

A MULTIVARIATE $I(2)$ COINTEGRATION ANALYSIS OF GERMAN HYPERINFLATION*

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Abstract

This paper re-examines the Cagan model of German hyperinflation during the 1920s under the twin hypotheses that the system contains variables that are $I(2)$ and that a linear trend is required in the cointegrating relations. Using the recently developed $I(2)$ cointegration analysis developed by Johansen (1992, 1995, 1997) extended by Paruolo (1996) and Rahbek *et al.* (1999) we find that the linear trend hypothesis is rejected for the sample. However, we provide conclusive evidence that money supply and the price level have a common $I(2)$ component. Then, the validity of Cagan's model is tested via a transformation of the $I(2)$ to an $I(1)$ model between real money balances and money growth or inflation. This transformation is not imposed on the data but it is shown to satisfy the statistical property of *polynomial* cointegration. Evidence is obtained in favor of cointegration between the two sets of variables which is however weakened by the sample dependence of the trace test that the application of the recursive stability tests for cointegrated VAR models show.

Key Words: $I(2)$ analysis, hyperinflation, cointegration, identification, temporal stability

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1. Introduction

Cagan's (1956) model on the demand for money under hyperinflation is by far the most widely used specification in studies of inflation dynamics and the money supply process and on issues of inflationary finance. Following this pioneering work, Sargent and Wallace (1973) and Sargent (1977) looked at the implications of letting Cagan's adaptive expectations scheme be rational and Salemi and Sargent (1979) tested the cross-equation parameter restrictions that the rational expectation imposes on a bivariate VAR model for inflation and money growth. Flood and Garber (1980), Hamilton and Whiteman (1985) and Casella (1989) used the Cagan model to investigate the existence of rational speculative bubbles, i.e. situations where the price level is partly driven by self-fulfilling expectations, independently of market fundamentals. Finally, Frenkel (1975, 1976) examined whether the authorities had expanded the money supply at much too high a rate to maximize the inflation tax revenue.

The major assumption in the works mentioned above is that the demand disturbances or shocks to velocity follow a random walk. This assumption implies that the deviations from the Cagan model have an infinite population variance, which substantially reduces the empirical content of the model from the outset (Taylor, 1991). Salemi and Sargent (1979) study some of the classic European hyperinflation episodes and they conclude that we can not reject the restrictions imposed by the Cagan model under rational expectations and a random walk error term. By contrast Goodfriend (1982) finds evidence supporting the Cagan model under rational expectations and no velocity shocks.

A main insight of Cagan's analysis is that under the conditions of hyperinflation, movements in prices are of a magnitude so much greater than movements in real macroeconomic aggregates that "relations between monetary factors can be studied, therefore, in what almost amounts to complete isolation from the real sector of the economy" (Cagan 1956, p.25). Taylor (1991), Engsted (1993, 1994) and Michael, Nobay and Peel (1994) suggest how this insight can be characterized formally in terms of the time properties of the data. Thus, they argued that the assumption that money demand disturbances follow a random walk could be tested explicitly using cointegration techniques, since the random walk assumption implies that real balances and inflation should not cointegrate. Therefore, when money and prices are integrated of order two, $I(2)$, and shocks to money demand or velocity

are stationary, then the Cagan (1956) monetary model of hyperinflation has the implication that real money balances cointegrate, in the sense of Engle and Granger (1987), with the rate of inflation. In addition, Engsted (1993, 1994) shows that given that velocity shocks are stationary, the Cagan model under rational expectations and no bubbles implies an additional cointegrating relationship between real money balances and money growth¹.

In the present paper we provide a re-examination of the empirical evidence on Cagan's model under rational expectations for the case of the German hyperinflationary period of the early 1920s, by applying recent contributions to the econometrics of non-stationarities. Several novel features are included in the paper. The first feature concerns the order of integration of the variables. We employ the recently developed testing methodology suggested by Johansen (1992, 1995, 1997) and extended by Paruolo (1996) and Rahbek *et al.* (1999) which allows us to reveal the existence of $I(2)$ and $I(1)$ components in a multivariate context. We depart from previous studies since we show that the cointegrating relationship between real money balances and inflation can be obtained through a testing procedure applied on a bivariate vector autoregressive, VAR, model of the monetary aggregate and the price level. So it could be claimed that our analysis is "data driven" instead of allowing theoretical results to determine *a priori* the empirical investigation. The property of cointegrated VAR models with $I(2)$ variables where a stationary relationship is derived from a linear combination of the levels of the variables and their first differences is known in the literature as *polynomial cointegration* or *multicointegration*, e.g. Granger and Lee (1990). Second, given that at least one statistically significant cointegrating vector has been found we examine the stability of the long-run relationships through time. Hansen and Johansen (1993, 1999) propose three tests for parameter stability in cointegrated-VAR systems that allow us to provide evidence for the sample independence of the cointegration rank as well as of parameter stability. Finally, following Engsted (1993, 1994) we test the Cagan specification under rational expectations and no velocity shocks by exploiting the interesting *cross-equation* parameter restrictions that those properties imply.

The main findings of the paper are: First the monetary aggregate and the price level are $I(2)$ variables and that they have a common $I(2)$ component. Furthermore we show that a stationary multicointegrating relationship is obtained by a linear combination of the real

money balances and inflation (or money growth). Second, the stability tests indicate that this relationship is established only when the last observations, of the “true” hyperinflation period, are included in the sample. Third, the exact rational expectations Cagan model is rejected, which provides further doubts that the velocity shocks were negligible.

The organization of this paper is as follows. Section 2 presents the Cagan model of hyperinflation. Section 3 discusses the cointegration methodology applied in this analysis. Section 4 reports our empirical results. Finally, section 5 presents our concluding remarks.

2. The Cagan model of hyperinflation

In this section we present the Cagan model following Engsted (1993, 1994)². Thus the Cagan model under money market clearing and rational expectations is given as

$$m_t - p_t = \alpha - \beta [E_t p_{t+1} - p_t] + u_t \quad (1)$$

where m_t and p_t are natural logarithms of the money stock and the price level, respectively, and α and β are parameters to be estimated. E_t is the expectations operator conditional on information at time t which includes at least lagged values of p_t and m_t and u_t is a stochastic variable representing velocity and/or demand shocks.

If the transversality condition $\lim_{i \rightarrow \infty} [\beta(1 + \beta)^{-1} E_t p_{t+i}] = 0$ holds, then by employing the forward solution of equation (1) it can be shown that

$$m_t - p_t = \alpha - \sum_{i=1}^{\infty} b^i E_t \Delta m_{t+i} + (1 - b) \sum_{i=0}^{\infty} b^i E_t u_{t+i} \quad (2)$$

where $b = \beta(1 + \beta)^{-1}$. The transversality condition rules out rational explosive bubbles, i.e. situations where the price level is driven by self-fulfilling expectations, independent of the evolution of the money supply (see e.g. Diba and Grossman, 1988)³. If $m_t - p_t$ rises, it is an indication of expectations of future decline in money growth, which leads to lower future inflation and therefore higher demand for real money balances today. According to eq. (2), the level of real balances is a predictor of future money growth and/or velocity shocks. In case of no velocity shocks, the level of balances is the *optimal* predictor of the money growth discounted by the elasticity parameter b . This is a general implication of the present value models. In the permanent income model of consumption, for example, savings predict future

labor income changes (Campbell, 1987), and in the expectations theory of the term structure the spread between long and short interest rates predicts future short interest rate changes (Campbell and Shiller, 1987).

