Irreversible Development of a Natural Resource:

Management rules and policy issues when direct use values and environmental values are uncertain

Anastasios Xepapadeas[1]
University of Crete, Department of Economics
University Campus
Rethymno 74100, Greece
e-mail: xepapad@econ.soc.uoc.gr

September 22, 2001

[1] I am grateful to an anonymous referee for valuable comments on an earlier draft of this paper. Support from ELKE 1266 is acknowledged.

Irreversible Development of a Natural Resource

Abstract

The paper analyzes resource management that entails the irreversible development of an exhaustible resource when the values of services generated by the resource in either the developed or the undeveloped state are uncertain. An exercise barrier approach is used to derive the privately-optimal and the socially-optimal free boundaries. The two boundaries are used to compare the pace of development under profit maximization and social optimization. Regulatory schemes on resource development that will induce the profit-maximizing decision maker to behave as the social planner, with regard to development choices under uncertainty and irreversibility, are also presented.

Keywords: Natural resource, irreversible development, uncertainty, exercise boundary, private optimum, social optimum, regulation.

JEL Classification numbers: Q0, Q2

1 Introduction

Uncertainty is an issue of considerable interest in the environmental and resource economics literature. When the analysis is carried out in a dynamic context the interactions between uncertainty and irreversibility are of special interest.[1] One fundamental proposition in the area of environmental economics, established by Arrow and Fisher (1974), is that an option value exists associated with refraining from an irreversible decision now, when next period benefits or losses resulting from the decision are uncertain, even if the decision-maker is risk neutral. Closely associated with the above concepts is the concept of timing of the irreversible decision, and the question of whether or not the decision-maker should postpone action until more information is acquired in the future.

Recent approaches to the solution of this type of stochastic control problem focus on the derivation of a free or exercise boundary derived from the solution of the associated Hamilton-Jacobi-Bellman (HJB) equation (e.g. Dixit and Pindyck 1994, Soner 1997). The basic property of the boundary is that it divides a certain strategy space into two regions. Depending on the region of the space in which a stochastic variable is realized, the decision-maker decides whether or not to undertake the irreversible action.

The purpose of this paper is to analyze the problem of resource management that entails the irreversible development of an exhaustible resource, when the benefits - or more precisely the values - of services generated by the resource in either the developed or the undeveloped state are uncertain. In analyzing the structure of the benefits associated with the resource, a broad division of values into direct use values and indirect values or non-use values is considered. Direct use values are associated with the value of services generated by the resource after it has been developed. On the other hand indirect or non-use values are associated with the value of services generated by the resource when it is in an undeveloped state. Indirect or non-use values can be associated with services generated by the biodiversity of an undeveloped part of an ecosystem, or with passive values related mostly to ethical positions.[2] These indirect or non-use values are referred to, in the rest of the paper, as the environmental or the intrinsic value of the undeveloped resource.

In the resource management problem, while the direct use values as defined above can in principle be approximated by market prices associated with the services generated by the developed resource, the non-use values are much harder to approximate because of the well-known missing market problems. In practice these values, especially for the cases of aesthetic or existence values, are approximated by state preference methods, such as contingent val-

[1] See for example Arrow and Fisher (1974), or Fisher and Haneman (1986, 1987). The same issue has also received considerable attention in the literature of finance and investment (e.g.Constandinidis 1986, Dixit and Pindyck 1994; Dixit, Pindyck and Sodal 1998) or public finance (Hassett and Metcalf 1999).

[2] For detailed definitions of these values and measurement issues see, for example, Perrings (1995).

uation, or revealed preference methods such as travel cost, hedonic models or random utility models. The difficulties associated with any attempt to approximate non-use values induce a relatively high degree of uncertainty in the estimates. On the other hand when environmental values are associated with services provided by intact ecosystems, which might undergo irreversible development, a more direct valuation might be possible. For example ecosystem services relate closely to the development of new products in biotechnology and pharmaceuticals. These services can be associated with the formation of market values for bioprospecting rights in locations representing biodiversity hot spots. Certain ecosystem services can have direct market values such as watershed services (Chichilnisky and Heal 1998) or ecotourism services based on the preservation of intact ecosystems (Heal 2000), which are ignored by private developers.

The problem of analyzing the irreversible development of an environmental resource under use value and non-use value uncertainty is thus considered, by explicitly taking into account the facts that: (i) the resource can be developed by a private profit-maximizing decision-maker that acquires profits by developing the resource involved in the problem or by a social planner or environmental regulator that seeks to maximize some appropriately-defined social welfare criterion; (ii) the undeveloped resource has an environmental value which is not taken into account by the individual developer, but is accounted for in the context of the social optimization problem faced by the social planner or environmental regulator; and (iii) there is simultaneous uncertainty both on profits from the resource development, that is market prices associated with direct use values, and the environmental value of the undeveloped resource, that is indirect non-use values.[3]

The resource management problem in this paper is analyzed by using an exercise barrier approach corresponding to the associated stochastic control problem and in particular by deriving the privately-optimal or unregulated free boundary and the socially-optimal free boundary. These boundaries characterize the optimal resource development in the sense that, depending on the region of the space in which a stochastic variable associated with profit or/and environmental uncertainty is realized, the decision-maker decides whether or not to undertake the irreversible resource development. The two boundaries are used to compare the pace of development under profit maximization or social optimization. It is shown that when uncertainty exists only with respect to the market prices associated with direct use values, the pace of development under social optimization is slower relative to the development pace under profit maximization, a result anticipated by the existing literature on

[3]Scheinkman and Zariphopoulou (1999) have recently analyzed a similar problem of resource management in a fairly general set up of simultaneous uncertainty on the returns from the resource development and the returns of the undeveloped resource. In the present paper by analyzing a simpler model it becomes possible to completely characterize and distinguish optimal policies, under private profit maximization and social optimization, and furthermore to address regulation issues under simultaneous uncertainty and irreversibility.

resource management. However when uncertainty is associated with both use and non-use values, the pace of development could be reversed under certain circumstances. This result is possible under a downward shift of the non-use values relative to use values. If this shift is sufficiently strong then the externality associated with non-use values that makes the socially-optimal development pace slower than the privately-optimal, works the other way round implying that in a given time interval the socially-optimal development should be faster than the privately-optimal one.

Having established the deviation between the unregulated and the socially-optimal free boundaries, the issue of policy design is addressed.[4] The idea is to introduce a regulatory scheme in the form of development taxes or command and control limits on development that will induce the profit-maximizing decision maker to behave in the same way as the social planner regarding development choices. We show that the optimal policy scheme is different in the case of use value uncertainty relative to the case of simultaneous use and non-use value uncertainty.

In the context described above, one contribution of this paper can be associated with using the concept of the optimal exercise boundary to compare development paths under socially-optimal and market solutions under simultaneous use and non-use values uncertainty. By using this exercise boundary comparison approach there is no need to compare expected equilibrium outcomes. Instead a deterministic function for the exercise boundary is used to compare development paths corresponding to the two solutions. Since the development paths are obtained by comparing the boundaries to the observed values of the stochastic variables, it is a straightforward process to compare the development paths by simply comparing the boundaries relative to the moves of the state variables. The comparison confirms the generally accepted results that the socially-optimal solution implies slower development than the market solution, but it also reveals the possibility, which to my knowledge has not been explored yet, that it might be socially optimal to develop the resource relatively faster at some time interval. This result is made possible by the use of the boundary comparison approach to the general problem of the simultaneous use and non-use value uncertainty.[5]

The paper contributes also to the design of policy schemes under uncertainty. The main idea is that since the private profit-maximizing agent uses a boundary to design management strategies, if a policy regime shifts the boundary so that it coincides with the socially-optimal boundary, then the privately-optimal development path will be the same as the socially-optimal development path. Thus under uncertainty the policy objective is to develop policy schemes that will make the optimal boundary of the profit-maximizing

[4] Uncertainty on the benefit and cost side in environmental problems has been related to the choice between price or quantity environmental policy instruments (e.g. Weitzman 1974). Stavins (1996) analyses this issue under correlated benefit and cost uncertainty.

[5] A boundary comparison method has been used by Xepapadeas (1998) in a simpler model of resource development with uncertainty only in prices.

agent the same as the optimal boundary for the social planner.

