On the Choice of Functional Form in Stochastic Frontier Modeling

Konstantinos Giannakas
Department of Agricultural Economics
University of Nebraska-Lincoln
216 H. C. Filley Hall
Lincoln, NE 68583-0922, USA

Kien C. Tran·
Department of Economics
University of Saskatchewan
9 Campus Drive
Saskatoon, Saskatchewan
S7N 5A5, Canada
(*E-mail: trank@sask.usask.ca*)

Vangelis Tzouvelekas
Department of Economics
University of Crete
University Campus, Rethymno
Crete, 74 100, Greece
(e-mail: vangelis@econ.soc.uoc.gr)

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Abstract
This paper examines the effect of functional form specification on the estimation of technical efficiency using a panel data set of 125 olive-growing farms in Greece for the period 1987-93. The generalized quadratic Box-Cox transformation is used to test the relative performance of alternative, widely used, functional forms and to examine the effect of prior choice on final efficiency estimates. Other than the functional specifications nested within the Box-Cox transformation, the comparative analysis includes the minflex Laurent translog and generalized Leontief that possess desirable approximation properties. The results indicate that technical efficiency measures are very sensitive to the choice of functional specification. Perhaps most importantly, the choice of functional form affects the identification of the factors affecting individual performance - the sources of technical inefficiency. The analysis also shows that while specification searches do narrow down the set of feasible alternatives, the identification of the most appropriate functional specification might not always be (statistically) feasible.

Keywords: stochastic frontiers, functional specifications, Box-Cox transformation, technical efficiency, Greek olive oil.

JEL Classification System Numbers: C12, C13, C23, C24

1. Introduction

The stochastic production frontier model, which was proposed independently by Aigner, Lovell and Schmidt and Meeusen and van den Broeck in 1977, has dominated the empirical literature of efficiency measurement. Within this framework, several alternative models for estimating productive efficiency have been progressively developed, extending the stochastic production frontier methodology to account for different theoretical issues in frontier modeling. Comparative studies to date have mainly focused on estimates of the degree of inefficiency in the samples under study within different production frontier model specifications (for detailed reviews of the theoretical and empirical work in this area see Coelli, Rao and Battese (1998), Greene (1999), and Kumbhakar and Lovell (2000)).

Apart from the choice of the appropriate production frontier model however, an important issue that arises, which is not unique to efficiency studies, concerns the functional specification of the estimated frontier - the features of the technology employed. Interestingly, empirical applications for the measurement of efficiency have traditionally focused on a single *ad hoc* imposed functional specification, mostly translog and Cobb-Douglas.

The choice of the appropriate functional form is not a trivial matter however. It is well known that functional forms are both data and model specific, and differ in their convergence properties and their ability to approximate alternative technologies. Simply put, there is no functional form that dominates under all circumstances - the appropriate functional specification is case specific. If the empirical estimates are contaminated with the imposition of an inappropriate functional form, predicted responses arising from the model may be biased and inaccurate, posing serious problems for policy design and/or policy implications. Therefore, when there are no strong theoretical or prior empirical reasons in favor of a specific functional specification, the exploration of the sensitivity of the economic optima, including efficiency, to the choice of functional form becomes crucial.

The objective of this study is to empirically evaluate the performance of different functional specifications in the estimation of technical efficiency for a panel data set of 125 olive-growing farms in Greece. The paper explores the sensitivity of obtained efficiency estimates to the choice of functional specification while maintaining an identical data set and retaining the same assumptions about the underlying technology and the structure of farm efficiencies. The effects of the choice of functional form on the estimates of production structure (such as production elasticities, returns to scale, and technological change) and the determination of the factors influencing farm efficiency are also examined. The latter is particularly important since determining the sources of technical efficiency provides policy makers with insight on the causes of inefficiency and can suggest potential policies that enhance the productivity of the sector under study.

The estimation of farm-specific technical efficiency is based on the stochastic frontier model of Battese and Coelli (1993; 1995). This stochastic frontier model allows for a more flexible intertemporal variation in efficiency ratings, and identifies the factors influencing the efficiency of sample participants directly from the estimated production frontier. The production frontiers utilized in this comparative study belong primarily to the generalized quadratic family of flexible functional forms. More specifically, technical efficiency measures obtained from the transcendental logarithmic, the generalized Leontief, the normalized quadratic, the squared-root quadratic, the non-homothetic constant elasticity of substitution (CES) and the Cobb-Douglas functional forms are analyzed and compared using the generalized quadratic Box-Cox transformation function that nests all these functional specifications (Appelbaum, 1979; Berndt and Khaled, 1979). In addition to the above functional forms, the comparative analysis includes the minflex Laurent translog and generalized Leontief functional specifications due to their attractive properties in approximating the production technology (Barnett, 1983; 1985).

The rest of the paper is organised as follows. Section 2 provides a review of studies on the effect of functional choice on efficiency measures derived from econometric frontier models. Section 3 presents the functional specification of the production frontiers used in the analysis. Section 4 outlines the stochastic production frontier model utilized for the measurement of technical efficiency. Section 5 provides data descriptions while empirical results are presented in Section 6. Section 7 summarizes and concludes the paper.

2. Background

To our knowledge, there are only few studies that examine the effect of functional choice on efficiency measures derived from econometric stochastic frontier models. Kopp and Smith (1980) compared efficiency estimates derived from the translog, the non-homothetic CES, and the Cobb-Douglas functional specifications using cross-sectional data from steam generating electric plants in the US. They found that plant level productive efficiency is less sensitive to the choice of functional form. Several years later, Gong and Sickles (1992) examined the relative performance of translog, CES-translog, and generalized Leontief functions under different model specifications using a Monte-Carlo simulation approach and panel data. Disagreeing with the earlier findings of Kopp and Smith (1980) they concluded that "the choice of functional form in stochastic frontier model appears to be crucial."

Zhu, Ellinger and Shumway (1995) applied the generalized quadratic Box-Cox transformation model to examine the relative performance of normalized quadratic, translog and generalized Leontief functions using cross-sectional data from rural US banks in the context of a stochastic cost frontier. In accordance with the earlier findings of Gong and Sickles (1992), Zhu, Ellinger and Shumway (1995) suggested that "the choice of inappropriate functional specification would substantially alter conclusions about both scale elasticities and inefficiencies." Finally, Battese and Broca (1997) compared the translog and Cobb-Douglas functional forms using panel data from wheat farms in Pakistan. They also concluded that the final efficiency measures are sensitive to the choice of both functional specification and inefficiency effects model.

When compared with Zhu, Ellinger and Shumway (1995) (who also utilized a generalized quadratic Box-Cox transformation), our study has four distinct features. *First*, it proceeds to the estimation of a production frontier and the subsequent measures of technical efficiency. Zhu, Ellinger and Shumway (1995) estimated a cost frontier assuming that any deviation from that frontier is due to technical inefficiency. However, this is questionable in the dual approach of estimation since such a specification assumes that individuals are allocatively perfectly efficient.[1] *Second*, the stochastic frontier model of Battese and Coelli (1993;1995) used in this paper does away with the need to impose restrictive assumptions regarding the inter- and intra-farm variation in efficiency ratings.[2] *Third*, the current study relies on a panel data set of 125 olive-growing farms observed in seven consecutive years. Efficiency measures derived from cross-sectional data (i.e. a single production period) may be distorted by period specific abnormalities, which questions the accuracy of the estimates and, perhaps more importantly, the relevance of the analysis (Dawson, Lingard and Woodford, 1991). *Finally*, our comparative analysis includes two more flexible functional forms with desirable approximation properties (i.e. the minflex translog and generalized Leontief) that are not nested within the generalized quadratic Box-Cox transformation.

3. Functional Specifications

Appelbaum (1979) and Berndt and Khaled (1979) generalized the application of the Box-Cox transformation function to allow for a variety of functional forms to be nested within this function and performed parametric tests to discriminate among them. Ever since, generalized quadratic Box-Cox models have been widely applied in problems of selecting among nested functional specifications in applied production analysis. The generalized quadratic Box-Cox model, assuming input-biased technical change, can be written as:

$$Y_{it}^{(\delta)} = \alpha_0 + \sum_{j=1}^{J} \alpha_j X_{jit}^{(\lambda)} + \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{J} \alpha_{jk} X_{jit}^{(\lambda)} X_{kit}^{(\lambda)} + \beta_1 t + \frac{1}{2} \beta_2 t^2 + \sum_{j=1}^{J} \gamma_j X_{jit}^{(\lambda)} t + \varepsilon_{it} \qquad (1)$$

[1] The estimation of technical efficiency in the context of the production frontier is conditional on the input combination. Whether that combination is allocatively efficient or not is a side issue, although an important one (Greene, 1993b), i.e. a technically efficient producer could still use an inappropriate (for given input prices) input mix.

