A strategic tax mechanism

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Abstract

We analyze a novel tax mechanism in imperfectly competitive markets. The government announces an excise tax rate and auctions-off a number of tax exemptions. Namely, it invites the firms in a market to acquire the right to be exempted from the excise tax. The highest bidders are exempted by paying their bids; and all other firms remain subject to it. The mechanism has a number of desirable features. First, it allows the government to collect more revenues than the standard tax policies (which is due to the competition among the firms to acquire the exemptions). Further it reduce distortions as fewer firms pay the excise tax. The mechanism reduces also the excess entry of firms that often occurs in oligopolistic markets. Lastly the mechanism creates no discrimination as all firms end up having the same net payoff, and it is also voluntary in the sense that the firms choose whether to participate in the auction or not and hence choose how to be taxed. Our mechanism can be seen as the product-market analogue of the income tax buyout fiscal mechanism (Del Negro et al. 2010, Journal of Monetary Economics).

Keywords: commodity tax; tax exemption; auction; entry.

JEL Classification: H25, L13.

1 Introduction

Is there a tax policy that can address simultaneously the following issues in a market: raise enough tax revenues; create few distortions; circumvent the excess entry of firms in the long-run; be voluntary in the sense that the firms may choose how to be taxed? The goal of the current paper is to propose such a policy mechanism. The mechanism combines

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an excise tax[1] with a tax-exemption auction: The government sets a tax rate τ per unit of firms' production, and also announces the auctioning of a number of tax exemptions. Namely, it invites the firms to bid in order to acquire, if they wish, the right to be excluded from paying τ. The firms that place the highest bids win this right and pay the government their bids (we assume a first-price auction); and all other firms remain subject to τ.

Via this mechanism each firm in the market essentially decides whether to transform a part of its marginal cost (i.e., the per unit tax) into a fixed cost (i.e., its bid in the tax exemption auction). If it does so, it may obtain a cost-efficiency advantage over its competitors in the product market. In this sense, the government acts as a "patent holder" who sells licenses to the use of a marginal cost-reducing "innovation" (the innovation being the avoidance of the per unit tax).

To fully capture the merits of the mechanism, which require competition among firms, we introduce it for markets operating as oligopolies, where firms' interactions are most prevalent. To boost competition for the exemptions, we assume that the government auctions-off a limited number of them. We show that the suggested mechanism has the desirable features spelled out earlier. Namely:

(i) It can create *more revenues* for the government in comparison to the excise tax policy for any policy rate τ.[2] This is due to the competition among the firms to acquire the exemptions and avoid competing with a marginal cost disadvantage. Among other things, this implies that if the government wants to collect revenues of magnitude, say, $R(\tau)$, it can do so by announcing a tax rate $\tau' < \tau$ and by inviting the firms to bid for tax exemptions.

(ii) Setting lower tax rates, and also having some firms being tax-exempted, *reduces distortions* in the market and thus raises welfare. Moreover, announcing low(er) tax rates is often more attractive from a political view point.

(iii) In markets with free entry of firms, the mechanism induces lower equilibrium entry compared to the excise tax policy, thus *correcting for the excess entry of firms* often observed in imperfectly competitive markets.

(iv) The suggested tax scheme takes place within a *voluntary mechanism*, namely the firms choose whether to participate in the auction or not and hence choose how to be taxed.[3]

Notice that there is an interesting relation of the current paper with the literature on tax buyouts. A tax buyout is a fiscal instrument according to which a citizen can choose to pay a fixed amount to the government in exchange for a reduction in their marginal tax rate (Del Negro et al. 2010; Goerke 2015). As the latter tax rate is distortionary with regards to labor supply decisions in the labor market, the tax buyout policy increases

[1]The mechanism we suggest could work in principle with an ad valorem tax too.

[2]Notice that the extra revenues may suffice to finance any costs resulting from setting up the tax-exemption auction.

[3]This may address a potential criticism against the mechanism: why setting up an auction and not impose mandatory lump sum taxes.

social welfare, without compromising on revenues, just as our mechanism does for the case of firms in product markets. The two mechanisms, therefore, complement one another.

