The Monetary Approach in the Presence of *I*(2) Components: A Cointegration Analysis of the Official and Black Market for Foreign Currency in Latin America

by

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Abstract

This paper re-examines the long-run properties of the monetary exchange rate model in the presence of a parallel or black market for U.S. dollars in two Latin American countries under the twin hypotheses that the system contains variables that are *I*(2) and that a linear trend is required in the cointegrating relations. Using the recent *I*(2) test by Rahbek *et al.* (1999) to examine the presence of *I*(2) and *I*(1) components in a multivariate context we find that the linear trend hypothesis could not be rejected and we find evidence that the system contains two *I*(2) variables for each country namely, Chile and Mexico, and this finding is reconfirmed by the estimated roots of the companion matrix (Juselius, 1995). The *I*(2) component led to the transformation of the estimated model by imposing long-run but not short-run proportionality between domestic and foreign money. Three statistically significant cointegrating vectors were found and, by imposing linear restrictions on each vector as suggested by Johansen and Juselius (1994) and Johansen (1995b), the order and rank conditions for identification are satisfied while the test for overidentifying restrictions was significant for either case. The main findings suggest that we reject the forward-looking version of the monetary model for each country, but the unrestricted monetary model is still a valid framework to explain the long-run movements of the parallel exchange rate in both countries. Furthermore, we test for parameter stability using the tests developed by Hansen and Johansen (1993) and it is shown that the dimension of the cointegration rank is sample dependent while the estimated coeffficients do not exhibit instabilities in recursive estimations.

Key words: *I*(2) cointegration, monetary model, parallel foreign exchange market, identification, temporal stability.

JEL Classification numbers: F31, F33, C32, C51, C52

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1. Introduction

Recently there has been a growing recognition of the importance of parallel or black markets for foreign currency. The evidence available suggests that black markets have recently increased in size and sophistication in many countries, in relation to capital movements, (Gupta 1981; Edwards 1989, 1999; Agenor 1992; Kiguel and O'Connell, 1995; and Phylaktis 1996 among others provide an extensive theoretical and empirical analysis of these markets as well as of the determinants of the black market premia in a variety of countries).

The emergence of parallel or black markets is a well known feature of many developing countries for several decades, with parallel exchange rates deviating, in some cases, considerably from official rates. Parallel markets for foreign currency are the result of direct and indirect government intervention in the foreign exchange market. When access to the official foreign-exchange market is limited and there are various foreign-exchange restrictions on international transactions of goods, services and assets, an excess demand develops for foreign currency at the official rate, which encourages some of the supply of foreign currency to be sold illegally, at a market price higher than the official rate. The size of of the market as well the black market premium, i.e. the amount by which the parallel market rate exceeds the official rate, varies from country to country and depends on the type of the exchange and trade restrictions imposed along with the degree to which these restrictions are implemented by the government agencies (see Edwards, 1989, 1999; Montiel, Agenor and Haque, 1993).

The main determinants of the demand for foreign currency in the parallel markets are the following. First, legal and illegal imports, the former resulting from the existence of rationing of foreign currency in the official market, and the latter from the different types of prohibitions of imports which give an incentive for smuggling when duties are greater than the black market premium. Second, domestic residents travelling abroad and facing limits on the amount of foreign currency they can buy. Third, portfolio diversification particularly in cases where the inflation is high and there is great uncertainty in economic activity, leading domestic residents to hold foreign currency as an efficient way of hedging against domestic inflation.[1] Finally, capital flight in the presence of political instability.

The main sources of the supply of currency are the following. The most significant source is smuggling and underinvoicing of exports; when there is an export tariff, underinvoicing allows the exporter to avoid the tariff and to sell the foreign currency which has been illegally obtained at a premium. When an export subsidy is considered, which is less than the black market premium, the sale of foreign currency in the parallel market could provide a compensation greater than the subsidy loss. Additional sources of supply of foreign currency to the parallel market is overinvoicing of imports when the tariff rate on imported goods is sufficiently lower than the premium, foreign tourists and diversion of remittances.

The purpose of this paper is to provide an insight on the relationship that exists between the exchange rate and several key monetary variables when a parallel or black market for dollars exists. The analysis is done by employing a popular model used to explain the movements of the exchange rate, namely the monetary model to the exchange rate first developed by Frenkel (1976).[2] This model suggests that the exchange rate is considered to be the price of relative monies and thus it should be explained by the movements of the monetary aggregates in the two countries, the corresponding real outputs and the interest rates. Blejer (1978) has extended the monetary approach to the exchange rate to emphasize the role of monetary factors as the main determinants of the black market rates. The importance of the monetary factors on the behaviour of the black market rate has been verified by several studies in addition to the empirical results presented by Blejer (1978) for Brazil, Chile and Colombia. Thus, Gupta (1980) and Biswas and Nandi (1986) have tested the model for India; Olgun (1984) for Turkey; and within the cointegration context Van den Berg and Jayanetti (1993) for Pakistan, Sri Lanka and India and recently Kouretas and Zarangas (1998) for Greece.

Our analysis is applied to two Latin American countries, Chile and Mexico and covers the recent period of floating exchange rates. Black market for foreign currency, and in particular for U.S. dollars, has operated continuously in most of the Latin America countries for the past decades. The experience of these countries with chronic high inflation rates and corresponding current account deficits since 1970s has led to the emergence of a strong black market for dollars, one that has become an integral part of the countries' infrastructure. Figure 1(a) gives a plot of the black market and official exchange rates for Chile. In January

1981 one U.S. dollar bought 41650 Chilean pesos on the black market and by January 1987 one U.S. dollar was buying 238100 pesos on the black market. At the same time in the official market the official exchange rate was 39000 pesos per dollar in January 1981 and 206290 in January 1987. Similar patterns emerge for the foreign exchange markets of Mexico and they are plotted in figure 1(b). Accordingly, figures 2(a)-(b) show the evolution of the black market premium in these Latin American countries. These plots show that there is significant variation in both countries and over time with respect to premia.

The analysis is conducted within the context of cointegration and therefore we examine the existence of a long-run relationship between the black market exchange rate, the official exchange rate and the monetary variables. Our approach is novel in a number of ways. First, we provide a new analysis for the determination of the order of integration of the variables. Although testing for unit roots has become a standard procedure it has been made clear that if the data are being determined in a multivariate framework, a univariate model is at best a bad approximation of the multivariate counterpart, while at worst, it is completely misspecified leading to arbitrary conclusions. Therefore, we employ the recently developed testing methodology suggested by Johansen (1992a, 1995a, 1997) extended by Paruolo (1996) and Rahbek *et al*. (1999) which allows us to reveal the existence of $I(2)$ and $I(1)$ components in a multivariate context. This analysis is done by testing successively less and less restricted hypotheses according to the Pantula (1989) principle. Additionally, we apply a recent test developed by Juselius (1995) that is based on the roots of the companion matrix and allows us to make firmer conclusions about the rank of the cointegration space. Second, since in a multivariate framework, such as the one given by the monetary model, a vector error correction model may contain multiple cointegrating vectors, a question arises as to whether one can associate all of them with the monetary model or otherwise which vector is identified with it and what is the interpretation given to the others. Thus, following Johansen and Juselius (1994) and Johansen (1995b) we impose independent linear restrictions on the coefficients of the accepted cointegrating vectors. Third, given that at least one statistically significant cointegrating vector has been found we examine the stability of the long-run relationships through time. Hansen and Johansen (1993) propose three tests for parameter

stability in cointegrated-VAR systems which allow us to provide evidence of the sample independence of the cointegration rank as well as of parameter stability.

There are several important findings which stem from our estimation approach. First, we find evidence of cointegration between the black market Chilean peso-dollar, and the Mexican peso-dollar exchange rates and the corresponding official rates and the monetary variables. Furthermore, in both cases we were able to establish the presence of a common $I(2)$ component which was assumed to be between the Chilean and U.S. money series and the Mexican an U.S. money series, respectively. Second, given the presence of an $I(2)$ stochastic trend we adopted a data transformation that allows us to move to the $I(1)$ model, which can simplify the empirical analysis considerably. Therefore, for both cases we tested whether long-run proportionality between domestic and foreign money could be imposed on the data. Third, given that three cointegrating vectors were found to be statistically significant, for both cases under investigation, we imposed independent linear restrictions so that we associated one vector with the monetary model, the second with the uncovered interest parity (UIP) condition and the third one was taken to describe a relationship between the official and black exchange rates. This joint structure is shown to be overidentified and the joint restrictions are rejected for both the Chilean peso-dollar and the Mexican peso-dollar exchange rates. This result implies that the monetary model in its forward-looking solution does not hold, an outcome which is attributed to the failure of the UIP condition in the long-run. Fourth, we find that the unconditional UIP condition version of the monetary model may still be a valid framework to explain the long-run movements of the black and official exchange rates in Chile and Mexico. Finally, the application of the recursive tests of Hansen and Johansen (1993) show that the dimension of the cointegration space may be sample dependent while the estimated coefficients do not exhibit instabilities in recursive estimations.

The plan of the remainder of the paper is as follows. Section 2 presents the monetary model in the presence of a parallel market for foreign exchange. In section 3 we discuss the econometric methodology for modelling and testing cointegration. The data used and the multivariate cointegration results are presented in section 4. The final section presents our concluding remarks.