Given that real balances and money growth need first differencing to become stationary, it will be useful to reparameterize (2) into

$$(m_t - p_t) + \beta \Delta m_t - \alpha = -(1-b)^{-1} \sum_{i=1}^{\infty} b^i E_t \Delta^2 m_{t+i} + (1-b) \sum_{i=0}^{\infty} b^i E_t u_{t+i} \quad (3)$$

Expression (3) shows that, if m_t and p_t are both integrated of order 2, $I(2)$, and the velocity shock, u_t , is stationary, then the Cagan model under rational expectations and no bubbles has the testable implication that real money balances, $m_t - p_t$, cointegrate in the sense of Engle and Granger (1987), with the growth rate of money, Δm_t . By contrast, equation (1) implies that stationarity of the velocity shocks is needed for real money balances to cointegrate with inflation, Δp_{t+1} .

Campbell and Shiller (1987) have constructed an appealing method to test the implications of present value models when the underlying time series are $I(1)$ processes. This can be done by first estimating β in a cointegrating regression between $(m_t - p_t)$ and Δm_t or Δp_{t+1} , and then setting up a bivariate VAR model for the two stationary variables $\Delta^2 m_t$ and $S_t = (m_t - p_t) + \beta \Delta m_t(p_{t+1})$. The overidentifying restrictions implied by rational expectations and no bubbles, which the Cagan model imposes on the parameter of this VAR system, can then be tested formally or informally. The VAR under consideration is as follows:

$$\begin{bmatrix} \Delta^2 m_t \\ S_t \end{bmatrix} = \begin{bmatrix} \alpha(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta^2 m_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \quad (4)$$

where $a(L), b(L), c(L)$ and $d(L)$ are lag polynomials of order p . Within the context provided by Cagan's model, the idea is to generate the unrestricted VAR forecast of the present value of future changes in the money growth rate or the expected inflation, which will be called S_t^* .

We can re-write the VAR model in first-order companion form as

$$Z_t = AZ_{t-1} + e_t,$$

where $Z_t = [\Delta m_t, \dots, \Delta^2 m_{t-p+1}, S_t, \dots, S_{t-p+1}]$ and A is the companion matrix of VAR parameters. It can then be shown that this forecast equals to:

$$\begin{aligned} S_t^* &= -(1-b)^{-1} \sum_{i=1}^{\infty} b^i E_t \Delta^2 m_{t+i} \\ &= -(1-b)^{-1} g \sum_{i=1}^{\infty} b^i A^i Z_t \\ &= -\beta g A (1-bA)^{-1} Z_t \end{aligned} \quad (5)$$

where g is a $(1 \times 2p)$ vector that picks out $\Delta^2 m_t$ from the VAR model and is assumed that $u_t = 0$, for all t .

Setting (5) equal to (3) the cross-equation parameter restrictions implied by the rational expectations Cagan model with no velocity shocks, the case investigated by Goodfriend (1982) who argued that velocity shocks were negligible during the German hyperinflation, are

$$\begin{aligned} c_i &= (1 + \beta) a_i, i = 1, \dots, p \\ d_1 &= (1 + \beta)(\beta^{-1} + b_1) \\ d_i &= (1 + \beta) b_i, i = 2, \dots, p \end{aligned} \quad (6)$$

These restrictions can be tested using a Wald or Likelihood ratio test (Johansen and Swensen, 1999). Equivalently the hypothesis can be tested by imposing the restrictions (5) on the VAR model (4). A simple transformation gives the variable $X_t = S_t - (1 + \beta)\beta^{-1}S_{t-1} - (1 + \beta)\Delta^2 m_t$ which is shown to be uncorrelated with information at time $t-1$ when velocity shocks are negligible. Next, by regressing X_t on lagged $\Delta^2 m_t$ and S_t we can test for the statistical significance of those variables. As Engsted (1993, 1994) argues, a problem with formal testing of these restrictions is that a statistical rejection of the restrictions is difficult to interpret *economically*. However the difference between S_t and S_t^* is easily shown to measure the noise in the model so that by plotting S_t together with S_t^* in a diagram one can get an informative picture of the Cagan model ability's to explain the data. Campell and Shiller (1987) argue that when deviations from the *exact* linear rational expectations model are transitory, S_t and S_t^* will be highly positively correlated.

Simultaneous statistical rejection of the parameter restrictions (6) and a close comovement of S_t and S_t^* therefore implies the presence of non-negligible but stationary velocity shocks. On the other hand, if S_t and S_t^* do not exhibit a high correlation this is indication that the velocity shock follows a random walk.

In much of the previous rational expectations literature on hyperinflation (e.g. Burmeister and Wall, 1982, Flood *et al.* 1984), money is assumed to be exogenous, in the sense that no feedback from prices to money is allowed. An important feature of the cointegrated VAR model described above is that such feedback is allowed (S_t Granger causes $\Delta^2 m_t$). The intuitive explanation for this is that, if agents use information besides current and lagged money growth to forecast future money growth, then, according to the exact Cagan model under rational expectations, S_t summarizes this additional information. To summarize, if velocity shocks are either negligible or stationary, (1) shows that the real balances cointegrate with the inflation rate. This holds regardless of the presence of bubbles in p_t . In contrast, real balances only cointegrate with money growth if the no bubble transversality condition holds. Therefore, if it is found that real balances cointegrate with both inflation and money growth, it in effect precludes bubbles. On the other hand, if real balances cointegrate with inflation, but not with money growth, it could be due to the presence of bubbles.

3. Econometric methodology

Our cointegration analysis is based on the Full Information Maximum Likelihood (FIML) multivariate cointegration technique developed by Johansen (1988, 1991), extended by Johansen (1992, 1992, 1997) and Paruolo (1996) and Rahbek *et al.* (1999) to incorporate the analysis of $I(2)$ variables.

Consider a p -dimensional vector autoregressive model which in error correction form is given by

$$\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + \gamma D_t + \mu + \varepsilon_t, \quad t=1, \dots, T \quad (7)$$

where $z_t = [m, p]$, z_{k+1}, \dots, z_0 are fixed and $\varepsilon_t \sim Nid_p(0, \Sigma)$. The adjustment of the variables to the values implied by the steady state relationship is not immediate due to a number of reasons like imperfect information or costly arbitrage. Therefore, the correct specification of the dynamic structure of the model, as expressed by the parameters $(\Gamma_1, \dots, \Gamma_{k-1}, \gamma)$, is important in order that the equilibrium relationship be revealed. The matrix $\Pi = \alpha\beta'$ defines the cointegrating relationships, β , and the rate of adjustment, α , of the endogenous variables to their steady state values. D_t is a vector of non-stochastic variables, such as centered seasonal dummies which sum to zero over a full year by construction and are necessary to account for short-run effects which could otherwise violate the Gaussian assumption, and/or intervention dummies; $\mu = \mu_0 + \mu_1 t$ account for the constant and the deterministic trend (Rahbek *et al.* 1999) and T is the sample size.

