\right]\theta_c}{4 - 3\gamma^2}, \tag{13}$$

[6]It should be noted that the difference between the indirect utility of the informed and the uninformed consumer decreases in the tax rate. The difference under taxation is $V_i(t) - V_n(t) = V_i^* - V_n^* - 2\Theta_d^p t$, where $V_i^* - V_n^* > 0$, has been defined in (9).

where, $t^d = d+(1-\mu)\theta_d$ is the optimal tax rate in the absence of market imperfections. The optimal tax, apart from the environmental externality, also corrects for the information asymmetry and the market imperfection. If the dirty good caused no damage at all, the tax (subsidy) corrects for the market imperfection. In the case of equal marginal cost of production that we examine, correction of the market imperfection requires a subsidy, that is, the tax rate without any damages is, $t_{|\theta_c=\theta_d=d=0} = \frac{-C}{4-3\gamma^2} < 0$.[7] The second term on the nominator in equation (13) captures the marginal social cost (individual and external) of the dirty good which should be added to the private marginal cost of producing the dirty good when the damages are present. That is, when the dirty good generates individual and external damages, the second term in C, as defined in footnote (7), becomes $-(4-3\gamma^2+\gamma^4)(a-c_d-d-\theta_d)$. The third term on the nominator in equation (13) captures the benefits from the increase appreciation of the clean good which should be added to the net value derived from the consumption of the clean good. That is, the first term in $-C$, as defined in footnote (7), becomes $2\gamma\left[(2-\gamma^2)(a-c_c)+\left(2-\frac{1+\mu}{2}\gamma^2\right)\theta_c\right]$, which for $\mu=1$, reduces to $2\gamma(2-\gamma^2)(a+\theta_c-c_c)$. ■

8.3 Proof of Proposition 2

Proof. (i) The first result is evident from direct comparison of equations (10), (11) to (6) and (7). From the same equations we also get that $q_{ci}^t - q_{cn}^t = q_{ci}^* - q_{cn}^*$, which yields, $q_{ci}^t - q_{ci}^* = q_{cn}^t - q_{cn}^* > 0$. Since $q_{ci}^* > q_{cn}^*$, we then have $\frac{q_{ci}^t - q_{ci}^*}{q_{ci}^*} < \frac{q_{cn}^t - q_{cn}^*}{q_{cn}^*}$.

(ii) Direct comparison of equation (12) to (8) reveals the result.

(iii) Substituting the optimal value of the tax rate from equation (13) into equations (10), (11) and (12) we get the equilibrium quantities of the two goods,

$$q_{ci}^t = \frac{D_c+(2-\gamma^2)\theta_c+(1-\mu)\Theta_c^{qt}}{(1-\gamma^2)(4-3\gamma^2)},\ q_{cn}^t = \frac{D_c+\gamma^2\theta_c-\mu\Theta_c^{qt}}{(1-\gamma^2)(4-3\gamma^2)}, \tag{14}$$

$$q_{di}^t = \frac{D_d-\gamma(3-2\gamma^2)\theta_c-(1-\mu)\Theta_d^{qt}}{(1-\gamma^2)(4-3\gamma^2)},\ q_{dn}^t = \frac{D_d-\gamma(2-\gamma^2)\theta_c+\mu\Theta_d^{qt}}{(1-\gamma^2)(4-3\gamma^2)}, \tag{15}$$

[7] If $c_c \neq c_d$ we get, $-C = 2\gamma(2-\gamma^2)(a-c_c)-(4-3\gamma^2+\gamma^4)(a-c_d)$. In this case $t_{|\theta_d=\theta_c=d=0} \lesseqgtr 0$ depending on the duopolists' marginal cost. If $c_d \gg c_c$ a positive tax on the D good would be required. The subsidy gets its highest value in the case that $\gamma=0$, which equals the subsidy to monopoly, and approaches zero as γ approaches 1, that is as we move towards perfect competition.

where, $D_c = (2-\gamma^2)\left[(a-c)-\gamma(a-c-d-\theta_d)\right]$, $\Theta_c^{qt} = (2-\gamma^2)\,\theta_c + \gamma\,(4-3\gamma^2)\,\theta_d$, $D_d = (2-\gamma^2)^2\,(a-c-d-\theta_d) - \gamma(3-2\gamma^2)(a-c)$ and $\Theta_d^{qt} = \gamma\,(3-2\gamma^2)\,\theta_c + (4-3\gamma^2)\,\theta_d$. The superscript t indicates equilibrium values under the tax policy. The informed consumer consumes more of the clean and less of the dirty good relative to the uninformed consumer, as we discussed above. The differences remain the same as under the unregulated equilibrium, that is, $q_{ci}^t - q_{cn}^t = \frac{\Theta_c}{1-\gamma^2} = q_{ci}^* - q_{cn}^* > 0$, and $q_{di}^t - q_{dn}^t = -\frac{\Theta_d}{1-\gamma^2} = q_{di}^* - q_{dn}^* < 0$. Assuming a positive tax, the consumption of the clean good increases and that of the dirty good decreases for both groups of consumers, that is, $q_{c\nu}^t > q_{c\nu}^*$ and $q_{d\nu}^t < q_{d\nu}^*$, $\nu = i, n$.[8]