2. The monetary exchange rate model

Since its conception in the 1970s the monetary exchange rate model has become the dominant theoretical model of exchange rate determination. The monetary model class of models is based on the assumption of perfect substitutability of non-money assets so that the exchange rate is determined only by relative excess money supplies. However, although this model is theoretically very appealing, its empirical validity has produced conflicting results. Furthermore, Meese and Rogoff (1983) show that a random walk model outperforms the monetary model in out-of-sample forecasting ability. Early studies for the recent floating exchange rate experience has shown that the monetary model is plagued by unstable regression coefficients in term of sign, magnitude and significance. Recently, attention has shifted towards the ability of the monetary model to adequately characterize long run movements in the exchange rate. In particular, following the work of Engle and Granger (1987), studies have been conducted to test the long run properties of the monetary model using cointegration analysis. Within this context, MacDonald and Taylor (1994), Kouretas (1997) and Diamandis *et al*. (1998) among others, provide evidence for the long-run validity of the model as well as its out-of-sample forecasting performance for a number of key currencies. Additionally, Diamandis, *et al*. (2000) provide further evidence in favour of the monetary model in the presence of variables that are *I*(2) processes, for the case of the drachma/dollar and drachma/mark exchange rates.[3]

The basic monetary model was developed by Frenkel (1976) and combines domestic and foreign money demand functions with purchasing power parity (PPP). Moreover, the UIP condition is invoked to derive the forward-looking version of the monetary model, under which the exchange rate depends on all the expected realizations of the forcing variables, that is, the monetary aggregates and the output variables.

Under these assumptions a typical monetary reduced form equation is obtained (see Baillie and MacMahon, 1989; and MacDonald and Taylor, 1992):

$$e_t = \beta_0 + \beta_1 m_t + \beta_2 m_t^* + \beta_3 y_t + \beta_4 y_t^* + \beta_5 i_t + \beta_6 i_t^* + u_t \qquad (1)$$

where e_t is the spot exchange rate (home currency price of foreign currency); m_t denotes the domestic money supply; y_t denotes domestic income; i_t denotes the short-term domestic interest rate; corresponding foreign magnitude are denoted by an asterisk; u_t is a disturbance error; and all variables apart from the interest rate terms, are expressed in natural logarithms.

The expected signs of the coefficients in (1) are: $\beta_0 > 0,\ \beta_1 > 0,\ \beta_2 < 0,\ \beta_3 < 0,\ \beta_4 > 0,\ \beta_5 > 0,\ \beta_6 < 0$. The Keynesian (sticky-price) model assumes opposite signs for the interest rates. Different signs of the interest rate coefficients in equation (1) will also be produced under imperfect substitutability between the assets of the two countries. Associated with equation (1) is a set of coefficients restrictions that are regularly imposed and tested. The most important restriction is whether proportionality exists between the exchange rate and relative monies ($\beta_1 = -\beta_2 = 1$). Moreover, the assumptions that the income and interest rate elasticities for money demand are equal in both countries, ($\beta_3 = -\beta_4$) and ($\beta_5 = -\beta_6$), are often being tested.

Blejer (1978) extended the monetary approach to emphasize the role of monetary factors as the main determinants of the black market rates. Blejer constructs a model of the black market exchange rate by incorporating a flow black market for foreign exchange into a monetary model in which the rate of devaluation of the official exchange rate is fixed by the authorities according to some reaction function aimed at maximizing a government utility function. In this section, we follow Phylaktis (1996) and Kouretas and Zarangas (1998) and we provide a simplified version of the model.

As a starting point we consider that the black market exchange rate depends on: (1) the underlying supply and demand for foreign currency, which according to PPP are in the long-run driven by countries' price levels, (2) the level of the official exchange rate, and (3) the diverse set of policies and institutions that govern the legal exchange market, e.g., rationing procedures, who is permitted to buy and sell there, and of course the severity and likelihood of penalties for dealing in the black market. If this latter set of policies and institutions is stable, we can then investigate whether there is a linear long-run equilibrium relationship between the parallel and official exchange rate as well as between the parallel exchange rate and the two price levels.

The black market rate is determined by the interaction between demand for and supply of foreign currency in the black market. The demand for foreign currency in this market depends positively on the return from holding this asset. Furthermore, this return is a function of the expected rate of appreciation of the foreign currency in the black market. If we assume that economic agents form their expectations by comparing the movements of the exchange rate with the movements of the ratio between domestic and foreign price level, then the demand for foreign currency can be described as follows:

$$D_b = \beta_0 + \beta_1(p - p^* - e_b), \qquad \beta_1 > 0, \tag{2}$$

where D_b is the demand for foreign currency in the black market, p and p^* are respectively the domestic and foreign price level, and e_b is the black market exchange rate. Therefore, in case that p rises faster than p^* and at the same time there is no corresponding increase in the parallel market exchange rate, the economic agents expect a depreciation of the parallel exchange rate by a percentage equal to the observed inflation rate differential.

The supply of foreign currency to the market is provided mainly through receipts from the overinvoicing of imports and underinvoincing of exports as well receipts from tourism, shipping and immigrants' remittances. These activities are positively related to the differential between the official and the black market exchange rates. As the differential increases, the profit possibilities increase leading to higher incentive to divert foreign exchange to the parallel market. The supply function of foreign currency to the black market is given as follows:

$$S_b = \gamma_0 + \gamma_1(e_b - e_0), \qquad \gamma_1 > 0, \tag{3}$$

where S_b is the supply of foreign currency to the black market, and e_0 is the official exchange rate. Both exchange rates are defined as domestic currency per one unit of foreign currency. All variables are in logarithms.

Equating the demand for and the supply of foreign currency in the black market and solving for e_b, we obtain

$$e_b = \alpha_0 + \alpha_1 e_0 + \alpha_2 p + \alpha_3 p^* \tag{4}$$

where $\alpha_1 = \frac{\gamma_1}{\gamma_1 + \beta_1}$ and $\alpha_2 = \frac{\beta_1}{\gamma_1 + \beta_1}$, and $\alpha_0 > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_3 < 0$.

The above formulation considers the black market exchange rate, is being a weighted average of the official exchange rate, e_0, and the price differential, which essentially is the PPP exchange rate. Absence of direct or indirect official intervention in the foreign exchange market through the imposition of capital controls the official exchange rate will converge to the PPP rate in the long-run while it will be equal to the black market rate leading to a gradual elimination of the black market for foreign currency. In case though, that intervention of some form exists, then the official exchange rate will be different from the PPP rate, and the black rate will be a function of the official rate and the equilibrium rate implied by PPP.

Substituting equation (4) in (1) we obtain the monetary model relationship in the presence of a black market rate

$$e_{pt} = \beta_0 e_{ot} + \beta_1 m_t + \beta_2 m_t^* + \beta_3 y_t + \beta_4 y_t^* + \beta_5 i_t + \beta_6 i_t^* + u_t \tag{5}$$

Model (5) implies that an increase in the domestic money supply results in a domestic money market disequilibrium. As economic agents get rid off their excess cash balances, domestic prices rise. This creates expectations of exchange rate depreciation and an increase in the demand for black market dollars. This in turn increases the differential between the official and the black market rate, increasing the incentive to underinvoice exports, to smuggle imports, or to divert remittances through the black market. Although this increase in the supply of foreign currency in the black market will reduce the upward pressure on the black market exchange rate, a higher stock of money will overall be associated with a depreciation of the parallel market rate.

3. Econometric Methodology

Our cointegration analysis is based on the multivariate cointegration technique developed by Johansen (1988, 1991) and extended by Johansen and Juselius (1990, 1992) which is a Full Information Maximum Likelihood (FIML) estimation method. It makes use of the information incorporated in the dynamic structure of the model and it also estimates the entire space of the long-run relationships among a set of variables, without imposing a normalization on the dependent variable *a priori*. Although the Johansen procedure is well known we discuss it briefly in light of some recent extensions of the methodology that are applied in this paper.

Consider a ***p***-dimensional vector time series z_t with an autoregressive representation (AR) which in its error correction form is given by

$$\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + \gamma D_t + \mu_0 + \mu_1 t + \varepsilon_t, \qquad t = 1,....,T \tag{6}$$

where $z_t = [e_p, e_0, m, m^*, y, y^*, i, i^*]_t$ as defined in section 2, $z_{k+1},\ldots\ldots,z_0$ are fixed and $\varepsilon_t \sim Niid_p(0,\Sigma)$. The adjustment of the variables to the values implied by the steady state relationship is not immediate due to a number of reasons like imperfect information or costly arbitrage. Therefore, the correct specification of the dynamic structure of the model, as expressed by the parameters $(\Gamma_1,.....,\Gamma_{k-1},\gamma)$, is important in order that the equilibrium be revealed. The matrix $\Pi = \alpha\beta'$ defines the cointegrating relationships, β, and the rate of adjustment, α, of the endogenous variables to their steady state values. D_t is a vector of nonstochastic variables, such as centered seasonal dummies which sum to zero over a full year by construction and are necessary to account for short-run effects which could otherwise violate the Gaussian assumption, and/or intervention dummies; μ is a drift and T is the sample size.

If we allow the parameters of the model $\theta = (\Gamma_1,\ldots\ldots,\Gamma_{k-1},\Pi,\gamma,\mu,\Sigma)$ to vary unrestrictedly, then model (6) corresponds to the *I*(0) model. The *I*(1) and *I*(2) models are

obtained if certain restrictions are satisfied. Thus, the higher-order models are nested within the more general *I*(0).

It has been shown (Johansen, 1991) that if $z_t \sim I(1)$, then that matrix Π has reduced rank $r < p$, and there exist $p\text{x}r$ matrices α and β such that $\Pi = \alpha\beta'$. Furthermore, $\Psi = \alpha_{\perp}^{'}(\Gamma)\beta_{\perp}$ has full rank, where $\Gamma = I - \sum_{i=1}^{\kappa}\Gamma_i$ and $\alpha_{\perp}$ and $\beta_{\perp}$ are $p\text{x}(p-r)$ matrices orthogonal to α and β, respectively.

Following this parameterization, there are r linearly-independent stationary relations given by the cointegrating vectors β and $p-r$ linearly-independent non-stationary relations. These last relations define the common stochastic trends of the system and the contribute to the various variables. By contrast the AR representation of model (6) is useful for the analysis of the long-run relations of the data.

The *I*(2) model is defined by the first reduced rank condition of the I(1) model and that $\Psi = \alpha_{\perp}^{'}\Gamma\beta_{\perp} = \varphi\eta^{'}$ is of reduced rank s_1, where φ and η are $(p-r)\text{ x }s_1$ matrices and $s_1 < (p-r)$.

