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1. Introduction

This Ph.D. thesis provides a synthesis of studies on different topics on consumer behaviour: it offers some insights concerning consumer demand for organic and non-organic commodities and addresses the issue of consumer inefficiency. Specifically, it (a) provides empirical evidence on the interrelationships between organic and non-organic commodities in the form of estimates of cross-quantity flexibilities, (b) analyses the effects on consumer welfare that arise from the substitution of non-organic commodities by organic ones, (c) develops a theoretical model for measuring efficiency in consumption, and (d) proposes an econometric framework for empirical measurement of consumer's inefficiency.

The first two topics addressed are those of consumer demand for organic and non-organic commodities and of changes in consumer welfare resulting from substitution of non-organic by organic commodities. Consumer demand for organic products was chosen as a subject of study due to the importance of organic agriculture, *i.e.*, of a production system which is based on locally or farm-produced renewable inputs in preference to external ones and which aims to promote and enhance ecosystem health (FAO, 2003). In the European Union (EU), the importance of the organic farming sector is reflected in the recent reforms of the EU Common Agricultural Policy (CAP), and in relevant Regulations. Since the 1992 CAP Reform, organic farming has been assigned an important role in the enhancement of environmental protection throughout the EU, while EU Regulations 2078/92 and 2092/91 provided specific incentives for conversion to and maintenance of organic farming and established organic products as distinctively different from their conventional counterparts (provision of standards and certification). However, higher prices received, is the most important incentive for farmers to convert to organic agriculture (Burton, Rigby, and Young, 1999, 2003; O'Riordan and Cobb, 2001). Farmers receive higher prices for organic products when consumers believe that there is a quality premium available in organic product's attributes (Loureiro, McCluskey, and Mittelhammer, 2001; Boland and Schroeder, 2002). Thus, consumers' acceptance of organic products is vital for the growth of the organic farming sector.

Most studies on the demand conditions for organic products examine consumer attitudes, identify their motivation for purchasing organic products, and elicit willingness to pay for them (see, for example, Yiridoe, Bonti-Ankomah, and Martin (2005) for a review of the literature on studies exploring these issues, and Gracia and de Magistris (2008) for a review of the recent empirical studies on the economic and demographic factors affecting consumer's demand for organic products). Such studies take into account socio-economic, demographic, psychological, and ethical factors which may have an effect on consumer's acceptance of organic commodities, but almost completely neglect the presence of organic product's competitors in the market, *i.e.*, their non-organic counterparts. In particular, there are very few empirical studies of consumer demand that employ actual quantity and price data in order to explore the interrelationships between organic and non-organic commodities in the form of estimates of cross-price elasticities. For example, Glaser and Thompson (1998) employed the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980*b*) and monthly scanner data from U.S. supermarkets, in order to estimate the demand for organic and conventional frozen vegetables, for the period from September 1990 to December 1996; Glaser and Thompson (2000) employed the AIDS and national scanner data from U.S. supermarkets in order to estimate the demand for organic, branded, and private-label milk, for the period from November 1996 to December 1999; and Wier, Hansen, and Smed (2001) employed the AIDS and time-series data (derived from a household panel data set) in order to estimate the demand for organic and conventional dairy products in Denmark, from the beginning of 1997 to the end of 1998. The limited number of empirical studies eliciting cross-price elasticities between organic and non-organic products is due to the fact that organic products are relatively new compared to their non-organic counterparts, and therefore to the paucity of sufficient historical data on retail prices and consumption.

In this context, the aim of Chapter 2 is to provide empirical evidence on the interrelationships between organic and non-organic commodities and on the changes in consumer welfare resulting from substitution of non-organic commodities by organic ones, by using a system of demand equations for both commodity types. To this aim, cross-sectional data and the Inverse Almost Ideal Demand System (IAIDS) of Eales and Unnevehr (1994), and Moschini and Vissa (1992) are employed for the empirical analysis of household demand for organic and non-organic milk & yoghurt, and fruits & vegetables, in London, UK. Inverse demand systems, *i.e.*, demand systems for which quantities are taken as predetermined, can be used when prices either do not exist or are artificially distorted (Deaton and Muellbauer, 1980*a*), or in the case of quickly perishable foods, agricultural, and fishery products for which quantities cannot adjust in the short-run (Barten and Bettendorf, 1989). In the latter case, the underlying assumption is that since supply of such commodities may be highly inelastic during short-intervals, price must adjust so that the available quantity is consumed. In this study, the commodities under analysis are quickly perishable ones and cannot be stored. In addition, the data at hand include information on purchased quantities and expenditure, but not on prices. Therefore, an inverse demand system seems more appropriate for the present analysis. Another issue that the use of inverse demand systems gives rise to is that of endogeneity of commodity quantities. Demand studies using cross-section data and direct demand systems, on the other hand, compute the commodity prices from the quantity and expenditure data. However, these computed prices (unit values) also reflect the quality of the commodities and are endogenous in the sense that they are the outcome of the households' purchasing decision (Deaton, 1990; Nelson, 1991).

The use of cross-section data in our analysis, however, is not without complications. It is common in micro-level analyses of consumer demand for many households to report zero purchases of certain commodities. The presence of these zero observations gives rise to two problems. Firstly, in the case of an inverse demand system, such as the IAIDS, where expenditure shares are functions of the logarithm of the quantities purchased, the logarithm of zero purchases cannot be defined. In order to overcome this problem, we extend the approach proposed by Battese (1997) in the context of stochastic production frontier estimation, which allows full-sample estimation and results in efficient and unbiased estimates. The second problem related to the presence of zero purchases is that standard systems estimation methods, e.g., seemingly unrelated regression or maximum likelihood, lead to biased parameter estimates. The Amemiya-Tobin model by Wales and Woodland (1983) is the approach adopted in the present study to account for the presence of zero purchases in our sample. The advantages of this approach for estimation of censored equation systems, relative to other approaches which are widely used in the literature, lie in that it can be applied to any demand system specification, addresses the problem of adding-up of observed expenditure shares adequately, and yields efficient parameter estimates. The Amemiya-Tobin model by Wales and Woodland has also been employed by Dong, Gould, and Kaiser (2004), for the estimation of Mexican household demand for 12 food categories, and by Dong, Kaiser, and Myrland (2007) for the assessment of the effects of advertising on Norwegian household demand for four fish and meat categories.