Some comments on the applicability or the real-world relevance of the mechanism are in order. First, our paper does not suggest that a tax-exemption auction mechanism should be built for all markets (that fit our theoretical framework). This would be practically impossible. What we suggest is that a government may choose a small number of markets, that are well organized, with financially sound firms,[4] etc, and therein run its auctions.

Secondly, in order to highlight the relevance of the mechanism for the real world we note that auctions are already used in a related policy issue in the field of international trade: the auctioning of import licenses. Via this policy the government of the country that imposes quotas on trade can uncover a part of the rents associated with the restriction. The mechanism has been used by countries like Australia, New Zealand, etc., (see Tan 2001) and its success depends on the structure of product markets (see Krishna 1988, 1991, 1993; Tan 2001). So, if an auction-backed government policy can work in international markets then a related mechanism could perhaps work in the current paper's context.

Thirdly, we refer to another real-world policy mechanism with a somehow similar structure: the auctioning of pollution permits. Through this mechanism firms purchase pollution permits which allow them to emit pollutants up to a certain degree. The auctions are designed so as to promote market efficiency and also tax revenue collection (see, for example, Cramton and Kerr 2002), i.e., goals our mechanism also sets. The success of the emission permit auctions hints that our mechanism could also work successfully in practice.

The paper connects to the literature on taxation in oligopoly. The roots of this literature go back to the mid 80's when economists started analyzing the design and optimality of various tax policies outside the two antipodean cases of perfect competition and monopoly. We refer the reader to Dierickx et al. (1988) for a review of the early literature on taxation under imperfect competition. The main issues examined in the literature include tax incidence, comparison between ad valorem and excise taxes, the comparison between ad valorem and excise taxation under uncertainty and/or asymmetric information, taxation in oligopolies under general equilibrium, taxation in vertical oligopolistic markets, etc.[5] Not all aspects related to taxation in oligopoly are, of course, examined in the current paper.[6] For example, the paper only focuses on excise and not on ad valorem taxation. Further, commodities in our model are not "sin" goods, so a potential increase in their consumption, due to our mechanism, is not a concern. Despite these restrictions though, our framework provides a first step towards introducing the tax mechanism and pointing out its merits.

[4]Such elements are silently assumed to hold true in our paper.

[5]Some key works include Katz and Rozen (1985), Seade (1985), Stern (1987), Hamilton (1999), Anderson et al. (2001) on tax incidence; Dierickx et al. (1988), Delipalla and Keen (1992), Skeath and Trandel (1994), Denicolo and Matteuzzi (2000) on the comparison between ad valorem and excise taxes; Myles (1989), Reinhorn (2005), Collie (2015) on taxation in oligopolies under general equilibrium; Kay and Keen (1983) on the optimal structure of excise and ad valorem taxes when firms maximize their joint profits; Colombo and Labrecciosa (2013), Azacis and Collie (2018) on the relative sustainability of collusion under ad valorem or excise taxation; Haufler and Schjelderup (2004) on the sustainability of collusion in international duopolies; Dickie and Trandel (1996), Goerke (2011), Goerke et al. (2014), Kotsogiannis and Serfes (2014) on the comparison between ad valorem and excise taxation under uncertainty and/or asymmetric information; Peitz and Reisinger (2014) on taxation in vertical oligopolistic markets.

[6]A companion paper examines a similar mechanism for the case of international markets operating under tariffs (Stamatopoulos 2021).

The rest of the paper is organized as follows. Section 2 presents the basic ingredients of the mechanism and the market environment. Sections 3 analyzes the main results. Section 4 examines some variations of the basic framework and the last section concludes.

2 Preliminaries

We consider a market with the set of firms $N = \{1, 2, \dots, n\}$. Firms produce the same product. The quantity of the product of firm i is denoted by x_i. The price of the good is denoted by p. The marginal cost of i is constant at the level of c, $i \in N$.[7]

There is a government which taxes the market. The tax mechanism consists of an excise or per unit tax τ and a tax-exemption auction. Namely, the government announces the tax rate τ that the firms should pay per unit of production and also invites the firms to bid in order to acquire exemption(s) from τ. If a firm acquires an exemption, it pays the government its bid and is not obliged to pay τ; and if a firm does not acquire one, it remains subject to it. If no firm decides to participate in the auction, then all firms are subject to the per unit tax. We assume that the government auctions-off k tax exemptions, where $1 \leq k \leq n-1$. Namely, the government sells fewer exemptions than the number of firms so as to boost competition among them to acquire the exemptions. For simplicity we assume a first-price auction.