The aggregate equilibrium output for the clean and dirty good are,

$$Q_c^t = \frac{D_c + \left[\gamma^2 + 2\,(1-\gamma^2)\,\mu\right]\theta_c}{(1-\gamma^2)\,(4-3\gamma^2)},\; Q_d^t = \frac{D_d - \gamma\left[(2-\gamma^2) + (1-\gamma^2)\,\mu\right]\theta_c}{(1-\gamma^2)\,(4-3\gamma^2)}\;. \tag{16}$$

The regulator, in trying to redirect the production away from the dirty good and towards the clean good, by applying a uniform tax, it misallocates the required effort between informed and uninformed consumers.

From equations (12) to (8) we get, $Q_c^t - Q_c^* = \frac{\gamma t}{(1-\gamma^2)(4-\gamma^2)} > 0$, and $Q_d^t - Q_d^* = -\frac{\left(2-\gamma^2\right)t}{(1-\gamma^2)(4-\gamma^2)} < 0$, $\forall\; t > 0$. Substituting the values of q_{ji}^t and q_{jn}^t, $j = c, d$ from equations (10) and (11) into the utility function we derive the indirect utility for both groups of consumers. We then derive the difference between the indirect utility of the informed and uninformed consumer in the unregulated case and the case of taxation ,

$$\begin{aligned} V_i^t - V_i^* &= \frac{(4-3\gamma^2)\,t^* - 2\,(4-\gamma^2)\left[(2-\gamma^2)\,d + (2-\gamma-\gamma^2)\,c + (1-\mu)\Theta_d^p\right]}{2\,(4-\gamma^2)^2\,(1-\gamma^2)}t^*\;, \\ V_n^t - V_n^* &= \left(V_i^t - V_i^*\right) + \frac{\Theta_d^p t^*}{(4-\gamma^2)\,(1-\gamma^2)}\;. \end{aligned}$$

Substituting the value of the optimal tax in the nominator of $V_i^t - V_i^*$ yields,
$-[(2-\gamma-\gamma^2)^2\,a\,(1-\gamma)\,(3+\gamma)\,(2+\gamma)^2\,c + (8-6\gamma^2+\gamma^4)\,(d+\theta_d) +$
$\gamma\,(4-\gamma^2-(8-3\gamma^2)\,\mu)\,\theta_c - (12-9\gamma^2+2\gamma^4)\,\mu\theta_d]$. This expression is negative except in cases that both γ and μ approach 1, cases in which we are not interested in. Thus,

[8] From equations (10) and (6) we get $q_{c\nu}^t - q_{c\nu}^* = \frac{\gamma t}{(1-\gamma^2)(4-\gamma^2)} > 0$, and from equations (11) and (7) we get $q_{d\nu}^t - q_{d\nu}^* = -\frac{\left(2-\gamma^2\right)t}{(1-\gamma^2)(4-\gamma^2)} < 0$, $\forall\; t > 0$, where $\nu = i, n$.

for the range of parameters that we are interested in, the utility of the informed consumer is lower at the taxation equilibrium relative to the unregulated equilibrium, that is $V_i^t < V_i^*$.

The welfare difference between the unregulated case and the case of taxation is

$$W^t - W^* = \mu\left(V_i^t - V_i^*\right) + (1-\mu)\left(V_n^t - V_n^*\right) - c\left(Q_c^t - Q_c^*\right) - (c+d)\left(Q_d^t - Q_d^*\right) \ .$$

Substituting into the above equation the indirect utility and aggregate output differences we found above, yields,

$$W^t - W^* = \frac{\left(4-3\gamma^2\right)t^2}{2\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)} > 0, \forall t \ . \tag{17}$$

■

8.4 Proof of Lemma 2

Proof. Substituting the values of p_c and p_d from equation (4) into the direct demand functions given in equation (1), yields the individual's demand as a function of ϕ,

$$q_{ci} = \frac{C + \left(4-\gamma^2\right)\Theta_c - m\Theta_c^q}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \ , \ q_{cn} = \frac{C - m\Theta_c^q}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \ , \tag{18}$$

$$q_{di} = \frac{C - \left(4-\gamma^2\right)\Theta_d + m\Theta_d^q}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \ , \ q_{dn} = \frac{C + m\Theta_d^q}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \ . \tag{19}$$