The estimated parameters of the IAIDS can be used for the derivation of implications regarding consumer behaviour towards organic and non-organic commodities, not only through the computation of the flexibilities (*i.e.*, the analogues to elasticities of direct demands), but also through computation of the effects of changes in consumed quantities on households' welfare levels. Measurement of consumer welfare using inverse demand functions has received much attention in the literature. However, most studies are based on the empirical measurement of the *Marshallian surplus*, which is derived from the uncompensated inverse demands. The Marshallian surplus is an *approximate* welfare measure since it cannot measure exactly the welfare effects resulting from quantity changes; it is a valid measure of welfare change when consumer preferences are homothetic. In the present study, on the other hand, the logarithmic distance function underlying the IAIDS can be used for the derivation of *exact* welfare measures for quantity changes. In particular, Palmquist (1988), and Kim (1997) have shown how the distance function underlying an inverse demand system can be used for computation of exact welfare measures for quantity changes, *i.e.*, of the *compensating variation* (CV) and *equivalent variation* (EV) for a change in commodity quantities. Despite their advantage of being exact welfare measures, however, it is only the consumer demand studies of Beach and Holt (2001), and Holt and Bishop (2002) that report these measures.

The third topic addressed in this Ph.D. thesis concerns the theoretical modelling of efficiency in consumption. Standard consumer demand analysis assumes *a priori* that consumers always behave optimally, that is, they do succeed in obtaining maximum utility from given purchased commodities, or they do succeed in choosing the minimum quantities required for the achievement of a utility level. However, optimality is a restrictive assumption to make for consumers' actual behaviour. As Afriat (1988) points out, "The ordinary theory of the consumer is based on utility – and unquestioned efficiency. Even when the utility is granted, perfect efficiency seems an extravagant requirement. The familiar volatilities of real consumers make such intolerance unsuitable." (p. 252). It is then more reasonable to assume that consumers may not behave optimally and employ theoretical and empirical models that accommodate any departure for optimality (*i.e.* inefficiency) and allow it to be measured.

The importance of studying inefficiency in consumption lies not only on the fact that optimal behaviour, and hence, efficiency, is a restrictive assumption to make for consumers' actual behaviour. It also lies on the fact that consumer's non-optimal behaviour has a negative impact on welfare levels. In particular, it has a negative impact on consumer's welfare levels in terms of budget that was wasted and which could have been allocated to the satisfaction of other needs. In addition, over-consumption leads to increased and more industrialised production, which itself fuels over-consumption, through, say, advertising. This circle implies excessive use of natural resources and/or wrong allocation of them in the production of commodities, increased waste from both consumption and production, and a negative impact on social welfare.

The assumption of consumer's non-optimal behaviour can be accommodated in the case of commodities, such as highly perishable foods, meat, fish and agricultural products. In such cases, consumers may be inefficient because they are making rough estimates of the volume of the commodities and the quantity combination of them that are enough for the achievement of some desired utility level: when consumers choose a commodity bundle, they choose it on the basis of their estimates of what commodity combination is the suitable one for their wants. Consumers may also be inefficient because they cannot predict the future exactly: since individuals' every day lives cannot be programmed to the detail, it is not unexpected that a portion of the purchased quantities of the commodities are not consumed but – in the case of highly perishable foods that cannot be stored – are disposed of instead. Or it could be lack of information, awareness and responsibility from the part of consumers with respect to the full social costs of their consumption decisions that lead to excess purchases and spending, and consumption inefficiency. Thus, consumers may purchase a commodity bundle that is later on proved to be non-optimal: they could have bought less of all the commodity quantities (*commodity inefficiency*), thus reducing expenditures, and/or they could have re-allocated their expenditures by choosing different quantity mix (*allocative inefficiency*), thus reducing expenditures even more. Reduction of consumer's inefficiency and mitigation of its negative impact on welfare levels could be accomplished though, say, advertising. If advertising plays an important role in creating and/or sustaining consumer's non-optimal behaviour, then advertising could perhaps be used as a means of awareness raising and initiation of changes in consumer's shopping, purchasing and consumption patterns.

In this context, the aim of the Chapter 3 is to propose a theoretical framework for analysing consumer's efficiency in price-quantity space. The theoretical model which is developed is based on the simple observation that consumer preferences are commonly defined over the consumption levels and no distinction is being made between the quantities of the commodities purchased and the consumption levels themselves, that is, it is implicitly assumed that the purchased quantities and the consumed quantities are the same. However, if it is assumed that consumers are free to dispose of any unwanted quantities of the commodities they have purchased, then it becomes possible to define a measure of efficiency of the consumers in their effort to mimimise expenditure for commodities. Past attempts to study consumption efficiency in price-quantity space have been based on revealed preference relations or money-metric utility functions in order to construct non-parametric or parametric efficiency indices (Afriat, 1967, 1988; Varian 1982, 1983, 1985, 1990). The focus of these studies, however, is on the examination of the goodness-of-fit of optimising models to actual data by measuring the departure from optimisation. Moreover, what is implied by these efficiency measures is that inefficiency occurs because a portion of the consumers' budget is wasted, and not a portion of the purchased quantities. However, it is this latter assumption that allows the construction of the measure of what we define in subsequent sections as *commodity efficiency*. Moreover, since these models do not allow for the possibility that an observed commodity bundle may also be commodity inefficient, no distinction is being made between what we will define as *allocative efficiency* and *overall efficiency*. As a result, the efficiency score that these models assign to consumers may be higher than it should.

Our analysis is carried out under the consumer's expenditure-minimisation framework, and the starting point is the assumption that the consumer's objective is to choose a feasible commodity vector in order to achieve a desirable utility level. Assuming also that the consumer need not make use of all the quantities of the purchased commodities and may dispose of any unwanted quantities of them, the quantities of the purchased commodities may well be higher than the ones required to just attain the desirable utility level, and the consumer may well have chosen an inefficient way of attaining this target utility level. This type of efficiency is what we are going to define as commodity efficiency. Another type of efficiency is what we call expenditure, or overall, efficiency, and which we describe as the consumer's ability to avoid wasting expenditures, by minimising the cost of purchased commodities in the achievement of a utility level. A third type of efficiency is allocative efficiency: assuming that the expenditure-minimising commodity vector and an observed commodity vector lie on the same indifference curve, allocative efficiency is concerned with how close this observed commodity vector is to the expenditure-minimising one. Finally, we show the relation between the three types of efficiency, *i.e.*, the decomposition of expenditure efficiency into commodity efficiency and allocative efficiency.