The interaction evolves as follows:

(i) The government announces the excise tax rate and the rules of the tax-exemption auction.

(ii) The firms decide whether to participate or not in the auction; the auction takes place (if any firm decides to participate) and the winners are determined. Any ties in the auction are resolved randomly.

(iii) The firms compete in the product market, given the outcome of the auction. Namely, firm i operates with marginal cost c if it has acquired the exemption, or with $c + \tau$ otherwise.

We will assume that a unique equilibrium exists in the product market for all values of the tax parameter τ and for all possible environments (which are described in the coming sections).

3 Main results

The first part of this section (subsection 3.1) analyzes the new mechanism in a short-run framework, where the number of firms is fixed. The second part (subsection 3.2) deals with entry of firms in the market. We state in advance that for the first part we don't need to specify the mode of competition among the firms. We only need an assumption on the relation between industry output and marginal costs. For second part, which requires more structure, we will focus on a Cournot market.

[7] Most of the analysis in the paper adopts the assumption of constant marginal costs; section 4 only extends the framework by considering increasing and decreasing marginal costs.

3.1 Short-run analysis

We denote by $x_w(k)$ and $\pi_w(k)$ the equilibrium quantity and profit of a firm that wins a tax exemption under the scenario that k firms in total are exempted; and by $x_l(k)$ and $\pi_l(k)$ the corresponding equilibrium quantity and profit of a firm that does not have an exemption. The equilibrium industry output is denoted by $X(k)$ and the corresponding market price by $p(k)$.

Consider the auction. To analyze the behavior of the firms, we may utilize a result from the patent licensing literature by Colombo et al. (2021). This paper shows that when a patent holder auctions-off k licenses, $1 \leq k \leq n-1$, to the use of a cost-reducing technology in a market with n symmetric firms, a firm submits a bid which is equal to the difference between its profit when it is one of the k licensees and its profit when it is not a licensee but k other firms are. The patent holder then earns k times this difference.

To see the analogy of the above framework with our model, we note that:

(a) The cost-reducing technology in the above paper reduces the marginal cost of production from c to $c - \varepsilon$; in our paper, the "technology" reduces the effective marginal cost from $c + \tau$ to c.

(b) The patent holder in Colombo et al. (2021) is the government in our paper.

Define

$$b(k) = \pi_w(k) - \pi_l(k) \tag{1}$$

Notice that $b(k)$ is the difference between the profit of a firm when it is one of the k firms that do not pay the per unit tax and its profit when itself pays the per unit tax and k other firms do not.

Proposition 1 *Assume the government auctions-off k tax exemptions, where $1 \leq k \leq n-1$. In the unique equilibrium outcome of the auction:*

(i) *At least $k+1$ firms bid $b(k)$.*

(ii) *The government collects in the auction $kb(k)$.*

Proof See Colombo et al. (2021, Theorem 1) and the analogy between this paper and the current paper described in (a)-(b). ■

To grasp the mechanics behind Proposition 1, and in particular part (i), let's assume that $n = 2$ which means $k = 1$. First notice that there cannot be an equilibrium in the auction where the firms bid differently, say $b_2 < b_1$: firm 1 has incentive to lower its bid, but maintain it above b_2, and still win the exemption. Notice that this includes the case where $b_2 = 0$.

Further there cannot be an equilibrium where $b_1 = b_2 < b(1)$. Consider say firm 1. If it sticks to such a b_1 its payoff is[8] $\pi' = \frac{1}{2}(\pi_w(1) - b_1) + \frac{1}{2}\pi_l(1)$. If firm 1 deviates to b' above b_1 and below $b(1)$, its payoff is $\pi_w(1) - b'$, which is higher than π' for b' sufficiently close to b_1.