Under these conditions we may re-write (6) as

$$\Delta^2 z_t = \Pi z_{t-1} - \Gamma\Delta z_{t-1} + \sum_{i=1}^{k-2}\Psi_i\Delta^2 z_{t-1} + \gamma D_t + \mu_0 + \mu_1 t + \varepsilon_t \qquad (7)$$

where $\Psi_i = -\sum_{j=i+1}^{k-1}\Gamma_i, \qquad i = 1,\ldots..,k-2$

Following Rahbek *et. al* (1999) we outline a representation of the restricted VAR (7) which allows the observed process z_t to have (at most) linear deterministic trends and some or all components *I*(2). In general if $z_t \sim I(2)$ then the unrestricted linear regressor, $\mu_1 t$, allows for cubic trends while the constant regressor, μ_0, allows for quadratic trends. Rahbek *et al*. (1999) show that to guarantee linear trends in all linear combinations of z_t we must

impose restrictions on both μ_1 and μ_0. First, the constant is decomposed into the spaces spanned by α and $\alpha_\perp$ respectively such that

$$\mu_0 = \alpha \bar{\alpha}' \mu_0 + \bar{\alpha}_\perp \alpha_\perp' \mu_0 \equiv \alpha \kappa_0' + \bar{\alpha}_\perp \alpha_\perp' \mu_0 \tag{8}$$

Then, the restrictions required to guarantee the linear trends correspond to

$$\mu_1 = \alpha \beta_0' \tag{9}$$

where $\beta_0' = -\beta' \tau_1$, and

$$\alpha_\perp' \mu_0 = -\xi \eta_0' - (\alpha_\perp' \Gamma \bar{\beta}) \beta_0' \tag{10}$$

where $\eta_0' = -(\bar{\beta}_\perp \eta)' \tau_1 = -\beta_\perp' \tau_1$. Note that κ_0', β_0' and η_0' are freely varying vectors of dimension s, r and s respectively.

Finally, Rahbek *et al.*, (1999) provide a likelihood ratio (LR) test to test whether the linear trend enters the cointegrating vector significantly. Thus, under H_r the hypothesis of no linear trend in $\beta' z_t$ and therefore in the *polynomial* or *multicointegrating* relations is given by $\beta_0 = 0$. The likelihood ratio test for this hypothesis is given by

$$Q_{\beta_0} = T \sum_{i=1}^{r} \ln \{ \frac{(1 - \hat{\lambda}_i^{\beta_0})}{(1 - \hat{\lambda}_i)} \} \tag{11}$$

where $\hat{\lambda}_i$ are the r largest eigenvalues solving the eigenvalue problem in (7) and likewise $\hat{\lambda}_i^{\beta_0}$ are the r largest eigenvalues solving (7) with z_t^* replaced by z_t. The test statistic for this likelihood ratio test is asymptotically $\chi^2(r)$ distributed.

Johansen (1997) shows that the space spanned by the vector z_t can be decomposed into r stationary directions, β, and $p - r$ nonstationary directions, $\beta_\perp$, and the latter into the directions $(\beta_\perp^1, \beta_\perp^2)$, where $\beta_\perp^1 = \beta_\perp \eta$ is of dimension $p \times s_1$ and $\beta_\perp^2 = \beta_\perp (\beta_\perp' \beta_\perp)^{-1} \eta_\perp$ is of dimension $p \times s_2$ and $s_1 + s_2 = p - r$. The properties of the process are described by:

$$I(2): \{\beta_{\perp}^{2} z_t\},$$

$$I(1): \{\beta' z_t\}, \{\beta_{\perp}^{1'} z_t\},$$

$$I(0): \{\beta_{\perp}^{1'} \Delta z_t\}, \{\beta_{\perp}^{2} \Delta^2 z_t\}, \{\beta' z_t + \omega' \Delta z_t\}$$

where ω is a p x r matrix of weights, designed to pick out the *I*(2) components of z_t (Johansen, 1995a). Thus, we have that the cointegrating vectors $\beta' z_t$ are actually *I*(1) and require a linear combination of the differenced process Δz_t to achieve stationarity, i.e. the *polynomial* or *multicointegration* cointegration (Haldrup, 1998).

Johansen (1991) shows how the model can be written in moving average form, while Johansen (1997) derives the FIML solution to the estimation problem for the *I*(2) model. Furthermore, Johansen (1995a) provides an asymptotically equivalent two-step procedure which computationally is simpler. It applies the standard eigenvalue procedure derived for the *I*(1) model twice, first to estimate the reduced rank of the Π matrix and then, for given estimates of α and β, to estimate the reduced rank of $\hat{\alpha}_{\perp}' \Gamma \hat{\beta}_{\perp}$, (Juselius, 1994, 1995, 1998).

In a multivariate context, such as the one given by the monetary model, a vector error correction model may contain multiple cointegrating vectors, and in such a case the individual cointegrating vectors are *underidentified* in the absence of sufficient linear restrictions on *each* of the vectors. The issue of identification in cointegrated systems has recently been addressed by Johansen and Juselius (1994) and Johansen (1995b).

Consider again the long run matrix $\Pi = \alpha\beta'$ and let Φ be any r x r matrix of full rank. Then $\Pi = \alpha\Phi^{-1}\Phi\beta' = \alpha^{*}\beta^{*}'$, where $\alpha^{*} = \alpha\Phi^{-1}$ and $\beta^{*} = \Phi\beta'$ and without imposing restrictions on α and β so that to limit the admissible matrices, Φ, the cointegrating vectors are not unique. In fact given the normalization under which both α and β are calculated, only the space spanned by the β vectors is uniquely determined. Thus, we need to impose restrictions implied by economic theory, for example homogeneity and zero restrictions, so that we are able to discriminate between them.

The necessary and sufficient conditions for identification in a cointegrated system in terms of linear restrictions on the columns of β are analogous to the classic identification problem that we face in the simultaneous equations problem. Thus, the order condition for identification of each of the r cointegrating vectors is that we can impose at least $r-1$, just identifying restrictions and one normalization on each vector without changing the likelihood function. This is a necessary condition. The necessary and sufficient condition for identification of the i*th* cointegration vector, *the Rank* condition, is that the rank$(R_i^{'}H_1,\ldots,R_i^{'}H_k)\geq k$, where i and $k=1,\ldots,r-1$ and $k\neq i$ (Johansen and Juselius, 1994). The linear restrictions of the model are of the form $R_i^{'}\beta_i=0$, where R_i is a $(p\times k_i)$ matrix, or equivalently by $R_i^{'}H_i=0, i=1,\ldots,r$, where H_i is a known $(p\times s_i)$ design matrix which satisfies $\beta_i=H_i^{'}\tau_i$ and τ_i is a $(s_i\times 1)$ vector of freely varying parameters $(k_i+s_i=p)$. For example, if there are three accepted cointegrating vectors among the eight variables of our model, the *exact* identification, according to the order condition requires one linear restriction on each cointegrating vector and the *Rank* condition is satisfied if rank $(R_i^{'}H_j)\geq 1, i\neq j$. Johansen and Juselius (1994) provide a likelihood ratio statistic to test for overidentifying restrictions that is distributed as a χ^2 with $\nu=\sum_i(p-r+1-s_i)$, where *p* and r are given by the dimension p x r of β, and s_i is the number of freely estimated parameters τ, in vector i, which comply with $\beta_i=H_i\tau_i$.

An equally important issue, along with the existence of at least one cointegration vector, is the issue of the stability of such a relationship through time as well as the stability of the estimated coefficients of such a relationship. Thus, Septhon and Larsen (1991) have shown that Johansen's test may be characterised by sample dependency. Hansen and Johansen (1993) have suggested methods for the evaluation of parameter constancy in cointegrated VAR models, formally using estimates obtained from the Johansen FIML technique. Three tests have been constructed under the two VAR representations. In the "Z-representation" all the parameters of model (8) are re-estimated during the recursions while under the "R-representation" the short-run parameters Γ_i, *i = 1...k*, are fixed to their full sample values and only the long-run parameters α and β are re-estimated.

The first test is called the ***Rank*** test and is used to examine the null hypothesis of sample independency of the cointegration rank of the system. This is accomplished by first estimating the model over the full sample, and the residuals corresponding to each recursive subsample are used to form the standard sample moments associated with Johansen's reduced rank. The eigenvalue problem is then solved directly from these subsample moment matrices. The obtained sequence of trace statistics is scaled by the corresponding critical values, and we accept the null hypothesis that the chosen rank is maintained regardless of the subperiod for which it has been estimated if it takes values greater than one.

A second test deals with the null hypothesis of constancy of the cointegration space for a given cointegration rank. Hansen and Johansen propose a likelihood ratio test that is constructed by comparing the likelihood function from each recursive subsample with the likelihood function computed under the restriction that the cointegrating vector estimated from the full sample falls within the space spanned by the estimated vectors of each individual sample. The test statistic is a χ^2 distributed with $(p-r)r$ degrees of freedom.

The third test examines the constancy of the individual elements of the cointegrating vectors β. However, when the cointegration rank is greater than one, the elements of those vectors can not be identified, except under restrictions. Fortunately, one can exploit the fact that there is a unique relationship between the eigenvalues and the cointegrating vectors. Therefore, when the cointegrating vectors have undergone a structural change, this will be reflected in the estimated eigenvalues. Hansen and Johansen (1993) have derived the asymptotic distribution as well as the asymptotic variance of the estimated eigenvalues.