2 Resource Development under Irreversibilities

A resource or environmental asset of fixed size S which can be developed into a new use is considered. In the undeveloped state the asset has an environmental or intrinsic value associated with indirect or non-use values equal to $q_t S$, where q_t is the unit environmental value of the resource at time t.[6] The asset can be some landscape that could for example undergo industrial, housing or agricultural development, or a scenic land that can potentially undergo tourist development. At the undeveloped stage the land provides indirect or non-use value services, such as services related to biodiversity of the undeveloped landscape, aesthetic values of the undeveloped land, or more general existence values.

Assume that one potential developer[7] of the site exists and at each point in time he or she develops $h_t \geq 0$. Thus total cumulative development at time t is defined as:

$$D_t = \int_0^t h(s)\, ds,\ D_t \leq S \tag{1}$$

Since $h_t \geq 0$, development is irreversible. After the development the environmental value of the asset is defined as:

$$q_t (S - D_t)$$

Thus the development of the site linearly reduces its intrinsic value.

However, the development generates a net flow of services for the developer, according to an increasing and strictly concave function:

$$f(D_t),\ f'(D_t) > 0,\ f''(D_t) < 0$$

We assume that the developer is small relative to the market for the resource services, and that these services can be sold at some exogenously determined world price.[8] This price, P_t, evolves stochastically following a geometric Brownian motion:

$$dP_t = a_1 P_t dt + \eta_1 P_t dz_{pt} \tag{2}$$

with $\{z_{1t}\}$being a Wiener process.[9],[10] Thus the developer's net revenues at

[6] This value could have been obtained by state preference or revealed preference methods.

[7] The qualitative nature of the results will not change if we assume n identical developers.

[8] This might be for example the case of services associated with the development of a scenic land which are sold in the world market of tourist services.

[9] See for example Malliaris and Brock (1982) for definitions and more details.

[10] It should be noted that the Brownian motion assumption causes price to move away from its starting point. If however price is related to long-run marginal costs, then a better assumption about price movements could be a mean-reverting process. Under this assumption price tends toward marginal costs in the long run and price movements can be

each instant of time are defined as

$$P_t f\left(D_t\right)$$

Let the cost of developing one unit of the resource be c. If the decision is to develop the site by ΔD from the existing development level D_0, then

$$\Delta D = D_0^+ - D_0$$

The cost of this change in the development is then defined as:

$$c\Delta D \tag{3}$$

Given this structure we seek an optimal development strategy which takes the form of a free boundary defined by an equation $P = P(D)$ relating price and cumulative development. When the realized price P^r at any point in time is such that $P^r < P(D)$, no development is undertaken, while when $P^r > P(D)$, enough development is undertaken to restore equality between the realized price and the value of the boundary.

3 Privately-Optimal Development

Consider the case of the developer when the initial market price of the services generated by the developed resource is P_0 and the initial development is D_0. Given a private discount rate $\rho > 0$,[11] with $\rho > a_1$, the developer seeks the nondecreasing process D_t, that will maximize the present values of net revenues less the cost of development.

Let $\mathcal{D} = \left\{\Delta D_t : \Delta D_t \geq 0,\ \forall t \geq 0 \text{ and } \int_0^t \Delta D_u du < L,\ \forall t \geq 0\right\}$. The set of admissible controls which represent resource development is defined as: $\mathcal{U} = \{\Delta D_t : \Delta D_t \in \mathcal{D},\ \forall D \in [0, L)\}$. Then the developer's problem is defined as

$$\max_{\mathcal{U}} J\left(P_t, D_t; \Delta D_t\right) = \mathcal{E}_0 \int_0^\infty e^{-\rho t}\left[P_t f\left(D_t\right) - c\Delta D_t\right] dt$$
$$\text{subject to (2).}$$

The value function associated with this problem can then be defined as:

$$V\left(D_t, P_t\right) = \sup_{\mathcal{U}} \mathcal{E}_0 \int_0^\infty e^{-\rho t}\left[P_t f\left(D_t\right) - c\Delta D_t\right] dt \tag{4}$$

By the concavity of $f(D)$ and the linear dynamics it can be shown that the value function is concave in D. The dynamic programming equation for the developer's problem takes the form (Soner 1997):

$$\rho V = \max_{\Delta D_t}\left\{\left[\mathcal{L}_{\Delta D}^{P,D}\right] V + f(D) - c\Delta D\right\}$$

modeled as

$$dP_t = a\left(\widetilde{P}_t - P_t\right) P_t dt + \sigma P_t dz_{pt}$$

where $\widetilde{P}_t$ can be interpreted as long-run marginal costs.

[11] For a small open economy with negligible country risk it could be assumed that firms can borrow at a risk-free world interest rate.

where $\mathcal{L}_{\Delta D}^{P,D} = \frac{1}{2}\eta_1^2 P^2 \frac{\partial^2}{\partial P^2} + a_1 P \frac{\partial}{\partial P} + \Delta D \frac{\partial}{\partial D}$ is the differential generator.

At each instant of time the developer has two choices: to preserve the site or to develop it. The time interval when no development is taking place and the previously acquired development is used to generate net revenues can be defined, following Dixit and Pindyck (1994), as the no action or the continuation interval. A stopping time is defined as a non-negative random variable τ at which new development is undertaken.

Let D_τ^* be the optimal development process at time τ. Following Fleming and Soner (1993) or Soner (1997), if τ is a stopping time then by the dynamic programming principle:

$$V(D,P) = \sup_{\mathcal{U}} \mathcal{E}_0 \left[\int_0^\tau e^{-\rho u} P_u f(D_u)\, du + e^{-\rho\tau} V(D_\tau^*, P_\tau) \right] \tag{5}$$

Assume that in the time interval $[0,\theta]$ the developer undertakes no new development, but keeps it constant at D_0. Then by the principle of dynamic programming, the value function should be no less that the payoff (continuation payoff) in the interval $[0,\theta]$, plus the expected value after θ, or:

$$\begin{aligned} V(D,P) \geq \mathcal{E}_0 \Bigg[\int_0^\theta & e^{-\rho u} P_u f(D_0)\, du \\ & + e^{-\rho\theta} V(D_\theta, P_\theta) \Bigg] \end{aligned} \tag{6}$$

with equality if D_0 is the optimal policy in $[0,\theta]$. Applying the Itô lemma to the value function on the right hand side of (6), dividing by θ and taking limits as $\theta \to 0$, we obtain :[12]

$$\rho V \geq \frac{1}{2}\eta_1^2 P^2 V_{PP} + a_1 P V_P + P f(D) \tag{7}$$

with equality if $D_t = D_0$ in the interval $[0,\theta]$.

Consider now the decision to develop instantaneously by $\Delta D = D_0^+ - D_0$. Then the right hand side of the dynamic programming equation becomes

$$\max_{\Delta D \geq 0} \left\{ \left[\mathcal{L}_{\Delta D}^{P,D}\right] V + f(D) - c\Delta D \right\} = \frac{1}{2}\eta_1^2 P^2 V_{PP} + a_1 P V_P + P f(D) + \widehat{\mathcal{H}}(V_D)$$

where

$$\widehat{\mathcal{H}}(V_D) = \max_{\Delta D \geq 0} \{V_D \Delta D - c\Delta D\}$$

which implies

$$V_D - c \leq 0,\ \Delta D \geq 0,\ \text{or}$$

$$\text{If } V_D - c < 0 \text{ then } \Delta D = 0 \tag{8}$$

$$\text{If } \ \Delta D > 0 \text{ the } V_D - c = 0, \tag{9}$$

[12] Subscripts associated with the value functions denote derivatives.

Thus when no development is optimal, (7) is satisfied as equality, while when development is optimal, (9) is satisfied as equality. Combining the two the HJB equation can be written as:

$$\min\left\{\left[\rho V-\frac{1}{2}\eta_1^2P^2V_{PP}-a_1PV_P-Pf\left(D\right)\right],-\left[V_D-c\right]\right\}=0 \tag{10}$$

The HJB equation can be used to derive the free boundaries at the unregulated and regulated equilibrium.