[8] Recall that ties are randomly resolved.

There cannot also be an equilibrium where $b_1 = b_2 > b(1)$. Take firm 1. Its payoff under the proposed profile of bids is $\pi' = \frac{1}{2}(\pi_w(1) - b_1) + \frac{1}{2}\pi_l(1) < \pi_l(1)$. If firm 1 deviates to a bid below b_1, it loses the auction with certainty but obtains payoff $\pi_l(1)$.

Let finally $b_1 = b_2 = b(1)$ and consider again firm 1. If it sticks to such a bid, its payoff is $\frac{1}{2}(\pi_w(1) - b(1)) + \frac{1}{2}\pi_l(1) = \frac{1}{2}(\pi_w(1) - \pi_w(1) + \pi_l(1)) + \frac{1}{2}\pi_l(1) = \pi_l(1)$. If firm 1 deviates to $b' < b(1)$ it loses the auction and achieves again $\pi_l(1)$. Finally if it deviates to $b' > b(1)$, it wins the auction with certainty but with payoff $\pi_w(1) - b' < \pi_l(1)$. We conclude that in the unique equilibrium, $b_1 = b_2 = b(1)$.

Given the outcome of the auction, the government collects in total the amount

$$B(k) = kb(k) + (n-k)\tau x_l(k)$$

Namely, it collects the bids of the k winners in the auction and the total per-unit tax payments from the $n-k$ other firms.

Consider next the excise tax policy where all n firms pay the rate τ. In analogy to the earlier notation, we denote by $x_l(0)$ the equilibrium quantity of each firm.[9] The government collects in this case the amount $B(0) = n\tau x_l(0) = \tau X(0)$.

We can now compare the revenues from the suggested mechanism to the revenues from the excise tax policy. We will show that the former revenues are higher for every τ for any market where industry output is a decreasing function of marginal costs. Clearly a Cournot market satisfies this assumption (see, for example, Bergstrom and Varian 1985).

Assumption A1. $X(k)$ is a decreasing function of the marginal cost of firm i, $i = 1, 2, \ldots, n$.

Proposition 2 *Assume A1 holds. Then $B(k) > B(0)$ for all $1 \leq k \leq n-1$.*

Proof We use the restriction that $\tau < p(k) - c$, as otherwise the non-exempted firms would exit the market. We have

$$\begin{aligned}
B(k) &= k\big(\pi_w(k) - \pi_l(k)\big) + (n-k)\tau x_l(k) \\
&= k(p(k) - c)(x_w(k) - x_l(k)) + n\tau x_l(k) \\
&> k\tau(x_w(k) - x_l(k)) + n\tau x_l(k) \\
&= k\tau x_w(k) + (n-k)\tau x_l(k) = \tau X(k),
\end{aligned}$$

where the inequality has used the restriction that $\tau < p(k) - c$ and all equalities follow from one another via simple rearrangements.

Given the above it suffices to have $X(k) \geq X(0)$, which holds as the latter is industry output when all firms pay the per-unit tax and the former is industry output when some only of the firms pay this tax, and by invoking A1. ■

The incentive of each firm to avoid competing in the product market with a marginal cost disadvantage drives its bid to a relatively high level, allowing thus the government to collect sufficiently high revenues. As Proposition 2 shows this holds irrespective of the

[9] I.e, zero firms are tax-exempted.

number of firms in the product market.

Remark 1 We note that all firms end up with the same net equilibrium profits. These net profits are $\pi_w(k) - b(k) = \pi_l(k)$ for a tax-exempted firm and $\pi_l(k)$ for a non-exempted firm. So the mechanism does not create ex post (i.e., after tax) asymmetries.

We next note that the suggested mechanism generates higher social welfare than the excise tax policy. By social welfare we mean the sum of consumer surplus, producer surplus and tax revenues. In particular, consumers are better-off under the auction mechanism. This holds as the total equilibrium output (which determines consumer surplus) goes up under the auction mechanism -as noted earlier.