4. Empirical results

The monthly data for this study, relating to the Chilean peso-dollar and Mexican peso-dollar official exchange rates and Chilean, Mexican and US macroeconomic variables, are all taken from the International Monetary's Fund *International Financial Statistics* CD-ROM while the data for the black market exchange rates were taken from the monthly series in various issues of the *World Currency Yearbook* and the relevant time periods are Chile (1973.10-1993.12) and Mexico (1976.09-1993.12). In particular, the black and official exchange rates are expressed in units of home currency per foreign currency and they are end-of-month

quotations; The money stock is M1 (line 34 for Chile and Mexico and line 59 for the U.S. and is seasonally unadjusted). Real output is proxied by manufacturing output (Chile; line 66) or industrial production (Mexico and U.S.; lines 66 and 66c, respectively). For the U.S. the interest rate is the Treasury bill rate (line 60c). Because sufficient interest rate date do not exist for Chile and Mexico, we measured the cost of holding money as the annualized three-month rate of consumer price inflation (line 64).[4,5]

4.1 Determination of the cointegration rank and the order of integration

The first step in the analysis is the determination of the cointegration rank index, r, and the order of integration of the variables. As a first check for the statistical adequacy of model (8) we report some multivariate and univariate misspecification tests in Table 1, in order to investigate that the estimated residuals do not deviate from being Gaussian white noise errors. A structure of three lags for each bilateral exchange rate was chosen based on these misspecification tests.

Specifically, the multivariate LB test for serial correlation up to the 42nd order and the multivariate LM tests for first and fourth order residual autocorrelations are not significant, whereas multivariate normality is clearly violated. Normality can be rejected as a result of skewness (third moment) or excess kurtosis (fourth moment). Since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis we report the univariate Jarque-Bera test statistics together with the third and fourth moment around the mean. It turns out that the rejection of normality is essentially due to excess kurtosis, and hence not so serious for the estimation results. The presence of non-normality may be attributed to the fact that both the Chilean peso-dollar and the Mexican peso-dollar official exchange rates were administratively determined throughout the period under consideration as well as to the short-term interest rates, signifying both the high volatility of money stock in both countries. The ARCH(3) tests for third order autoregressive heteroscedasticity and is rejected for all equations. Again cointegration estimates are not very sensitive to ARCH effects.[6] The R^2 measures the improvement in explanatory power relative to the random walk (with drift) hypothesis, i.e. $\Delta x_t = \mu + \varepsilon_t$. They show that with this information set we can explain quite a large proportion of the variation in the exchange rates

and the money supply, but to a much lesser extent the variation in the output and the interest rates.

The Johansen-Juselius multivariate cointegration technique, as explained in section 3, is applicable only in the presence of variables that are realizations of *I*(1) processes only or a mixture of *I*(1) and *I*(0) processes, in systems used for testing for the order of cointegration rank. Until recently the order of integration of each series was determined via the standard unit root tests. However, it has be made clear by now that if the data are being determined in a multivariate framework, a univariate model is at best a bad approximation of the multivariate counterpart, while at worst, it is completely misspecified leading to arbitrary conclusions. Thus, in the presence of *I*(1) series, Johansen and Juselius (1990) developed a multivariate stationarity test which has become the standard tool for determining the order of integration of the series within the multivariate context.

Additionally, when the data are *I*(2) one also has to determine the number of *I*(2) trends, s_2, among the $p-r$ common trends. The two-step procedure discussed in section 3 is used to determine the order of integration and the rank of the two matrices. The hypothesis that the number of *I*(1) trends = s_1 and the rank = r is tested against the unrestricted H_0 model based on a likelihood ratio test procedure discussed in Paruolo (1996) and Rahbek *et.al* (1999).

Table 2(a) reports the evidence from the application of the two step procedure discussed in section 3. The numbers refer to the value of the trace test statistics for all values of r and $s_1 = p-r-s_2$, under the assumption that the data contain linear but no quadratic trends. The 95% critical test values reported in italics below the calculated test values are taken from the asymptotic distributions reported in Rahbek *et.al* (1999, Table 1). Starting from the most restricted hypothesis {r=0, s_1 = 0, s_2 = 8} and testing successively less and less restricted hypotheses according to the Pantula (1989) principle, it is shown that the case in favour of one *I*(2) component can not be rejected in both cases. Specifically, we are unable to reject the hypothesis {r=3, s_1 = 4, s_2 = 1} for both the Chilean peso – dollar and the Mexican peso – dollar case.[7,8.]

In addition to the formal test, Juselius (1995) offers further insight into the *I*(2) and *I*(1) analysis as well as the correct cointegration rank. She argues that the results of the trace and

maximum eigenvalue test statistics of the *I*(1) analysis, i.e. from the estimation of the model without allowing for *I*(2) trends, should be interpreted with some caution for two reasons. First, the conditioning on intervention dummies and weakly exogenous variables is likely to change the asymptotic distributions to some (unknown) extent. Second, the asymptotic critical values may not be very close approximations in small samples. Juselius (1995) suggests the use of the additional information contained in the roots of the characteristic polynomial. Table 2(b) provides the $p\mathrm{x}k$ roots of the companion matrix. If there are *I*(2) components in the vector process, then the number of unit roots in the characteristic polynomial is $s_1 + 2s_2$. The results of this test are consistent with the estimated roots of the companion matrix since for both the Chilean peso-dollar case and the Mexican peso-dollar case there are six unit roots in the process, four of which are *I*(1) components and one of which is the *I*(2) component, and given that we have a system of eight variables three additional smaller roots are left in the process associated with the three stationary long-run relationships[9].

Finally, we allow for the presence of a linear trend following the work by Dornbusch (1989) who suggests that due to both differing productivity trends in the tradeable and non-tradeable goods sectors and inter-country differences in consumption patterns, a decline in domestic prices relative to foreign prices could appear as a linear trend in the purchasing power parity (PPP) relationship underlying the monetary model. We tested for the significance of the deterministic trend in the multicointegrating relation by applying the likelihood ratio statistic discussed in (12). The test statistic in the Chilean peso – dollar case is and in the Mexican peso – dollar case is 13.69 and 15.21 respectively, with a p-value (0.00) and thus we reject the null hypothesis that the linear trend does not enter significantly in the cointegration vector of the multicointegrating relation.

4.2. A data transformation from I(2) to I(1)

Since the statistical inference of the *I*(2) model is quite complicated relative to that of *I*(1) model, a data transformation that allows us to move to the *I*(1) model will simplify the empirical analysis considerably without any loss of substance, and this transformation is needed for both the Chilean peso-dollar and the Mexican peso–dollar cases. A possible hypothesis which could be extracted from presence of an *I*(2) component in the system is that

the variable $\{m_t - m_t^*\}$ is a first-order nonstationary process.[10] If accepted, the implication is that the domestic and foreign money aggregates are cointegrating from *I*(2) to *I*(1), and use of the transformed data vector $z_t^{'} = [e_p, e_o, m_t - m_t^*, \Delta m_t, y_t^*, i_t, i_t^*]$, would then allow us to move to the *I*(1) model. The validity of this transformation is based on the assumption that $\{m_t - m_t^*\} \sim I(1)$, $\{e_p, e_o, y_t, y_t^*, i_t, i_t^*\} \sim I(1)$, and that $\{m_t - m_t^*\}$ is a valid restriction on the long-run structure, but not necessarily on the short-run structure.

To test whether long-run proportionality between the domestic and foreign money could be imposed on the data and the test statistic which is asymptotically distributed as $\chi^2(1)$, is equal to 0.64 for the Chilean peso – dollar case and 0.89 for the Mexican peso – dollar case and hence was not significant. Therefore, long-run proportionality between the Chilean and US money stock and the Mexican and US money stock could not be rejected. Furthermore, the *I*(2) test confirmed that this transformation removes all signs of the *I*(2) components from the data.

The remaining analysis for Chile and Mexico will be performed in the *I*(1) model, containing long-run but not short-run proportionality between the domestic and foreign money, based on the vector $[e_p, e_o, m - m^*, \Delta m, y, y^*, i, i^*]$. Alternatively we could have chosen to analyze the vector $[e_p, e_o, m - m^*, \Delta m^*, y, y^*, i, i^*]$ as it corresponds to the same likelihood function. Since, we are interested in how the exchange rate reacts to disequilibrium positions in the domestic money we choose the first alternative.

To assess the statistical properties of the chosen variables for both cases the test statistics reported in Table 3 are useful. The test of long-run exclusion is a check of the adequacy of the chosen measurements and show that none of the variables can be excluded from the cointegration space. The test for stationarity indicate that none of the variables can be considered stationary under any reasonable choice of r. Finally, the test of weak exogeneity shows that the output and possibly the domestic interest variables can be considered weakly exogenous for the long-run parameters β. All three tests are χ^2 distributed and are constructed following Johansen and Juselius (1990, 1992). Furthermore, table 3 presents diagnostics on the residuals from the cointegrated VAR model which indicate

that they are *i.i.d*. processes since no evidence of serial correlation or non-normality was detected. This provides further support for the hypothesis of a correctly specified model.

4.3. The empirical analysis of the transformed I(1) *model*

All results discussed in this section are based on the analysis of model (2) with the reduced rank condition on Π imposed for $k = 3$ and $r = 3$ applied to the transformed vector for Chile and Mexico $z_t = [e_p, e_o, m - m^*, \Delta m, y, y^*, i, i^*]$.

Table 4 reports the unrestricted estimates of the normalized cointegrating vectors which are based upon eigenvectors obtained from an eigenvalue problem resulting from Johansen's reduced rank regression approach. The estimated parameters, in both cases, carry signs which are in line with those that the monetary model predicts in (1) (expressed in implicit form that the estimates correspond to the elements of an eigenvector).