[2] The model of Battese and Coelli (1993; 1995) was also used by Battese and Broca (1997).

where $i=1, \ldots, N$ represents cross sectional units; $t=1, \ldots, T$ denotes time; $j,k=1, \ldots, J$ are the applied inputs, and ε_{it} is a random error. The variables $Y_{it}^{(\delta)}$ and $X_{it}^{(\lambda)}$ are the Box-Cox transformations of output and inputs, respectively, defined as (Box and Cox, 1964):

$$Y_{it}^{(\delta)} = \frac{Y_{it}^{2\delta} - 1}{2\delta} \quad \text{and} \quad X_{jit}^{(\lambda)} = \frac{X_{jit}^{\lambda} - 1}{\lambda} \tag{2}$$

where δ and λ are the transformation parameters to be estimated. Under appropriate parametric restrictions for the values of δ and λ, the generalized quadratic Box-Cox transformation yields the four locally flexible functional forms (i.e. translog, generalized Leontief, normalized quadratic, squared-root quadratic) as well as the non-homothetic CES and Cobb-Douglas specifications.

More specifically, by utilizing *l'Hôpital's* rule the power transformations are continuous around zero. Thus, for $\delta = \lambda = 0$ the generalized quadratic Box-Cox becomes the non-homothetic translog functional form:

$$\begin{aligned} \ln Y_{it} &= \alpha_0 + \sum_{j=1}^{J} \alpha_j \ln X_{jit} + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{J} \alpha_{jk} \ln X_{jit} \ln X_{kit} + \beta_1 t + \frac{1}{2}\beta_2 t^2 \\ &+ \sum_{j=1}^{J} \gamma_j \ln X_{jit} t + \varepsilon_{it} \end{aligned} \tag{3}$$

It becomes the non-homothetic generalized Leontief when $\delta = \lambda = 0.5$:

$$\begin{aligned} Y_{it} &= (\alpha_0 + 1) + 2\sum_{j=1}^{J}\left(\sum_{k=1}^{J} \alpha_{jk} - \alpha_j\right) + 2\sum_{j=1}^{J}\left(\alpha_j - 2\sum_{k=1}^{J}\alpha_{jk}\right) X_{jit}^{0.5} + \\ &+ 2\sum_{j=1}^{J}\sum_{k=1}^{J} \alpha_{jk} X_{jit}^{0.5} X_{kit}^{0.5} + \left(\beta_1 - 2\sum_{j=1}^{J}\gamma_j\right)t + \frac{1}{2}\beta_2 t^2 + 2\sum_{j=1}^{J}\gamma_j X_{jit}^{0.5} t + \varepsilon_{it} \end{aligned} \tag{4}$$

The generalized quadratic Box-Cox results in the non-homothetic normalized quadratic when $\delta = 0.5$ and $\lambda = 1$:

$$Y_{it} = (\alpha_0 + 1) + \sum_{j=1}^{J}\left(\sum_{k=1}^{J}\frac{\alpha_{jk}}{2} - \alpha_j\right) + \sum_{j=1}^{J}\left(\alpha_j - \sum_{k=1}^{J}\alpha_{jk}\right)X_{jit} + + \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{J}\alpha_{jk}X_{jit}X_{kit} + \left(\beta_1 - \sum_{j=1}^{J}\gamma_j\right)t + \frac{1}{2}\beta_2 t^2 + \sum_{j=1}^{J}\gamma_j X_{jit}t + \varepsilon_{it} \tag{5}$$

and becomes the squared-root quadratic when $\delta = \lambda = 1$:

$$Y_{it} = \left[2(\alpha_0 + 1) + 2\sum_{j=1}^{J}\left(\sum_{k=1}^{J}\frac{\alpha_{jk}}{2} - \alpha_j\right) + 2\sum_{j=1}^{J}\left(\alpha_j - \sum_{k=1}^{J}\alpha_{jk}\right)X_{jit} + + \sum_{j=1}^{J}\sum_{k=1}^{J}\alpha_{jk}X_{jit}X_{kit} + 2\left(\beta_1 - \sum_{j=1}^{J}\gamma_j\right)t + \beta_2 t^2 + 2\sum_{j=1}^{J}\gamma_j X_{jit}t + 2\varepsilon_{it}\right]^{0.5} \tag{6}$$

All the above functional forms are second-order Taylor series expansions[3] and provide equally plausible *a priori* approximations of a true but unknown production technology. However, all four functions maintain important restrictions in modeling production relationships. More specifically, the generalized Leontief, normalized quadratic and squared-root quadratic maintain quasi-homotheticity of the underlying technology even at the point of approximation. Even though the translog does not maintain this restriction, it is less separable flexible than the other three functional forms. Nevertheless, these functional specifications do satisfy the requirement of parametric parsimony since the number of free parameters is adequate for ensuring flexibility (for a detailed discussion on the properties of several functional specifications used in applied production analysis see Fuss, McFadden and Mundlak (1978), Griffin, Montgomery and Rister (1987) and Thompson (1988) among others). Since all parametric models display these features, there is no *a priori* reason to favor any one of them.

Apart from these locally flexible functional specifications, the generalized quadratic Box-Cox also nests the restrictive but widely applied non-homothetic CES and Cobb-Douglas. More specifically, the generalized quadratic Box-Cox production function in equation (1) becomes a non-homothetic CES function when the second-order parameters (α_{jk}) equal zero $\forall$ *j,k,*, i.e.,

[3] The concept of linear-in-parameters functional forms and the property of second-order approximation at a point are due to Diewert (1971), who introduced the generalized linear and generalized Leontief forms.

$$Y_{it} = \left[(2\delta\alpha_0 + 1) - \frac{2\delta}{\lambda}\sum_{j=1}^{J}\alpha_j + \left(2\delta\beta_1 - \frac{2\delta}{\lambda}\sum_{j=1}^{J}\gamma_j\right)t + \delta\beta_2 t^2 + \right.$$
$$\left. + \frac{2\delta}{\lambda}\sum_{j=1}^{J}\alpha_j X_{jit}^{\lambda} + \frac{2\delta}{\lambda}\sum_{j=1}^{J}\gamma_j X_{jit}^{\lambda} t + 2\delta\varepsilon_{it} \right]^{\frac{1}{2\delta}} \tag{7}$$

Finally, the familiar Cobb-Douglas functional form (or, equivalently, a strongly separable translog when input-biased technical change is maintained) is obtained either from the CES function when $\delta = \lambda = 0$ (by utilizing *l'Hôpital's* rule) or from the translog by setting $\alpha_{jk} = 0 \ \forall j,k$.

The selection of the appropriate functional form within the generalized quadratic Box-Cox transformation function can be based on nested hypothesis testing (i.e. *likelihood ratio test*). Whereas the alternative specifications can be tested by likelihood support against the generalized quadratic Box-Cox, they cannot be tested against each other however. This problem can be surmounted using the *likelihood dominance criterion* that ranks models based on their adjusted likelihood values (Pollak and Wales, 1991). Specifically, the likelihood dominance criterion assures an unambiguous ordering of the functional forms nested within the generalized quadratic Box-Cox no matter the number of estimated parameters in each model (Anderson *et al.*, 1996).