3.1 The free boundary at the private optimum

The HJB equation (10) determines the conditions under which a profit-maximizing agent will undertake new development or not. Thus the HJB divides the (P, D) space into two regions. The curve $P = P(D)$ for the boundary between the two regions determines the profit-maximizing development process. This optimal exercise or free boundary will divide the (P, D) space into two regions: the 'no development' region, called region I, and the 'development region', called region II. In region I the first term of the HJB equation is zero since $\Delta D = 0$ and the second term of the HJB equation is positive by (8), thus

$$\rho V-\frac{1}{2}\eta_1^2P^2V_{PP}-a_1PV_P-Pf\left(D\right)=0 \tag{11}$$

The general solution of (11) can be obtained as:

$$V\left(D,P\right)=A_1\left(D\right)P^{\beta_1}+A_2\left(D\right)P^{\beta_2}+P\frac{f\left(D\right)}{\rho-a_1} \tag{12}$$

where $\beta_1=\frac{1}{2}-\frac{a_1}{\eta_1^2}+\sqrt{\left(\frac{a_1}{\eta_1^2}-\frac{1}{2}\right)^2+\frac{2\rho}{\eta_1^2}}>1$ is the positive route of the fundamental quadratic $Q=\frac{1}{2}\eta_1^2\beta\left(\beta-1\right)+a_1\beta-\rho=0$. We need to disregard the negative root in order to prevent the value from becoming infinitely large when the market size becomes very small, thus we set $A_2(D) = 0$. [13]

In region II the second term of (10) is satisfied as zero and $\Delta D > 0$, or

$$V_D\left(D,P\right)-c=0 \tag{13}$$

Using (12) and (13), the constant $A_1(D)$ and the function $P = P(D)$ can be determined. To obtain this the 'value matching' and the 'smooth pasting' conditions are used.[14]

The value matching condition means that on the boundary separating the two regions the value functions should be equal. Solving (13) for P we can obtain the yet unspecified function for the boundary $P = P(D)$. Then we have, combining (12) and (13) and substituting for P,

[13] See Dixit and Pindyck (1994).

[14] For a presentation of these conditions, see Dixit and Pindyck (1994).

$$V_D(D,P) = A_1'(D)\,P^{\beta_1} + P\frac{f'(D)}{\rho - a_1} = c\,,\ P = P(D) \tag{14}$$

The smooth pasting condition means that the derivatives of the value functions with respect to P on the boundary are equal or:

$$V_{DP}(D,P) = \beta_1 A_1'(D)\,P^{\beta_1 - 1} + \frac{f'(D)}{\rho - a_1} = 0 \text{ with } P = P(D) \tag{15}$$

Combining (14) and (15) we can solve for the unknown functions $P(D)$ and $A_1'(D)$ to obtain:

$$P(D) = \frac{\beta_1}{\beta_1 - 1}\frac{\delta c}{f'(D)} \tag{16}$$

$$A_1'(D) = -\left(\frac{\beta_1 - 1}{c}\right)^{\beta_1 - 1}\left(\frac{f'(D)}{\beta_1(\rho - a_1)}\right)^{\beta_1} \tag{17}$$

The optimal boundary is increasing in D by the assumption of diminishing returns and the convexity of the cost function,

$$\frac{dP}{dD} = \frac{-f''(D)\,\beta_1(\rho - a_1)\,c}{(\beta_1 - 1)\,[f'(D_i)]^2} > 0$$

In region I, (6) holds as a strict inequality and no development is undertaken. For any given D, random price fluctuations move the point (D,P) vertically upward or downward. If the point goes above the boundary then development is immediately undertaken so that the point shifts on the boundary. Thus optimal development proceeds gradually. In the terminology of Dixit and Pindyck, this is a 'barrier control' policy. The free boundary is shown in figure 1. If the price P moves from P_0 to P_1, the private developer will undertake $D_1 - D_0$ new development.

[Figure 1]

The equation describing the boundary can be interpreted in the following way. For a small development dD, its expected present value is defined as:

$$E[PV] = \left(\frac{Pf'(D)}{\delta}\right) dD$$

where $\rho - a_1$ is the real discount rate defined as the difference between ρ and the expected rate of growth of the price. Define the incremental cost of this development as $\Delta C = c$, then the benefit cost ratio for this investment is defined as $BC = \frac{E[PV]}{c}$. As seen from the definition of the incremental benefits and the strict concavity of f, the benefit cost ratio is lower, the higher the development level is. Using the equation of the boundary to substitute for P, the optimal investment rule requires:

$$BC = \frac{\beta_1}{\beta_1 - 1} > 1$$

The fact that the benefit cost ratio for the marginal project exceeds one, as compared to the traditional rule of $BC = 1$ in the no uncertainty/irreversibility case, reflects the option value of keeping the status quo development level.

By equation (14) the incremental development is justified if the discounted value of the incremental development marginal value product, $\frac{Pf'(D)}{\rho - a_1}$, covers development costs, c, plus the opportunity cost of the option to wait $A_1'(D) P^{\beta_1}$. By (17) the marginal option value $A_1'(D)$ is negative, that is, it represents a cost.[15]

4 Socially-Optimal Development

Consider the case of a social planner or an environmental regulator that seeks to optimally develop the site by taking into account - in addition to the development benefits, that is the direct use values - the environmental losses arising from the irreversible destruction of the site during the development process.

In addition to price uncertainty, the regulator also faces uncertainty regarding the environmental or intrinsic value of the undeveloped resource. We assume that the environmental values associated with the resource in the undeveloped state evolve stochastically following a geometric Brownian motion

$$dq_t = a_2 q_t dt + \eta_2 q_t dz_{qt} \tag{18}$$

The use of a geometric Brownian motion to model the evolution of environmental values fits better when these values are interpreted as reflecting values of ecosystem services which can be associated with market values, such as land values for bioprospecting,[16], watershed or ecotourism services.[17]

[15] Integrating (17) we obtain $A_1(D)$ as:

$$A_1(D) = \int_D^{\infty} \left[-A_{1s}'\right] ds =$$

$$\left(\frac{\beta_1 - 1}{c}\right)^{\beta_1 - 1} \int_D^{\infty} \left(\frac{f'(D)}{\beta_1 \delta}\right)^{\beta_1}$$

As shown by Dixit and Pindyck (1994) for the integral to converge with a Cobb-Douglas development function, $f(D) = D^{\kappa}$, $0 < \kappa < 1$, κ should be sufficiently small so that $\beta_1 > \frac{1}{1-\kappa}$.

[16] Heal (2000) mentions that bioprospecting rights might be worth as much as \$9000 per hectare in biodiversity hot spots.

[17] This interpretation of the evolution of environmental values is related to the diffusion formulation of environmental benefits at the undeveloped state used by Scheinkman and Zariphopoulou (1999). On the other hand, when environmental values are interpreted as unobserved existence values which are currently uncertain, the Brownian motion assumption might not be the most appropriate because of the dependency of its drift and volatility on uncertain current values.

The instantaneous benefits of the regulator can then be defined as:

$$B(P_t, q_t, D_t) = P_t f(D_t) + q_t (S - D_t)$$

with the value function defined as:

$$w(P_t, q_t, D_t) = \sup_{\mathcal{U}} \mathcal{E}_0 \int_0^\infty e^{-\rho^s t} \left[P_t f(D_t) + q_t (S - D_t) - c\Delta D_t\right] dt$$
$$\text{subject to (2) and (18)}$$

In the value function definition above ρ^s denotes the social discount rate used by the regulator. With perfect private capital markets and no divergence between private and social costs and benefits, the private and the social discount rates coincide.[18] The case of environmental losses due to irreversible development of a natural resource examined in this paper, introduce a source of deviation between private and social benefits implying potential differences between the private and the social discount rates. In this context, Weitzman (1994) shows that environmental effects imply a lower social discount rate relative to the private one. Following Weitzman the environmental effect is modeled here by introducing a correction factor $\delta > 0$, and defining the social discount rate as

$$\rho^s = \rho(1 - \delta)$$

Following Davis and Norman (1990), the linear homogeneity of the benefit function in P and q implies that the value function is also linearly homogeneous.[19] Then the dimensionality of the social planner's problem can be reduced from three to two for $q_t \neq 0$. By taking the non-use values as the numeraire, the social planner's benefit function can be written as:

$$v_t f(D_t) + (L - D_t) \ , \ v_t = \frac{P_t}{q_t}$$

Linear homogeneity of the value function implies that

$$w(P, q, D) = q w(v, 1, D)$$

Then, it can be defined that:

$$W(v, D) = w(v, 1, D) = q^{-1} w(P, q, D)$$

where $W(v, D)$ is the value function associated with the problem:

$$W(v, D) = \sup_{\mathcal{U}} \mathcal{E}_0 \int_0^\infty e^{-\widehat{\rho} t} \left[v_t f(D_t) + (S - D_t) - c\Delta D_t\right] dt$$

subject to

[18] The comparison between the private and the social discount rate is an issue that has been discussed extensively. See for example, Arrow and Kurz (1970, Ch. I.2), Lind (1982, 1990).