If we allow the parameters of the model $\theta = (\Gamma_1, \dots, \Gamma_{k-1}, \Pi, \gamma, \mu, \Sigma)$ to vary unrestrictedly then model (7) corresponds to the $I(0)$ model. The $I(1)$ and $I(2)$ models are obtained if certain restrictions are satisfied. Thus, the higher-order models are nested within the more general $I(0)$ model.

It has been shown (Johansen, 1991) that if $z_t \sim I(1)$, then that matrix Π has reduced rank $r < p$, and there exist $p \times r$ matrices α and β such that $\Pi = \alpha\beta'$. Furthermore, $\Psi = \alpha'_\perp (\Gamma) \beta_\perp$ has full rank, where $\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i$ and α_\perp and β_\perp are $p \times (p-r)$ matrices orthogonal to α and β , respectively.

Following this parameterization, there are r linearly-independent stationary relations given by the cointegrating vectors β and $p-r$ linearly-independent non-stationary relations. These last relations define the common stochastic trends of the system and the MA representation shows how they contribute to the various variables. By contrast the AR representation of model (7) is useful for the analysis of the long-run relations of the data.

The $I(2)$ model is defined by the first reduced rank condition of the $I(1)$ model and that $\Psi = \alpha'_\perp \Gamma \beta_\perp = \varphi \eta'$ is of reduced rank s_1 , where φ and η are $(p-r) \times s_1$ matrices and $s_1 < (p-r)$.

Under these conditions we may re-write (7) as

$$\Delta^2 z_t = \Pi z_{t-1} - \Gamma \Delta z_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 z_{t-1} + \gamma D_t + \mu + \varepsilon_t \quad (8)$$

$$\text{where } \Psi_i = - \frac{\sum_{j=i+1}^{k-1} \Gamma_j}{\Gamma_i}, \quad i=1, \dots, k-2$$

Johansen (1997) shows that the space spanned by the vector z_t can be decomposed into r stationary directions, β , and $p-r$ nonstationary directions, β_{\perp} , and the latter into the directions $(\beta_{\perp}^1, \beta_{\perp}^2)$, where $\beta_{\perp}^1 = \beta_{\perp} \eta$ is of dimension $p \times s_1$ and $\beta_{\perp}^2 = \beta_{\perp} (\beta_{\perp}^1 \beta_{\perp}^1)^{-1} \eta_{\perp}$ is of dimension $p \times s_2$ and $s_1 + s_2 = p - r$. The properties of the process are described by:

$$\begin{aligned} I(2): & \{\beta_{\perp}^2 z_t\}, \\ I(1): & \{\beta' z_t\}, \{\beta_{\perp}^1 z_t\}, \\ I(0): & \{\beta_{\perp}^1 \Delta z_t\}, \{\beta_{\perp}^2 \Delta^2 z_t\}, \{\beta' z_t + \omega' \Delta z_t\} \end{aligned}$$

where ω is a $p \times r$ matrix of weights, designed to pick out the $I(2)$ components of z_t (Johansen, 1992, 1995). Thus, we have that the cointegrating vectors $\beta' z_t$ are actually $I(1)$ and require a linear combination of the differenced process Δz_t to achieve stationarity.

Johansen (1991) shows how the model can be written in moving average form, while Johansen (1997) derives the FIML solution to the estimation problem for the $I(2)$ model. Furthermore, Johansen (1995) provides an asymptotically equivalent two-step procedure which computationally is simpler (Paruolo, 2000). It applies the standard eigenvalue procedure derived for the $I(1)$ model twice, first to estimate the reduced rank of the Π matrix,

and then for given estimates of α and β , to estimate the reduced rank of $\alpha_{\perp}' \Gamma \beta_{\perp}$, (Juselius, 1994, 1995, 1998). In both steps a likelihood ratio test for the associated rank of

either Π or $\alpha_{\perp}' \Gamma \beta_{\perp}$ are provided. The sum of the two likelihood ratio tests for all possible values of r forms the basis of the testing procedure. Paruolo (1996) and Rahbek *et. al.* (1999)

have extended model (7) to allow for linear deterministic trends. Both of them apply restrictions so that quadratic trends are excluded in the solution for z_t , the difference however between them is that in the specification followed by Paruolo (1996) deterministic trends in the cointegrating vector are not allowed while in the Rahbek *et al.* (1999) study this is feasible. Another interesting result of those two studies is that the joint test for $I(1)$ and $I(2)$ cointegrating ranks is asymptotically similar with respect to the drift terms which implies that it is not necessary to determine the rank together with the trend specification.

An equally important issue, along with the existence of at least one cointegration vector, is the issue of the stability of such a relationship through time as well as the stability of the estimated coefficients of such a relationship. Thus, Septhon and Larsen (1991) have shown that Johansen's test may be characterized by sample dependency. Hansen and Johansen (1993, 1999) have suggested methods for the evaluation of parameter constancy in cointegrated VAR models, formally using estimates obtained from the Johansen FIML technique. Three tests have been constructed under the two VAR representations. In the "Z-representation" all the parameters of model (7) are re-estimated during the recursions while under the "R-representation" the short-run parameters $\Gamma_i = 1, \dots, k-1$ are fixed to their full sample values and only the long-run parameters α and β are re-estimated.

The first test is called the **Rank** test and we examine the null hypothesis of sample independency of the cointegration rank of the system. This is accomplished by first estimating the model over the full sample, and the residuals corresponding to each recursive subsample are used to form the standard sample moments associated with Johansen's reduced rank. The eigenvalue problem is then solved directly from these subsample moment matrices. The obtained sequence of trace statistics is scaled by the corresponding critical values, and we accept the null hypothesis that the chosen rank is maintained regardless of the subperiod for which it has been estimated if it takes values greater than one.

A second test deals with the null hypothesis of constancy of the cointegration space for a given cointegration rank. Hansen and Johansen (1993, 1999) propose a likelihood ratio test that is constructed by comparing the likelihood function from each recursive subsample to the likelihood function computed under the restriction that the cointegrating vector estimated from the full sample falls within the space spanned by the estimated vectors of

each individual sample. The test statistic is a χ^2 distributed with $(p-r)r$ degrees of freedom.

The third test examines the constancy of the individual elements of the cointegrating vectors β and the loadings α . However, when the cointegration rank is greater than one, the elements of those vectors can not be identified, except under restrictions. Fortunately, one can exploit the fact that there is a unique relationship between the eigenvalues and the cointegrating vectors. Therefore, when the cointegrating vectors or the loadings have undergone a structural change this will be reflected in the estimated eigenvalues. Hansen and Johansen (1993, 1999) have derived the asymptotic distribution of the estimated eigenvalues.