Recognizing certain deficiencies in the ability of Taylor-series expansion to generate flexible functional forms, Barnett (1983, 1985) utilized the Laurent-series expansion to provide more desirable approximations of the underlying production technology. The clear advantage of the Laurent-series expansion is the fact that its remainder term varies less over the interval of convergence for the same fixed order of expansion. A special case of a second-order Laurent series expansion that includes both the translog and generalized Leontief functional specifications is:

$$Y_{it}^{*} = \alpha_0 + 2\sum_{j=1}^{J}\alpha_j X_{jit}^{*} + \sum_{j=1}^{J}\alpha_{jj} X_{jit}^{*\,2} + \sum_{j=1}^{J}\sum_{k=1}^{J}\left(\alpha_{jk} X_{jit}^{*} X_{kit}^{*} - \zeta_{jk} X_{jit}^{*\,-1} X_{kit}^{*\,-1}\right) + \varepsilon_{it} \tag{8}$$

Specifically, equation (8) becomes the minflex Laurent generalized Leontief when $X_{jit}^{*} = X_{jit}^{1/2}$ while when $Y_{it}^{*} = \ln Y_{it}$, $X_{jit}^{*} = \ln X_{jit}$ equation (8) generates the minflex Laurent translog. Whereas both minflex Laurent translog and generalized Leontief are flexible functional forms they do not possess greater parametric freedom than is needed to attain local flexibility. Note that the specification in (8) is not nested within the generalized quadratic Box-Cox model, and the statistical discrimination among these non-nested models can be performed by means of non-nested hypothesis testing. In our case we use the P_E test developed by MacKinnon, White and Davidson (1983).[4] While the P_E test follows the same analytical approach as the Andrews test (1971), it is based on a Gauss-Newton artificial regression (for details on non-nested hypothesis testing see Davidson and MacKinnon, 1993, pp. 505-507).

4. Modeling Technical Efficiency

Each functional specification presented in the previous section is used to estimate technical inefficiency by utilizing the stochastic production frontier model of Battese and Coelli (1993; 1995). Technical inefficiency is expressed as a linear function of explanatory variables associated with farm specific characteristics (inefficiency effects) to allow for the investigation of inter-farm efficiency variation. In this formulation every farm in the sample faces its own frontier (given the current state of technology and the physical endowments of the farm) rather than a sample norm. In addition, modeling technical inefficiency as a function of farm specific characteristics allows for the consistent estimation of the stochastic frontier and the inefficiency effects model in a single stage (Reifschneider and Stevenson, 1991; Battese and Coelli, 1995).[5]

Specifically, the model of Battese and Coelli (1993; 1995) in the presence of technical change and panel data has the following general form:

$$y_{it} = f(x_{it}, t; \mathrm{B}) \cdot \exp(\varepsilon_{it}) \tag{9}$$

where y_{it} is the output of farm i ($i=1,2, \ldots, N$) at time t ($t=1, 2, \ldots, T$); x_{it} is the corresponding matrix of J inputs; t is a time index that serves as a proxy for technical change; B is the vector of parameters to be estimated; and ε_{it} is the error term composed of two independent elements v_{it}

[4] We choose the P_E test because of it is simple to compute and more importantly, it has sufficient power for applied research.

[5] Other than simultaneously predicting and explaining technical inefficiency, this model formulation has two important advantages: (*a*) it identifies separately time-varying output-oriented technical efficiency and technical change as long as the inefficiency effects are stochastic and have a known distribution and; (*b*) it does not require that technical efficiencies follow a specific time pattern common to all farms in the sample.

and u_{it} such that $\varepsilon_{it} \equiv v_{it} - u_{it}$. The component v_{it} is a symmetric identically and independently distributed (*iid*) error term that represents random variation in output due to factors outside the control of the farmer (weather, diseases etc.) as well as the effects of measurement errors, left-out explanatory variables, and statistical noise.

The component u_{it} is a non-negative error term representing the stochastic shortfall of farm *i*'s output from its production frontier due to technical inefficiency. Thus, technical efficiency is defined in an output-expanding manner (*Debreu-type*) and reveals the maximum amount by which output can be increased using the same level of inputs.[6] It is obtained by truncation of the normal distribution with mean $\mu_{it} = \theta_0 + \sum_m^M \theta_m z_{mit}$ and variance σ_μ^2, where z_{mit} is the m^{th} explanatory variable associated with technical inefficiencies of farm *i* over time and θ_0 and θ_m are the unknown coefficients to be estimated.[7]

The parameters of both the stochastic frontier and the inefficiency effects model can be consistently estimated by the maximum likelihood procedure. The likelihood function and estimation issues are explicitly discussed in Battese and Coelli (1993). The variance parameters of the likelihood function are estimated in terms of $\sigma^2 \equiv \sigma_v^2 + \sigma_u^2$ and $\gamma \equiv \sigma_u^2 / \sigma^2$. Farm- and time-specific estimates of output-based technical efficiency are obtained using the expectation of u_{it} (or function of u_{it}, depending on whether the dependent variable is in level or in logs), conditional upon the observed value of ε_{it}.

A $(1-\alpha)100$ percent confidence interval for the predicted technical efficiencies can be determined as (Horrace and Schmidt, 1996, pp. 261-2):[8]

$$exp\left(-\mu_{it}^0 - \sigma_0 Z_{it}^U\right) \le TE_{it} \le exp\left(-\mu_{it}^0 - \sigma_0 Z_{it}^L\right) \tag{10}$$

[6] In a similar manner, an input-conserving measure of technical inefficiency (*Shephard-type*) is defined as the ratio of best practice input usage to actual usage, with output held constant (Kumbhakar and Lovell, 2000, p. 6). Färe and Lovell (1978) have shown that these two measures of technical inefficiency are equal only under constant returns to scale. Under decreasing (increasing) returns to scale the output-oriented measure is greater (less) than the input-oriented measure of technical inefficiency. Output-oriented measures of technical efficiency are seem more appropriate in agricultural frontier modeling, since input choices are made prior to farm production.

[7] Exclusion of the intercept parameter θ_0 may result in biased estimates of θ_m since in such a case the shape of the distribution of the inefficiency effects is being unnecessarily restricted (see Battese and Coelli (1995)).

[8] These confidence intervals are based on monotonic transformations of the $\alpha/2$ and $(1-\alpha/2)$ quantiles of the distribution $(u_{it}|\varepsilon_{it})$. Since, however, these intervals are conditioned on known values of the parameters (ignoring therefore any variation in the parameter estimates used to construct them), they should be regarded as minimal width intervals (Greene, 1999, pp. 108).

where

$$Z_{it}^{L} = \Phi^{-1}\left(1 - (1-\alpha/2)\left(1 - \Phi\left(-\mu_{it}^{0}/\sigma_0\right)\right)\right) \quad (10a)$$

and

$$Z_{it}^{U} = \Phi^{-1}\left(1 - (\alpha/2)\left(1 - \Phi\left(-\mu_{it}^{0}/\sigma_0\right)\right)\right) \quad (10b)$$

are the lower and upper limits of the standard normal variable Z respectively. Obviously, the variables introduced to explain inter-farm efficiency differentials have an effect on the range of the confidence interval; they affect the variability of the conditional mean of u_{it} which, in turn, influences the spread of the lower and upper limits of technical efficiency (Hjalmarsson Kumbhakar and Heshmati, 1996, p. 320).

5. Data and Variables Definition

The data used in this study were extracted from a survey undertaken by the Institute of Agricultural Economics and Rural Sociology in Greece. Our analysis focuses on a sample of 125 olive-growing farms, located in the four most productive olive-growing regions of Greece (Peloponissos, Crete, Sterea Ellada and Aegean Islands). The sample was selected with respect to production area, the total number of farms within the area, the number of olive trees on the farm, the area of cultivated land, and the share of olive oil production in farm output. Observations were obtained on an annual basis for the period 1987-93.

The dependent variable is annual olive-oil production measured in kilograms (kgs). The aggregate inputs included as explanatory variables are: (a) total *labor*, comprising hired (permanent and casual), family and contract labor which includes all farm activities such as plowing, fertilization, chemical spraying, harvesting, irrigation, pruning, transportation, administration and other services and is measured in working hours; (b) *fertilizers*, including nitrogenous, phosphate, potash, complex and others, measured in kgs; (c) *other cost* expenses, consisting of pesticides, fuel and electric power, irrigation taxes, depreciation,[9] interest payments, fixed assets interest, taxes and other miscellaneous expenses, measured in Greek drachmas (GDR) (constant 1990 prices); and (d) *land*, including only the area devoted to olive-tree cultivation, measured in stremmas (one stremma equals 0.1 ha). Summary statistics of these

[9] The rate of depreciation applied to machinery was between 10 and 13% depending on the size of the farm, while for buildings and inventories it was 7% of the stock value.

variables are presented in Table 1. Aggregation over the various components of the above input categories was conducted using *Divisia* indices with cost shares serving as weights (Vogt and Barta, 1997, pp. 29-33). Finally, the explanatory variables in the inefficiency effects model include: (a) farmer's *age* (in years) and *age squared*; (b) farmer's *formal education* in years of schooling; (c) the existence of an *improvement plan*[10] in the farm (1 = Yes, 0 = No); (d) farm's *location* (1 = Less Favored Area, 0 = More Favored Area); (e) farm *size* in stremmas; and (f) a single *time-trend* that captures intertemporal variation in efficiency ratings.