The theoretical model which was just described is based on the assumption that any unwanted quantities of the purchased commodities can be disposed off. However, this assumption gives rise to the definition of consumer's inefficiency, not only in terms of quantities and budget that were wasted, but also in terms of utility that could have been, but was not, attained at the end of the day. More formally, assuming that the consumer's objective is to obtain maximum utility from a given commodity vector, if the consumer does not achieve this objective, *i.e.*, if he/she does not use the purchased commodities as efficiently as he/she could, then the utility attained may well be lower than the maximum attainable one. We will use the notion of *utility efficiency*, in order to describe consumer's ability to obtain maximum utility from given purchased commodities. This type of efficiency applies not only to commodities the quantities of which the consumer may dispose off. Since the starting point is the consumer's objective of obtaining maximum utility from a given commodity vector, the notion of utility efficiency can be used to study non-foods as well. For instance, it could be lack of information about the commodities' characteristics, the way(s) they could and/or should be used, etc., that may lead the consumer to obtain from a given commodity bundle a utility level that is lower than the one the commodities could potentially allow him/her to. Examples are cars and lack of information regarding "green driving" (car use and driving behaviour that reduces the negative externalities produced by cars), foods and lack of information about the preparation of healthy meals, etc. Furthermore, a consumer facing exogenous market prices for commodities and having a fixed budget to spend on the purchase of them behaves optimally as long as the commodity bundle he/she spends his/her budget on is the utility-maximising one. However, due to lack of information about the commodities' characteristics, the way(s) the commodities could and/or should be used, whims, accidents, or other factors that may affect the consumer's decisions, the consumer may spend his/her budget on a commodity mix which is not the utility-maximising one. This type of efficiency is what we will define as *budget allocative efficiency*, whereas the notion of *utility overall efficiency* will be used in order to describe the consumer's ability to obtain the highest utility level that the utility-maximising commodity vector is capable of generating. Finally, as in the case of expenditure efficiency, we propose a relation for the decomposition of utility overall efficiency into its constituent parts, *i.e.*, utility efficiency and budget allocative efficiency.

The fourth topic addressed in this Ph.D. thesis is that of empirical measurement of efficiency in consumption. Our empirical analysis of consumer's efficiency focuses on the empirical measurement of commodity, allocative and expenditure efficiency, whereas the empirical measurement of utility, budget allocative and utility overall efficiency is left for future research. The empirical methodology that is adopted for the econometric estimation of the former set of consumption efficiency indices lies on the employment of approaches that are used not only in consumer demand analysis, but also in other fields. In particular, as shown in Chapter 3, the index which is proposed for the measurement of commodity efficiency is based on a distance function representation of consumer preferences. Hence, calculation of the commodity efficiency index requires knowledge of the value of the distance function, which can be acquired though econometric estimation of the latter. However, the difficulty in estimation of a distance function representation of consumer preferences lies on that it is a function, not only of observed commodity quantities, but also of consumer's utility level which is unobserved. Chapter 4 illustrates how this problem can be dealt with, by estimating a translog distance function with a panel of household consumption data for milk & yoghurt, fruits, and vegetables, and the use of two different approaches: a proxy for utility, and treatment of consumer's utility as a random error term. Lewbel and Pendakur (2006) invented Implicit Marshallian Demand systems, which are systems of Hicksian demands where utility u is substituted by *implicit utility*, a simple function of observables. Following Lewbel and Pendakur (2006), Färe, Grosskopf, Hayes, and Margaritis (2008) proxy utility with household annual income in order to estimate and assess systems of demand equations which are derived from expenditure and benefit functions. The advantage of such an approach is that, once observable variables are used as a proxy for utility, the distance function can be estimated using standard frontier-estimation techniques which are widely used in production efficiency analysis (see, for example, Kumbhakar and Lovell, 2000, Ch. 3). Its drawback, however, is that consumer's utility level is assumed to be affected by the set of observable variables used, while any other factors that may effect consumer's preferences are ignored. The second methodology that is adopted for estimation of the translog distance function lies on treating consumer's unobserved utility level as a random error term. Specifically, treatment of the terms associated with the utility level and the distance as one-sided positive error terms gives rise to a density for the composite error term which resembles the two-tiered frontier framework by Polachek and Yoon (1987, 1996). The estimated distance function can then be used as an index to measure the consumer's commodity inefficiency. As far as calculation of the measure of allocative efficiency is concerned, knowledge of either the expenditure function or the expenditure-minimising commodity vector is required; standard procedures employed in production efficiency analysis for computing the index of allocative efficiency are employed for computation of the measure of allocative efficiency in consumption. Finally, the measure of expenditure efficiency can be computed with the use of the proposed relation for the decomposition of expenditure efficiency into commodity and allocative efficiency.

In dealing with these topics on consumer behaviour, the present Ph.D. thesis contributes to the literature of consumer behaviour by:

- providing empirical evidence on the interrelationships between organic and non-organic commodities, in the form of cross-quantity flexibilities, and on the changes in consumer welfare resulting from substitution of non-organic commodities by organic ones,
- developing indices of commodity-oriented efficiency for measuring consumer's efficiency in price-quantity space, which allow consumer's efficiency to be studied not only in terms of budget that is wasted, but also in terms of quantities that are wasted,
- extending the *output distance function* from the producer theory context to the consumer theory context and developing indices of utility-oriented efficiency in price-quantity space, which allow consumer's inefficiency to be studied in terms of utility that could have been, but is not, attained,

- proposing an approach for empirical measurement of commodity-oriented efficiency, under which consumer's unobserved utility level is treated as a random error term in the estimation of a distance function representation of consumer's preferences.