The aggregate output for each good is,

$$Q_c = \frac{C + m\Theta_c^p}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)}, \ Q_d = \frac{C - m\Theta_d^p}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)} \ . \tag{20}$$

As in the case of taxation, the regulator evaluates both the informed and the uniformed consumers' utility using the true values of θs. The regulator derives the optimal fraction of consumers ϕ, that receive information from the social welfare maximization problem,

$$\max_{\phi} W = mV_i(\phi) + (1-m)V_n(\phi) - cQ_c(\phi) - (c+d)Q_d(\phi) - \omega\frac{\phi^2}{2} \ .$$

The solution of the above maximization problem yields,

$$\phi^* = (1-\mu)\frac{\left(4-\gamma^2\right)\left[2d\Theta_d^p - \gamma\Theta_2^\phi\right] + 2\mu\Theta_1^\phi - 2(2+\gamma)(1-\gamma)(\theta_d-\theta_c)C}{2\left[(1-\gamma^2)(4-\gamma^2)^2\omega - (1-\mu)^2\Theta_1^\phi\right]}, \tag{21}$$

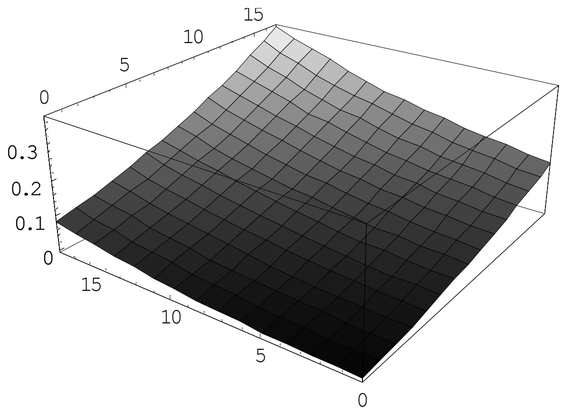

Figure 1: Optimal ϕ as a function of θ_c and θ_d.

where $\Theta_1^{\phi} = (12 - 5\gamma^2 + \gamma^4)(\theta_c^2 + \theta_d^2) + 2\gamma(16 - 9\gamma^2 + \gamma^4)\theta_c\theta_d$ and $\Theta_2^{\phi} = \gamma(\theta_c^2 + \theta_d^2) + 2(2 - \gamma^2)\theta_c\theta_d$. Assuming that the denominator is positive, that is, the second order condition for welfare maximization, $W'' < 0$, holds, the fraction of the consumers that the regulator targets with advertisement is related positively to the marginal external damage, that is, $\frac{\partial \phi^*}{\partial d} > 0$, and negatively to the slope of the marginal cost of advertisement, $\frac{\partial \phi^*}{\partial \omega} < 0$. These relationships are evident from equation (21). The value of ϕ^* is also decreasing when the value of γ is low and the difference between the θs is large. In such cases the first term on the nominator gets larger. In the case of symmetric response, that is, $\theta = \theta_c = \theta_d \neq 0$, the demand and cost parameters do not affect the optimal ϕ. In such case, equation (21) reduces to,

$$\phi^*_{|\theta_c = \theta_d} = (1 - \mu)\theta \frac{(2 + \gamma)(d - \gamma\theta) + 2(3 + 4\gamma + \gamma^2)\mu\theta}{(1 - \gamma)(2 + \gamma)^2\omega - 2(3 + 4\gamma + \gamma^2)(1 - \mu)^2\theta}.$$

Although the expressions for the derivatives with respect to θs are cumbersome and we do not present them here, they are clearly positive for the range of values we are interested in. Figure 1 presents the optimal ϕ as a function of both θs using simulations. We have used the following set of parameter values: $a = 60$, $c = 5$, $\gamma = 0.6$, $\mu = 0.6$, $d = 10$, $\omega = 1000$, $\theta_c \in \{0, 17\}$ and $\theta_d \in \{0, 17\}$. The optimal ϕ is increasing in both θs.

Although the problem is not well defined in the extreme case that $\gamma = 1$, we present the values of the derivatives in order to illustrate their sign. For $\gamma = 1$ we have that, $\frac{\partial \phi^*}{\partial \theta_c} = \frac{\partial \phi^*}{\partial \theta_d} = \frac{3d}{8(1-\mu)(\theta_c+\theta_d)^2} > 0$. ■

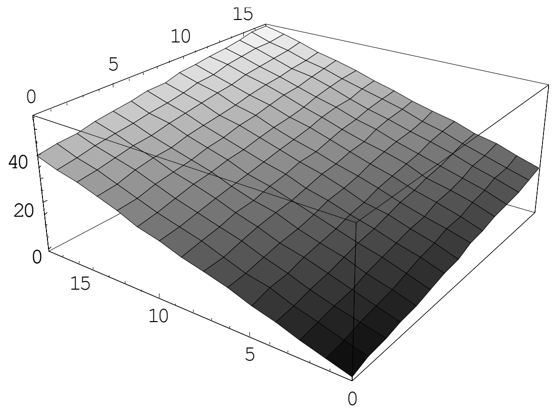

Figure 2: The sign of the condition determining the sign of $Q_c(t) - Q_c(\phi)$.