[19] See also Scheinkman and Zariphopoulou (1999).

$$dv_t = \mu v_t dt + \sigma v_t dz\,(t)$$

where:

$$\begin{aligned}
\widehat{\rho} &= \rho^s - a_1 = \rho\,(1-\delta) - a_1 > 0 \\
\mu &= a_1 - a_2 \\
\sigma^2 &= \eta_1^2 - 2\gamma\eta_1\eta_2 + \eta_2^2
\end{aligned}$$

and γ is the correlation coefficient of the Wiener processes dz_p, dz_q.

The stopping time for the regulator's problem satisfies:

$$\begin{aligned}
W\,(D,v) = \max_{\mathcal{U}} \mathcal{E}_0 \Big[\int_0^\tau e^{-\widehat{\rho} u}\,[v_u f\,(D_u) - D + L]\,du \\
+ e^{-\rho\tau} W\,(D_\tau^*, v_\tau) \Big) \qquad (19)
\end{aligned}$$

Then, following the same steps as in the section above, the HJB equation is defined as:

$$\min\left\{\left[\widehat{\rho} W - \frac{1}{2}\sigma^2 v^2 W_{vv} - \mu v W v - (v f\,(D) - D + L)\right], -[W_D - c]\right\} = 0 \qquad (20)$$

4.1 The free boundary at the social optimum

As before, the socially-optimal free boundary will divide the (v, D) space into two regions: the 'no development' region I and the 'development' region II. In region I the first term of the HJB is zero and the second term of the HJB equation is positive, thus the general solution for the value function is defined as:

$$W\,(D,v) = R_1\,(D)\,P^{\beta_1^s} + v\frac{f\,(D)}{\phi} - \frac{(D-L)}{\hat{\rho}} \qquad (21)$$

where $\phi = \hat{\rho} - \mu$ and β_1^R is the positive roote of the fundamental quadratic, $Q^s = \frac{1}{2}\sigma^2\beta^s\,(\beta^s - 1) + \mu\beta^s - \hat{\rho} = 0$, with solution: $\beta_1^s = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\hat{\rho}}{\sigma^2}} > 1$

In region II the second term of the HJB equation is zero and $\Delta D > 0$ or

$$W_D\,(D,v) - c = 0 \qquad (22)$$

The constant $R_1\,(D)$ and the function $v = v\,(D)$ can be determined as before, using the value matching and the smooth pasting conditions, or

$$W_D\,(D,v) = R_1'\,(D)\,v^{\beta_1^s} + v\frac{f'\,(D)}{\phi} - \frac{1}{\hat{\rho}} = c\,,\;\; v = v\,(D) \qquad (23)$$

and

$$W_{Dv}(D,v) = \beta_1^s R_1'(D)\, v^{\beta_1^s - 1} + \frac{f'(D)}{\phi} = 0 \text{ with } v = v(D) \tag{24}$$

Combining (22) and (23) we obtain:

$$v(D) = \frac{\beta_1^s}{\beta_1^s - 1} \frac{\phi\left(\frac{1}{\hat{\rho}} + c\right)}{f'(D)} \tag{25}$$

$$R_1'(D) = -\left(\frac{\beta_1^s - 1}{1+c}\right)^{\beta_1^s - 1} \left(\frac{f'(D)}{\beta_1^s \phi}\right)^{\beta_1^s} \tag{26}$$

$$\phi = \hat{\rho} - \mu = \rho(1-\delta) + a_2 - 2a_1 > 0 \tag{27}$$

As before the optimal boundary is increasing in D by the assumption of diminishing returns.

By equation (23) the incremental development is justified if the discounted value of the incremental development marginal value product adjusted for non-use values, $\frac{vf'(D)}{\phi}$, covers development costs, c, plus the present value of one unit of resource irreversibly developed, $\frac{1}{\hat{\rho}}$, plus the opportunity cost of the option to wait, $R_1'(D)\, v^{\beta_1^s}$. By (26) the marginal option value $R_1'(D)$ is negative, that is, it represents a cost.[20]

The benefit cost rule for the regulated development is also determined as:

$$BC^s = \frac{\beta_1^s}{\beta_1^s - 1} > 1$$

5 Comparison of Exercise Boundaries and Management Rules

The privately-optimal and the socially-optimal exercise boundaries determine the profit-maximizing and the socially-optimal management rules respectively, for the resource development. They are defined as:

$$P(D) = \frac{\beta_1}{\beta_1 - 1} \frac{(\rho - a_1)\, c}{f'(D)}$$

$$v(D) = \frac{\beta_1^s}{\beta_1^s - 1} \frac{(\hat{\rho} - \mu)\left(\frac{1}{\hat{\rho}} + c\right)}{f'(D)}$$

[20] Furthermore,

$$R_1(D) = \int_D^\infty \left[-R_1'(s)\right] ds$$
$$= \left(\frac{\beta_1^s - 1}{1+c}\right)^{\beta_1^s - 1} \int_D^\infty \left(\frac{f'(D)}{\beta_1^s \phi}\right)^{\beta_1^s}$$

where for the same Cobb-Douglas production function we must have $\beta_1^s > \frac{1}{1-\kappa}$.

$$\mu = a_1 - a_2$$
$$\beta_1 = \frac{1}{2} - \frac{a_1}{\eta_1^2} + \sqrt{\left(\frac{a_1}{\eta_1^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\eta_1^2}}$$
$$\beta_1^s = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\hat{\rho}}{\sigma^2}}$$
$$\sigma^2 = \eta_1^2 - 2\gamma\eta_1\eta_2 + \eta_2^2\,,\ \hat{\rho} = \rho\,(1-\delta) - a_1$$

For $\lim_{D\to 0} f'(D) = \infty$, $P(0) = v(0) = 0$ and both boundaries pass through the origin; furthermore they are both increasing in D.

To compare the two boundaries we start with the simplest case where the environmental value of the undeveloped resource is fixed and normalized to one, implying $a_2 = \eta_2 = 0$. In this case $\mu = a_1, \sigma^2 = \eta_2^2$ while for $\delta \to 0$, $\beta_1 \to \beta_1^s$. In this case the difference $P(D) - v(D)$ is defined as:

$$v(D) - P(D) = \frac{\beta_1}{\beta_1 - 1}\frac{(\rho - a_1)}{f'(D)} > 0$$

Thus the socially-optimal boundary is above the privately-optimal one. This means that there is a need for a higher upsurge in price in order to undertake development under social optimization. Thus the socially-optimal management rule in this case implies slower development of the resource. This slowdown in the development occurs because the non-use values are taken into account under maximization of social benefits, but are ignored under maximization of private profits. The result is shown in figure 1. If the price moves from P_2 to P^*, the new private development is $D_2^U - D_2$, while the socially-optimal development is $D_2^S - D_2$.

When δ is greater than zero, so that the social discount rate takes into account the environmental correction factor and becomes lower than the private discount rate, the $v(D) - P(D)$ is more complex. However numerical simulations indicate that $\frac{\partial v(D,\delta)}{\partial \delta} > 0$.Figure 1a depicts the derivative $\frac{\partial v(D,\delta)}{\partial \delta}$ as a function of δ, for $a_1 = 0.01$, $\rho = 0.1$, $c = 1$, $n_1 = 0.1$. Thus an increase in δ shifts the socially-optimal boundary upwards, implying further slowing down in the development of the resource. This result is in agreement with the central proposition in the theory of exhaustible resources that a reduction in the discount rate leads to greater conservation.

[Figure 1a]

When however we consider the general case with $(a_2, \eta_2) \neq 0$, the comparison is not straightforward. To obtain some idea of the relative positions between the two exercise boundaries, we consider the following parameter values:

- $a_1 = 0.01$

- $a_2 = 0$, this assumption implies that we do not expect the expected value of the resource's environmental value to change as compared to the current level.