On the other hand, the firms are worse-off: the net profit of each firm, tax-exempted or not, is equal to $\pi_l(k)$, which is the profit of a firm that pays the per-unit tax while k firms do not pay it. Under the excise tax policy, all firm pay the excise tax, so the corresponding profit of each firm is necessarily higher than $\pi_l(k)$. Finally the government's revenues are higher, as Proposition 2 shows. In total, social welfare goes up under the auction mechanism as tax distortions are lower (fewer firms pay the distortionary tax).[10]

We summarize below.

Proposition 3 *The auction mechanism creates higher welfare than the excise tax policy.*

3.2 Entry

In this part we study a market with free entry. We assume that each firm that enters the market faces a fixed cost F. We are interested in two questions: (a) how is equilibrium entry affected by the suggested mechanism vis-a-vis the excise tax policy; (b) given equilibrium entry, do the revenues under the mechanism still surpass the revenues under the standard tax policy.

Consider the auction mechanism. The government auctions again k tax exemptions, with $1 \leq k \leq n-1$. The profit of a firm that has a tax exemption along with $k-1$ others is denoted by[11] $\pi_w(k,n) - F$, whereas that of a firm that doesn't have an exemption by $\pi_l(k,n) - F$.

The equilibrium bid in the auction will be as in (1). This means that the net payoff of any firm, be it a winner or not in the auction, will be $\pi_l(k,n) - F$. Hence entry will take place up to a number of firms n such that $\pi_l(k,n) - F = 0$. We denote this number by n_k.

Consider next the excise tax policy. Using the notation of the previous section, the profit of each firm under this policy is $\pi_l(0,n)$. Entry will take place up to a number of firms n such that $\pi_l(0,n) - F = 0$. Denote the corresponding number of firms by n_0.

Clearly, and as noted already, $\pi_l(0,n) > \pi_l(k,n)$, for any $k \geq 1$, as a firm that is not tax-exempted prefers others not to be exempted too. Given also that $\pi_l(k,n)$ and $\pi_l(0,n)$ are both decreasing in n (as entry lowers profitability) the following holds -for which we need not specify further details in the market.

[10] Recall that the government sets tax rates that allow the non-exempted firms to stay active in the market. So competition is not artificially weakened.

[11] Since we are examining entry it will be useful to index everything by "n".

Lemma 1 *The auction mechanism generates lower entry than the excise tax policy, i.e.,* $n_k < n_0$, *for* $1 \leq k \leq n-1$.

It is known that the equilibrium entry in, say, a homogeneous Cournot industry is higher than the socially optimal (e.g., Mankiw and Whinston 1986). The suggested mechanism therefore has one more merit as it induces lower entry.

We now turn to the comparison of the revenues between the two policy schemes. Consider the auction mechanism. As noted already, the equilibrium bid under free entry is as in (1). Hence $b(k,n) = \pi_w(k,n) - \pi_l(k,n)$, $1 \leq k \leq n-1$. In particular, under equilibrium entry

$$b(k,n_k) = \pi_w(k,n_k) - \pi_l(k,n_k)$$

The government thus collects the amount

$$B(k,n_k) = kb(k,n_k) + (n_k - k)x_l(k,n_k) \tag{2}$$

On the other hand, the revenues of the excise tax policy under equilibrium entry in the market are

$$B(0,n_0) = n_0 \tau x_l(0,n_0) \tag{3}$$

To compare the revenues of the government under the two alternative policies we will resort to a linear inverse demand $p = a - X$, where $a > c$ and $X = \sum\limits_{i \in N} x_i$. This is done for reasons of analytical tractability only. We restrict again attention to tax rates such that a firm that is not tax-exempted does not exit the market. This is met if $\tau < (a-c)/(k+1)$, which is assumed to hold.

Under linear demand, equilibrium entry is (see proof of Proposition 4 in the Appendix)

$$n_k = \frac{\sqrt{F}(a-c-\tau(1+k))}{F} - 1, \;\; n_0 = \frac{\sqrt{F}(a-c-\tau)}{F} - 1$$

Given the above, we have the following result.

Proposition 4 *Assume* $p = a - X$. *Then* $B(k,n_k) > B(0,n_0)$, *for* $1 \leq k \leq n-1$.