4. Empirical results for the German hyperinflation

The data used in our analysis are monthly observations on prices and money from the German hyperinflation episode, 1920-23 and are depicted on diagrams 1 and 2. The wholesale price index (1913-14 = 100) is a monthly average reported in *International Economic Statistics 1919-1930* (1934, pp. 82-84, International Conference of Economic Services, London), and is the same series used to construct the inflation series reported in Cagan (1956). The time series on nominal money is taken directly from the appendix in Flood and Garber (1980). The sample runs from January 1920 to June 1923, giving a total of forty-two observations. This is the same sample period used by Casella (1989) and Engsted (1993).

4.1 Determination of the cointegration rank and the order of integration

The first step in the analysis is the determination of the order of integration of German money and prices. Since the data employed in our study have been subjected to careful scrutiny before we present the main findings of previous studies. Taylor (1991) and Engsted (1993), among others, have tested real balances, money growth and inflation for being non stationary stochastic processes and they concluded that all variables appear to be $I(1)$ processes. The problem however with the sequential use of the Dickey – Fuller test statistic,

for the identification of the number of unit roots, is that it has a low power against the explosive alternative which have properties mimicking those of the $I(2)$ processes (Haldrup, 1998). It has been suggested that the Hazda and Fuller (1979) $I(2)$ -test statistic is more appropriate in those cases since it tests jointly for double unit roots by applying a two sided test where the alternative hypothesis is quite general as it covers situations where z_t is either explosive, $I(0)$ or $I(1)$. Haldrup (1998) has applied this methodology and he failed to reject the $I(2)$ null hypothesis for money and prices on the same period as the one used in the present paper.

As a first check of the statistical adequacy of model (7) we report some multivariate and univariate misspecification tests in Table 1. We note that our chosen VAR model with six lags is well specified. The multivariate Ljung-Box test and the multivariate LM test for first and fourth order residual autocorrelations as well as the multivariate normality test are not significant. The univariate residual tests also show no signs of misspecification. ARCH(6) tests for sixth order autoregressive heteroscedasticity and could not be rejected for both equations. The R^2 measures show that with the chosen specification we can explain quite a large proportion of the variation in the money growth and the inflation rate.

The Johansen - Juselius multivariate cointegration technique, as explained in section 3, is applicable only in the presence of variables that are realizations of $I(1)$ processes and/or a mixture of $I(1)$ and $I(0)$ processes, in systems used for testing for the order of cointegration rank. Until recently the order of integration of each series was determined via the standard unit root tests. However, it has been made clear by now that if the data are being determined in a multivariate framework, a univariate model is at best a bad approximation of the multivariate counterpart, while at worst, it is completely misspecified leading to arbitrary conclusions. Thus, in the presence of $I(1)$ series, Johansen and Juselius (1990) developed a multivariate stationarity test which has become the standard tool for determining the order of integration of the series within the multivariate context.

Additionally, when the data are $I(2)$, one also has to determine the number of $I(2)$ trends, s_2 , among the $p-r$ common trends. The two-step procedure discussed in section 3 is used to determine the order of integration and the rank of the two matrices. The hypothesis that the number of $I(1)$ trends = s_1 and the rank = r is tested against the unrestricted H_0

model based on a likelihood ratio test procedure discussed in Johansen (1992, 1995, 1997) and extended by Paruolo (1996) and Rahbek *et al.* (1999).

Table 2 reports the trace test statistics for all possible values of r and $s_1 = p - r - s_2$, under the assumption that the data contain linear but no quadratic trends. We have included in the estimation eleven centered seasonal dummies, which are necessary to account for short-run effects which could otherwise violate the Gaussian assumption. The 95% critical test values reported in italics below the calculated test values are taken from the asymptotic distributions reported in Rahbek *et al.* (1999, Table 1). Starting from the most restricted hypothesis $\{r = 0, s_1 = 0, s_2 = 2\}$ and testing successively less and less restricted hypotheses according to the Pantula (1989) principle, it is shown that the case in favor of the presence $I(2)$ components can not be rejected at the 5% level, since the hypothesis of $\{r = 1, s_1 = 0, s_2 = 1\}$ can not be rejected.^{4,5,6}

We then tested for the significance of the deterministic trend in the multicointegrating relation. Rahbek *et al.* (1999) have shown that this hypothesis can be tested with a likelihood ratio test constructed from the r largest eigenvalues of two models; in the first the deterministic trend appears in z_t while in the second it is excluded. The null hypothesis is that the linear trend does not enter significantly in the cointegration vector and the test statistic under the null is a likelihood ratio test is asymptotically distributed as $\chi^2(r)$. The test statistic in our case is equal to 1.4 with a p-value of 0.25 and thus we reject the presence of a deterministic trend in the multicointegrating relation.

4.2. Interpreting the $I(2)$ results

In Table 2 the estimated results are reported for the case $r = 1, s_1 = 0$, and $s_2 = 1$. We have decomposed the vector z_t into the cointegrating (β) and the non-cointegrating components (β_{\perp}) and moreover the cointegrating components into the $r_0 = 0$ directly stationary relations, and the $r_1 = 1$ multicointegrated relation, and the remaining nonstationary components into the $I(1)$ and the $I(2)$ relations as discussed in section 3.

Given that there are only two variables under consideration, there are no directly stationary relations, but only one multicointegration, $\beta' z_t + \omega' \Delta z_t$, which is homogeneous relation between money supply, prices and money growth or inflation. This seems to support the choice of $p - r = 1$ common stochastic trend, which is second-order nonstationary, being in line with the evidence from the roots of the characteristic polynomial.

Given that $s_1 = 0$, no estimates of α_{\perp}^1 and β_{\perp}^1 are provided since these elements are associated with the $I(1)$ stochastic trend. The estimate of β_{\perp}^2 shows how the $I(2)$ stochastic trend affects nominal money stock and prices. Thus, this estimate describes the weight with which the $I(2)$ trend component influences the variables of the system. Hence, a condition that the variable z_t is $I(2)$ is, therefore, $\beta_{\perp ij}^2 \neq 0$ for $j = 1, \dots, s_2$. The estimate of α_{\perp}^2 shows that the twice cumulated shocks to the nominal money stock are of importance for the $I(2)$ trend $\alpha_{\perp}^2' \sum \sum \varepsilon_i, i = 1, 2$. This indicates that the second order stochastic trend in nominal prices derives from unanticipated shocks to the nominal money supply.