- $\rho = 0.1$, $\delta = 0.2$

- $\kappa = 0.5$, $c = 1$

- $\eta_1 = 0.1$, $\eta_2 = 0.2$, these values reflect the assumption that the volatility of the resource's non-use values is likely to be higher than the price volatility associated with direct use values.

- it is difficult to make *a priori* assumptions regarding the sign and the size of the correlation coefficient γ between profit and environmental uncertainty, since the value of γ is most likely to depend on the specific problem. For example an increase in the price of the resource's services indicating stronger demand might be accompanied by an increase in the undeveloped resource's indirect use values because at the same time more people might want to experience and preserve the undisturbed resource, yielding a *positive* correlation between profit and environmental uncertainty.[21]

Figures 2a and 2b show the private and social exercise boundaries as functions of the cumulative resource development D and the correlation coefficient γ. Figure 2c shows the difference $P(D) - v(D)$. As shown for the chosen parameter constellation, the socially-optimal exercise boundary is uniformly above the privately-optimal exercise boundary.

[Figure 2]

However the relative pace of development can not be inferred from the relative positions of the exercise boundaries as in the case of profit uncertainty alone ($a_2 = \eta_2 = 0$). This is because the private agent chooses his/her development plan according to the movements of P_t, while the social planner chooses his/her development plan according to the movements of $v_t = \frac{P_t}{q_t}$. Thus unless $\frac{P_t}{q_t}$ behaves exactly as P_t, we expect different responses, which can be summarized in the following proposition:

Proposition 1 *Assume simultaneous uncertainty in the market price of resource services (use values) and the environmental value (non-use values) of the undeveloped resource. Then, if the relative price of resource services with the environmental value of the undeveloped resource as numeraire,* $\frac{P_t}{q_t}$*, evolves*

[21] When $\gamma = 0$ implying that the social returns of the resource are uncorellated with the private returns, then the Arrow and Lind (1970) result - that for small public investment uncorellated with the previous national income, the government should be risk neutral - implies that the environmental regulator should use a risk free discount rate, along with the potential environmental correction.

differently from the unadjusted price P_t, the development of the resource under social planning could be slower or faster relative to the profit-maximizing behavior.

This proposition can be shown with reference to figure 3, where the socially-optimal exercise boundary is drawn uniformly above the privately-optimal exercise boundary. Assume that from a given development level D_0, and market price of resource services P_0, the price moves to P_1, while the relative price moves from v_0 to $v_1 = \frac{P_1}{q_1}$. Then the social planner will undertake new development $D^s - D_0$, while the private developer will undertake development $D^U - D_0 > D^s - D_0$. Thus development is slower under the socially-optimal rule. Assume now that instead of moving to P_1 the price moves to P_2, while relative price still moves to v_1. Then while the social planner still undertakes new development $D^s - D_0$, the private developer undertakes *no* new development and development is *faster* under the socially-optimal rule.

[Figure 3]

This result goes contrary to the generally accepted view that taking into account environmental or intrinsic values, which are not taken into account under profit maximization, implies greater conservation. The driver of the result is the possibility of different evolution of market prices relative to non-use values. Under plausible parameter values, the socially-optimal exercise boundary is uniformly above the privately-optimal exercise boundary. This indicates that under similar movement of market prices and environmental values or under relatively faster growth of environmental values, the socially-optimal development is always slower than the privately-optimal development. However if there is a downward movement of environmental values, then the externality effect goes the other way and for a certain time interval, development at the social optimum is faster relative to the private optimum. This could happen for example if new information pushes down bioprospecting values of undeveloped ecosystems, or changes in preferences reduce the eco-tourism value of the undeveloped landscape. In this case it would be socially desirable to develop the resource faster. This is because the private developer ignores, say the drop in bioprospecting values, and thus the loss from keeping an asset whose return is going down relative to the returns of the alternative (which is to develop the resource) is not internalized. This loss is internalized at the social optimum which takes into account the relative returns of both assets. Socially desirable development is also faster if the undeveloped part is associated with negative intrinsic values, for example existence of a disease in the undeveloped state of the resource. In this case the private optimization problem might not fully internalize the external cost of the disease, which is however internalized at the social optimum indicating relatively faster development, which eliminates the negative externality.

This result indicating the possibility of faster development at the social optimum can be related to a similar result obtained by Farzin (1984), where a reduction in the discount rate might lead to a faster and *not* slower development of an exhaustible resource, depending on the capital requirement for the production of a substitute and the size of the resource stock. In our case the *slow-down* effect induced by the environmental correction might be counterbalanced by the movement of environmental values relative to the market prices.

6 Policy Design

Since development at the private optimum is determined with reference to the privately-optimal boundary, the policy scheme should be chosen so that the private exercise boundary following the introduction of regulation coincides with the socially-optimal exercise boundary.

The case in which non-use values are fixed at the level $\overline{v}$, and uncertainty exists only with respect to the use values is considered first. In this case the socially-optimal development is always slower than the privately-optimal development and a possible policy instrument could be a development tax that will slow down private development. Consider a fixed tax $\mathcal{T}$ per unit of incremental development so that the unit development cost is $c + \mathcal{T}$. Then, the regulated privately-optimal exercise boundary is defined as:

$$P^R(D) = \frac{\beta_1}{\beta_1 - 1} \frac{(\rho - a_1)(c + \mathcal{T})}{f'(D)}$$

while the socially-optimal exercise boundary becomes:

$$P(D) = \frac{\beta_1^s}{\beta_1^s - 1} \frac{(\rho^s - a_1)\left(c + \frac{\overline{v}}{\rho^s}\right)}{f'(D)}$$

It is clear that the optimal development tax which makes the two boundaries identical is defined for $\delta = 0$ as:

$$\mathcal{T} = \frac{\overline{v}}{\rho}$$

Thus the optimal development tax is equal to the present value of the flow of one unit non-use value services which are lost by the irreversible development of the resource. It is interesting to note that by looking at the policy design problem as an issue of equating privately-optimal and socially-optimal free boundaries, the optimal tax is the same as the tax that would have been used under certainty. Since the private developer compares the regulated boundary, which is now identical to the socially-optimal boundary, with observed market prices, the regulation problem is solved by a simple deterministic tax without the need to use contingent instruments.

The appealing characteristic of what might be called *a barrier control policy design*, is that instead of choosing the optimal tax according to the realization of a random variable, the tax is set at a level such that the private developer is induced to behave like the social planner for *any* realization of the random variable. So the system is decentralized and once the tax is set the private developer is left to respond to price changes.

When $\delta > 0$ then the development tax is defined as:

$$\mathcal{T}^{s} = \frac{(\beta_1 - 1)}{\beta_1(\rho - a_1)} \left[v(D) - P(D) \right]$$

The larger δ the larger is the difference $v(D) - P(D)$ and the higher is the tax is. This is expected since the larger the difference $v(D) - P(D)$ is, the slower the development at the social optimum and the higher the required tax to induce the private developer to slow down is.

The general case with simultaneous use and non-use value uncertainty is now considered. In this case the development tax that equates the two boundaries is determined as:

$$\mathcal{T}^{*} = \frac{(\beta_1 - 1)\, \beta_1^s}{(\beta_1^s - 1)\, \beta_1} \frac{(\hat{\rho} - \mu)\left(c + \frac{1}{\hat{\rho}}\right)}{(\rho - a_1)} - c$$

However, under simultaneous uncertainty the private developer responds to changes in P_t while the social planner responds to changes in $v_t = \frac{P_t}{q_t}$. Thus although the regulated boundary coincides with the socially-optimal boundary, it is not certain that the private developer will follow the socially-optimal development. The socially-optimal development will be followed if P_t evolves in the same way as $\frac{P_t}{q_t}$. In all other cases the development tax will not induce the socially-optimal behavior. This is summarized in the form of a non-existence result.

Proposition 2 *Assume, under simultaneous use and non-use values uncertainty, that market prices for use values P_t evolve differently than the adjusted prices $\frac{P_t}{q_t}$. Then there is no development tax that can induce the private developer to undertake the socially-optimal development.*

The result can be shown with reference to figure 4. Let the private boundary $P(D)$ shift after regulation to $v(D)$, and observed prices move from (P_0, v_0) to (P_1, v_1). The socially-optimal development is $D^s - D_0$, but since the private developer responds to the unadjusted price signal, the privately-optimal development after regulation is $D^R - D_0$. The unregulated development would have been $D^U - D_0$. Thus although the tax does not achieve the socially-optimal development, it restricts the unregulated development towards the social optimum. This development tax can be regarded as a fixed second-best tax.