8.5 Proof of Proposition 3

Proof. (i) The only difference between equations (18), (19) and (6), (7) is in the fraction of consumers that belong to each of the two groups, that is, the fraction of informed consumers has increased by $\phi(1-\mu)$. Compared to the case of taxation, individuals' consumption of the clean good is lower and that of the dirty good higher. Formally, from equations (18), (19) and (10), (11) we get, $q_{c\nu}(t) - q_{c\nu}(\phi) = \frac{\gamma t+(m-\mu)\Theta_c^q}{(1-\gamma^2)(4-\gamma^2)} > 0$, and $q_{d\nu}(t) - q_{d\nu}(\phi) = \frac{-(2-\gamma^2)t-(m-\mu)\Theta_d^q}{(1-\gamma^2)(4-\gamma^2)} < 0$, $\nu = i, n$, $\forall\ t > 0$ and $\phi > 0$.

(ii) From equations (20) to (8) we get, $Q_c^\phi - Q_c^* = \frac{(1-\mu)\phi\Theta_c^p}{(1-\gamma^2)(4-\gamma^2)} > 0$, and $Q_d^\phi - Q_d^* = -\frac{(1-\mu)\phi\Theta_d^p}{(1-\gamma^2)(4-\gamma^2)} < 0$, $\forall\ t > 0$.

(iii) Equations (12) and (20) yield, $Q_c(t) > Q_c(\phi)$ and $Q_d(t) < Q_d(\phi)$ if $t > (1-\mu)\frac{\Theta_c^p}{\gamma}\phi > (1-\mu)\frac{\Theta_d^p}{2-\gamma^2}\phi$.[9] This condition holds for high values of γ, and significant external damages, that is, when the market distortion is not prominent and the tax rate is positive. Figure 2 presents the simulation results of the difference $t-(1-\mu)\frac{\Theta_c^p}{\gamma}\phi$ using the same parameter values as above ($a = 60$, $c = 5$, $\gamma = 0.6$, $\mu = 0.6$, $d = 10$, $\omega = 1000$, $\theta_c \in \{0, 17\}$ and $\theta_d \in \{0, 17\}$). The difference is always positive indicating that taxation is more effective in shifting the demand towards the clean good.

(iv) Substituting the values of $q_{ji}^\phi(\phi)$ and $q_{ji}^\phi(\phi)$, $j = c, d$ from equations (23) and

[9] The inequality $(1-\mu)\frac{\Theta_c^p}{\gamma}\phi > (1-\mu)\frac{\Theta_d^p}{2-\gamma^2}\phi$, holds always since $\frac{\Theta_c^p}{\gamma} > \frac{\Theta_d^p}{2-\gamma^2}$ requires $(2-\gamma^2)^2 - \gamma^2 > 0$, which is true $\forall\ \gamma < 1$.

(19) into the utility function we derive the indirect utility for both groups of consumers under information provision, as a function of ϕ. We then derive the difference between the indirect utility of the informed and uninformed consumer in the unregulated case and the case of taxation ,

$$\begin{aligned}
V_i^\phi - V_i^* &= (1-\mu)\frac{2\left(1-\gamma\right)^2\left(2+\gamma\right)\left(\theta_d-\theta_c\right)B-\left(\mu+m\right)\Theta_3^\phi}{2\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\phi^* \ , \\
V_n^\phi - V_n^* &= \left(V_i^\phi - V_i^*\right)-(1-\mu)\frac{2\left[\left(\theta_d^2+\theta_c^2\right)+\gamma\left(3-\gamma^2\right)\theta_d\theta_c\right]}{\left(4-\gamma^2\right)\left(1-\gamma^2\right)}\phi^* \ ,
\end{aligned}$$

where $\Theta_3^\phi = (4+\gamma^2-\gamma^4)\left(\theta_d^2+\theta_c^2\right)+2\gamma\left(8-5\gamma^2+\gamma^4\right)\theta_d\theta_c$.