Given the presence of three cointegrating vectors we continue now with the economic identification of our system. On the first cointegrating vector we impose five restrictions, namely proportionality between the exchange rate and relative monies and exclusion of the official exchange rate, the growth of domestic money as well as of the two interest rates. This long-run relationship is necessary to hold in the forward looking solution of the monetary model when the variables are *I*(1) processes, the UIP condition is invoked and no bubbles are present in the foreign exchange market. In fact, the imposition of these five restrictions overidentifies this relationship. Identification of the second cointegration vector requires a set of restrictions that is independent of the one imposed on the first one. This implies that from the accepted cointegrated vectors only one can possibly describe the long-run monetary model and this is in variance with the cointegrating results on the monetary model which other researchers report (e.g. MacDonald and Taylor, 1992, 1994), where they conclude that as many four vectors can be considered as possibly explaining the monetary model, but in line with the recent results of Kouretas (1997) and Diamandis *et al*. (1998, 2000). The second vector can be interpreted as a particular variant of the UIP condition for countries like Chile and Mexico, which has been suffering from chronic budget deficits and have adopted a policy

of high interest rates to finance these deficits with increasing capital inflows while at the same time the Central Bank of Chile and the Central Bank of Mexico had been using the exchange rate as a target for the monetary policy in an effort to combat double digit inflation rates, (Edwards, 1988, 1989; Edwards and Montiel, 1989). During the period under examination Latin American countries have experienced serious financial imbalances and a quite contrasting behaviour of net capital flows. In late 1970s those capital flows were associated mainly with foreign direct investments while in early 1990s there was a tremendous surge in portfolio funds following the market oriented reforms adopted by almost all the countries in the region. In the meantime, the 1980s, the area experienced a drying up of private international financing which resulted to significantly negative net resource transfers.

A common feature for the majority of Latin American countries had been the restrictions on international capital mobility through a variety of means like administrative controls, outright prohibitions etc. However, the true degree of capital mobility was substantially higher than what the legal restrictions would imply. This has been clearly documented either by examining the historical events following the 1982 debt crisis and the ensuing massive private capital outflows and/or from several recent papers (Edwards, 1994, 1998 and 1999; Phylaktis, 1991). The foreign exchange restrictions initially were adopted in order to defend the domestic currency from devaluation pressures. In fact we observed a significant increase in capital controls prior to the abandonment of the fixed peg and a substantial increase of the black market premium (Edwards, 1998; Montiel and Reinhart, 1999). In the early 1990s several Latin American countries- with the exception of Mexico- resorted to exchange controls in order to prevent the real appreciation of their currencies. This appreciation was the outcome that the capital inflows had on the monetary base with a resulting negative impact on inflation. Most countries tried to deal with this situation through the imposition of controls on capital inflows and sterilized interventions. The latter mechanism has been used by almost all countries in the region although its effectiveness in the medium to long run is very doubtful due to the high cost it imposes on the central bank and the higher interest rates it generates. Chile has been an exception to this situation through the adoption

of a policy towards higher exchange rate flexibility based on a crawling band system which helped it to maintain the real appreciation of peso to controlled levels.

Finally, on the third vector we impose the proportionality hypothesis between the black and official exchange rates and zero restrictions on all other coefficients, as well as on the constant term, and this set up provides a direct test of long-run informational efficiency between the two markets (Moore and Phylaktis, 1996).

Imposing the above restrictions on the transformed vector for Chile and Mexico, the matrix of the linear and homogeneous restrictions is the following.

$$\beta = \begin{bmatrix} 1 & 0 & -1 & 0 & \beta_5 & \beta_8 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

where β_3 is expected to be negative.

The results of the estimated restricted vectors along with the likelihood test for the acceptance of the overidentifying restrictions, for both the Chilean peso-dollar and the Mexican peso-dollar exchange rates, are given in Table 4. According to the evidence we reject the joint restrictions for both cases which implies that for both countries we reject the forward-looking version of the monetary model.

In order to uncover which of the three structures, the monetary model in its forward-looking version (i. e. the interest rates are excluded) or the UIP condition in the long-run (i.e the interest rate differential is stationary) or the proportionality hypothesis between the black and official exchange rates, is responsible for the afore-mentioned result, we tested each one of them separately. This can be accomplished by imposing the same restrictions on all three cointegrating vectors (Johansen and Juselius, 1992) and the test statistics is distributed as χ^2 with $(p-s)\mathrm{x}r$ degrees of freedom. The test results imply that we are unable to reject the coefficient restrictions implied by the monetary model given in equation (1). On the contrary in both cases we rejected the result that the UIP condition is encompassed in the cointegrating space we have estimated. This finding may be attributed to the extensive foreign exchange controls which still exist in both countries and cause a continuous deviations from the UIP

condition. Finally, long-run informational efficiency holds in both countries implying the black and official exchange markets have the ability to process information efficiently.

Figures 3-5 present the evidence from the Hansen-Johansen (1993) recursive analysis on the sample independence of the Johansen procedure results. The overall conclusion drawn from the three tests is mixed and it may suggest that there is evidence of sample dependency of the cointegration results. Specifically, Figures 3(a) and 3(b) show that the rank of the cointegration space depends on the sample size from which it has been estimated, since the null hypothesis of a constant rank is rejected for both the Chilean peso-dollar and the Mexican peso-dollar cases. Figures 4(a) and 4(b) clearly indicate that we are always unable to reject the null hypothesis for the sample independence of the cointegration space for a given cointegration rank for both cases. Finally, the last two figures 5(a) and 5(b) in each case provide overwhelming evidence in favour of the constancy of the cointegrating vectors since no substantial drift was detected on the time paths of the eigenvalues. The last finding seems to indicate that the maximum likelihood estimates do not display considerable instabilities in recursive estimates. These results further reinforce our conclusion that the unrestricted monetary model of exchange rate determination is a valid framework to analyze movements of the Chilean and Mexican black market exchange rates from a long-run perspective.

5. Conclusions

In this paper we have examined the long-run properties of the monetary exchange rate model modified to incorporate the existence of a substantial black market for U.S. dollars for two Latin America countries, Chile and Mexico under the twin hypotheses that the system contains variables that are *I*(2) and that a linear trend is required in the cointegrating relations. The data used are monthly and are Chile (1973.10-1993.12) and Mexico (1976.09-1993.12). Several recent developments in the econometrics of non-stationarity and cointegration were applied and a number of novel results stem from our analysis. First, this paper makes use of the recently developed testing methodology developed by Johansen (1992a, 1995a, 1997) and extended by Paruolo (1986) and Rahbek *et al*. to test for the existence of *I*(2) and *I*(1) components in a multivariate context. Additionally, we estimated the roots of the companion

matrix as suggested by Juselius (1995) in order to make firmer conclusions about the rank of the cointegration space. The joint hypothesis of three cointegration vectors and one I(2) component could not be rejected for both countries an outcome that led to the transformation of the basic monetary model to contain *I*(1) variables and in which the rate of growth of domestic money plays a significant role. Second, given that three cointegration vectors were accepted, we formally imposed independent linear restrictions on each vector as suggested by Johansen and Juselius (1994) and Johansen (1995) in order to identify our system. Based on a likelihood ratio test for overidentifying restrictions (Johansen and Juselius, 1994) we rejected the joint restriction that the system represents the forward looking version of the monetary model for either case. Given this negative result we then tested whether independently the unrestricted version of the monetary model, the UIP condition and the proportionality between the black and official exchange rates could be considered and the results show that the UIP condition is not valid as a long-run relationship while the unrestricted version of the monetary model can still be a valid framework to investigate the long-run movements of the Chilean and Mexican black market exchange rates. There is also evidence of long-run informational efficiency in the black market which implies that this market processes information efficiently to the official market. Finally, we tested for parameter stability and it is shown that the dimension of the cointegration rank is sample dependent while the estimated coefficients do not exhibit instabilities in recursive estimations.

Footnotes

1. Gulati (1988) estimated that during the period 1977-83 underinvoicing of exports as a percentage of official exports was 20% for Argentina, 13% for Brazil and 34% for Mexico.

2. Apart for the monetary class of models, two other group of models have been developed to explain the behaviour of black market exchange rates. One group of models evolved from the theory of international trade and emphasize the transactions demand for foreign currency, (see, for example, Sheikh, 1976; Pitt, 1984). Another class of models, the portfolio balance of models, combines the characteristics of real trade models, by taking into account the flow considerations for black market dollars with the characteristics of monetary approach models by emphasizing the role of asset composition in the determination of the black market exchange rates (see, for example, Dornbusch, *et al*., 1983; Phylaktis, 1991).

3. The important link between exchange rates and fundamentals and the relevance of the monetary model to the exchange rate determination was again discussed in a series of recent papers Rogoff (1999), Flood and Rose (1999) and MacDonald (1999).

4. Availability of data is a major problem with all Latin American countries and this fact restricts our choices of measures. For Mexico, there is a treasury bill rate series available that begins in January 1978 and it could be used as a proxy for short-term interest rate. However when this series is compared to the inflation rate series the latter is smoother, which may be the result of continuous intervention of the Central Bank of Mexico. Furthermore, the treasury bill market in Mexico was substantially thin for most of the period. Similarly, for Chile a deposit rate series exist from January 1977 but there is also doubt about its usefulness. Finally, we note that we need to use as much as long data series following Hakkio and Rush (1991) who demonstrate the difficulties of detecting cointegration over short periods.

5. For an early justification of inflation as a measure of the cost holding money see Cagan (1956) and Wong (1977).

6. Gonzalo (1994) shows that the performance of the maximum likelihood estimator of the cointegrating vectors is little affected by non-normal errors. Lee and Tse (1996) have shown similar results when conditional heteroskedasticity is present.

7. The calculations of all tests as well as the estimation of the eigenvectors have been performed using the program CATS 1.1 in RATS 4.20 developed by Katarina Juselius and Henrik Hansen, Estima Inc. Illinois, 1995.

8. A small sample adjustment has been made to the Trace test statistics, Q_r, for the *I*(1) analysis

$$-2\ln Q = -(T - kp)\sum_{i=r_{0+1}}^{k}\ln(1-\hat{\lambda}_i) \text{ as suggested by Reimers (1992)}$$

9. Madhavi and Zhou (1994) have shown that the Mexican peso-dollar official exchange rate is *I*(2) and McNown and Wallace (1994) have shown that the Chilean peso-dollar official exchange rate and the Chilean money stock are also *I*(2) variables. Both these works have use univariate tests to reach their conclusion and we have already discussed how inappropriate these tests are.