[Figure 4]

The socially-optimal development can be secured by a proportional tax on the market prices equal to $\frac{1}{q_t}$ that will make the effective market price equal to v_t, and a subsidy for the undeveloped resource equal to $L - D_t$ per unit time.

In this case the private solution exactly reproduces the social planner's value function. This scheme however is contingent on the realizations of the random variables and dependent on the current development state, and may be hard to implement, since it should be updated for changes in non-use values and development levels.

Another way to implement the socially-optimal rule is to use a system of quantity instruments, with a possible subsidy to correct for cases where the socially-optimal development should be faster than the privately-optimal one.

By inverting (24) the socially-optimal exercise boundary can be written as:

$$D^s = g(v) = v^{-1}(D)$$

Then the quantity instrument can be set as the limit $D_t \leq D_t^s$. Under the limit the HJB equation implies for the development region that:

$$V_D(D, P) - c - \lambda \leq 0\ ,\ \Delta D \geq 0$$
$$\lambda(D^s - D) = 0\ ,\ \lambda \geq 0$$

where λ is the Lagrangian multiplier associated with the quantity limit. Consider figure 4, and suppose that prices move to (P_1, v_1). Then $D^s < D^U$, the constraint is binding and the limit secures the socially-optimal development. Suppose now that the market price moves to P_2, while the adjusted price remains at v_1. To secure the socially-optimal development, a subsidy s should be given, such that

$$(1 + s) P_2 = v_1$$

Under the subsidy the private developer responds to the correct, from the social point of view, signal v' and undertakes the socially-optimal development D^s. The policy scheme can therefore be defined as follows:

- If $D_t^U > D_t^s$ then the development limit is set at D_t^s.
- If $D_t^U < D_t^s$ then the private developer receives a subsidy $s_t : (1 + s_t) = \frac{1}{q_t}$.

The above scheme is also contingent on the realization of the random variable, q_t, and dependent on the development state.

The analysis of the policy schemes seems to suggest that under simultaneous use and non-use value uncertainty, a fixed development tax can not achieve the social optimum but could have only a second-best character. The social optimum can be achieved by contingent, state development schemes, which however might be difficult to implement.

7 Concluding Remarks

In this paper an exercise boundary approach has been used to analyze the problem of irreversible development of a natural resource under uncertainty in the use and non-use values associated with the resource.

This approach seems to have a number of advantages that could be useful in the analysis of dynamic management problems, not only in the field of environmental and resource economics. In particular it allows the characterization and comparison of development paths under privately-optimal and socially-optimal management rules. The comparison of development paths made possible by this approach provides a basis for exploring situations where, contrary to the generally accepted intuition, socially-optimal development might be faster than privately-optimal development for a certain time interval.

Furthermore the use of exercise boundaries indicates that regulation can be designed in such a way that the regulated boundary coincides with the socially-optimal boundary. For the case of uncertainty in use values only, this approach shows that a simple fixed deterministic development tax can induce the socially-optimal path. This is clearly an advantage since there is no need for contingent instruments. When however uncertainty affects both use and environmental values, the fixed tax can bring the regulated development closer to the socially-optimal one, but can not achieve the socially-optimal path. To do this contingent and state dependent schemes are required. It is an open issue however, whether or not a fixed tax that approaches the optimal path should not be preferred, on implementability and acceptability grounds, to a complicated scheme that achieves the social optimum.

References

Arrow, K.J and A. Fisher (1974), ‘Environmental Preservation, Uncertainty and Irreversibility’, *Quarterly Journal of Economics*, 88, 312-319.

Arrow, K.J. and M. Kurz (1970), *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Baltimore: Johns Hopkins University Press for Resources for the Future.

Arrow, K.J. and .C. Lind (1970), Uncertainty and he Evaluation of Public Investment Decisions, *American Economic Review*, 60, 364-378.

Chichilniski, G. and Heal, G. (1998), “Economic Returns from the Biosphere.” *Nature*, 629-630.

Constandinidis, G. M. (1986), ”Capital market Equilibrium with Transaction Costs”, *Journal of Political Economy*, 94, 842-862.

Davis, M. and A. Norman (1990), ”Portfolio selection with transaction costs”, *Mathematics of Operations Research*, 15, 676-713

Dixit, A.K. and R.S. Pindyck (1994), *Investment under Uncertainty,* Princeton New Jersey, Princeton University Press.

Dixit,A. K., R.S. Pindyck and S. Sondal, (1998), “A Markup Interpretation of Optimal Investment Rules”, *The Economic Journal*, 109, 179-189.

Farzin, Y. H., (1984), “The Effect of Discount Rate on Depletion of Exhaustible Resources”, *Journal of Political Economy*, 92, 841-851.

Fisher, A.C. and M. Hanemann (1986), ‘Environmental Damages and Option Values’, *Natural Resource Modelling*, 1, 111-124

Fisher, A.C. and M. Hanemann (1987), ‘Quasi-Option Value: Some Misconceptions Dispelled’, *Journal of Environmental Economics and Management*’, 14, 183-190.

Fleming, W. and H. M. Soner (1993), *Controlled Markov Process and Viscosity Solutions*, Springer-Verlag.

Hassett, K. A. and G. Metcalf (1999), “Investment with Uncertain Tax Policy: Does Random Tax Policy Discourage Investment?”, *The Economic Journal,* 109, 372-393.

Heal, G. (2000) “Biodiversity as a Commodity”, in S. Levin, G.C. Daily, J. Lubchenco, and D. Tilman, eds., *Encyclopedia of Biodiversity.* Academic Press.

Lind, R. (1982), “Discounting for Time and Risk in Energy Policy”, Resources for the Future, Washington DC.

Lind, R. (1990), “Reassessing the Government’s Discount rate Policy in Light of New Theory an Data in a World Economy with High Capital Mobility”, *Journal of Environmental Economics and Management,* 18 ,S9-S28.

Malliaris, A.G. and W.A. Brock (1982), *Stochastic Methods in Economics and Finance*, Amsterdam: North-Holland.

Perrings, C, (1995), “The Economic Value of Biodiversity”, in H. Heywood and R. Watson, (eds) *Global Biodiversity Assessment*, UNEP, Cambridge University Press.

Scheinkman, J. A. and T. Zariphopoulou (1999), 'Optimal Environmental Management in the Presence of Irreversibilities', *Journal of Economic Theory*, forthcoming.

Soner, H. M. (1997), "Controlled Markov Processes, Viscosity Solutions and Applications to Mathematical Finance", in I. Capuzzo Dolcetta and P.L. Lions (eds) *Viscosity Solutions and Applications*, Springer-Verlag, Berlin.

Stavins, R.N. (1996), 'Correlated Uncertainty and Policy Instrument Choice', *Journal of Environmental Economics and Management*, 30, 218-33.

Weitzman, M.L. (1974), 'Prices vs. Quantities', *Review of Economic Studies*, 41, 477-91.

Weitzman, M. (1994), On the Environmental' Discount Rate', *Journal of Environmental Economics and Management,* 26 (3), 200-209.

Xepapadeas, A. (1998), "Optimal Resource Development and Irreversibilities: Co-operative and Noncooperative Solutions", *Natural Resource Modelling*, 11, 357-378.