The welfare difference between the unregulated case and the case of information provision is,

$$\begin{aligned}
W^\phi - W^* &= \mu\left(V_i^\phi - V_i^*\right)+(1-\mu)\left(V_n^\phi - V_n^*\right)+(1-\mu)\phi\left(V_i^\phi - V_n^\phi\right)- \\
&\quad c\left(Q_c^\phi - Q_c^*\right)-(c+d)\left(Q_d^\phi - Q_d^*\right)-\omega\frac{\phi^2}{2} \ .
\end{aligned}$$

Substituting into the above equation the indirect utility and aggregate output differences we found above, yields,

$$W^\phi - W^* = \frac{(1-\mu)\left[Kd+L-2\left(2-\gamma-\gamma^2\right)^2\left(\theta_d-\theta_c\right)\left(a-c\right)\right]-(4-\gamma^2)(1-\gamma^2)\omega\phi^*}{2\left(4-\gamma^2\right)\left(1-\gamma^2\right)}\phi^* \ ,$$

where $K = 2(4-\gamma^2)\Theta_d^p$, and $L = \Theta_1^\phi\left(m+\mu\right)-\gamma(4-\gamma^2)\Theta_2^\phi$. Simple manipulations yield,

$$W^\phi - W^* = \frac{(4-\gamma^2)(1-\gamma^2)\omega-(1-\mu)^2\Theta_1^\phi}{2\left(4-\gamma^2\right)^2\left(1-\gamma^2\right)}\phi^{*2} > 0, \forall\phi \ , \qquad (22)$$

since $(4-\gamma^2)(1-\gamma^2)\omega-(1-\mu)^2\Theta_1^\phi > 0$, is the the denominator of ϕ^* in equation (21).

Note that $W^t - W^\phi = W^t - W^* - \left(W^\phi - W^*\right)$. Substituting the values of the welfare difference from (17) and above we immediately get that $W^t - W^\phi > 0$ if $\left(4-3\gamma^2\right)t^2 - \left[(4-\gamma^2)(1-\gamma^2)\omega-(1-\mu)^2\Theta_1^\phi\right]\phi^2 > 0$. Figure 3 presents the simulation results of the welfare difference using the same parameter values as above ($a = 60$, $c = 5$, $\gamma = 0.6$, $\mu = 0.6$, $d = 10$, $\omega = 1000$, $\theta_c \in \{0, 17\}$ and $\theta_d \in \{0, 17\}$). The difference is always positive and increasing in both θs.

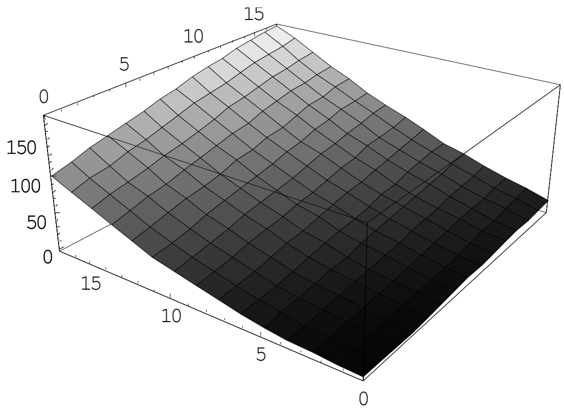

Figure 3: Welfare difference $W^t - W^\phi$.

If the cost of providing information is very small, the government could provide information to all consumers. This case cannot be examined with the above analysis which deals with interior solutions only. As the cost of providing information decreases, the second order conditions of the welfare maximization problem fail and the welfare function becomes convex. In order to examine what happens at low cost, we have to consider corner solutions. Assuming zero cost of information, the government could provide information to the entire population, that is if $\omega = 0$, then $\phi = 1$. The welfare difference between information provision and taxation in this extreme case is,

$$W^{\phi}_{|\omega=0,\ \phi=1} - W^t = \frac{(1-\mu)\,\Phi - (4-3\gamma^2)\,t^2}{2\,(4-\gamma^2)^2\,(1-\gamma^2)},$$

where $W^{\phi}_{|\omega=0,\ \phi=1}$ is social welfare under information provision, evaluated at $\omega = 0$ and $\phi = 1$, and $\Phi = 2d\,[\gamma(4-\gamma^2)\theta_c + (8-6\gamma^2+\gamma^4)\theta_d] + [(12-9\gamma^2+2\gamma^4) + (12-5\gamma^2+\gamma^4)\,\mu]\left(\theta_c^2+\theta_d^2\right) + \gamma\,[(8-3\gamma^2) + (16-9\gamma^2+\gamma^4)\,\nu]\,\theta_c\theta_d - 2(2-\gamma-\gamma^2)(\theta_d - \theta_c)(a-c)$. For the range of parameter values that result in a positive tax rate, we get that $W^{\phi}_{|\omega=0,\ \phi=1} - W^t > 0$. Figure 4 presents the simulation results of the above welfare difference using the same parameter values as above ($a = 60$, $c = 5$, $\gamma = 0.6$, $\mu = 0.6$, $d = 10$, $\omega = 1000$, $\theta_c \in \{0, 17\}$ and $\theta_d \in \{0, 17\}$). ■