Proof Appears in the Appendix. ■

Recall from Proposition 2 that for a *fixed* number of firms n, the auction-backed mechanism creates higher government revenues than the excise tax policy. Allowing for entry turns out to give one more boost to the auction mechanism in terms of how it performs on revenues. Entry intensifies competition in the market, lowers the profitability of firms and thus lowers their "ability" to pay for taxes, either via their bids in an auction or via excise taxation. Since equilibrium entry is lower under the auction policy, this reduction in the ability is lower when the latter policy is used compared to the excise tax policy. This, together with Proposition 2, explain the finding of Proposition 4.

4 Variations

In this section we analyze the suggested mechanism in some variations of the basic framework introduced in the previous sections. The variations focus on cost asymmetries and general cost functions. Further a market with product differentiation is briefly discussed. We note that in all these cases we deal with duopolistic markets where a sole tax exemption is auctioned.

We consider first a duopoly with cost-asymmetric firms where, say, $c_1 < c_2$. The first question that arises, in relation to our mechanism, is whether the efficient or the inefficient firm wins the auction. Define by $\pi_{wi}(1)$ the profit of firm i when i has the tax exemption; and by $\pi_{li}(1)$ the profit of firm i when i's rival has the exemption. Notice that the maximum amount firm 1 is willing to bid in the auction is $b_1^{max}(1) = \pi_{w1}(1) - \pi_{l1}(1)$; likewise the maximum amount for firm 2 is $b_2^{max}(1) = \pi_{w2}(1) - \pi_{l2}(1)$. The firm with the highest such amount will win the auction by bidding the other firm's maximum amount.

Lemma 2 *Assume that $c_1 < c_2$. Then firm 1 wins the auction bidding $b_2^{max}(1)$.*

Proof We need to show that $b_1^{max}(1) \geq b_2^{max}(1)$. This holds if and only if $\pi_{w1}(1) + \pi_{l2}(1) \geq \pi_{l1}(1) + \pi_{w2}(1)$. The l.h.s of this inequality is the industry profit at the marginal cost profile $(c_1, c_2 + \tau)$ and the r.h.s is industry profit at the marginal cost profile $(c_1 + \tau, c_2)$. The former profile has higher variance than the latter (this holds as $c_1 < c_2$). Then we can use Proposition 1 of Van Long and Soubeyran (2001) that industry profit (at a Cournot equilibrium where all firms are active) is an increasing function of the variance of individual marginal costs. ■

The next task is to compare the revenues under the policies of tax exemption and excise tax. To facilitate the analytical comparison we restrict attention to a linear inverse demand function, $p = a - X$.

Remark 2 We assume that $c_2 < (a + c_1)/2$ which guarantees that, in the absence of any government policy, the less efficient firm is active in the market.

As in the symmetric case, we will assume that the tax rate is such that the firm that loses the auction remains active in the market. For the linear demand case this requires that $\tau < (a - 2c_2 + c_1)/2$.[12]

Proposition 5 *Assume that $p = a - X$. The following hold:*

(i) If $c_2 \leq (a + 7c_1)/8$ then $B(1) \geq B(0)$.

(ii) If $c_2 \geq (a + 4c_1)/5$ then $B(0) \geq B(1)$.

(iii) If $(a + 7c_1)/8 < c_2 < (a + 4c_1)/5$ then $B(1) \geq B(0)$ for $\tau \leq a + 4c_1 - 5c_2$; and $B(1) < B(0)$ for $a + 4c_1 - 5c_2 < \tau$.

Proof Appears in the Appendix. ■

[12] Imposing this bound on τ further guarantees, in conjunction with Remark 2, that the thresholds presented in Proposition 6, and their relations, are well-defined.

If c_2 is sufficiently small (but always higher than c_1) then we are "close" to the symmetric case, so the result of Proposition 2 is essentially repeated. On the other hand, if c_2 is large enough, namely if the cost disadvantage of firm 2 is large, then firm 1 outbids firm 2 relatively easy. As a result, the government collects relatively low revenues from the auction and the standard policy outperforms it. For these two polar cases, the role of c_2 is dominant, so the tax rate plays no role in the comparison of the policies. Naturally, for intermediate values of c_2 other factors, such us τ, come into place. Essentially part (iii) repeats parts (i) and (ii) by saying that the auction mechanism (the standard policy) generates more revenue than the standard policy (the auction mechanism) if the cost asymmetry adjusted by τ is low (high). This we can see by re-writing $\tau \lesseqgtr a + 4c_1 - 5c_2$ (which combines the two inequalities in the statement of part (iii)) as $\tau + 5c_2 \lesseqgtr a + 4c_1$.