4.3. A data transformation from $I(2)$ to $I(1)$

Since the statistical inference of the $I(2)$ model is not yet as developed as that of the $I(1)$ model, a data transformation that allows us to move to the $I(1)$ model will simplify the empirical analysis considerably. A natural hypothesis which follows from the $I(2)$ -ness of the money supply and price level is that the real money balances $\{m_t - p_t\}$ is a first-order nonstationary process. The implication of this hypothesis is that the money supply and prices are cointegrating from $I(2)$ to $I(1)$, and use of the transformed data vector $\tilde{z}_t' = [m_t - p_t, \Delta m_t (\Delta p_{t+1})]$ would then allow us to move to the $I(1)$ model. The validity of this transformation is based on the assumption that $\{m_t - p_t\} \sim I(1)$, $\{\Delta m_t \text{ or } \Delta p_{t+1}\} \sim I(1)$, and that $\{m_t - p_t\}$ is a valid restriction on the long-run structure, but not necessarily on the short-run structure. Price homogeneity is directly tested by imposing the linear restriction $(1, -1)$ to the accepted cointegrating vector. The test statistic which is asymptotically distributed as $\chi^2(1)$ is equal to 1.04 and therefore we fail to reject the hypothesis of long run price

homogeneity and this approach contrast with some of the literature on the subject that takes this hypothesis as given, (Cagan, 1956; Taylor, 1991).⁷

The second requirement needed for the transformation to the $I(1)$ model is that $\{m_t - p_t\} \sim I(1)$, $\{\Delta m_t \text{ or } \Delta p_{t+1}\} \sim I(1)$. This will be directly checked in the remaining analysis that will be performed in the $I(1)$ model, containing long-run but not short-run price homogeneity, based on the transformed vector $[m - p, \Delta m(\Delta p_{t+1})]$.

To assess the statistical properties of the chosen variables in the transformed model the test statistics reported in Table 3 are useful. The test of long-run exclusion is a check of the adequacy of the chosen measurements and show that none of the variables can be excluded from the cointegration space. The tests for stationarity indicate that none of the variables can be considered stationary under any reasonable choice of r . Finally, the test of weak exogeneity shows that none of the variables can be considered weakly exogenous for the long-run parameters β independently of the choice of r . All three tests are χ^2 distributed and are constructed following Johansen and Juselius (1990, 1992). Furthermore, Table 3 presents diagnostics on the residuals from the cointegrated VAR model which indicate that they are *i.i.d.* processes, since no evidence of serial correlation or non-normality was detected. This provides further support for the hypothesis of a correctly specified model.⁸

Finally, Table 3 also reports the estimated cointegrating vectors of the transformed model which are based upon eigenvectors obtained from an eigenvalue problem resulting from Johansen's reduced rank regression approach. The estimates of the elasticity parameter β in the Cagan model are 7.826 and 6.616 respectively, which are very reasonable and within the expected theoretical range.

The final stage of the cointegration analysis involves the stability analysis of our cointegration results. Figures 3(a)-(b), and 4(a)-(b) present the Hansen-Johansen (1993,1999) recursive analysis on the parameter stability of the cointegrated-VAR models. The first set of graphs shows that the rank of the cointegration space depends on the sample size from which it has been estimated, since the null hypothesis of a constant rank is rejected. This result is quite important for making inference about the validity of the Cagan model under rational expectations and no speculative bubbles, provided that we require cointegration between real money balances and money growth and between real money

balances and inflation, in order to preclude bubbles and to show that the velocity shocks were negligible. Thus, from these figures we note that in fact cointegration in both cases is established in the last two months and even in this case this has occurred marginally. The evidence of the recursive analysis is in agreement with the cointegration results reported above and we argue that the results show very weak evidence in favour of cointegration. The result we present here indicates that, for the Cagan model to be established, we need the observations of the late months of 1923 when the inflationary pressures are exacerbated due to the suspension of reparations by Germany and the French invasion of the Ruhr. The previous works by Taylor (1991) and Engsted (1993, 1994) do not consider a small sample adjustment in the estimated trace test statistic. In a sample of 42 observations, as in this case, this is a requirement. The second set of graphs indicates that we are always unable to reject the null hypothesis for the sample independence of the cointegration space for a given cointegration rank. Therefore, we can conclude that the estimated coefficients do not display instabilities in recursive estimates.

Since we have found cointegration between the real money balances and money growth and real money balances and inflation, the next step of our analysis is to estimate the VAR model in (4), which takes into account the cointegrating properties of the data. Table 4 reports the results of estimating a sixth-order VAR model for the two $I(0)$ variables $S_t = (m_t - p_t) + 7.826\Delta m_t$ and $\Delta^2 m_t$. This estimation yields two main findings. First, it is shown that S_t strongly-causes $\Delta^2 m_t$. Second, the *exact* rational expectation restrictions (6) which are imposed on the VAR are strongly rejected providing further evidence against the view taken by Goodfriend (1982), that velocity shocks were negligible in the German hyperinflation episode.

Finally, to examine whether the rejection of the Cagan model is due to a model misspecification or is caused by transitory deviations from the model, we construct the variable S_t^* given in (5) using the unrestricted VAR-parameter and a β -value of 7.826. Then, regressing S_t^* and S_t gives a slope coefficient of 1.95 which is statistically significantly different from unity. Thus, it is obvious that there are significant differences between the two variables, implying that the deviations from the model are non-negligible.

5. Conclusions

In this paper we provided a re-examination of the Cagan model of hyperinflation for the German case of the 1920s by applying recent contributions in the econometrics of non-stationarities and cointegration. First, we examined the order of integration and the cointegration rank in a multivariate context using the recently developed testing methodology suggested by Johansen (1992, 1995, 1997) and extended by Paruolo (1995) and Rahbek *et al.* (1999) it was shown that both money supply and prices are $I(2)$ processes while we were able to identify one statistically significant cointegrating vector as well as one $I(2)$ component between money supply and prices. Second, given that the variables of interest are $I(2)$ we estimated the multi-cointegrating relationship of the transformed $I(1)$ model between real money balances and money growth and real money balances and inflation. This is proved to be the necessary and sufficient condition for excluding the presence of rational bubbles. Third, although cointegration was established on both cases, the evidence is rather weak given that the rank of the cointegration space exhibits sample dependence, a result obtained from the application of the recursive tests of Hansen-Johansen (1993,1999). This evidence implies that the Cagan model is statistically established only when the “true” hyperinflation period, which began in June 1922 and lasted until the end of 1923 when stabilization was achieved, is included in the sample. Finally, the *exact* rational expectations restrictions implied by the Cagan model with rational expectations and no transitory shocks were rejected, providing further evidence against this model specification of hyperinflation.

Footnotes

1. Phylaktis and Taylor (1992, 1993), Frenkel and Taylor (1993), Engsted (1996, 1998), Petrovic and Yujosevic (1996), and Choudhry (1998) are among the recent studies which apply cointegration methods to test the Cagan model for several other countries that have experienced hyperinflation in different periods of time. Recently, Lee *et al.* (2000) exploited the main insight of Cagan's model in order to examine the relationship between stock returns and inflation during the German hyperinflation period.

2. Recently Laidler and Stadler, (1998) provide evidence that there was a small minority of German economists who at the time of the Weimar hyperinflation favored a monetary explanation of the phenomenon.