8.6 Proof of Lemma 3

Proof. Substituting the values of p_c and p_d from equation (5) into the direct demand

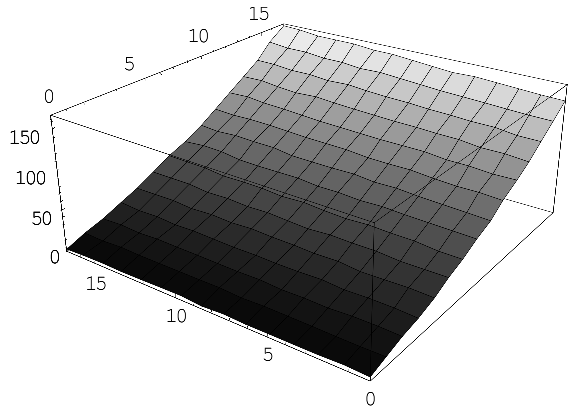

Figure 4: Welfare difference $W^{\phi}_{|\omega=0,\ \phi=1} - W^{t}$

functions given in equation (1), yields the optimal output as a function of t_σ and ϕ_σ,

■

Proof.

$$q_{ci} = \frac{C + (4-\gamma^2)\,\Theta_c - m_\sigma \Theta_c^q + \gamma t_\sigma}{(1-\gamma^2)\,(4-\gamma^2)}\ ,\ q_{cn} = \frac{C - m_\sigma \Theta_c^q + \gamma t_\sigma}{(1-\gamma^2)\,(4-\gamma^2)}\ , \tag{23}$$

$$q_{di} = \frac{C - (4-\gamma^2)\,\Theta_d + m_\sigma \Theta_d^q - (2-\gamma^2) t_\sigma}{(1-\gamma^2)\,(4-\gamma^2)}\ ,\ q_{dn} = \frac{C + m_\sigma \Theta_d^q - (2-\gamma^2) t_\sigma}{(1-\gamma^2)\,(4-\gamma^2)} \tag{24}$$

The total output for each good is,

$$Q_c = \frac{C + m_\sigma \Theta_c^p + \gamma t_\sigma}{(1-\gamma^2)\,(4-\gamma^2)},\ Q_d = \frac{C - m_\sigma \Theta_d^p - (2-\gamma^2) t_\sigma}{(1-\gamma^2)\,(4-\gamma^2)}\ . \tag{25}$$

As in both previous cases, the regulator evaluates both the informed and the uniformed consumers' utility using the true value of θs. The regulator derives the optimal level of the choice variables t_σ and ϕ_σ from the social welfare maximization problem,

$$\max_{t_\sigma, \phi_\sigma} W = m_\sigma V_i + (1-m_\sigma) V_n - c Q_c - (c+d) Q_d - \omega \frac{\phi_\sigma^2}{2}\ .$$

The solution of the above maximization problem yields,

$$\phi_\sigma^* = (1-\mu)\,\frac{2\theta_c F + 2\gamma(1-\gamma^2)\theta_c d - \Theta_2^{\phi_\sigma} + 2\mu \Theta_1^{\phi_\sigma}}{2\left[(4-3\gamma^2)(1-\gamma^2)\omega - (1-\mu)^2 \Theta_1^{\phi_\sigma}\right]}\ , \tag{26}$$

$$t_\sigma^* = t^* - (1-\mu)\,\frac{(4-3\gamma^2)\theta_d + \gamma^3 \theta_c}{4-3\gamma^2} \phi_\sigma^*\ , \tag{27}$$

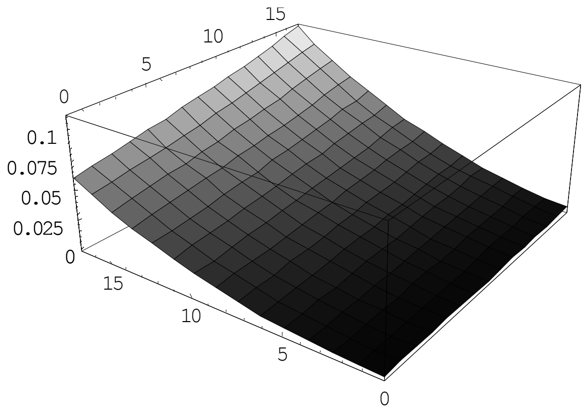

Figure 5: Difference between ϕ_σ^* and ϕ^*.

where, $F = (1-\gamma^2)(1-\gamma)(a-c)$, $\Theta_1^{\phi_\sigma} = (3-2\gamma^2)\theta_c^2 + (4-3\gamma^2)\theta_d^2 + 2\gamma(4-3\gamma^2)\theta_c\theta_d$ and $\Theta_2^{\phi_\sigma} = \gamma^2\theta_c^2 + (4-3\gamma^2)\theta_d^2 + 2\gamma(3-2\gamma^2)\theta_c\theta_d$. The optimal fraction of consumers informed by the government is positive, $\phi_\sigma^* > 0$, if $\mu < 1$ and $\theta_d > 0$. Thus, from (27) we get that $t^* > t_\sigma^*$. The weighted sum of the two policy instruments equals the equilibrium tax level under taxation.