All data (except for the dummy variables) were normalized around the sample mean to define the point of approximation and wash out the effect of different units of measurement. The generalized quadratic Box-Cox model in equation (1) was estimated using the maximum likelihood method, after doing the necessary transformation in the dependent and independent variables using a bi-dimensional grid search around the 0-2 range for the values of δ and λ (Greene, 1993a, pp. 329-334). Since the employed data set was generated by an unknown technology the regularity conditions, apart from symmetry, were assumed rather than imposed.[11]

6. Empirical Results

Production Frontier Estimates

The maximum likelihood[12] estimates of the generalized quadratic Box-Cox, translog, generalized Leontief, normalized quadratic, squared-root quadratic, CES and Cobb-Douglas stochastic production frontier and inefficiency effects models are reported in Tables 2 and 3. The corresponding parameter estimates for minflex translog and generalized Leontief frontier models are reported in Table 4. All functional forms satisfy monotonicity since, at the point of approximation, marginal products are positive and diminishing - all estimated first-order coefficients (α_j) fall between zero and one. The bordered Hessian is positive semi-definite for all locally flexible functional specifications except for normalized quadratic and squared-root quadratic, indicating that these two models do not support concavity.

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[10] Within Reg. 1278/88, some farms in the sample were receiving financial aid from the Greek Ministry of Agriculture during the 1987-93 period to improve their infrastructure and introduce certain technological innovations like new tractors, genetically improved seeds etc.

[11] Symmetry restrictions do not affect the flexibility of any of the flexible functional forms examined.

[12] The maximum likelihood estimation of the model was carried out using the FRONTIER (version 4.1a) computer program, kindly provided by T.J. Coelli.

The ratio parameter, γ, is positive and significant at the 1% level in all models, implying that farm specific technical efficiency is important in explaining the total variability of output produced. The value of γ ranges from a minimum of 0.547 in normalized quadratic to a maximum of 0.879 in minflex generalized Leontief models. The statistical significance of modeling farm effects within the stochastic frontier model is further examined using *likelihood ratio* tests (the results of statistical testing are presented in Table 5).[13]

The null hypothesis that the traditional average response model adequately represents the structure of Greek olive-growing farms is rejected. This is true regardless of whether farm inefficiency effects are present or absent from the production frontier model.[14] The hypothesis that inefficiency effects are not a linear function of the variables considered herein is rejected at the 5% level of significance. As well, the specification of the model in equation (9) cannot be reduced to neither Aigner, Lovell and Schmidt (1977) nor Stevenson's (1980) formulations, as the null hypotheses of $\theta_0 = \theta_m = 0$ and $\theta_m = 0$ (for m = 1,2,…,M), respectively, are rejected at 5% level of significance. Finally, the hypothesis that technical inefficiency is time-invariant $(\theta_T = 0)$ is rejected for all but the square-root quadratic and the minflex model specifications. Hence, no sub-hypothesis of the stochastic frontier model is justified apart of the temporal patterns of technical inefficiencies in squared-root quadratic, minflex translog and minflex generalized Leontief models.

Several hypotheses concerning the structure of the underlying technology were also examined using *likelihood ratio* test. Both homogeneity and linear homogeneity (constant returns to scale) are rejected by all functional specifications at the 5% level of significance. Technical change is present in almost all models. The hypothesis of zero technical change is not rejected at the 5% level of significance for the normalized quadratic and minflex generalized Leontief models (see table 5). There is no consistency regarding the nature of technical change. While the underlying technological change is characterized as Hicks-neutral according to translog, generalized Leontief and normalized quadratic models, this hypothesis is rejected under all other

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[13] The *likelihood ratio* test statistic is calculated as $LR = 2\left[\ln L(\delta^*, \lambda^*) - \ln L(\delta, \lambda)\right]$ where * denotes estimates from the unrestricted model. The test-statistic has asymptotic distribution that is chi-square or mixed chi-square with degrees of freedom equal to the number of restrictions (Coelli, 1995; Coelli and Battese, 1996).

[14] If the parameter γ equals zero the model reduces to a mean response function in which the variables in the inefficiency effects model (θ_m) are included directly in the production function. In this case the constant θ_0 and the time parameter θ_T are not identified while the LR-test has a mixed chi-square distribution, the appropriate critical values of which are obtained from Kodde and Palm (1986, table 1).

functional specifications. The rate of technical change follows an increasing trend over time, with the time-pattern being model specific (Figure 2).

Average estimates over farms and time of production elasticities, returns to scale (RTS), and the rate of technical change are presented in Table 6. Estimates of production elasticities indicate that land has contributed the most to olive-oil production, followed by labor, according to all functional specifications.[15] However, the relative contributions of fertilizers and other capital inputs differ across models. Whereas point elasticity estimates in translog, normalized quadratic, CES and Cobb-Douglas are very close, the rest of the models generate significantly different average values. For instance, the land elasticity takes values between 0.509 and 0.913 in generalized quadratic Box-Cox and squared-root quadratic respectively, and labor elasticity varies between 0.124 in translog and 0.415 in squared-root quadratic.

The time development of production elasticities is also similar across models. However, the estimated elasticities of scale show that the magnitude of production elasticities is model specific. Specifically, olive-growing farms in the sample exhibit, on average, decreasing returns to scale according to generalized quadratic Box-Cox, translog, CES, Cobb-Douglas and minflex generalized Leontief functional forms, and increasing returns according to generalized Leontief, normalized quadratic, squared-root quadratic and minflex translog. The higher average value is 1.488 in squared-root quadratic and the lower is 0.750 in translog. In addition, while the value of scale elasticities follows a decreasing trend according to all functional specifications, the time-pattern differs among them (Figure 1).

Technical Efficiency

Mean technical efficiencies over farms and the corresponding confidence intervals for the alternative functional specifications are presented in Table 7. The results indicate a significant variation in estimated efficiency scores with the mean values ranging from 67.37% in squared-root quadratic to 86.82% in normalized quadratic. Furthermore, the choice of functional form seems to have a significant effect on the confidence interval of technical efficiency estimates; the difference between the lower and upper bounds varies from 7.1 to 11.7% in generalized quadratic Box-Cox while in squared-root quadratic it takes values between 18.3 and 21.4%.

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[15] Since the land input also appears in the inefficiency effects model, the corresponding point elasticity estimates were computed using the formulas set forth by Huang and Liu (1994) and Battese and Broca (1997).

Regarding the average values over farms and time, the efficiency interval varies between 7.09 and 21.07% in minflex generalized Leontief and squared-root quadratic, respectively. On the other hand, there was no intertemporal pattern of the width of the confidence intervals present during the study period.

The temporal patterns of technical efficiency ratings are also very sensitive to the choice of functional specification. Table 7 shows that while technical efficiencies in normalized quadratic model follow a decreasing trend over time, they are rather stable in generalized Leontief, Cobb-Douglas, squared-root quadratic, minflex translog and minflex generalized Leontief, and increasing in CES model. For the generalized quadratic Box-Cox and translog models the corresponding pattern show a decreasing trend for the first three periods and then an increasing trend thereafter. These results are consistent with both Spearman's correlation coefficients (Table 9) and the corresponding coefficient estimates in the inefficiency effects model (i.e. θ_T in Table 3). The differences in the estimated temporal patterns of technical efficiency can significantly affect the results in studies of total factor productivity growth.