What makes our study on consumer demand for organic products different from the majority of studies in the relevant literature is the use of actual data for household purchases (as opposed to survey data), and the adoption of a demand system for both organic and non-organic commodities whose functional form is consistent with known household-budget data. Also, the technique proposed by Battese (1997) for dealing with the problem of the logarithm of explanatory variables which may take on zero values is extended from to the stochastic production frontier context to the context of inverse demand systems estimation. As far as the proposed commodity efficiency measures are concerned, they allow consumer's efficiency to be studied not only in terms of budget that is wasted (*i.e.*, as in the models developed by Afriat and Varian), but also in terms of quantities of the purchased commodities that the consumer may waste. As regards to the proposed utility-oriented efficiency measures, their construction is based on the use of the output distance function. The output distance function is well-established in the producer theory context, but it has never been used before in the consumer theory context. The utility-oriented efficiency measures allow consumer's inefficiency to be studied in terms of utility that could have been, but is not, attained. This side of consumer's non-optimal behaviour has not been explicitly dealt with in the literature of consumer's efficiency. Specifically, an index which resembles our index of utility overall efficiency can be found in Russell (1998). However, the issue of consumer's efficiency is not studied by Russell (1998); it is only implied that consumer's efficiency could be studied in terms of utility that is wasted. Finally, as far as the empirical measurement of consumer's commodity-oriented efficiency is concerned, we propose a new approach for dealing with consumer's unobserved utility level in the estimation of consumer models. In particular, we propose that estimation of a distance function representation of consumer' preferences be carried out via treatment, and estimation, of consumer's unobserved utility level as a random error term. As already stated, the proposed empirical approach yields an empirical model for consumer's efficiency which resembles, but is not similar to, the two-tiered frontier framework by Polachek and

Yoon (1987, 1996). In the present Ph.D. thesis, however, we propose estimators for the one-sided error terms which are distinctively different from the ones proposed by Polachek and Yoon (1987, 1996).

The structure of this Ph.D. thesis is as follows. Chapter 2 is concerned with the study on consumer demand for organic and non-organic commodities. This chapter begins with a presentation of the inverse demand system that is adopted, *i.e.*, the IAIDS, as well as its extensions and applications. The adaptation of the IAIDS to the purposes of our study, and the econometric framework adopted are then presented. The final section of this chapter provides a description of the data and an analysis of the empirical results. The theoretical model which is proposed for measuring efficiency in consumption is the subject-matter of Chapter 3. Firstly, a historical background on consumption efficiency measurement is provided: the past attempts of measuring efficiency in consumption are presented in detail. Then, a somewhat more involved presentation of the expenditure, and distance functions is offered, and existing results, which are derived from the producer's cost-minimisation problem and used in applied production analysis, are also derived in the context of consumer theory. Next, a detailed presentation of the proposed theoretical model is provided: the notions of commodity, allocative, and expenditure efficiency are described, measures for these types of inefficiency are derived, and a decomposition of expenditure efficiency into commodity and allocative efficiency is also proposed and illustrated. Moreover, the notion and a measure of utility efficiency are provided, and its relation to the measure of commodity efficiency is explained. Finally, the notions and measures of budget allocative efficiency and utility overall efficiency are provided and a decomposition of utility overall efficiency into utility efficiency and budget allocative efficiency is proposed. In Chapter 4, two alternative approaches for empirical application of the proposed theoretical model for measuring efficiency in consumption are proposed, and described in detail. Then, the data used for estimation of the model are described, and the empirical results are analysed. Finally, Chapter 5 summarises and concludes.

2. The Demand for Organic Foods in the UK: A Censored Inverse Almost Ideal Demand System

2.1 INTRODUCTION

The present chapter is concerned with consumer demand for organic commodities and contributes to the relevant literature by providing empirical evidence on the interrelationships between organic and non-organic commodities, in the form of cross-quantity flexibilities, and by exploring the effects of substitution of non-organic commodities by organic ones on consumers' welfare levels. As already stated in Chapter 1, what makes the present study on consumer demand for organic products different from the majority of studies in the relevant literature is the use of actual data for household purchases (as opposed to survey data), and the adoption of a demand system for both organic and non-organic commodities whose functional form is consistent with known household-budget data. Also, in the present study, the technique proposed by Battese (1997) for dealing with the problem of the logarithm of explanatory variables which may take on zero values is extended from to the stochastic production frontier context to the context of inverse demand systems estimation.

The demand model that is adopted for modelling consumer demand for organic and non-organic commodities is the inverse AIDS. This model has almost all of the properties of its widely used direct counterpart, and, as a result, its appearance in the literature triggered a number of studies encompassing empirical applications, extensions, and modifications of it, as well as studies proposing new inverse demand systems which nest it. The econometric model that was chosen for estimation of the IAIDS is the Amemiya-Tobin model by Wales and Woodland (1983). The direct and inverse AIDS, other inverse demand systems associated with the IAIDS, and econometric models that compete the one that is adopted here are presented in Section 2.2. The Battese (1997) approach that is adopted for tackling the problem that in the IAIDS the logarithm of zero quantities cannot be defined, the final model used for estimation and further issues related to its estimation, are the subject-matter of Section 2.3. The same section also presents the framework that is employed for economic interpretation of the parameter estimates, that is, the methodology for computing the flexibilities and analysing the effects of changes in consumed quantities on households' welfare levels. The last section of this chapter provides a description of the data used for estimation, as well as the analysis of the empirical results in terms of the consumption scale, uncompensated and compensated flexibilities, and of the changes in household welfare induced by changes in consumed quantities. In particular, the focus of the welfare analysis carried out in the present study is on the changes in the welfare levels of the households which report zero purchases of organic commodities, when a portion of consumed non-organic commodities is substituted by their organic counterparts.[1]

2.2 LITERATURE REVIEW

2.2.1 The Inverse Almost Ideal Demand System: The Model, Extensions, Generalisations

As mentioned in the introductory section, the empirical study presented in this chapter employs the IAID system of Eales and Unnevehr (1994), and Moschini and Vissa (1992) for the estimation of demand conditions for organic and non-organic foods. The IAIDS is the inverse counterpart of, but not dual to, the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980*b*). The latter model is widely used in empirical studies of consumer demand as it has several advantages. Specifically, the cost function underlying the AIDS represents, not an arbitrary, but a specific class of preferences known as Price Independent Generalised Logarithmic (PIGLOG) preferences (Muellbauer, 1975, 1976). PIGLOG preferences have the property of exact aggregation over consumers, without invoking parallel linear Engel curves. Moreover, the PIGLOG parameterisation of the AIDS cost function is flexible, in that it has enough parameters to be considered as a local second-order approximation to the true unknown cost function. In addition, the functional form of the AIDS is

[1] The study in this chapter was presented at the *XIIth Congress of the European Association of Agricultural Economists*, Ghent, Belgium, August 26-29, 2008, under the title "The Demand for Organic Foods in UK: A Censored Inverse Almost Ideal Demand System" (with M. Genius, P. Midmore, and V. Tzouvelekas).

consistent with known household-budget data, and, as discussed below, it also allows linear estimation methods to be employed.