3. As Timmerman, (1994) has shown in present value models with feedback relations, rational explosive bubbles can be ruled out without invoking the transversality condition. The intuition is that in the presence of a feedback from prices to money, the bubble component in the endogenous price process which grows asymptotically will come to dominate the forcing variable (money) such that a growing difference between those two variables cannot exist. This result implies that cointegration tests for the presence of explosive bubbles make no sense once the presence of feedback from prices to money is established. Since there is no unanimity in the literature on the issue of exogeneity or not of the money supply (Flood *et al.*, 1984, Sargent and Wallace, 1973) we proceed with the assumption that bubbles are excluded if the transversality condition holds.

4. The estimation of the eigenvectors as well as the stability tests have been performed using the program CATS in RATS 4.20 developed by Katarina Juselius and Henrik Hansen, Estima Inc., Illinois, 1995.

5. A small sample adjustment has been made in all the likelihood ratio statistics, equal to

$$-2 \ln Q = -(T - kp) \sum_{i=j_0+1}^k \ln(1 - \hat{\lambda}_i)$$

as suggested by Reimers (1992).

6. Gonzalo (1994) shows that the performance of the maximum likelihood estimator of the cointegrating vectors is little affected by non-normal errors. Lee and Tse (1996) have shown similar results when conditional heteroskedasticity is present.

7. Johansen (1995) shows that such an analysis is valid when m_t and p_t are $I(2)$ variables. Specifically, the tables of Osterwald-Lenum (1992) can be used to test for cointegration, and inference concerning the cointegrating vectors can be conducted using the chi-squared distribution.

8. The application of the Rahbek *et al.* (1999) test on the two systems shows that the adopted transformation removes all signs of the $I(2)$ component from the data since it is shown that all $I(2)$ hypotheses can be rejected at the 5% critical level. To save space these results are available upon request.

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Table 1. Residual misspecification tests of the model with $k = 6$

(a) Univariate Residual Misspecification Tests:

	Δm_t	Δp_t
ARCH(6)	11.82	5.27
NORM	0.95	2.33
η_3	0.34	-0.38
η_4	2.68	2.30
LB(36)	26.28	30.10
σ_ε	0.02	0.05
R^2	0.98	0.91

Notes: σ_ε is the standard error of the residuals, η_3 and η_4 are the skewness and kurtosis statistics. The LB is the test for serial correlation, ARCH is the test for the presence of conditional heteroskedasticity, and NORM the Jarque-Bera test for normality. The ARCH and NORM statistics are distributed as χ^2 with 6 and 2 degrees of freedom, respectively and the LB statistic is distributed as χ^2 with 36 degrees of freedom.

(b) Multivariate Residual Misspecification Tests

Residual autocorrelation. LB(9)	$\chi^2(138) = 36.9$	p-value = 0.01
Residual autocorrelation. LM(1)	$\chi^2(4) = 4.23$	p-value = 0.38
Residual autocorrelation. LM(4)	$\chi^2(4) = 10.72$	p-value = 0.03
Normality	$\chi^2(4) = 3.75$	p-value = 0.48

LB is the multivariate version of the Ljung-Box test for autocorrelation based on the estimated auto and cross – correlations of the first ($T/4=9$) lags distributed as a χ^2 with 138 degrees of freedom. LM(1) and LM(4) are the tests for the first and fourth-order autocorrelation distributed as a χ^2 with 4 degrees of freedom. The Normality test is a multivariate version of the Shenton-Bowman test and is distributed with 4 degrees of freedom.

Table 2. Testing the Rank in the I(1) and I(2) Model

Testing the joint hypothesis $H(s_1 \cap r)$

$p-r$	r	$Q(s_1 \cap r / H_0)$		Q_r
2	0	61.2*	40.6*	26.3*
		47.6	34.4	25.4
1	1		9.7	2.1
			19.9	12.5
s_2		2	1	0

Notes: p is the number of variables, r denotes the number of cointegrating vectors, s_1 and s_2 denote the number of I(1) and I(2) components respectively. In performing the Johansen test, a structure of six lags was chosen according to a likelihood ratio test, corrected for the degrees of freedom, (Sims, 1980) and the Ljung-Box Q statistic for detecting serial correlation in the residuals of the equations of the VAR. A model with an unrestricted constant and a linear trend in the cointegrating vector is estimated according to the Johansen (1992) testing methodology. The numbers in italics are the 95% critical values (Rahbek *et. al.*, 1999, Table 1). (*) denotes statistical significance at the five percent critical level.

Vector z_t decomposed into the I(0), I(1), and I(2) directions

	Cointegration Space			
	$\hat{\beta}_1^0$	$\hat{\beta}_2^0$	$\hat{\beta}_1$	ω
m	-	-	- 4.080	102.1
p	-	-	7.664	154.2
	The space spanned by β_{\perp}		The space spanned by α_{\perp}	
	β_{\perp}^1	β_{\perp}^2	α_{\perp}^1	α_{\perp}^2
m	-	3.1	-	0.05
p	-	2.2	-	-0.08

m_t and p_t , are, respectively, the nominal money supply and price level

Table 3. Estimation of the Transformed I(1) Model

(a) Tests for Long-Run Exclusion, Stationarity, and Weak Exogeneity

Test	p-r	$m_t - p_t$	Δm_t	Δp_{t+1}
Long-Run Exclusion	1	14.80*	20.46*	
		15.62*		15.20*
Stationarity	1	20.46*	14.80*	
		15.20*		15.62*
Weak Exogeneity	1	12.16*	6.70*	
		7.06*		4.96*

Notes: $m_t - p_t$, Δm_t , Δp_{t+1} , are respectively, the real money balances, the money growth and the next period's inflation rate. The long-run exclusion restriction test is χ^2 distributed with r degrees of freedom, the multivariate stationarity test is χ^2 distributed with $(p - r)$ degrees of freedom and the weak-exogeneity test is χ^2 distributed with r degrees of freedom. In our case all the tests are distributed with one degree of freedom and the 5% critical value is 3.84. (*) denotes statistical significance at the five percent critical level.

(b) Multivariate Residuals Diagnostics

Case	L-B(9)	LM(1)	LM(4)	$\chi^2(4)$
$m_t - p_t, \Delta m_t$	34.28(0.05)	0.93(0.92)	5.69(0.22)	3.73(0.44)
$m_t - p_t, \Delta p_{t+1}$	35.19(0.05)	3.91(0.42)	4.67(0.36)	2.89(0.58)

Notes: as in Table 1.