10. The assertion that the domestic and foreign money are *I*(2) comes from recent empirical work on modeling money demand functions which suggest that nominal money stocks are *I*(2), (see Johansen, 1992c; Haldrup, 1994; Paruolo, 1996; and Rahbek *et al*. (1999) for UK monetary data and Juselius, 1994 for Danish data).

References

Agenor, P.R., 1992, Parallel currency markets in developing countries: Theory, evidence, and policy implications, *Essays in International Finance*, No. 188, Princeton University.

Baillie, R. and P.C. McMahon, P.C., 1989, Foreign Exchange Market: Theory and Econometric Evidence, Cambridge: Cambridge University Press.

Biswas, B. and Nandi R, the black market exchange rate in a developing economy: the case of India, Indian Economic Journal, 33, 1986, 23-34.

Blejer, M.L., 1978, Exchange restrictions and the monetary approach to the exchange rate, in the *Economics of Exchange Rates: Selected Studies*, eds. J.A. Frenkel and H.G. Johnson, Reading, M.A.: Addison-Wesley.

Cagan, P., 1956, The monetary dynamics of hyperinflation, in *Studies in the Quantity Theory of Money*, edited by Milton Friedman, Chicago: University of Chicago Press, 25-117.

Diamandis, P.F., D.A. Georgoutsos and G.P. Kouretas, 1998, The monetary approach to the exchange rate: Long-run relationships, identification and temporal stability, Journal of Macroeconomics, 20, 441-466.

Diamandis, P.F., D.A. Georgoutsos and G.P. Kouretas, 2000, The monetary model in the presence of *I*(2) components: Long-run relationships, short-run dynamics and forecasting of the Greek drachma, Journal of International Money and Finance, 19, 917-941.

Dornbusch, R., 1989, Real exchange rates and macroeconomics: A selected survey, Scandinavian Journal of Economics, 91, 401-432.

Dornbusch, R., D.V. Dantas, C. Pechman, R. Rocha, and D. Simoes, 1983, The black market for dollars in Brazil, The Quarterly Journal of Economics, 98, 25-40.

Edwards, S., 1988, Real and monetary determinants of real exchange rate behaviour: Theory and evidence from developing countries, Journal of Development Economics, 29,

Edwards, S., 1989, Exchange controls, devaluations, and real exchange rates: The Latin American experience, Economic Development and Cultural Change, 37, 457-494.

Edwards, S., 1994, The political economy of inflation and stabilization in developing countries, Economic Development and Cultural Change, 42, 235-266.

Edwards, S., 1998, Capital inflows into Latin America : A stop-go story, NBER Working Paper 6441.

Edwards, S., 1999, How effective are capital controls?, Journal of Economic Perspectives, 13, 65-84.

Edwards, S. and P. J. Montiel, 1989, Devaluation crises and the macroeconomic consequences of postponed adjustment in developing countries, International Monetary Fund Staff Papers, 36, 875-904.

Engle, R. F., and C. W. J. Granger, 1987, Co-integration and error correction: Representation, estimation and testing, Econometrica, 55, 251-276.

Frenkel, J.A., 1976, the monetary approach to the exchange rate: doctrinal aspects and empirical evidence, Scandinavian Journal of Economics, 78, 255-276.

Flood, R.P. and A. K. Rose, 1999, Understanding exchange rate volatility without the contrivance of macroeconomics, The Economic Journal, 109, F660-F672.

Gonzalo, J., 1994, Comparison of five alternative methods of estimating long-run equilibrium relationships, Journal of Econometrics, 60, 203-233.

Gulati, S., 1988, Capital flight: causes, concenquences and cures, Journal of International Affairs, 42, 165-185.

Gupta, S., 1980, An application of the monetary approach to black markets exchange rates, Weltwirtschaftliches Archiv, 116, 235-251.

Gupta, S., 1981, Black Markets Exchange Rates, Tubingen: J.C.B. Mohr.

Haldrup, N., 1994, The asymptotics of single-equation cointegrating regressions with I(1) and I(2) variables, Journal of Econometrics, 63, 153-181.

Haldrup, N. 1998, An econometric analysis of I(2) variables, Journal of Economic Surveys, 12, 595-650.

Hansen, H. and S. Johansen, 1993, Recursive estimation in cointegrated VAR-models, Working Paper, University of Copenhagen, Institute of Mathematical Statistics.

Johansen, S., 1988, Statistical analysis of cointegrating vectors, Journal of Economic Dynamics and Control, 12, 231-254.

Johansen, S., 1991, Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, Econometrica, 59, 1551-1580.

Johansen, S., 1992a, A representation of vector autoregressive processes integrated of order 2, Econometric Theory, 8, 188-202.

Johansen, S., 1992b, Determination of cointegration rank in the presence of a linear trend, Oxford Bulletin of Economics and Statistics, 54, 383-397.

Johansen, S., 1992c, Testing weak exogeneity and the order of cointegration in UK money demand data, Journal of Policy Modeling, 14, 313-334.

Johansen, S., 1995a, A statistical analysis of cointegration for I(2) variables, Econometric Theory, 11, 25-29.

Johansen, S., 1995b, Identifying restrictions of linear equations with applications to simultaneous equations and cointegration, Journal of Econometrics, 69, 111-132.

Johansen, S., 1997, A Likelihood analysis of the I(2) model, Scandinavian Journal of Statistics, 24, 433-462.

Johansen, S. and K. Juselius, 1990, Maximum likelihood estimation and inference on cointegration - with applications to the demand for money, Oxford Bulletin of Economics and Statistics, 52, 169-210.

Johansen, S. and K. Juselius, 1992, Testing structural hypotheses in a multivariate cointegration analysis of the PPP and the UIP for UK, Journal of Econometrics, 53, 211-244.

Johansen, S. and K. Juselius, 1994, Identification of the long-run and the short-run structure: An application to the ISLM model, Journal of Econometrics, 63, 7-36.

Juselius, K., 1994, On the duality between long-run relations and common trends in the I(1) and the I(2) case: An application to the aggregate money holdings, Econometric Reviews, 13, 151-178.

Juselius, K., 1995, Do purchasing power parity and uncovered interest rate parity hold in the long-run? An example of likelihood inference in a multivariate time-series model, Journal of Econometrics, 69, 211-240.

Juselius, K., 1998, A structured VAR under changing monetary policy, Journal of Business and Economic Statistics, 16, 400-411.

Kiguel, M., and M. O'Conell, 1995, Parallel exchange rates in developing countries, The World Bank Research Observer, 10, 22-52.

Kouretas, G.P., 1997, Identifying linear restrictions on the monetary exchange rate model and the uncovered interest parity: Cointegration evidence from the Canadian-U.S. dollar, Canadian Journal of Economics, 30, 875-890.

Kouretas, G.P., and L.P. Zarangas, 1998, A cointegration analysis of the official and parallel foreign exchange market for dollars in Greece, International Journal of Finance and Economics, 3, 261-276.

Lee, T.-H. and Y. Tse, 1996, Cointegration tests with conditional heteroskedasticity, Journal of Econometrics, 73, 401-410.

MacDonald, R., 1999, exchange rate behaviour: Are fundamentals important?, The Economic Journal, 109, F673-F691.

MacDonald, R. and M.P. Taylor, 1992, Exchange rate economics: A survey, International Monetary Fund Staff Papers, 39, 1-57.

MacDonald, R. and M.P. Taylor, 1994, The monetary model of the exchange rate: Long-run relationships, short-run dynamics and how to beat a random walk, Journal of International Money and Finance, 13, 276-290.

Mandavani, S., and S. Zhou, 1994, Purchasing power parity in high inflation countries: Further evidence, Journal of Macroeconomics, 16, 403-422.

McNown, R., and M.S. Wallace, 1994, Cointegration tests of the monetary exchange rate model for three high-inflation economies, Journal of Money, Credit and Banking, 26, 396-411.

Meese, R. and K. Rogoff, 1983, Empirical exchange rate models of the seventies: Do they fit out of sample?, Journal of International Economics, 14, 3-24.

Montiel, P.J., P-R. Agenor, and N. Ul Haque, 1993, Informal Financial Markets in Developing Countries, Oxford: Blackwell Publishers, 26-41.

Montiel, P. and C. M. Reinhart, 1999, Do capital controls and macroeconomic policies influence the volume and composition of capital flows? Evidence from the 1990s, Journal of International Money and Finance, 18, 619-635.

Moore, M.J. and K. Phylaktis, 1996, Black and official exchange rates in the pacific basin, mimeo.

Olgun, H., 1984, an analysis of black markets, exchange rates in a developing country-The case of Turkey, Wert, 120, 327-347.

Pantula, S. G., 1989, Testing for unit roots in time series data, Econometric Theory, 5, 256-271.

Paruolo, P., 1996, On the determination of integration indices in I(2) systems, Journal of Econometrics, 72, 313-356.

Phylaktis, K., 1991, The black market for dollars in Chile, Journal of Development Economics, 37, 155-172.

Phylaktis, K., 1996, Black markets for foreign currency: A survey of theoretical and empirical issues, Financial Markets, Institutions and Instruments, 5, 210-226.

Pitt, M., 1984, Smuggling and the black market for foreign currency, Journal of International Economics, 16, 243-257.

Rahbek, A.H., C. Konsted and C. Jorgensen, 1999, Trend-Stationarity in the I(2) cointegration model, Journal of Econometrics, 90, 265-289.

Reimers, H-E., 1992, Comparison of Tests for Multivariate Co-integration, Statistical Papers, 33, 335-359.

Rogoff, K., 1999, Monetary models of dollar/yen/euro nominal exchange rates: dead or undead?, The Economic Journal, 109, F655-F659.

Sephton, P.S. and Larsen K.H., 1991, Tests of exchange market efficiency: fragile evidence from cointegration tests, Journal of International Money and Finance, 10, 561-570.

Sheikh, M.A., 1976, Black market for foreign exchange, capital flows and smuggling, Journal of Development Economics, 3, 9-26.