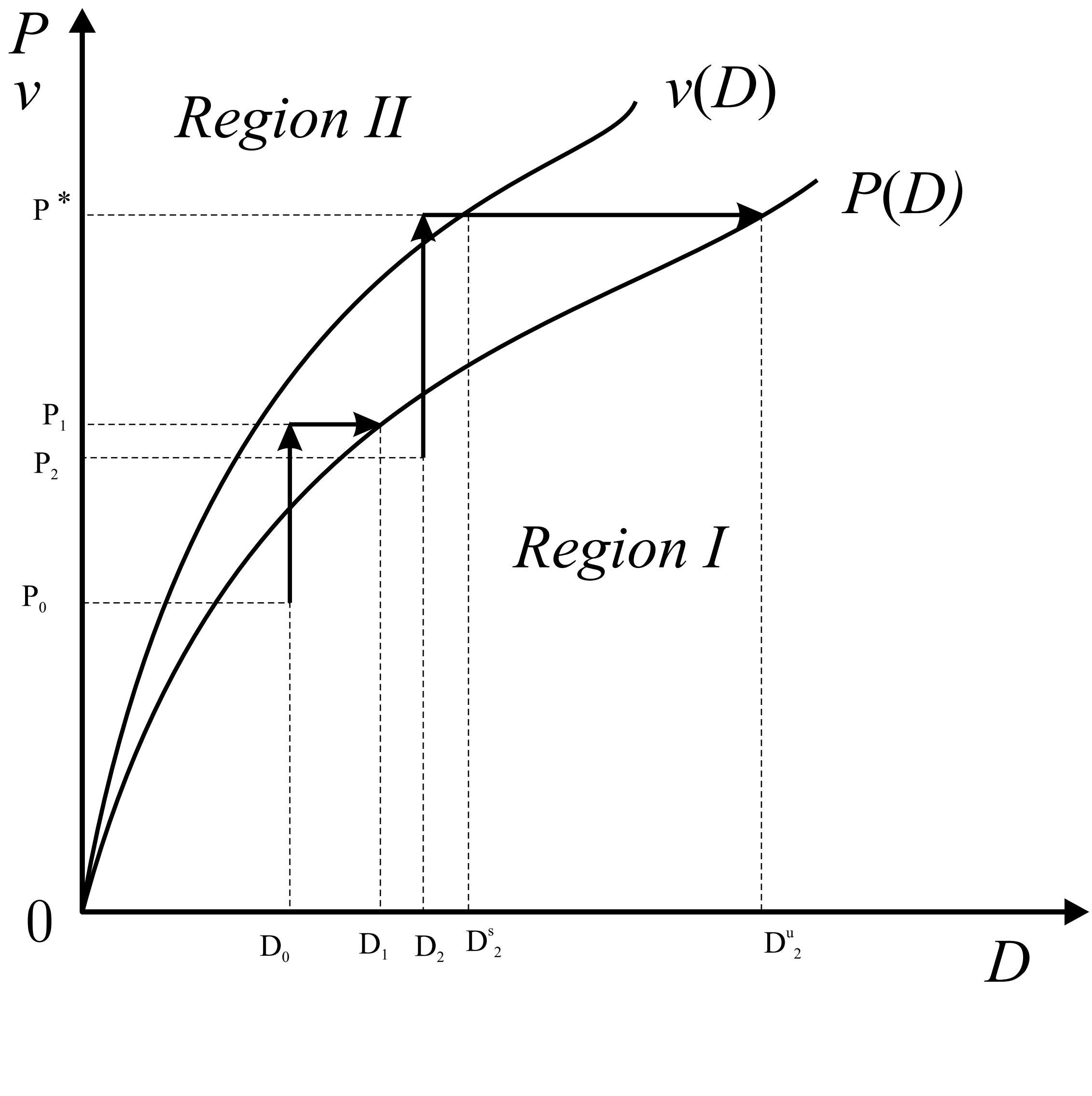

Figure 1: Privately-optimal and socially optimal exercise boundaries

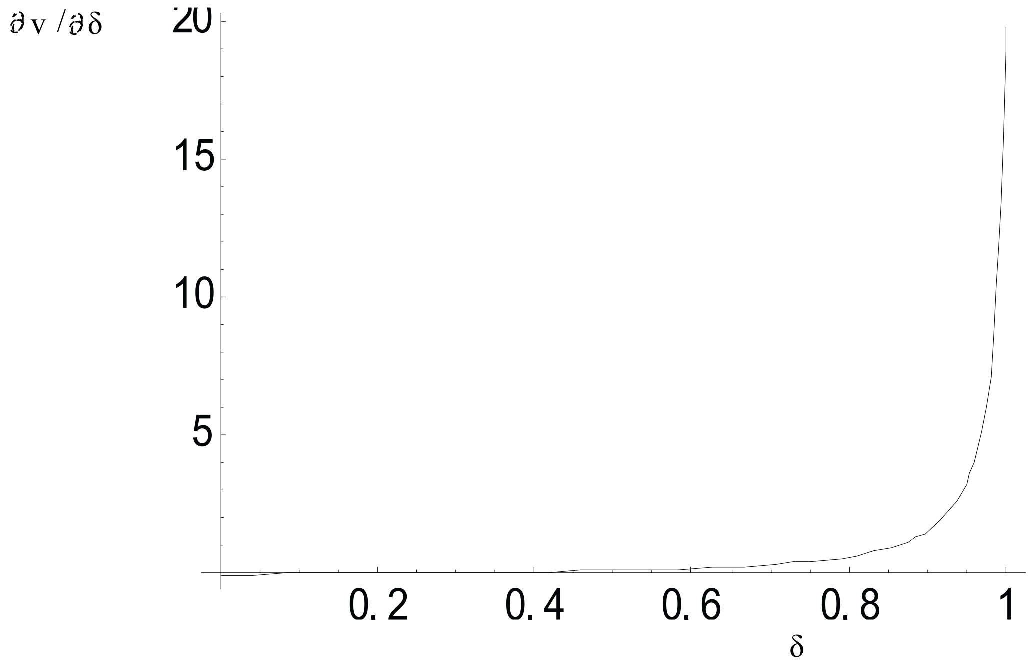

Figure 1a: The derivative $\partial v / \partial \delta$

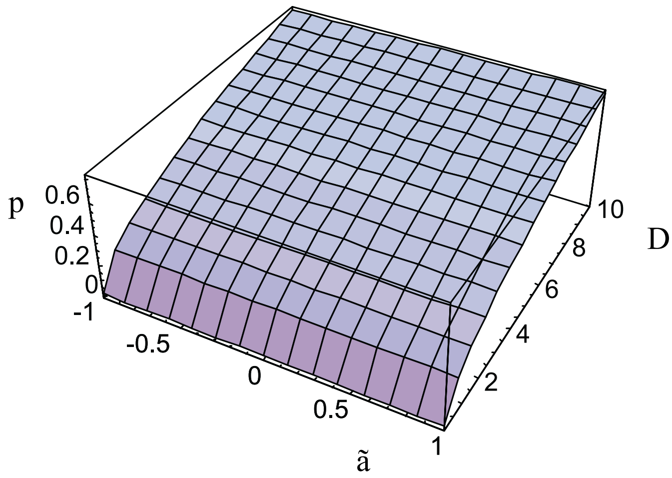

Figure 2a: The privately-optimal exercise boundary

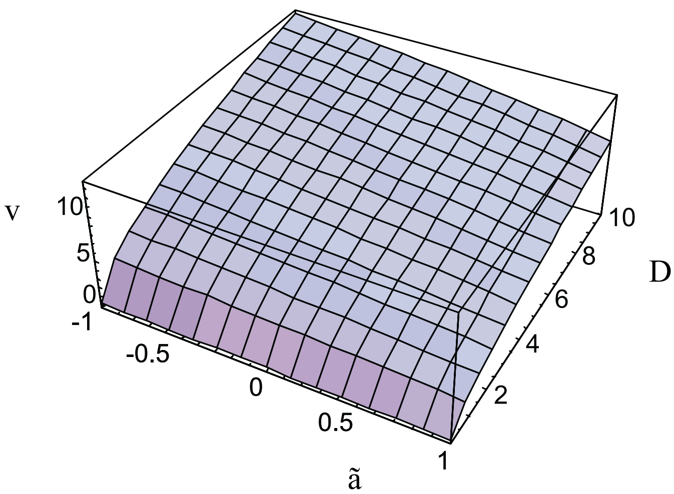