The starting point for the derivation of the AIDS is the following logarithmic-cost-function representation of PIGLOG preferences:

$$\log C(u,\mathbf{p}) = (1-u)\log a(\mathbf{p}) + u\log b(\mathbf{p}), \tag{2.1}$$

where,

$$\log a(\mathbf{p}) = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2}\sum_i \sum_j \gamma_{ij}^* \log p_i \log p_j, \tag{2.2}$$

$$\log b(\mathbf{p}) = \log a(\mathbf{p}) + \beta_0 \prod_j p_j^{\beta_j}, \tag{2.3}$$

and $i, j = 1,\ldots,N$ denote commodities, $\mathbf{p} = (p_1,\ldots,p_N)$ is a vector of commodity prices, and *u* denotes a utility level. As Deaton and Muellbauer (1980*b*) state, except for a few cases, *u* lies between zero and unity. Therefore, the function $\log a(\mathbf{p})$ can be thought of as the cost of subsistence, while the function $\log b(\mathbf{p})$ can be thought of as the cost of bliss. Substituting the functions $\log a(\mathbf{p})$ and $\log b(\mathbf{p})$ into the AIDS logarithmic cost function, the latter can be written as

$$\log C(u, \mathbf{p}) = \alpha_0 + \sum_j \alpha_j \log p_j + 0.5\sum_i \sum_j \gamma_{ij}^* \log p_i \log p_j + u\beta_0 \prod_j p_j^{\beta_j}, \tag{2.4}$$

where $i, j = 1,\ldots,N$ indicate commodities. Consumer theory requires that a consumer's cost function be homogeneous of degree 1 in prices, implying that the parameters of the AIDS logarithmic cost function should satisfy the following restrictions: $\sum_i \alpha_i = 1,\ \sum_i \gamma_{ij}^* = \sum_j \gamma_{ij}^* = 0,$ and $\sum_j \beta_j = 0$. Derivation of the AID system of expenditure shares starts with application of *Shephard's lemma* (Shephard, 1953) to the cost function (2.4), to obtain the compensated demands.[2] At the

[2] Shephards' lemma allows the compensated direct demand functions to be recovered from the expenditure function. In particular, Shephards' lemma states that, if the

optimum, consumer's total expenditure x is equal to $C(u, \mathbf{p})$. Inversion of this equality yields u as a function of prices and total expenditure, *i.e.*, the indirect utility function, which can then be used to uncompensate the compensated demands. This way, Deaton and Muellbauer (1980*b*) derive the following system of expenditure share equations:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(P/x), \qquad (2.5)$$

where, $\gamma_{ij} = (1/2)(\gamma_{ij}^* + \gamma_{ji}^*)$, and P is a price index defined by

$$\log P = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \log p_i \log p_j . \qquad (2.6)$$

The system of expenditure share equations in (2.5) and (2.6) should represent a system of demand functions that satisfy the budget constraint, are homogeneous of degree zero in prices and total expenditure, and yield a symmetric Slutsky matrix. This implies the following restrictions on the parameters of the AIDS: $\sum_i \alpha_i = 1$, $\sum_i \gamma_{ij} = 0$, $\sum_i \beta_i = 0$; $\sum_j \gamma_{ij} = 0$; and $\gamma_{ij} = \gamma_{ji}$. Moreover, the underlying preference structure is homothetic if $\beta_i = 0$ for all i. Finally, as it stands, the system represented by equations (2.5) and (2.6) is non-linear in its parameters. Deaton and Muellbauer (1980*b*) proposed the use of a price index such as the *Stone* price index, $\ln P^* = \sum_i w_i \ln p_i$, as an approximation to the AIDS price index in (2.6). The resulting model, often called as the Linear Approximate AIDS (LA/AIDS), does not require non-linear estimation. In addition, Deaton and Muellbauer (1980*b*) suggest

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expenditure function $C(u,\mathbf{p})$ is differentiable with respect to prices at a given point **p**, then its first partial derivatives with respect to prices yield the compensated direct demands. That is, $\partial C(u,\mathbf{p})/\partial p_i = q_i(u,\mathbf{p})$ for all $i = 1,\ldots,N$.

that the AIDS price index can also be substituted by an index such as the Stone price index when there is evidence of high collinearity in time-series data.[3]

Eales and Unnevehr (1994) applied the PIGLOG parameterisation of the cost function to the distance function, and specified the following logarithmic distance function as a starting point for the derivation of the IAIDS model

$$\ln D(u,\mathbf{q}) = (1-u)\ln a(\mathbf{q}) + u\ln b(\mathbf{q}), \tag{2.7}$$

where $\mathbf{q} = (q_1,\ldots,q_N)$ is a vector of commodity quantities, and $\ln a(\mathbf{q})$ and $\ln b(\mathbf{q})$ are defined as

$$\ln a(\mathbf{q}) = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2}\sum_i\sum_j \gamma_{ij}^* \ln q_i \ln q_j , \tag{2.8}$$

$$\ln b(\mathbf{q}) = \log a(\mathbf{q}) + \beta_0 \prod_j q_j^{-\beta_j} . \tag{2.9}$$

Hence, the IAIDS logarithmic distance function can be written as[4]

$$\ln D(u,\mathbf{q}) = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2}\sum_i\sum_j \gamma_{ij}^* \ln q_i \ln q_j + u\beta_0 \prod_j q_j^{-\beta_j} . \tag{2.10}$$

Application of the *Shephard-Hanoch lemma* (Shephard, 1953; Hanoch, 1978) to the distance function in (2.10) yields the compensated inverse demands.[5] At the optimum,

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[3] Apart from the Stone price index, other indices, which have better approximation properties (e.g., a log-linear Laspeyres index), have been proposed as approximations to the AIDS price index (see, for example, Buse (1994), and Moschini (1995)).