Another discrepancy between the different functional specifications relates to the frequency distribution of mean technical efficiencies over farms and time. As it is clearly shown in Table 8, translog and Cobb-Douglas models are characterized by increased variation among farms when compared to the other five models. More specifically, means technical efficiencies range from a minimum of 30.74% to a maximum of 97.54% in translog, and in Cobb-Douglas the corresponding estimates are 32.35 and 99.99%, respectively. On the other hand, the relevant range in normalized quadratic is considerably smaller, 67.42 and 98.91%. Put in a different way, while the results from the translog specification indicate that 16.8% of the farms are less than 60% technically efficient, estimates derived from the CES, normalized quadratic and minflex translog models suggest that there is no farm in the sample operating below that level. In general, the frequency distribution of mean technical efficiencies is quite similar between normalized quadratic, minflex translog and CES, and between translog and Cobb-Douglas models, while it differs between all other models. The Spearman's correlation coefficients reported in Table 9 further confirm this finding.

Besides the differences in the frequency distribution of mean technical efficiencies, alternative functional specifications reveal significantly different efficiency rankings of individual farms. Table 10 shows the discrepancy in the efficiency ranking of (the same) 20 farms under the different functional specifications of the production frontier.

Perhaps more importantly, the results indicate that the choice of functional form affects significantly the identification of sources of these efficiency differentials among producers - the relevant estimates of the inefficiency effects models reveal considerable differences between the alternative functional specifications (Table 3). According to all seven models, farmers' education affects positively their efficiency levels though at a different rate, while the coefficient of age (age-squared) is negative (positive) in all but the minflex translog model, supporting the hypothesis of decreasing returns to human capital. However, for the rest of the explanatory variables there are significant differences among models. More specifically, farm size seems to have a negative effect on farm efficiency based on generalized quadratic Box-Cox, translog, generalized Leontief, CES, Cobb-Douglas, minflex translog and minflex generalized Leontief models while the normalized quadratic and squared-root quadratic models indicate the opposite. Location in less-favored areas positively affects the efficiency of a farm in CES and Cobb-Douglas models and negatively in all other models, while the existence of an improvement plan in the farm has a positive effect on efficiency in generalized quadratic Box-Cox, generalized Leontief, normalized quadratic, CES, Cobb-Douglas, minflex translog and minflex generalized Leontief models.

Selection of Functional Form

The empirical results presented in the previous section show that different functional specifications of the stochastic production frontier model result in different conclusions concerning both the production structure and the estimated technical (in)efficiencies of the production units. Since the empirical results are model specific, the question that naturally arises is what is the model formulation that fits the data the best. For the functional forms that are nested within the generalized quadratic Box-Cox transformation function, the relative statistical fitness can be determined relatively easily using the standard *likelihood ratio* (LR) test. Specifically, each functional form can be tested against the generalized form (generalized quadratic Box-Cox) using the estimated values of the likelihood function. Results of nested hypotheses testing are presented in Table 11. The likelihood ratio test statistic for all restricted models is higher than the corresponding critical value of the chi-square distribution at the 95% significance level, indicating that all functional forms are rejected against the generalized quadratic Box-Cox function.

After adjusting the logarithm of the likelihood function by the Jacobian transformation (see table 11), the *likelihood dominance criterion* suggests that the generalized quadratic Box-Cox is the preferred functional form followed by the translog, generalized Leontief, normalized quadratic and squared-root quadratic. Regarding the two non-flexible functional specifications, the CES is clearly preferred to the Cobb-Douglas. Pairwise comparisons between CES, Cobb-Douglas and the flexible functional forms indicate that Cobb-Douglas is the least preferred functional form against translog, generalized Leontief, normalized quadratic but it outperforms squared-root quadratic (Pollak and Wales, 1991).

The above statistical testing is only able to determine the best alternative model among the functional specifications that are nested within generalized quadratic Box-Cox however. To statistically examine the relative performance of generalized quadratic Box-Cox model against the family of minflex flexible functional forms (minflex translog and minflex generalized Leontief) we used the P_E test. Specifically, the P_E test is used to conduct pairwise comparisons between the generalized quadratic Box-Cox, minflex translog and minflex generalized Leontief models. The relevant test statistics are also presented in Table 11.

Interestingly, the results are inconclusive in choosing between these three functional specifications. While the P_E test clearly favors the minflex flexible functional forms over the generalized quadratic Box-Cox function,[16] it provides no clear statistical evidence about the relative performance (fitness) of minflex translog and minflex generalized Leontief models – both functional specifications fit the data set comparably. The P_E test statistic is lower than the corresponding critical value no matter the null hypothesis tested – both the null hypothesis that minflex translog is the correct specification and the null hypothesis that minflex generalized Leontief is the correct specification cannot be rejected by the current data set.

The statistical infeasibility to determine the functional specification that approximates the underlying production technology more accurately is bothersome given that the (statistically indistinguishable) minflex translog and minflex generalized Leontief models reveal different

[16] The superior performance of minflex translog and minflex generalized Leontief relative to generalized quadratic Box-Cox can be explained by the relatively larger regular regions of Laurent-series expansions compared to regular regions provided by Taylor-series expansions.

conclusions concerning the production possibilities of olive-growing farms and the efficiency in the use of their resources[17]. One possible explanation for this counterintuitive finding is that, albeit both the minflex translog and the minflex generalized Leontief models fit the data equally well, they can have very different disturbance distributions. Since the technical efficiency predictor in (11) is conditioned upon specific distribution of the disturbances, different distributional assumptions can result in different technical efficiency measures.

Lau (1986) and Thompson (1988) suggest that a choice among the various flexible functional specifications available for applied production analysis can be made on either theoretical or empirical grounds. The former refers to *a priori* restrictions regarding the algebraic form and the maintaining assumptions on the underlying production technology, while the latter refers to an *ex post* evaluation of functional specifications given the peculiarities of any particular empirical application. However, as Griffin, Montgomery and Rister (1987) point out, theoretical criteria could lead to contradictory conclusions about the choice between the currently available functional forms. On the other hand, our comparative analysis reveals that the empirical *ex post* evaluation does not always lead to the determination of "the superior" functional specification. Put in a different way, unless a more general composite model is developed, the search for the appropriate functional specification will always involve non-nested hypothesis testing which, however, entails the possibility of statistically indistinguishable results.

The inability to achieve the "first best" should not be perceived as an *anathema* to specification searches. Since the efficiency estimates are sensitive to the choice of functional form, one should always attempt to statistically discriminate among the viable alternatives. Despite its drawbacks, in our case statistical testing did narrow down the set of suitable alternatives from eight to two functional specifications.

The natural question that arises is then what is the best way to proceed in cases where the determination of the appropriate functional form is not statistically feasible. A potential solution can be borrowed from the time-series forecasting literature where many authors suggest that composite predictions quite often outperform any particular predictive model (see Coelli and Perelman (1999)). Palm and Zellner (1992, p. 699) argue that "in many cases a simple average of forecasts achieves a substantial reduction in variance and bias." On this basis, when different functional specifications are statistically indistinguishable and give different predictions of

[17] We would like to thank the associate editor for pointing out this issue.

technical efficiency and inference on its determinants, a composite measure can provide a solution reducing the bias of the obtained efficiency estimates.[18]

7. Summary and Concluding Remarks

In recent years several attempts have been made to measure technical efficiency in both developed and developing countries. Since policy recommendations could be drawn from such studies, the design of the employed methodology is of great importance. Although several studies have examined the impact of estimation techniques on final efficiency estimates, only a few can be pinpointed as dealing with the effect that the choice of functional specification has on these estimates.

This paper utilizes recent advances in stochastic production frontier modeling and a panel data set of 125 olive growing farms in Greece during the period 1987-93 to examine the effect that the choice of functional form has on measures of farm efficiency. The relative performance of six popular functional specifications (i.e. translog, normalized quadratic, squared-root quadratic, generalized Leontief, non-homothetic CES, and Cobb-Douglas) was evaluated using the generalized quadratic Box-Cox transformation model. In addition, our comparative analysis included the minflex translog and generalized Leontief flexible functional specifications due to their desirable approximation properties.

The results show that estimates of both production structure and measures of farm efficiency are sensitive to the functional form used. The choice of functional specification significantly affects the measures obtained, implying that the selection of a particular parametric specification cannot be a matter of indifference. Not only are estimation results of overall inefficiency sensitive to functional choice, but different functional specifications also render significantly different conclusions regarding the potential sources of these inefficiencies. The latter is crucial for the design of policies aimed at improving the economic performance of the farms.