Assuming that the denominator in (26) is positive, that is, the second order condition for welfare maximization, $W'' < 0$, holds, we get, $\frac{\partial\phi_\sigma^*}{\partial d} > 0$, $\frac{\partial\phi_\sigma^*}{\partial\theta_c} > 0$, $\frac{\partial\phi_\sigma^*}{\partial\theta_d} > 0$ and $\frac{\partial\phi^*}{\partial\omega} < 0$. The expression for the difference between ϕ^* and ϕ_σ^* is cumbersome, and for this reason we resort to simulations. Figure 4 presents the simulation results for $\phi^* - \phi_\sigma^*$ using the same values as above ($a = 60$, $c = 5$, $\gamma = 0.6$, $\mu = 0.6$, $d = 10$, $\omega = 1000$, $\theta_c \in \{0, 17\}$ and $\theta_d \in \{0, 17\}$) . ■

8.7 Proof of Proposition 4

Proof. (i) From equations (23), (24) and (6), (7) we get, $q_{ck}(\phi_\sigma, t_\sigma) - q_{ck}^* = \frac{\gamma t_\sigma - (1-\mu)\phi_\sigma\Theta_c^q}{(1-\gamma^2)(4-\gamma^2)}$, and $q_{dk}(\phi_\sigma, t_\sigma) - q_{dk}^* = \frac{(1-\mu)\phi_\sigma\Theta_d^q - (2-\gamma^2)t_\sigma}{(1-\gamma^2)(4-\gamma^2)}$, $k = i, n$. Compared to the unregulated case, individual consumers' demand for the clean (dirty) good increases (decreases) if $t_\sigma^* > \frac{(1-\mu)\Theta_c^q}{\gamma}\phi_\sigma^* > \frac{(1-\mu)\Theta_d^p}{2-\gamma^2}\phi_\sigma^*$.

From equations (23), (24) and (10), (11) we get, $q_{c\nu}(\phi_\sigma, t_\sigma) - q_{c\nu}(t) = -\frac{\gamma(t-t_\sigma)+(1-\mu)\phi_\sigma\Theta_c^q}{(1-\gamma^2)(4-\gamma^2)} < 0$, and $q_{d\nu}(\phi_\sigma, t_\sigma) - q_{d\nu}(t) = \frac{(1-\mu)\phi_\sigma\Theta_d^q + (2-\gamma^2)(t-t_\sigma)}{(1-\gamma^2)(4-\gamma^2)} > 0$, $\nu = i, n$, $\forall\ t > t_\sigma$. Compared to the case of taxation, individual consumers' demand for the clean (dirty) good is lower (higher) for $\phi^*_\sigma > 0$, which implies $t^*_\sigma < t^*$.

From equations (23), (24) and (18), (19) we get, $q_{c\nu}(\phi_\sigma, t_\sigma) - q_{c\nu}(\phi) = \frac{(m-m_\sigma)\Theta^q_c + \gamma t_\sigma}{(1-\gamma^2)(4-\gamma^2)} > 0$, and $q_{d\nu}(\phi_\sigma, t_\sigma) - q_{d\nu}(\phi) = -\frac{(m-m_\sigma)\Theta^q_d + (2-\gamma^2) t_\sigma}{(1-\gamma^2)(4-\gamma^2)} < 0$, $\nu = i, n$, $\forall\ m > m_\sigma$. Compared to the case of information provision, individual consumers' demand for the clean (dirty) good is higher (lower) for $\phi^* > \phi^*_\sigma$, which implies $m_\sigma < m$.

In summary, $q_{c\nu}(\phi) < q^*_{ck} < q_{c\nu}(\phi_\sigma, t_\sigma) < q_{c\nu}(t)$, $\nu = i, n$, $\forall\ t^* > t^*_\sigma$, $m^* > m^*_\sigma$, and $t^*_\sigma > \frac{(1-\mu)\Theta^q_c}{\gamma}\phi^*_\sigma$.