Sims, C.A., 1980, Macroeconomics and Reality, Econometrica, 48, 1-48.

Wong, C-H., 1977, Demand for money in developing countries: Some theoretical and empirical results, Journal of Monetary Economics, 3, 59-86.

Van den Berg H. and Jayanetti, S.C., A novel test of the monetary approach using black market exchange rates and Johansen-Juselius cointrgration method, Economics Letters, 41, 1993, 413-418.

Table 1. Residual misspecification tests of the model with $k = 3$

Chilean Peso-Dollar

Eq.	σ_ε	LB(42)	ARCH(3)	NORM(3)	R^2
Δe_b	0.0023	42.34	3.35	6.98	0.69
Δe_o	0.0018	44.32	2.89	12.34	0.55
Δm	0.0044	54.67	4.45	5.57	0.77
Δm^*	0.0056	44.82	5.56	4.56	0.62
Δy	0.0012	46.98	3.46	8.67	0.73
Δy^*	0.0145	61.22	0.99	1.23	0.45
Δi	0.0033	53.45	2.32	22.34	0.46
Δi^*	0.0125	37.28	1.98	5.97	0.42

Mexican Peso-Dollar

Eq.	σ_ε	LB(42)	ARCH(3)	NORM(3)	R^2
Δe_b	0.0037	47.00	2.68	7.89	0.72
Δe_o	0.0015	43.00	1.99	13.55	0.54
Δm	0.0032	53.93	2.31	9.23	0.80
Δm^*	0.0127	55.42	2.42	3.38	0.84
Δy	0.0013	34.56	4.28	9.76	0.85
Δy^*	0.0009	51.28	1.34	0.98	0.50
Δi	0.0022	26.78	0.21	19.34	0.41
Δi^*	0.0008	33.24	2.47	5.78	0.49

Notes: LB is the Ljung-Box test statistic for residual autocorrelation, ARCH is the test for heteroscedastic residuals, and NORM the Jarque-Bera test for normality. All test statistics are distributed as χ^2 with the degrees of freedom given in parentheses.

Multivariate Residuals Diagnostics

Case	L-B(2938)	LM(1)	LM(4)	χ^2 (14)
CP/USD	1933.29(0.20)	64.09(0.07)	59.79(0.14)	35.82(0.00)
MP/USD	2009.33(0.15)	66.22(0.06)	62.78(0.12)	37.67(0.00)

Notes: L-B is the multivariate version of the Ljung-Box test for autocorrelation based on the estimated auto- and cross - correlations of the first [T/4=51] lags with 2938 degrees of freedom. LM(1) and LM(4) are the tests for first and fourth-order autocorrelation distributed as a χ^2 with 24 degrees of freedom and χ^2 is a normality test which is a multivariate version of the Shenton-Bowman test with 14 degrees of freedom.

Table 2. Testing the Rank of the I(2) and the I(1) Model

Testing the joint hypothesis $H(s_1 \cap r)$

Chilean Peso-Dollar

p-r	r	$Q(s_1 \cap r / H_0)$								Q_r
8	0	1152.9 *441.5*	942.4 *397.4*	795.0 *356.5*	676.4 *317.9*	576.8 *283.3*	489.4 *252.3*	425.1 *225.6*	368.3 *202.2*	350.6 *182.6*
7	1		895.4 *351.6*	725.7 *311.2*	586.8 *274.0*	469.9 *241.2*	378.2 *211.6*	309.2 *186.1*	252.1 *164.6*	239.5 *146.8*
6	2			691.8 *269.2*	528.1 *233.8*	401.9 *202.8*	304.1 *174.9*	219.3 *151.3*	156.2 *130.9*	146.7 *115.4*
5	3				504.2 *198.2*	362.75 *167.9*	253.0 *142.2*	169.3 *119.8*	93.5 *101.5*	79.2 *87.2*
4	4					322.4 *137.0*	209.5 *113.0*	125.8 *92.2*	70.2 *75.3*	58.6 *62.8*
3	5						204.9 *86.7*	*107.7* *68.2*	47.3 *53.2*	30.6 *42.7*
2	6							99.3 *47.6*	*39.6* *34.4*	16.0 *25.4*
1	7								45.1 *19.9*	4.4 *12.5*
s_2		8	7	6	5	4	3	2	1	

Mexican Peso - Dollar

p-r	r	$Q(s_1 \cap r / H_0)$								Q_r
8	0	989.4 *441.5*	813.5 *397.4*	688.3 *356.5*	566.7 *317.9*	471.8 *283.3*	392.7 *252.3*	334.7 *225.6*	284.2 *202.2*	265.4 *182.6*
7	1		710.6 *351.6*	564.2 *311.2*	445.4 *274.0*	362.9 *241.2*	285.0 *211.6*	231.1 *186.1*	194.2 *164.6*	177.3 *146.8*
6	2			519.1 *269.2*	401.4 *233.8*	313.0 *202.8*	241.8 *174.9*	193.7 *151.3*	148.1 *130.9*	124.7 *115.4*
5	3				345.9 *198.2*	237.5 *167.9*	166.0 *142.2*	117.5 *119.8*	94.1 *101.5*	85.3 *87.2*
4	4					206.2 *137.0*	116.3 *113.0*	78.1 *92.2*	55.8 *75.3*	48.2 *62.8*
3	5						128.5 *86.7*	53.5 *68.2*	32.4 *53.2*	25.0 *42.7*
2	6							89.2 *47.6*	31.3 *34.4*	14.0 *25.4*
1	7								28.7 *19.9*	5.4 *12.5*
s_2		8	7	6	5	4	3	2	1	

Notes: *p* is the number of variables, *r* is the rank of the cointegration space, s_1 is the number of *I*(1) components and s_2 is the number of *I*(2) components. The numbers in italics are the 95% critical values (Rahbek, *et al*., 1999, Table 1). For all tests a structure of three lags for both black exchange rates was chosen according to a likelihood ratio test, corrected for the degrees of freedom (Sims, 1980) and the Ljung-Box Q statistic for detecting serial correlation in the residuals of the equations of the VAR. A model with an unrestricted constant in the VAR equation is estimated for all three cases according to the Johansen (1992b) testing methodology.

Table 2. Continues

Modulus of 9 largest roots

Chilean peso									
Unrestricted model	0.98	0.98	0.97	0.97	0.95	0.88	0.71	0.55	0.42
r = 3	1.00	1.00	1.00	1.00	1.00	0.94	0.65	0.48	0.33
Mexican peso									
Unrestricted model	0.99	0.99	0.95	0.95	0.90	0.90	0.81	0.72	0.62
r = 3	1.00	1.00	1.00	1.00	1.00	0.96	0.70	0.60	0.33

Table 3. Tests for long-run exclusion, stationarity, and weak exogeneity

	Long-run exclusion		Stationarity		Weak exogeneity	
Variable	**CP/USD**	**MP/USD**	**CP/USD**	**MP/USD**	**CP/USD**	**MP/USD**
e_p	11.34*	9.46*	33.87*	32.01*	15.45*	13.11*
e_o	11.55*	13.97*	43.15*	31.93*	40.22*	41.58*
m-m*	22.45*	21.08*	21.56*	25.45*	11.23*	53.88*
Δm	35.67*	24.29*	33.23*	20.45*	16.45*	20.20*
y	8.99*	7.99*	21.44*	20.49*	6.68	7.11
y*	10.23*	8.87*	25.23*	19.71*	3.24	2.32
i	9.67*	9.64*	17.77*	17.60*	17.56*	16.45*
i*	19.67*	21.03*	19.02*	23.75*	14.22*	13.67*

Notes: e_o, e_p, (m-m*), Δm, y and i are respectively the spot exchange rate, the relative monies, the first difference of the domestic money supply, the real output and the short-term interest rate, with the U.S. magnitudes denoted with an asterisk. The long-run exclusion restriction and the weak exogeneity tests are χ^2 distributed with three degrees of freedom and the 5% critical level is 7.81, and the stationarity test is a χ^2 distributed with six degrees of freedom and the 5% critical level is 12.59.

Multivariate Residuals Diagnostics

Case	L-B(3072)	LM(1)	LM(4)	χ^2 (16)
GRD/USD	1696.12(0.26)	52.20(0.35)	61.12(0.09)	726.29(0.00)
GRD/DM	1453.60(0.28)	58.71(0.09)	63.50(0.08)	288.12(0.00)

Notes: L-B is the multivariate version of the Ljung-Box test for autocorrelation based on the estimated auto- and cross - correlations of the first [T/4=51] lags with 3072 degrees of freedom. LM(1) and LM(4) are the tests for first and fourth-order autocorrelation distributed as a χ^2 with 64 degrees of freedom and χ^2 is a normality test which is a multivariate version of the Shenton-Bowman test with 16 degrees of freedom.