[4] As Eales and Unnevehr (1994) note, there is no closed-form solution for the dual of the AIDS logarithmic cost function. Hence, the logarithmic distance function underlying the IAIDS is not the dual of the AIDS logarithmic cost function.

[5] The *Shephard-Hanoch lemma* (also known as the *dual Shephard's lemma*) states that the first partial derivatives of the distance function with respect to quantities yield the compensated inverse demand functions, that is, $\partial D(u,\mathbf{q})/\partial q_i = r_i(u,\mathbf{q})$ for all

the distance is equal to unity, and, hence, $\ln D(u,\mathbf{q}) = 0$. Solving this equation for u yields the direct utility function which can be used to uncompensate the inverse demand equations. This procedure yields the following system of IAIDS expenditure shares:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln Q, \tag{2.11}$$

where $\gamma_{ij} = (1/2)\left(\gamma_{ij}^* + \gamma_{ji}^*\right)$, and Q is a quantity index defined by

$$\ln Q = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln q_i \ln q_j . \tag{2.12}$$

The IAID system represented by relations (2.11) and (2.12) must add-up to total expenditure, be homogeneous of degree zero in quantities and satisfy symmetry. These imply adding-up restrictions $\sum_i \alpha_i = 1$, $\sum_i \gamma_{ij} = 0$, and $\sum_i \beta_i = 0$; homogeneity restrictions $\sum_j \gamma_{ij} = 0$; and symmetry restrictions $\gamma_{ij} = \gamma_{ji}$ on the parameters of the IAIDS model. Finally, a linearised version of the IAIDS can be obtained by using a Stone quantity index in order to approximate the IAIDS quantity index. Since linear estimation methods could then be applied to the resulting model, Eales and Unnevehr (1994) call the latter as the Linear Approximate IAIDS (LA/IAIDS). They do note, however, that, in contrast to prices, quantities are not highly correlated. Hence, the justification employed for the use of the Stone price index in the AIDS is not as relevant for the IAIDS.

As the IAIDS retains all of the desirable theoretical properties of the AIDS, except of the property of aggregation over consumers (*i.e.*, aggregation from the micro to the market level), it is not surprising that consumer demand researchers

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$i = 1,\ldots,N$. The compensated inverse demand functions express expenditure-normalised prices as a function of a fixed utility level and a reference consumption bundle. These prices have been called by Hicks (1956) as *marginal valuation functions*, and are also known in the literature as *marginal willingness to pay*, or *shadow prices*.

welcomed the emergence of the IAIDS in the literature by adopting it or adapting it. For example, Rickertsen (1998) employed the IAIDS in order to investigate the effects of advertising on vegetable demand in Norway, using time-series data, while Holt and Goodwin (1997) extended the IAIDS to include nonlinear, nonadditive habit effects and habit stock terms (long memory) in order to estimate the demand for meat in the U.S. using quarterly meat expenditures data.

A generalisation of the IAIDS is provided by Moro and Sckokai (2002), who derived the Quadratic Inverse Almost Ideal Demand System (IQAIDS). The IQAIDS is the inverse analogue of (but not dual to) the Quadratic Almost Ideal Demand System (QAIDS) of Banks, Blundell, and Lewbel (1997), and it is a generalisation of the IAIDS in that it allows non-linear scale curves.[6] In particular, Banks, Blundell, and Lewbel (1997) proposed an indirect utility function which provides a more general representation of PIGLOG preferences by allowing quadratic Engel curves. Moro and Sckokai (2002) inverted this indirect utility function in order to derive the cost function, and extended the parametric representation of the latter to the distance function in order to obtain

$$\ln D(u,\mathbf{q}) = \ln a(\mathbf{q}) - \left[\frac{u\, b(\mathbf{q})}{1 - u\,\lambda(\mathbf{q})} \right], \tag{2.13}$$

where

$$\ln a(\mathbf{q}) = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln q_i \ln q_j \,, \tag{2.14}$$

$$\ln b(\mathbf{q}) = \prod_j q_j^{\beta_j} \,. \tag{2.15}$$

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[6] Scale curves are, for inverse demands, an analogue of Engel curves for direct demands. However, scale curves are not equivalent to Engel curves, in the same way that scale flexibilities (or scale elasticities) of inverse demands are an analogue of, but not equivalent to, expenditure elasticities of direct demands. For the relation between scale flexibilities and expenditure elasticities, see, for example, Park and Thurman (1999).

$$\ln \lambda(\mathbf{q}) = \sum_j \lambda_j \ln q_j \ . \tag{2.16}$$

Application of the Shephard-Hanoch lemma to the distance function in (2.13) yields the compensated inverse demands. Inversion of the distance function at the optimum yields the direct utility function which can be used to uncompensate the compensated inverse demand equations. Then, the following system of uncompensated inverse demands is obtained:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln q_j - \beta_i \ln a(\mathbf{q}) - \lambda_i \frac{1}{b(\mathbf{q})} \left[\ln a(\mathbf{q}) \right]^2 . \tag{2.17}$$

The homogeneity property of the distance function with respect to quantities and the symmetry property of the demand functions imply the following restriction on the parameters of the IQAID system of share equations: $\sum_i \alpha_i = 1$, $\sum_i \beta_i = 0$, $\sum_i \gamma_{ij} = 0$, $\sum_j \gamma_{ij} = 0$, $\sum_i \lambda_i = 0$, and $\gamma_{ij} = \gamma_{ji}$. Finally, the IQAIDS represented by (2.17) nests the IAIDS: the IQAIDS is reduced to the IAIDS by setting $\lambda_i = 0 \ \forall i$.