(ii) Aggregate demand for the clean (dirty) good increases (decreases) relative to the unregulated equilibrium, that is, $Q_c(\phi_\sigma, t_\sigma) > Q^*_c$ and $Q_d(\phi_\sigma, t_\sigma) < Q^*_d$. Compared to the case of taxation, aggregate quantity demanded of the clean (dirty) good is higher (lower), that is, $Q_c(\phi_\sigma, t_\sigma) > Q_c(t)$ $(Q_d(\phi_\sigma, t_\sigma) < Q_d(t))$ since $t^* - t^*_\sigma < (1-\mu)\frac{\Theta^p_d}{2-\gamma^2}\phi^*_\sigma < (1-\mu)\frac{\Theta^p_c}{\gamma}\phi^*_\sigma$.[10] Compared to the case of information provision, aggregate quantity demanded of the clean (dirty) good is higher (lower), that is, $Q_c(\phi_\sigma, t_\sigma) > Q_c(\phi)$ $(Q_d(\phi_\sigma, t_\sigma) < Q_c(\phi))$ if $t^*_\sigma > (1-\mu)\frac{\Theta^p_c}{\gamma}(\phi^* - \phi^*_\sigma) > (1-\mu)\frac{\Theta^p_d}{2-\gamma^2}(\phi^* - \phi^*_\sigma)$.

(iii) From equations (25) to (8) we get, $Q^\sigma_c - Q^*_c = \frac{\gamma t_\sigma + (1-\mu)\phi_\sigma\Theta^p_c}{(1-\gamma^2)(4-\gamma^2)} > 0$, and $Q^\sigma_d - Q^*_d = -\frac{\left(2-\gamma^2\right)t_\sigma + (1-\mu)\phi_\sigma\Theta^p_d}{(1-\gamma^2)(4-\gamma^2)} < 0$, $\forall\ t_\sigma, \phi_\sigma > 0$. Substituting the values of $q^\sigma_{ji}(\phi_\sigma, t_\sigma)$ and $q^\sigma_{ji}(\phi_\sigma, t_\sigma)$, $j = c, d$ from equations (28) and (24) into the utility function we derive the indirect utility for both groups of consumers under information provision, as a function of ϕ_σ and t_σ. We then derive the difference between the indirect utility of the informed and uninformed consumer in the unregulated case and the case of combined policy instruments, that is $V^\sigma_i - V^*_i$ and $V^\sigma_n - V^*_n$. The welfare difference between the the unregulated case and the case of information provision is,

$$\begin{aligned} W^\sigma - W^* &= \mu\left(V^\sigma_i - V^*_i\right) + (1-\mu)\left(V^\sigma_n - V^*_n\right) + (1-\mu)\phi_\sigma\left(V^\sigma_i - V^\sigma_n\right) - \\ &\quad c\left(Q^\sigma_c - Q^*_c\right) - (c+d)\left(Q^\sigma_d - Q^*_d\right) - \omega\frac{\phi^2_\sigma}{2}\ . \end{aligned}$$

Substituting the output and indirect utility differences yields,

$$W^\sigma - W^* = \frac{\left(4-3\gamma^2\right)t^2_\sigma + \left[(4-\gamma^2)(1-\gamma^2)\omega - (1-\mu)^2\Theta^\phi_1\right]\left(2\phi^* - \phi^*_\sigma\right)\phi^*_\sigma}{2\left(4-\gamma^2\right)\left(1-\gamma^2\right)}\ .$$

[10] It is evident that this inequality holds since, $\frac{(4-3\gamma^2)\theta_d + \gamma^3\theta_c}{4-3\gamma^2} < \frac{\Theta^p_d}{2-\gamma^2}$. Notice that assuming $\theta_c = 0$, yields $t^* - t^*_\sigma = (1-\mu)\frac{\Theta^p_d}{2-\gamma^2}\phi^*_\sigma$ and therefore, $Q^\sigma_c = Q^t_c$ and $Q^\sigma_d = Q^t_d$.

A sufficient condition for the above to be positive is $2\phi^* - \phi^*_\sigma > 0$, which always holds. Therefore, $W^\sigma > W^*$.

Note that $W^\sigma - W^t = W^\sigma - W^* - (W^t - W^*)$. Substituting the values of the welfare differences from (17) and above we immediately get that $W^\sigma - W^t > 0$ if $\left[(4-\gamma^2)(1-\gamma^2)\omega - (1-\mu)^2\Theta_1^\phi\right](2\phi^* - \phi^*_\sigma)\,\phi^*_\sigma > (4-3\gamma^2)\,(t^{*2} - t^{*2}_\sigma)$, a condition that always holds.

Note that $W^\sigma - W^\phi = W^\sigma - W^* - \left(W^\phi - W^*\right)$. Substituting the values of the welfare differences from (22) and above we immediately get that $W^\sigma - W^\phi > 0$ if $(4-3\gamma^2)\,t^{*2}_\sigma > \left[(4-\gamma^2)(1-\gamma^2)\omega - (1-\mu)^2\Theta_1^\phi\right](\phi^* - \phi^*_\sigma)^2$, a condition that always holds. ■