Table 4. Estimated Coefficients and Hypothesis Testing

$$e_b = \beta_1 e_o + \beta_2 (m - m^*) + \beta_3 \Delta m + \beta_4 y + \beta_5 y^* + \beta_6 i + \beta_7 i^* + \beta_0 + \gamma_1 t$$

CP/USD	1.00	-1.46	3.45	-93.82	-7.88	5.34	0.03	-0.15	-0.23	-1.34
	1.00	-6.55	11.24	-86.46	77.87	-33.24	0.01	-0.11	0.34	0.02
	1.00	-1.14	10.98	-34.56	22.56	-12.55	0.02	-0.16	0.09	0.23
MP/USD	1.00	-1.22	4.33	-3.32	-8.01	4.21	0.04	-0.11	-0.79	-1.45
	1.00	-3.52	9.23	-6.22	7.21	-3.89	0.03	-0.17	1.25	0.11
	1.00	-1.15	4.66	-4.77	-6.22	-2.55	0.19	-0.24	-011	-0.55

Notes: The eigenvectors have been normalized with respect to the estimated coefficient on the black market exchange rate

A. Tests for overidentifying restrictions

CP/USD

$$\beta = \begin{bmatrix} 1 & 0 & -1 & 0 & 5.58(2.16) & -3.65(1.22) & 0 & 0 & -6.6(1.34) & -3.56(2.56) \\ 0 & 0 & 0 & -29.16(3.22) & 0 & 0 & 1 & -1 & 0.03(0.013) & 0.23(0.89) \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.88(1.01) \end{bmatrix}$$

Q(7)=43.79[0.00]

MP/USD

$$\beta = \begin{bmatrix} 1 & 0 & -1 & 0 & 3.87(0.93) & -2.77(1.08) & 0 & 0 & -4.6(1.1) & -2.89(3.01) \\ 0 & 0 & 0 & -24.35(2.99) & 0 & 0 & 1 & -1 & 0.07(0.03) & -0.78(1.23) \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99(0.65) \end{bmatrix}$$

Q(7)=39.42[0.00]

Notes : Q denotes a likelihood ratio test for overidentifying restrictions as suggested by Johansen and Juselius (1994) and is distributed as a with the corresponding degrees of freedom given in parentheses. Numbers in brackets denote marginal significance levels. Numbers in parentheses below the coefficient estimates report estimated asymptotic standard errors which are the square roots of the computed Wald test statistics developed by Johansen (1991).

Case	H_1 $(\beta_1 = 0, \beta_2 = 1, \beta_4 = -\beta_5, \beta_6 = \beta_7)$	H_2 $(\beta_1 = 0, \beta_4 = \beta_5 = 0, \beta_6 = -\beta_7)$ e_b is excluded	H_3 $\beta_1 = 1$
CP/USD	0.23	0.00	0.11
MP/DM	0.12	0.00	0.09

Notes: Numbers correspond to marginal significance levels of the H_5 test statistic (Johansen and Juselius, 1992) distributed as a χ^2 with five degrees of freedom, $(p-r)xr_1$, [p = number of variables, r = cointegration rank, r_1 = number of vectors on which the restrictions are imposed]. The coefficient estimates necessary for the construction of the test are those given above.

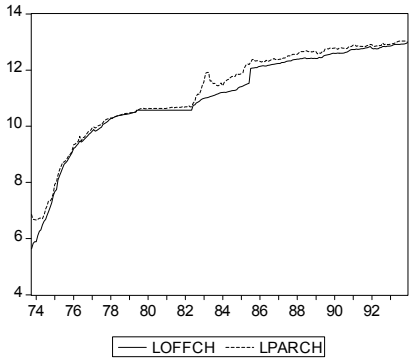


Figure 1(a) : Official and black exchange rates
Chile-U.S. case

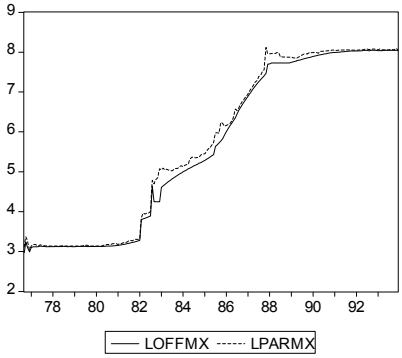


Figure 1(b) : Official and black exchange rates
Mexico-U.S. case

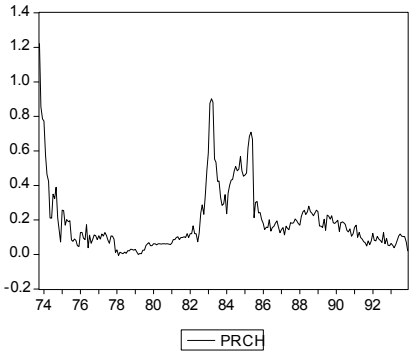


Figure 2(a) : The black market premium
Chile-U.S. case

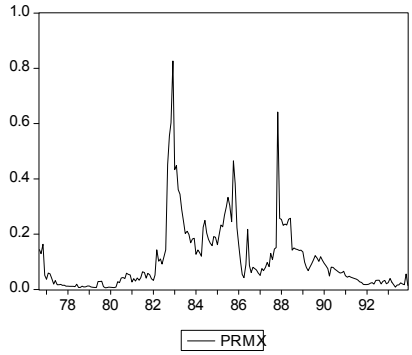


Figure 2(b) : The black market premium
Mexico-U.S. case

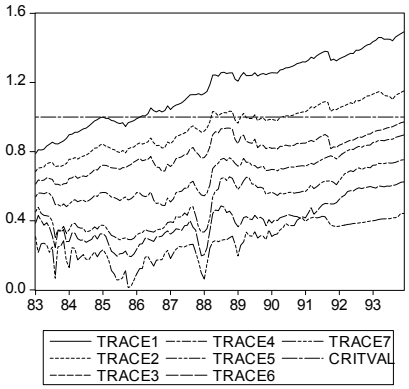


Figure 3(a) : The Trace Test
Chile-U.S. case

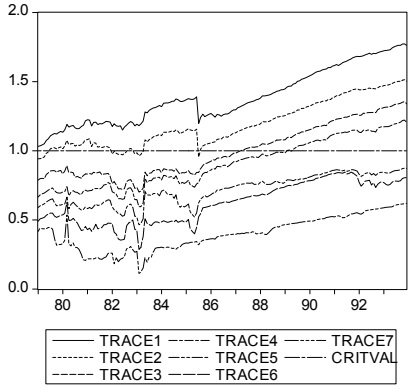


Figure 3(b) : The Trace Test
Mexico-U.S. case

1 is the 5% significance level

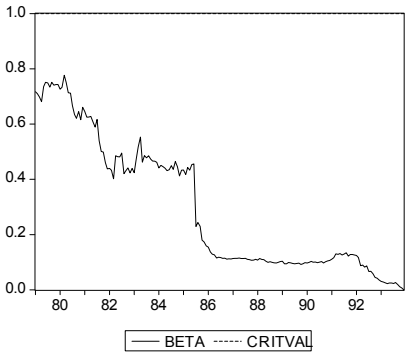


Figure 4(a) : The test for the constancy of beta
Chile-U.S.case

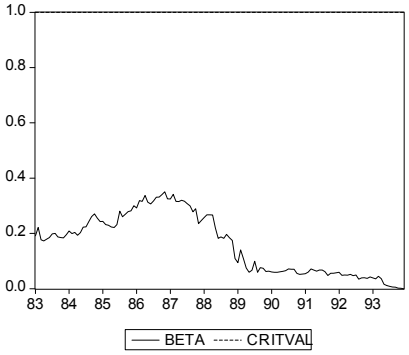
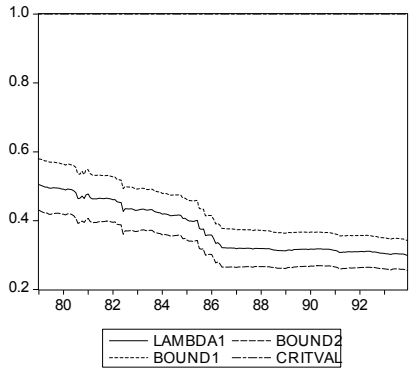
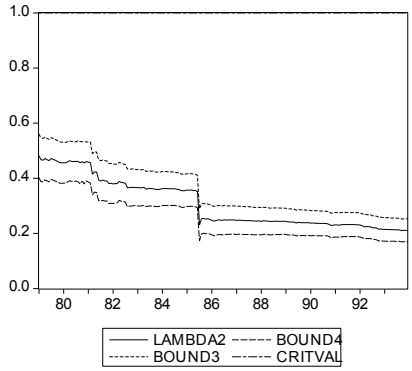


Figure 4(b) : The test for the constancy of beta
Mexico-U.S.case

1 is the 5% significance level





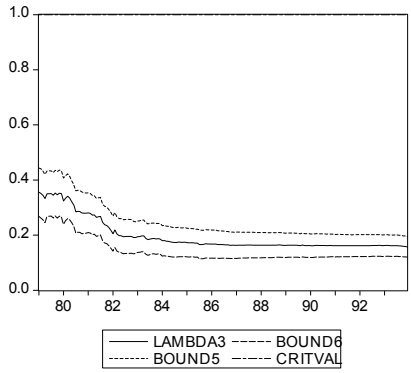
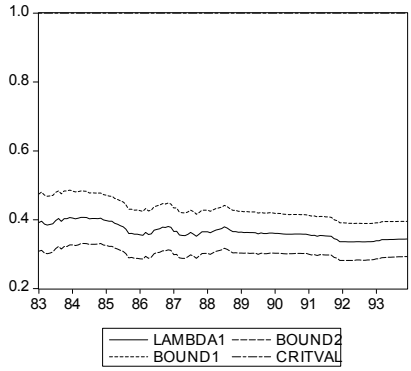
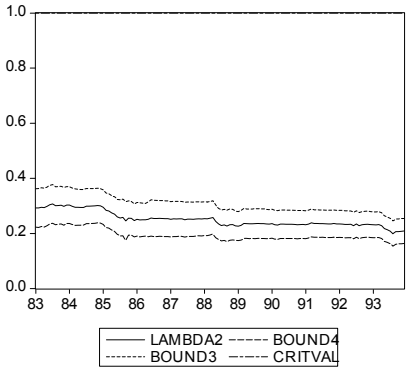


Figure 5 (a) : The eigenvalue test : Chile-U.S. case





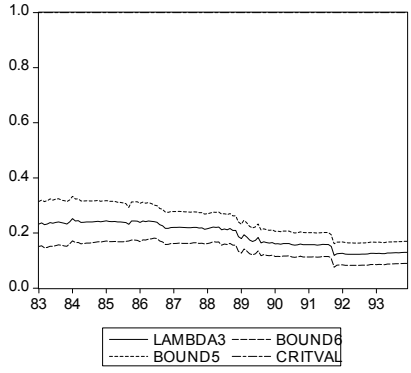


Figure 5 (b) : The eigenvalue test : Mexico-U.S. case

1 is the 5% significance level