We next turn to a duopoly with a general cost function. Let $C(x_i)$ be cost function of firm i, $i = 1, 2$, so symmetry is assumed. We will again compare our mechanism vis-a-vis the standard excise tax policy in terms of revenues when one exemption is auctioned-off.

As in other parts of the paper where symmetry is imposed, the bid of each firm in the action is equal to the difference between its profit when it is the sole firm exempted and its profit when its opponent is the only firm exempted. We have the following result.

Proposition 6 *Assume that either (i) or (ii) holds:*

(i) $C(x_i)$ *is concave,* $i = 1, 2$.

(ii) $C(x_i)$ *is convex and* τ *is sufficiently low,* $i = 1, 2$.

Then $B(1) > B(0)$.

Proof (i) The proof will follow the steps of the proof of Proposition 2 adjusted when necessary. We again assume that the non-exempted firm remains active in the market. For this we require $\tau < p(1) - \dfrac{C(x_l(1))}{x_l(1)}$.

Using the analysis of section 3.1, adjusted for the new framework, we need to show that

$$p(1)(x_w(1) - x_l(1)) - C(x_w(1)) + C(x_l(1)) + 2\tau x_l(1) > 2\tau x_l(0) \tag{4}$$

We will first show that

$$p(1)(x_w(1) - x_l(1)) - C(x_w(1)) + C(x_l(1)) \geq \tau(x_w(1) - x_l(1)) \tag{5}$$

To show the above, notice first by the constraint on the tax rate that

$$\tau(x_w(1) - x_l(1)) < \big(p(1) - \frac{C(x_l(1))}{x_l(1)}\big)(x_w(1) - x_l(1))$$

Furthermore,

$$\big(p(1) - \frac{C(x_l(1))}{x_l(1)}\big)(x_w(1) - x_l(1)) \leq p(1)(x_w(1) - x_l(1)) - C(x_w(1)) + C(x_l(1))$$

$$\Leftrightarrow \frac{C(x_w(1))}{x_w(1)} \leq \frac{C(x_l(1))}{x_l(1)} \tag{6}$$

But the last inequality holds by the concavity of the cost function: the concavity of cost implies that the marginal cost is lower than the average cost, which means that the latter is decreasing in quantity (recall that $x_l(1) < x_w(1)$). Hence (6) holds and thus (5) holds.

By (5), expression (4) holds if $\tau(x_w(1) - x_l(1)) + 2\tau x_l(1) \geq 2\tau x_l(0)$. But this holds, as we can see with an argument identical to that in the last part of the proof of Proposition 2.

(ii) We may use the same steps to reach (5). But to continue under cost convexity, we cannot use (6), as it does not hold anymore. Instead, by simple rearrangements we can show that (5) holds iff τ is low enough, and in particular if

$$\tau \leq p(1) - \frac{C(x_w(1) - C(x_l(1))}{x_w(1) - x_l(1)}$$

Then the rest of the proof is exactly similar to the last part of the proof of part (i). ■

Notice by Proposition 6(ii) that we do not characterize the comparison between the two policies when cost is convex. When τ is high, in particular, we don't know how the two policies area ranked. Still if we restrict attention to the case of quadratic cost and linear demand, it is easy to show that the auction mechanism generates higher revenues for all values of τ.[13]

Finally we note that the suggested mechanism outperforms the standard excise tax policy also when the firms produce differentiated goods. Denote by $p_i = a - x_i - \gamma x_j$ the price function of firm i, $i, j = 1, 2$, $i \neq j$, where $\gamma \in (0, 1)$ is the degree of product differentiation. Then assuming common and constant marginal costs, $c_1 = c_2 = c$, it is easy to show that $B(1) > B(0)$ for all γ. The reasoning is as that under product homogeneity.