To the extent that an empirical analysis seeks to be relevant, these results strongly reject the *ad hoc* imposition of a (any) functional specification and underline the importance of

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[18] Coelli and Perelman (1999) used a similar approach in analyzing technical efficiency estimates obtained from the non-parametric and parametric estimation of an output distance function. Specifically, they argued that since there is no *a priori* reason for choosing among these two techniques, one should construct geometric means of the obtained technical efficiency estimates for each data point.

specification searches. When estimation procedures and the data set are adequate, formal empirical hypotheses may be tested to help narrow the range of viable alternatives; that is, one may proceed with a general-to-specific modeling approach to determine the appropriate functional specification. When data and/or estimation/testing procedures are not adequate, a range of relevant alternative functional specifications should at least be explored to determine how sensitive empirical findings, such as efficiency, are to these specifications. The current study shows that the inappropriate choice of functional form could result in significantly biased efficiency estimates and misleading policy recommendations regarding efficiency improvements.

Finally, a potential solution for cases where statistically indistinguishable functional specifications yield significantly different results could involve the construction of composite efficiency measures that reduce the bias of the final efficiency predictions.

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Table 1. Summary Statistics of the Variables.

Variable	Mean	Standard Deviation	Min	Max
Output (Kgs)	1,212	1,047	50	9,897
Labor (hours)	607	522	21	3,715
Fertilizer (Kgs)	1,475	1,254	50	16,984
Other Cost (GDR)	23,523	18,522	2,100	544,900
Land (stremmas)	24	14	2	105
Age (years)	55	17	26	74
Education (years of schooling)	8	3	6	12

Exchange rate 1US$ ≅ 387 GDR; 1 stremma equals 0.1 ha.

Table 4. Parameter Estimates of the Minflex Laurent Translog (MTL) and Minflex Generalized Leontief (MGL) Production Frontier and Inefficiency Effects Models.

Variable	MTL		MGL	
Stochastic Production Frontier				
Constant	0.2682	(3.5151)	1.3796	(3.7236)
Labour	0.2147	(5.4492)	0.1735	(3.3494)
Fertilizers	0.1603	(1.7481)	0.0987	(2.0351)
Other Cost	0.0446	(1.7698)	0.0541	(3.0914)
Area	0.6378	(7.2313)	0.3867	(5.8502)
LabXLab	-0.0260	(4.9776)	-0.0894	(0.9933)
FertXFert	0.1133	(1.8727)	0.5583	(2.4158)
CostXCost	-0.0065	(3.0952)	0.0049	(0.1317)
AreaXArea	-0.1069	(2.4406)	-0.1498	(0.4064)
LabXFert	-0.0787	(1.5931)	-0.1211	(0.4408)
$Lab^{-1}XFert^{-1}$	-0.0000	(0.8571)	-0.0037	(1.4601)
LabXCost	0.0443	(4.3010)	0.1641	(1.4770)
$Lab^{-1}XCost^{-1}$	-0.0001	(0.8104)	-0.0011	(1.5714)
LabXArea	0.2039	(4.6553)	1.7231	(4.9245)
$Lab^{-1}XArea^{-1}$	-0.0001	(0.4710)	-0.0460	(1.7557)
FertXCost	-0.0495	(1.2103)	0.0816	(0.4780)
$Fert^{-1}XCost^{-1}$	-0.0013	(1.8571)	-0.0001	(4.7619)
FertXArea	0.0761	(0.6315)	0.4211	(0.8624)
$Fert^{-1}XArea^{-1}$	-0.0006	(0.6667)	-0.0069	(1.9714)
CostXArea	0.0183	(0.5562)	-0.2011	(0.9393)
$Cost^{-1}XArea^{-1}$	-0.0005	(1.6667)	0.0014	(1.5556)
Time	0.0139	(1.8784)	0.0161	(1.5333)
$Time^2$	0.0280	(0.5611)	0.0284	(0.5420)
LabXTime	0.0208	(0.6582)	0.0104	(3.2379)
FertXTime	-0.0866	(1.7637)	-0.0288	(0.3051)
CostXTime	0.0088	(0.3296)	-0.0019	(0.0720)
AreaXTime	-0.0836	(1.8174)	-0.0674	(1.4433)
Inefficiency Effects Model				
Constant	-0.4368	(1.5561)	-0.3709	(2.3929)
Time	0.0075	(0.1773)	0.0095	(0.8796)
Age	0.0359	(3.1491)	-0.0218	(2.7595)
Age^2	-0.0004	(3.2415)	0.0014	(1.8843)
Education	-0.0202	(2.5570)	-0.0166	(1.8864)
ImpPlan	-0.2290	(2.2992)	-0.1902	(3.0335)
Location	0.0730	(1.5974)	0.0560	(1.4698)
Size	0.0011	(0.7857)	0.0012	(0.9231)
σ^2	0.2541	(19.851)	0.2657	(19.115)
γ	0.7325	(71.116)	0.8799	(59.937)
LnL	-464.145		-481.425	

In parentheses are the corresponding t-ratios.

Table 5. Model Specification Tests

Null Hypothesis[1]	Calculated LR-Test									CV
	GQBC	TL	GL	NQ	SRQ	CES	C-D	MTL	MGL	(α=0.05)
$\gamma = 0$[2]	28.3	43.8	28.1	31.1	248	45.7	49.3	43.9	35.91	7.05
$\gamma = \theta_0 = \theta_m = 0$[2]	58.3	62.2	55.9	57.6	88.3	72.3	76.4	55.3	62.3	16.3
$\theta_0 = \theta_m = 0$	34.6	43.6	39.7	32.0	65.9	49.2	67.3	45.7	51.7	15.5
$\theta_m = 0$	31.2	37.9	28.5	24.6	59.2	41.3	58.1	41.0	40.3	14.1
$\theta_T = 0$	7.7	8.2	5.4	6.2	3.5	9.2	8.5	2.3	2.4	3.84
ZTC	16.2	13.2	15.0	11.4	13.7	12.8	17.1	13.2	11.9	12.6
HNTC	12.3	9.1	8.3	7.2	14.3	9.8	15.6	11.9	8.9	9.49

[1] For every *m=1, 2, …, 7*.
[2] The corresponding critical values were obtained from Kodde and Palm (1986, table 1).
ZTC: zero technical change, HNTC: Hicks-neutral technical change.
GQBC: generalized quadratic Box-Cox; TL: translog; GL: generalized Leontief; NQ: normalized quadratic, SRQ: squared-root quadratic; CES: constant elasticity of substitution; C-D: Cobb-Douglas; MTL: minflex translog; MGL: minflex generalized Leontief.

Table 6. Production Elasticities, Elasticities of Scale and Rate of Technical Change for Alternative Functional Forms (average values of the 1987-93 period).

	GQBC	TL	GL	NQ	SRQ	CES	C-D	MTL	MGL
Production Elasticities									
Labor	0.260	0.124	0.281	0.276	0.415	0.244	0.148	0.231	0.173
	(0.025)	(0.068)	(0.101)	(0.145)	(0.278)	(0.111)	(0.012)	(0.098)	(0.064)
Fertilizer	0.101	0.024	0.155	0.053	0.104	0.051	0.016	0.114	0.082
	(0.041)	(0.012)	(0.077)	(0.041)	(0.084)	(0.025)	(0.011)	(0.056)	(0.022)
Other	0.052	0.033	0.051	0.056	0.056	0.054	0.020	0.092	0.041
	(0.021)	(0.009)	(0.038)	(0.047)	(0.051)	(0.031)	(0.006)	(0.023)	(0.019)
Area	0.509	0.569	0.654	0.703	0.913	0.518	0.622	0.762	0.527
	(0.162)	(0.244)	(0.283)	(0.352)	(0.407)	(0.211)	(0.092)	(0.212)	(0.103)
RTS	0.922	0.750	1.141	1.088	1.488	0.867	0.807	1.119	0.823
	(0.217)	(0.321)	(0.441)	(0.302)	(0.507)	(0.372)	(0.242)	(0.321)	(0.212)
Technical Change									
Total	0.642	3.029	0.077	0.853	-1.335	-0.401	1.887	2.098	1.532
	(0.217)	(0.816)	(0.033)	(0.289)	(0.857)	(0.325)	(0.523)	(0.534)	(0.653)
Neutral	0.244	1.234	-0.612	-0.780	-3.493	-0.922	0.180	0.342	0.863
	(0.107)	(0.326)	(0.214)	(0.136)	(0.847)	(0.428)	(0.082)	(0.099)	(0.342)
Biased	0.398	1.795	0.689	1.633	2.157	0.522	1.706	1.756	0.669
	(0.147)	(0.298)	(0.147)	(0.458)	(1.748)	(0.396)	(0.754)	(0.532)	(0.231)

In parentheses are the corresponding standard errors.
GQBC: generalized quadratic Box-Cox; TL: translog; GL: generalized Leontief; NQ: normalized quadratic, SRQ: squared-root quadratic; CES: constant elasticity of substitution; C-D: Cobb-Douglas; MTL: minflex translog; MGL: minflex generalized Leontief.