Another model that nests the IAIDS is the Inverse Lewbel Demand System (ILDS) introduced by Eales (1994). The direct Lewbel Demand System (LDS) (Lewbel, 1989) itself parametrically nests both the AIDS and the indirect translog demand model of Christensen, Jorgenson, and Lau (1975) as specialisations. The ILDS nests the IAIDS and the direct translog demand system of Christensen, Jorgenson, and Lau (1975), and allows testing whether consumer preferences are better represented by one of the two more restrictive models. Eales (1994) shows that the ILDS share equations can be derived by application of the *Hotelling-Wold identity*, (Hotelling, 1935; Wold, 1944). to the following direct logarithmic utility function[7]

[7] The Hotelling-Wold identity allows the uncompensated inverse demand functions to be derived from the direct utility function via the following relation: $\left(\partial U(\mathbf{q})/\partial q_i\right) \Big/ \sum_{j=1}^{N} q_j (\partial U(\mathbf{q})/\partial q_j) = r_i(\mathbf{q})$, $\quad i = 1, ..., N$. This identity gives the expenditure-normalised prices that would induce the consumer to choose the utility-maximising consumption bundle.

$$\ln U(\mathbf{q}) = \sum_i \beta_i \ln q_i + \ln[\ln Q], \tag{2.18}$$

where

$$\ln Q = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln q_i \ln q_j . \tag{2.19}$$

Alternatively, as Eales (1994) shows, the ILDS can be obtained from the distance function that corresponds to (2.18). The utility function is implicitly defined by $u = U(\mathbf{q}/D(u,\mathbf{q}))$. Thus, the logarithmic distance function, $\ln D(u,\mathbf{q})$, that corresponds to (2.18) can be derived by solving the implicit equation

$$\ln U(\mathbf{q}) = \sum_i \beta_i \ln\left(\frac{q_i}{D}\right) + \ln\left\{\alpha_0 + \sum_j \alpha_j \ln\left(\frac{q_j}{D}\right) + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln\left(\frac{q_i}{D}\right)\ln\left(\frac{q_j}{D}\right)\right\}, \tag{2.20}$$

for $\ln D$. Then, the following logarithmic distance function is obtained:

$$\ln D(u,\mathbf{q}) = \frac{\ln Q - u\prod_i q_i^{-\beta_i}}{1 + \sum_j \sum_k \gamma_{jk} \ln q_k}, \tag{2.21}$$

where $\ln Q$ is given by equation (2.19), and the following restrictions apply: $\sum_j \alpha_j = 1$, $\sum_j \beta_j = 0$, $\sum_i \sum_j \gamma_{ij} = 0$, and $\gamma_{ij} = \gamma_{ji}$. Application of the Shephard-Hanoch lemma to the logarithmic distance function in (2.21) yields the compensated inverse demands:

$$w_i = \frac{\alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i u \prod_k q_k^{-\beta_k}}{1 + \sum_i \sum_j \gamma_{ij} \ln q_j} - \frac{\sum_i \gamma_{ij}\left(\ln Q - u\prod_k q_k^{-\beta_k}\right)}{\left(1 + \sum_i \sum_j \gamma_{ij} \ln q_j\right)^2}. \tag{2.22}$$

Observing that at the optimum $\ln D(u,\mathbf{q}) = 0$, and hence, the last right-hand term in (2.22) is equal to zero, and substituting for the unobservable utility level, *u*, from equation (2.18), yields the ILDS share equations

$$w_i = \frac{\alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln Q}{1 + \sum_i \sum_j \gamma_{ij} \ln q_j}. \tag{2.23}$$

The IAIDS is obtained from the ILDS in (2.23) by imposing the restrictions $\sum_j \gamma_{ij} = 0 \; \forall i$, while the direct translog demand system is obtained by imposing the restrictions $\beta_i = 0 \; \forall i$.

The ILDS, in turn, is nested within a more general inverse demand system, the Hybrid Inverse Demand System (HIDS) of Holt (2002). In particular, the HIDS is a synthetic demand system that nests not only the ILDS, and, hence, the IAIDS and the direct translog demand system, but also a model which Holt (2002) calls as the Inverse Nonseparable Linear Expenditure System (INLES).[8] The HIDS is given by

$$w_i = (1-\lambda)\frac{\alpha_i + \sum_j \gamma_{ij} \ln q_j + \eta_i \ln Q}{1 + \sum_j \sum_k \gamma_{jk} \ln q_k} + \lambda \left\{ \sum_j \upsilon_{ij} q_i^{1/2} q_j^{1/2} + \beta_i \left(1 - \sum_j \sum_k \upsilon_{jk} q_j^{1/2} q_k^{1/2} \right) \right\}, \tag{2.24}$$

where $\ln Q$ is given by (2.19). The properties of adding-up, homogeneity and symmetry imply that the following restrictions should be imposed on the parameters of the HIDS: $\sum_i \alpha_i = \sum_i \beta_i = 1$, $\sum_i \eta_i = 0$, $\sum_i \sum_j \gamma_{ij} = 0$, $\gamma_{ij} = \gamma_{ji}$, and $\upsilon_{ij} = \upsilon_{ji}$. The IAIDS is obtained from the HIDS by imposing the restrictions $\sum_k \gamma_{ik} = 0 \; \forall i$, and $\lambda = 0$.

Finally, a differential form of the LA/IAIDS is nested within the model of Brown, Lee, and Seale (1995). The latter model also nests the inverse Rotterdam (RIDS) (Barten and Bettendorf, 1989), the Laitinen-Theil (Laitinen and Theil, 1979)

[8] The direct Nonseparable Linear Expenditure System is attributed to Ray (1985).

and the RAIIDS (Brown, Lee, and Seale, 1995).[9] The model of Brown, Lee, and Seale (1995), although called as "synthetic" by the authors themselves, was later proved by Matsuda (2005) to have the same scale effects as the Box-Cox scale curves do, and hence, to be a demand model in its own right (Matsuda, 2005).

2.2.2 Econometric Models for Censored Equation Systems

The previous section provided a presentation of the IAIDS, its direct counterpart, some extensions and generalisations of it, and inverse demand systems within which the IAIDS is nested. In empirical applications of consumer demand systems, however, where household-level data are employed, the presence of zero purchases in the sample requires adoption of an appropriate econometric model for estimation of censored demand equations. If the problem of zero purchases is not properly addressed, then standard system-estimation methods, such as maximum likelihood (ML), or seemingly unrelated regression (SUR), will yield biased parameter estimates. Two main approaches have emerged in the literature for the estimation of micro-level demand systems, namely, the Amemiya-Tobin model by Wales and Woodland (1983), and the Kuhn-Tucker model and its dual (Wales and Woodland, 1983; and Lee and Pitt, 1986). An alternative approach concerns two-step estimators such as the ones proposed by Heien and Wessells (1990), Shonkwiler and Yen (1999), Perali and Chavas (2000), etc. The Amemiya-Tobin model by Wales and Woodland (1983) is the approach adopted here for the estimation of our censored demand system. The advantages of this approach and, hence, the reasons why it was chosen, are discussed at the end of this section.