5 Conclusions

This paper has introduced a novel tax mechanism in oligopoly. The proposed policy resembles a patent licensing mechanism. Under the latter, the firms purchase from an innovator licenses to the use of a technology that allows them to reduce their marginal cost. In our mechanism, the firms purchase from the government the right to avoid an increase in their marginal cost.

The suggested mechanism works complementarily to the existing commodity tax policies. One of the merits of the mechanism is that it is not mandatory: each firm may choose how to be taxed. In this sense, the mechanism improves also upon the policy of mandatory lump-sum taxes on firms' profits.

One of the features of the paper is that the government and the firms know the characteristics of the market. A natural extension of the paper is to consider markets where the firms' characteristics are private information and analyze their bidding behavior within that framework. The analysis of the mechanism under an ad valorem tax policy is also of

[13]The calculations are available by the author.

interest. Further, allowing for transferable exemptions, in the sense that the winners of the auction are free to sell their tax exemptions to rivals is also worth to be examined. Finally, the analysis of markets characterized by uncertainty could also enhance the significance of the suggested mechanism.

Appendix

Proof of Proposition 4 Under linear demand, the equilibrium quantities of exempted and non-exempted firms are

$$x_w(k,n) = \frac{a-c+(n-k)\tau}{n+1}, \quad x_l(k,n) = \frac{a-c-(1+k)\tau}{n+1} \tag{7}$$

A non-exempted firm survives in the market if $\tau < \dfrac{a-c}{1+k}$. Define $\theta = a-c-\tau$. Using the above formulas we have that the profit (net of the fixed cost of entry) of a firm that is not tax-exempted is $\pi_l(k,n) = \dfrac{(\theta-k\tau)^2}{(1+n)^2}$. Hence

$$\pi_l(k,n) - F = 0 \Leftrightarrow n = \frac{\sqrt{F}(a-c-\tau(1+k))}{F} - 1 \equiv n_k$$

On the other hand, the profit of a firm (net of the fixed cost of entry) under the excise tax policy is $\pi_l(0,n) = \dfrac{\theta^2}{(1+n)^2}$. Hence

$$\pi_l(0,n) - F = 0 \Leftrightarrow n = \frac{\sqrt{F}(a-c-\tau)}{F} - 1 \equiv n_0 > n_k$$

Using again the formulas in 7 and the equilibrium entries n_k and n_0 we get that

$$B(k,n_k) = \frac{(\theta\sqrt{F}(\theta-k\tau) + F(a-c)(k-1) + F\tau(1-k^2))\tau}{(\theta-k\tau)\sqrt{F}}$$

$$B(0,n_0) = \frac{(\theta\sqrt{F} - F)\theta\tau}{\theta\sqrt{F}}$$

By straightforward computations $B(k,n_k) > B(0,n_0)$ iff

$$\theta\sqrt{F}[(a-c)(k-1) + \tau(1-k^2) + \theta - k\tau] > 0 \tag{8}$$

Since $\tau < \dfrac{a-c}{1+k}$, we have that $\theta > 0$. Moreover the sum of the terms inside the brackets in (8) is positive iff $a-c-\tau(1+k) > 0$. But this holds by the condition that a non-exempted firm survives in the market. ■

Proof of Proposition 5 We have that $B(0) = \tau(\frac{a-2c_1+c_2-\tau}{3} + \frac{a-2c_2+c_1-\tau}{3})$ and $B(1) = \tau\frac{2a-4c_2+2c_1-\tau}{3} + \tau\frac{a-2c2+c_1-2\tau}{3}$. It is easy to show that $B(1) > B(0)$ iff $a+4c_1-5c_2-\tau > 0$. We notice that if $c_2^{max} > c_2 > (a+4c_1)/5 \equiv c_2'$ then $B(1) < B(0)$ for all τ. And if $c_2 < c_2'$ then

$B(1) > B(0)$ iff $\tau < a+4c_1-5c_2$. We conclude the proof noting $a+4c_1-5c_2 > (a-2c_2+c_1)/2$ iff $c_2 < (a+7c_1)/8$. ■

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