Figure 1. Time Development of Returns to Scale for Alternative Functional Forms (average values over farms).

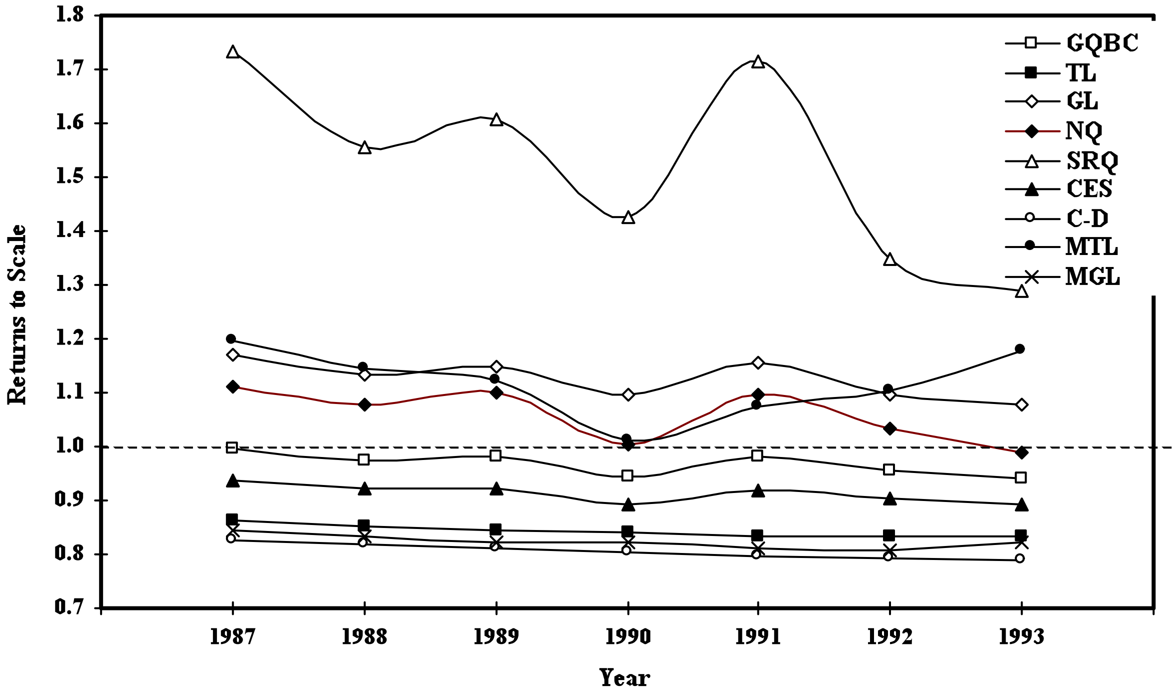

Figure 2. Time Development of Technical Change for Alternative Functional Forms (average values over farms).

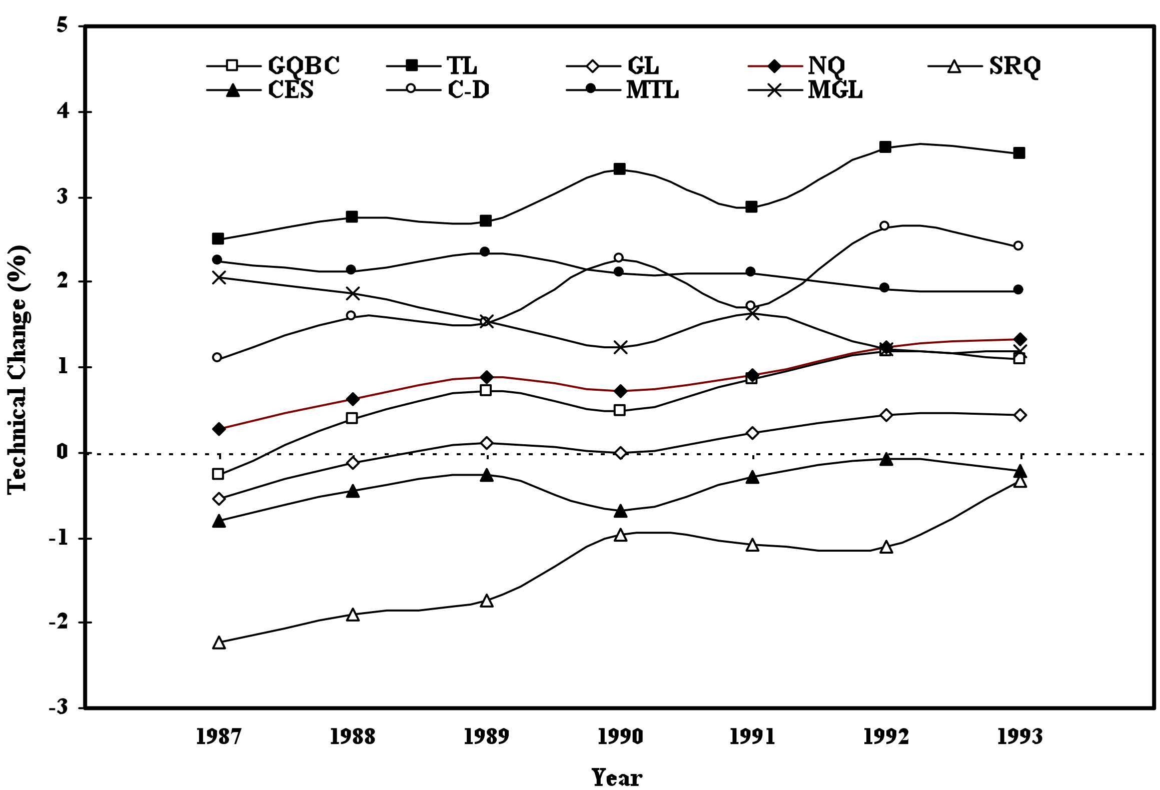

Table 7. Confidence Intervals and Mean Technical Efficiency over Farms for the Alternative Functional Forms.

Year	GQBC				TL				GL			
	L	M	U	R	L	M	U	R	L	M	U	R
1987	78.31	82.82	86.77	8.46	68.30	75.66	80.51	12.21	64.58	72.02	77.36	12.78
1988	78.10	82.75	86.69	8.59	68.91	75.53	81.26	12.35	63.25	71.93	78.65	15.40
1989	75.58	82.51	87.24	11.66	67.23	74.88	81.59	14.36	62.36	72.05	78.47	16.11
1990	76.69	81.91	88.10	11.41	67.39	75.18	81.89	14.50	61.58	71.69	79.28	17.70
1991	76.68	81.14	87.69	11.01	66.92	74.82	82.00	15.08	60.28	71.35	77.39	17.11
1992	78.62	82.16	85.69	7.07	67.28	76.14	81.47	14.19	61.89	72.14	78.97	17.08
1993	76.57	82.55	84.65	8.08	68.23	79.42	83.33	15.10	62.25	72.38	77.14	14.89
Av	*77.22*	*82.26*	*86.69*	*9.47*	*67.75*	*75.95*	*81.72*	*13.97*	*62.31*	*71.94*	*78.18*	*15.87*