(c) Estimated coefficients

$m_t - p_t$	Δm_t
1.0	7.826
$m_t - p_t$	Δp_{t+1}
1.0	6.616

Table 4. Summary Statistics from VAR model

Sample : January 1920 - June 1923

Sims likelihood ratio criterion selects five - lags VAR

$\Delta^2 m_t$ equation: $\bar{R}^{-2} = 0.89$

F test for the null hypothesis that S_t Granger-causes $\Delta^2 m_t$: $F = 15.26(0.00)$

S_t equation: $\bar{R}^{-2} = 0.86$

F test of rational expectations restriction (eq. 6) on VAR parameters : $F = 35.66(0.00)$

Correlation coefficient (S_t, S_t^*) = 0.99

Slope coefficient in regression of S_t^* on S_t : 1.95 (0.034)

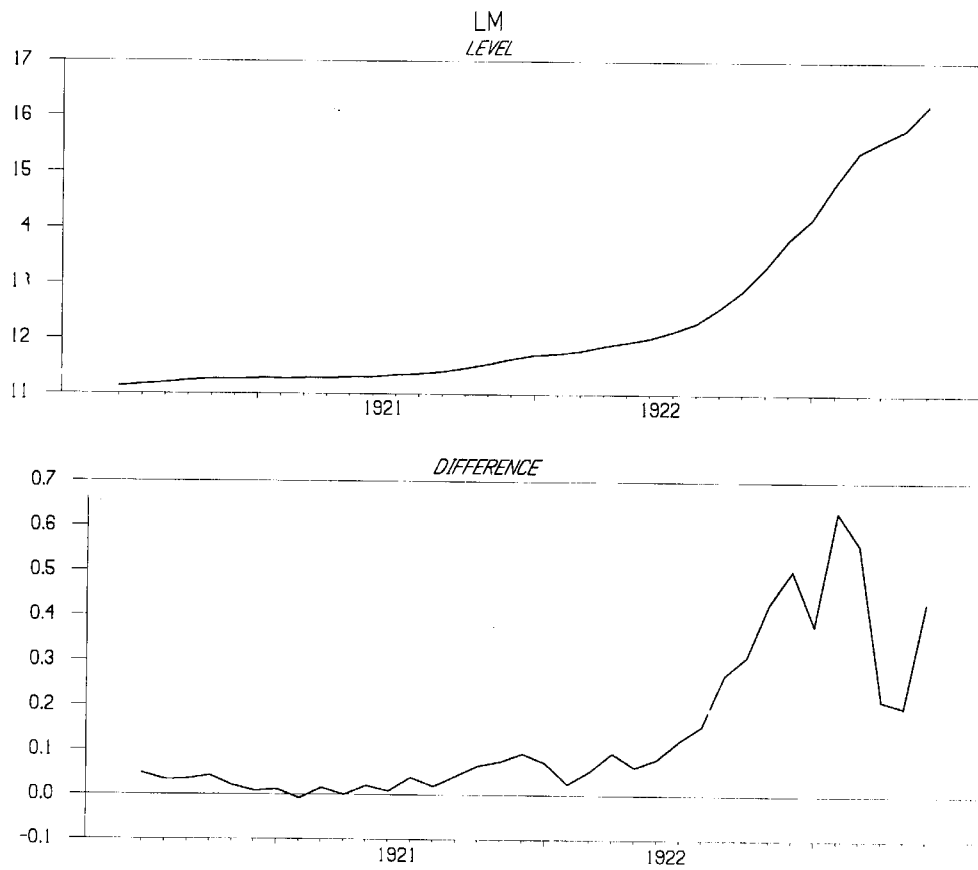


Figure 1. The German money supply in levels and differences

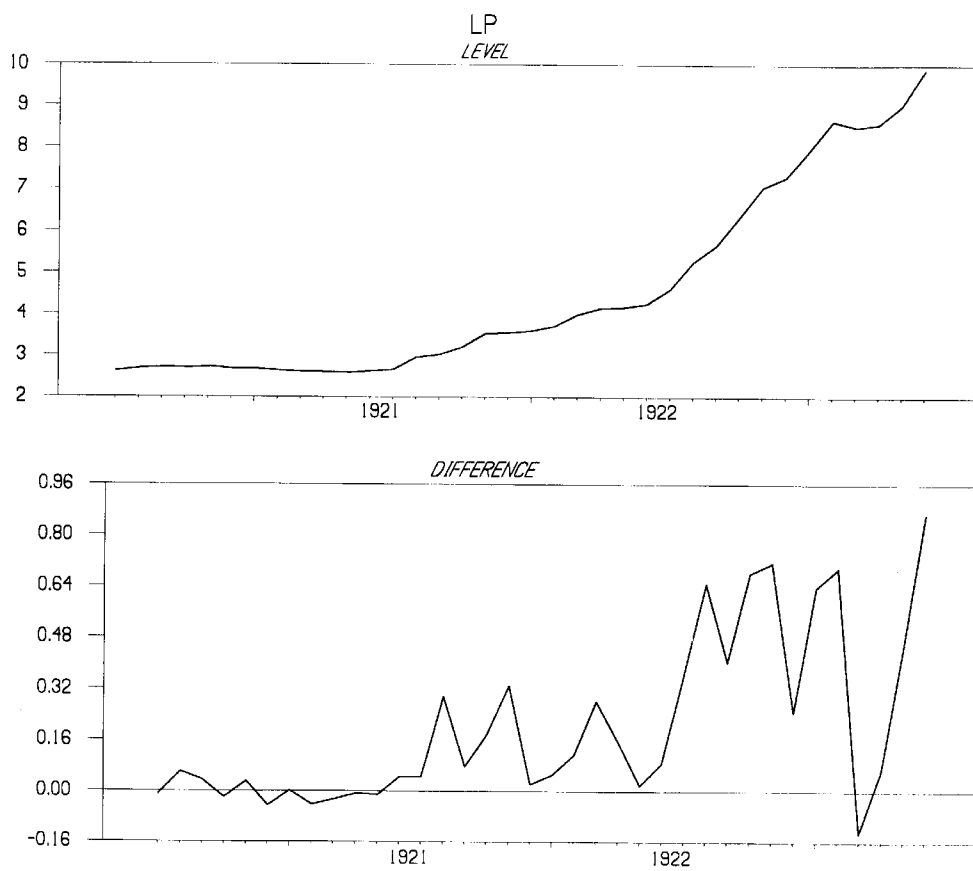


Figure 2. The German WPI in levels and differences

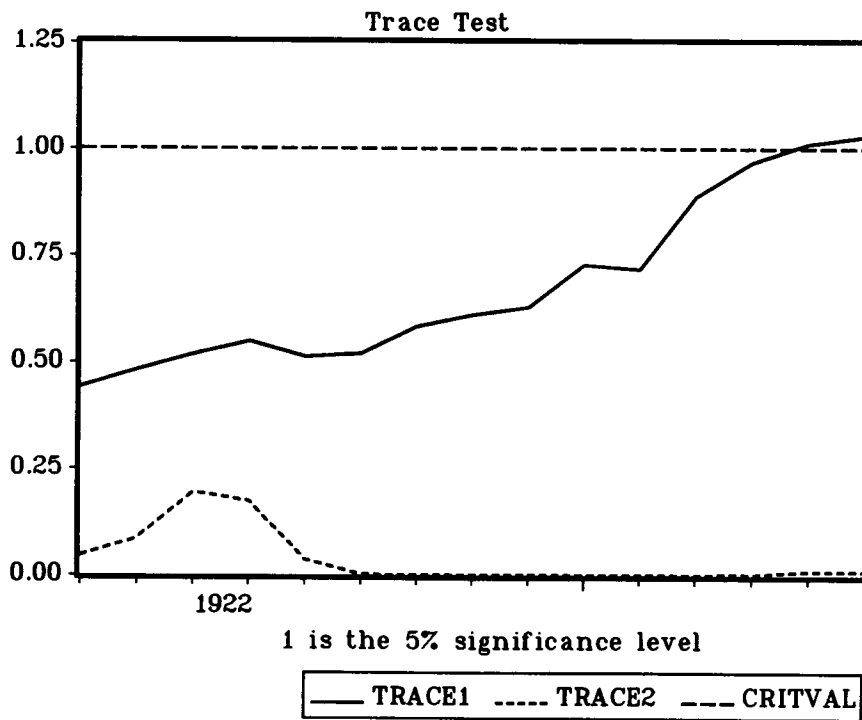


Figure 3a. The Trace Test : Money Growth case

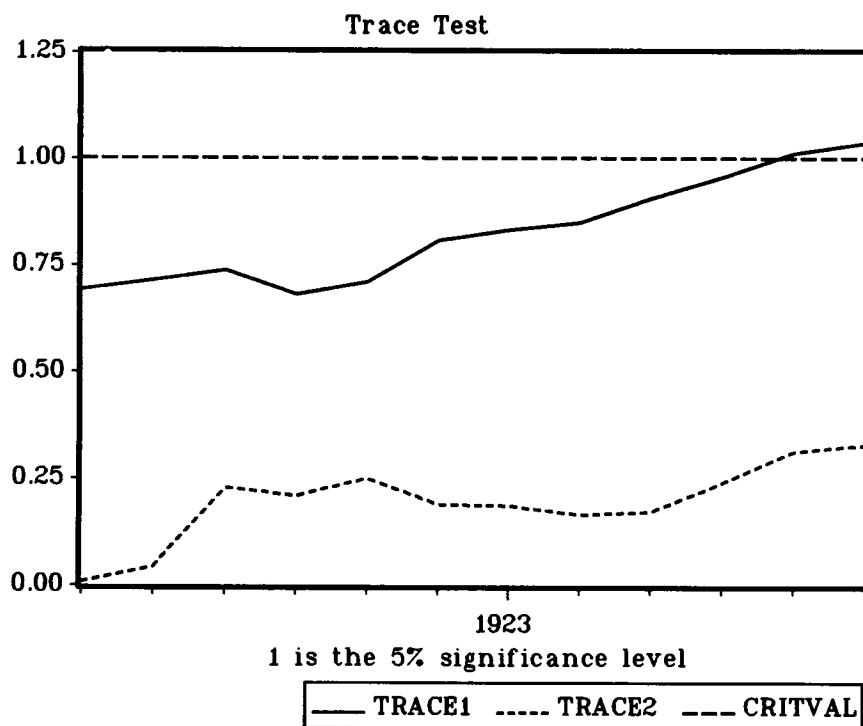


Figure 3b. The Trace Test : Inflation case

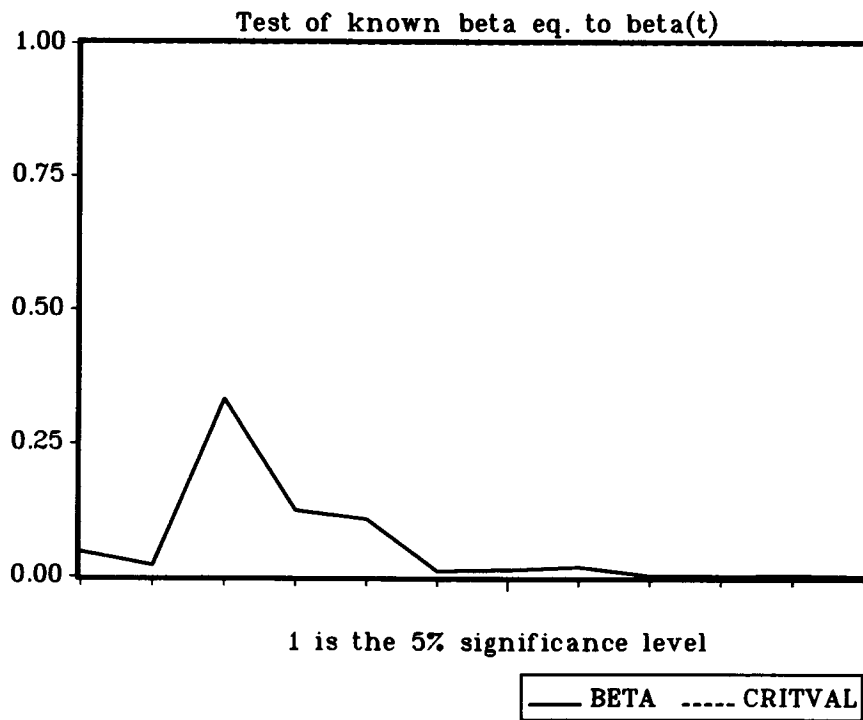


Figure 4a. The test of the constancy of beta : Money Growth case

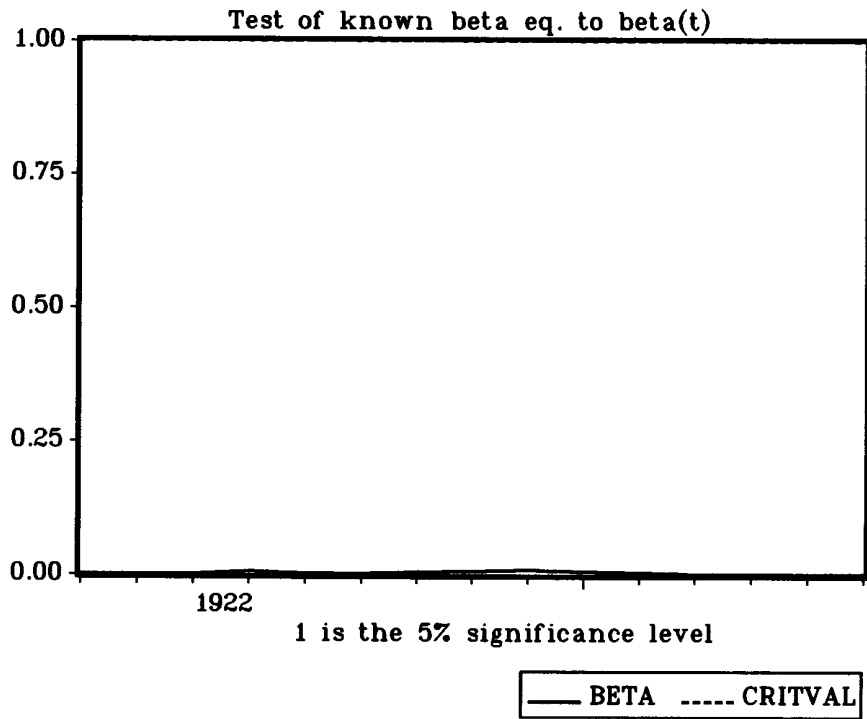


Figure 4b. The test of the constancy of beta : Inflation case