Figure 2b: The socially-optimal exercise boundary

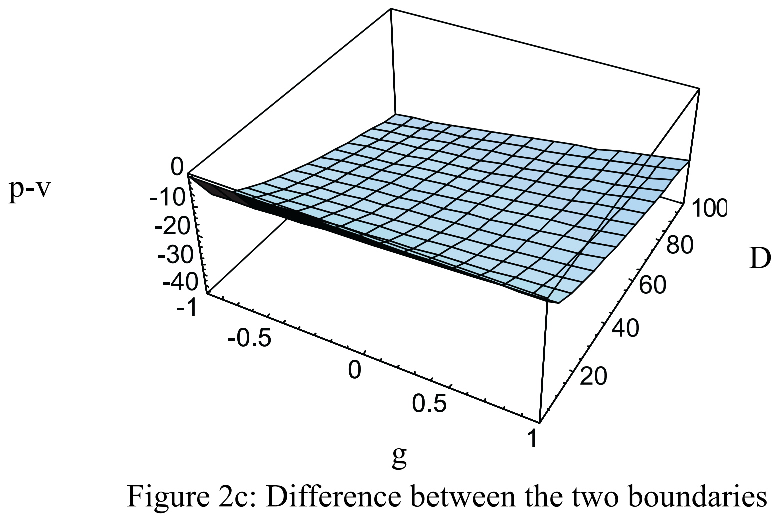

Figure 2c: Difference between the two boundaries

Figure 2: Comparison of optimal exercise boundaries

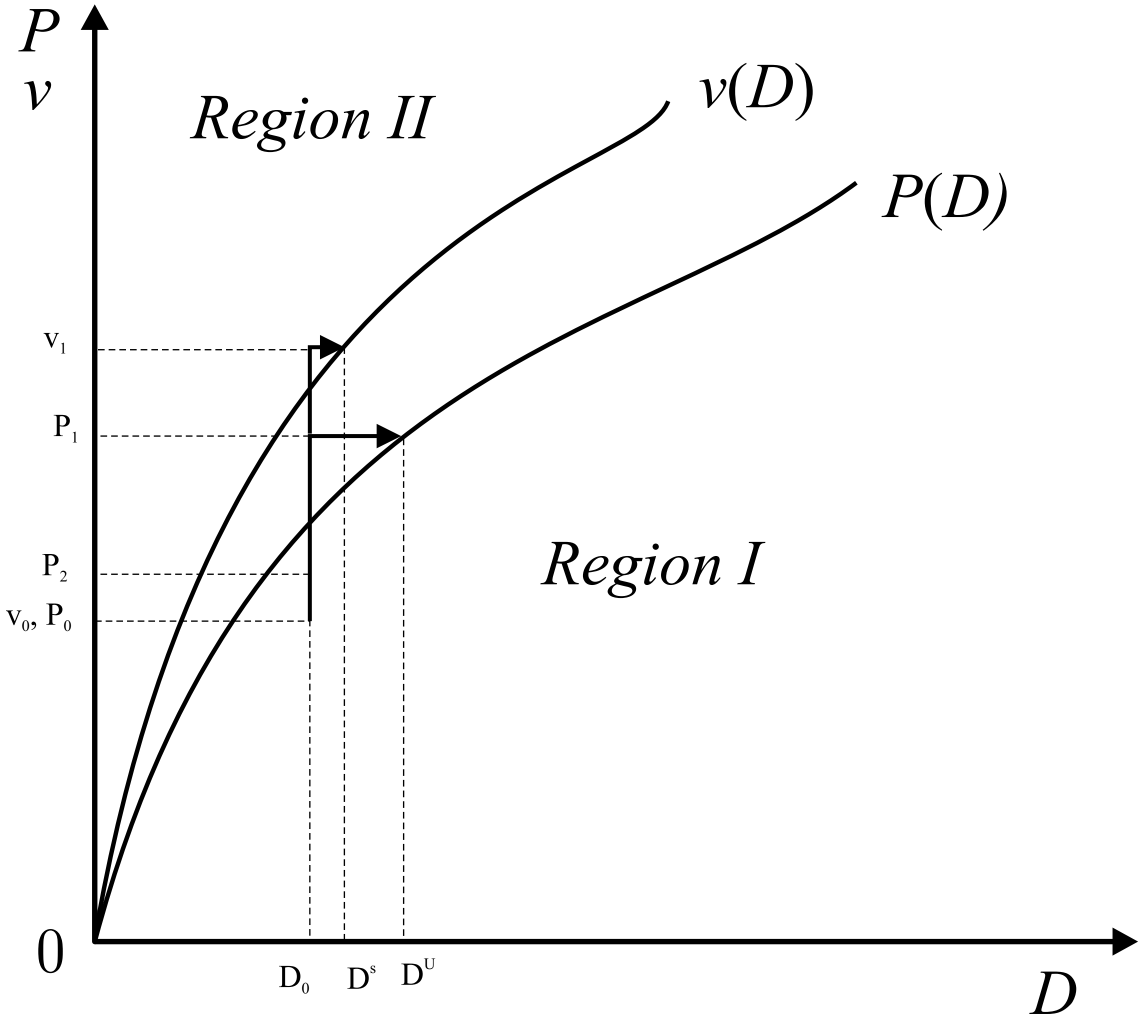

Figure 3: Development pace at the private and social optimum

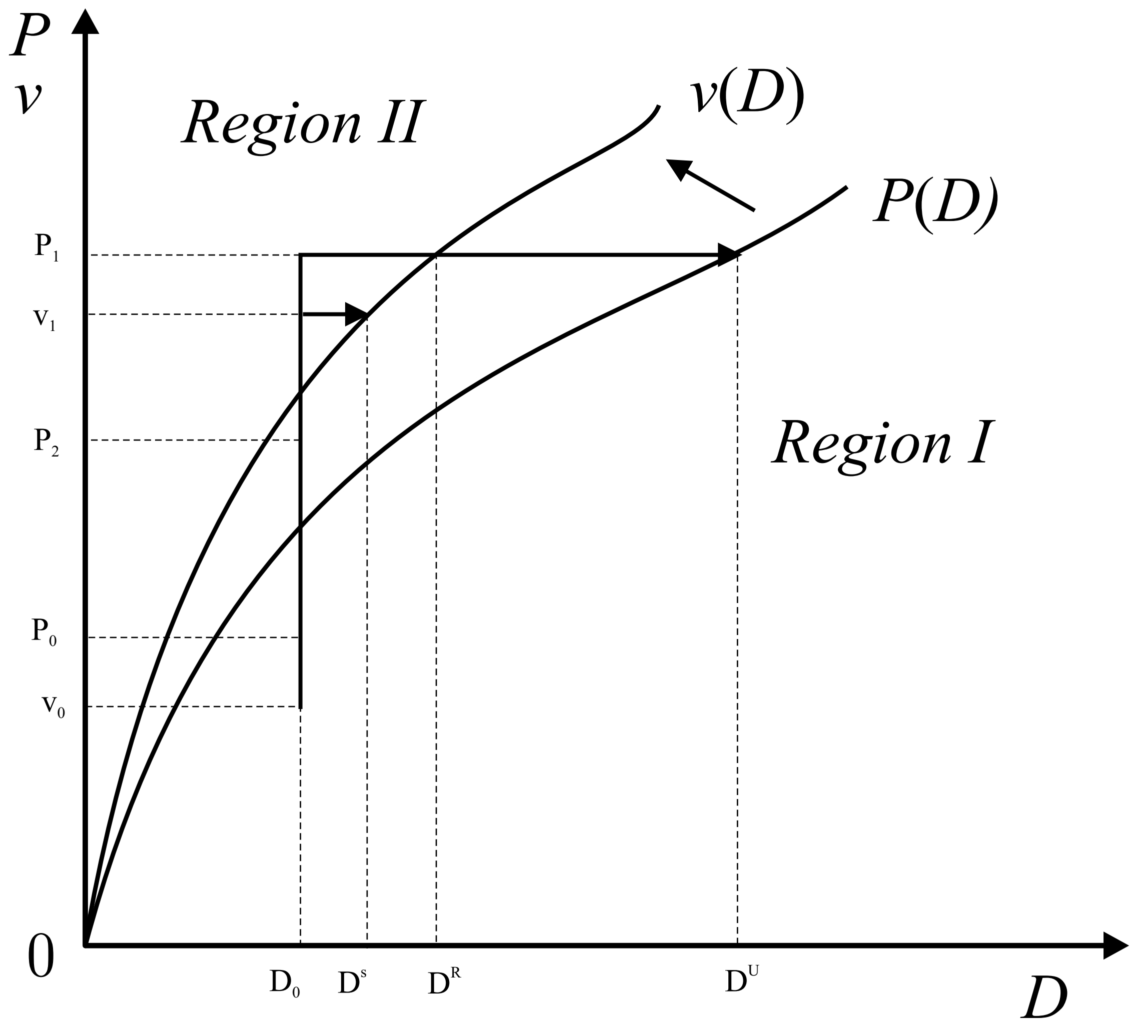

Figure 4: Regulation