Year	NQ				SRQ				CES			
	L	M	U	R	L	M	U	R	L	M	U	R
1987	79.45	87.69	95.32	14.87	53.87	67.15	75.25	20.38	71.25	75.90	80.36	9.11
1988	78.98	87.40	94.68	14.70	54.25	67.38	74.58	19.33	70.25	76.36	81.36	11.11
1989	80.24	87.18	96.57	15.33	54.69	67.48	73.97	18.28	69.89	76.81	82.3	12.41
1990	79.06	86.69	97.36	17.30	52.95	67.38	74.99	21.04	70.25	77.03	81.98	11.73
1991	78.86	86.11	96.68	16.82	53.87	67.18	76.25	21.38	71.33	77.29	82.35	11.02
1992	78.25	86.39	97.57	18.32	54.36	67.49	75.36	20.00	72.63	78.70	83.36	10.73
1993	77.36	86.30	95.35	16.99	55.20	67.54	76.25	20.05	72.02	79.13	85.69	13.67
Av	*78.89*	*86.82*	*96.22*	*17.33*	*54.17*	*67.37*	*75.24*	*21.07*	*71.09*	*77.32*	*82.49*	*11.40*

Year	C-D				MTL				MGL			
	L	M	U	R	L	M	U	R	L	M	U	R
1987	68.02	74.22	81.03	13.01	77.32	81.47	85.21	7.89	70.12	74.16	76.32	6.20
1988	67.12	74.72	81.42	14.30	77.09	81.11	86.12	9.03	68.31	72.61	75.54	7.23
1989	65.32	75.21	82.02	16.70	76.12	80.93	83.42	7.30	68.91	73.35	77.63	8.72
1990	65.21	74.55	81.43	16.22	74.23	80.57	84.23	10.00	69.54	72.06	75.03	5.49
1991	66.13	74.12	82.36	16.23	75.43	80.26	85.21	9.78	68.93	72.58	77.31	8.38
1992	67.14	75.88	80.13	12.99	76.36	81.12	84.33	7.97	69.15	72.86	75.43	6.28
1993	65.76	76.03	80.22	14.46	76.12	80.96	84.97	8.85	68.75	72.94	76.05	7.30
Av	*66.39*	*74.96*	*81.23*	*14.84*	76.10	80.92	84.78	8.69	69.10	72.94	76.19	7.09

L: lower bound; M: mean value; U: upper bound; R: range
GQBC: generalized quadratic Box-Cox; TL: translog; GL: generalized Leontief; NQ: normalized quadratic, SRQ: squared-root quadratic; CES: constant elasticity of substitution; C-D: Cobb-Douglas; MTL: minflex translog; MGL: minflex generalized Leontief.

Table 8. Frequency Distribution of Mean Technical Efficiencies over Farms and Time

(%)	GQBC	TL	GL	NQ	SRQ	CES	C-D	MTL	MGL
20-30	0	0	0	0	0	0	0	0	0
30-40	0	2	0	0	0	0	1	0	0
40-50	0	8	0	0	0	0	6	0	0
50-60	5	11	11	0	6	0	11	0	4
60-70	15	17	56	5	97	23	14	14	49
70-80	32	48	33	21	20	65	51	54	45
80-90	59	35	11	49	2	26	39	31	22
90-100	14	4	14	50	0	11	3	26	5
Mean	82.26	75.95	71.94	96.22	67.37	77.32	75.73	80.92	72.94
Min	54.63	30.74	55.68	67.42	55.42	63.50	32.35	63.92	56.26
Max	94.18	97.54	93.29	98.91	84.80	98.11	99.99	99.27	91.05
N	125	125	125	125	125	125	125	125	125

GQBC: generalized quadratic Box-Cox; TL: translog; GL: generalized Leontief; NQ: normalized quadratic, SRQ: squared-root quadratic; CES: constant elasticity of substitution; C-D: Cobb-Douglas; MTL: minflex translog; MGL: minflex generalized Leontief.

Table 9. Spearman's Correlation Coefficients of Efficiency Ratings (mean values over time above the diagonal and mean values over farms below the diagonal).

	GQBC	TL	GL	NQ	SRQ	CES	C-D	MTL	MGL
GQBC		0.536	0.429	0.750*	-0.072	-0.536	0.282	0.898**	0.568
TL	0.448		-0.786**	0.071	0.523	0.321	0.532	0.248	0.065
GL	0.902*	0.654*		0.000	0.775**	0.393	0.813**	0.732	0.422
NQ	0.866*	-0.122	0.940*		-0.396**	-0.893*	-0.112	0.696	0.586
SRQ	-0.032	-0.003**	-0.036	-0.246*		0.721	0.898*	0.413	-0.093
CES	0.864*	0.423	0.957*	0.960*	-0.136		0.498	-0.177	-0.309
C-D	-0.334	0.921*	0.754	0.651	0.007	0.041		0.375	-0.042
MTL	0.566	0.618**	0.377	0.264	-0.082	0.789**	0.347		0.721
MGL	0.259	0.301	0.649**	0.098	-0.175	0.133	0.265	0.522	

* significant at the 1% level; ** significant at the 5% level.
GQBC: generalized quadratic Box-Cox; TL: translog; GL: generalized Leontief; NQ: normalized quadratic, SRQ: squared-root quadratic; CES: constant elasticity of substitution; C-D: Cobb-Douglas; MTL: minflex translog; MGL: minflex generalized Leontief.

Table 10. Ranking of the Ten Most and Least Technical Efficient Farms According to GQBC Model.

GQBC	TL	GL	NQ	SRQ	CES	C-D	MTL	MGL
Ten most efficient farms								
1	5	1	1	52	1	1	1	9
2	13	2	2	43	3	2	13	6
3	4	15	3	85	2	6	4	22
4	34	4	10	90	10	7	8	8
5	44	5	6	31	9	8	6	13
6	29	3	5	57	6	3	28	5
7	31	10	14	64	20	10	16	17
8	7	16	4	61	4	12	4	7
9	3	18	7	53	7	9	9	16
10	2	25	20	55	22	4	25	28
Ten least efficient farms								
116	111	108	111	125	115	116	113	96
117	115	122	122	111	118	117	120	117
118	107	125	123	123	123	113	123	119
119	122	118	113	10	113	123	117	112
120	120	110	116	68	116	120	114	111
121	125	88	68	89	70	125	60	92
122	119	111	114	73	114	119	112	104
123	123	107	118	121	93	122	102	110
124	121	114	117	118	117	121	116	120
125	124	105	112	46	103	124	101	115

GQBC: generalized quadratic Box-Cox; TL: translog; GL: generalized Leontief; NQ: normalized quadratic, SRQ: squared-root quadratic; CES: constant elasticity of substitution; C-D: Cobb-Douglas; MTL: minflex translog; MGL: minflex generalized Leontief.

Table 11. Nested and Non-Nested Hypotheses Testing for the Alternative Functional Forms.

Functional Forms	Transformation (δ, λ)	LR-Test[1]
Nested hypotheses testing		
Generalized Box-Cox	(0.191, 0.559)	
Translog	(0, 0)	189.34
Generalized Leontief	(0.5, 0.5)	286.42
Normalized Quadratic	(0.5, 1)	246.34
Squared-Root Quadratic	(1, 1)	1394.82
CES	(0.191, 0.559)	107.26
Cobb-Douglas	(0, 0)	368.32
Non-nested hypotheses testing		
	P_E-test[2]	
GQBC vs MTL	3.854	
MTL vs GQBC	0.932	
GQBC vs MGL	2.976	
MGL vs GQBC	1.123	
MTL vs MGL	0.782	
MGL vs MTL	0.564	

[1] The critical value of the LR test statistic is obtained from the chi-square distribution with the number of restrictions equal to 2 $\left(\chi^2_{2,95} = 5.99\right)$. In the cases of CES and Cobb-Douglas functional forms, the number of restrictions is 12 since it is further assumed that $\alpha_{ij}=0$ for all i,j $\left(\chi^2_{12,95} = 21.03\right)$.

[2] The P_E-test is based on an artificial compound model to form a Gauss-Newton regression (GNR) for each pair of non-nested models. Then a usual t-test for the compound parameter of the GNR is used to validate the null hypothesis (MacKinnon, White and Davidson, 1983; Davidson and MacKinnon, 1993, p. 507).

GQBC: generalized quadratic Box-Cox; CES: constant elasticity of substitution; MTL: minflex translog; MGL: minflex generalized Leontief.