The Amemiya-Tobin model by Wales and Woodland (1983) is a non-trivial modification of Amemiya's (1974) extension of the Tobit model (Tobin, 1958)) for a system of equations. In the Amemiya-Tobin model preferences are non-random, *i.e.*, the consumers have the same utility function. Furthermore, the consumer is assumed to maximise a utility function subject to the budget constraint, and the utility

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9 The Laitinen-Theil model is the inverse counterpart of the Central Bureau of Statistics (CBS) model by Keller and van Driel (1985), while the RAIIDS is the inverse counterpart of the National Bureau of Research (NBR) demand model by Neves (1987).

maximising expenditure shares add-up to unity. The observed expenditure shares, however, may deviate from the utility-maximising ones due to errors of maximisation, errors of measurement of the observed shares, and other random disturbances which influence the consumer's decisions (Wales and Woodland, 1983, p. 267). The model then allows for the non-negativity restrictions by assuming that the observed shares are the sum of the utility-maximising shares plus a vector or random error terms which follow a truncated multivariate normal distribution. However, in the Amemiya-Tobin model, the adding-up restrictions hold for the latent expenditure shares but not for the observed (*i.e.*, censored) expenditure shares.[10] The modification that Wales and Woodland (1983) proposed in the context of the Amemiya-Tobin model concerns an alternative mapping of the latent to the observed expenditure shares, such that the budget constraint is satisfied, not only by the latent, but also by the observed expenditure shares. In order to present this approach, let us define the system of latent expenditure share equations as:

$$w_i^* = h_i(\mathbf{x};\mathbf{b}_i) + \varepsilon_i, \tag{2.25}$$

where $i = 1,\ldots,N$ denotes commodities, w_i^* is the demand system's (in our case, the IAIDS) *i*-th latent expenditure share, $\mathbf{x}$ is a vector of the demand system's explanatory variables, $\mathbf{b}_i$ is a conformable vector of demand parameters, and ε_i is the error term associated with the *i*-th equation. The adding-up restrictions for the parameters of the demand system imply that the latent expenditure shares satisfy the budget constraint, that is, $\sum_i w_i^* = 1$ and $\sum_i \varepsilon_i = 0$. As a consequence, the joint density function of the latent expenditure shares is degenerate and one of them, say the *N*-th expenditure share, may be dropped during estimation as redundant. The error terms associated with the remaining *N*-1 latent share equations can now be assumed to be distributed as multivariate normal with mean **0** and variance-covariance matrix **Σ**.

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[10] Censored expenditure shares, or observed expenditure shares, are dependent variables that take on the value of zero with positive probability but are continuous random variables over strictly positive values (they have a mixed discrete-continuous distribution). The (unobserved) latent expenditure shares summarise the influence of the explanatory variables on the outcome of the observed variable.

In order to ensure that the observed expenditure shares will also satisfy the budget constraint, Wales and Woodland (1983) used the following mapping of the latent to the observed expenditure shares:

$$w_i = \frac{w_i^*}{\sum_{j\in\mathbf{M}} w_j^*} \qquad \text{if } w_i^* > 0, \tag{2.26.a}$$

$$w_i = 0 \qquad \text{if } w_i^* \le 0, \tag{2.26.b}$$

where w_i^* and w_i are the latent and observed expenditure shares of good i, respectively, and $\mathbf{M}$ is the set of positive latent expenditure shares. Equation (2.26.a) defines each observed expenditure share as the ratio of the respective latent share to the sum the positive latent expenditure shares, thus forcing the observed expenditure shares to add-up to unity. This mapping of the latent to the observed expenditure shares then generates a density for expenditure shares which has the form of a partially-integrated mixed discrete-continuous multivariate distribution, *i.e.*, it is a continuous probability density function with respect to the positive observed shares and a discrete probability mass with respect to the zero observed shares. In particular, assuming that the first m goods are purchased, while consumption for the remaining N-m goods is zero, and dropping one of the latent shares as redundant, say, the last of the zero shares, w_N^*, Wales and Woodland (1983) provide the following expression for the probability density/mass function for $\mathbf{w} = \left(w_1,\ldots,w_m,0,\ldots,0\right)$:[11]

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[11] In the Amemiya-Tobin model, the mapping of latent to observed expenditure shares is: $w_i = w_i^*$ if $w_i^* > 0$, and $w_i = 0$ if $w_i^* \le 0$. The resulting probability/density mass function for $\mathbf{w} = \left(w_1,\ldots,w_m,0,\ldots,0\right)$ is less complicated than the one described in (2.27), and has the form

$$f\left(w_1,\ldots,w_m,w_{m+1},\ldots,w_{N-1};\mathbf{h},\mathbf{\Sigma}\right)$$
$$= \int_{-\infty}^{0}\cdots\int_{-\infty}^{0} f^*\left(w_1,\ldots,w_m,w_{m+1}^*,\ldots,w_{N-1}^*;\mathbf{h},\mathbf{\Sigma}\right)dw_{N-1}^*\ldots dw_{m+1}^*.$$

Nevertheless, as shown by the mapping of latent to observed expenditure shares and the expression for the probability/density mass function for the shares, there is nothing in the Amemiya-Tobin model which ensures that the observed expenditure

$$f\left(w_1,\ldots,w_m,w_{m+1},\ldots,w_{N-1};\mathbf{h},\mathbf{\Sigma}\right)$$

$$=\begin{cases}\int\limits_{w_1}^{\infty}\int\limits_{\alpha_{m+1}}^{0}\cdots\int\limits_{\alpha_{N-1}}^{0} g^*\left(w_1^*,\dfrac{w_1^* w_2}{w_1},\ldots,\dfrac{w_1^* w_m}{w_1},w_{m+1}^*,\right. \\ \qquad\qquad \left.\ldots,w_{N-1}^*;\mathbf{h},\mathbf{\Sigma}\right)J\left(\mathbf{w}\right)dw_{N-1}^*\ldots dw_{m+1}^*dw_1^* & 1\le m<N \\ g^*\left(w_1^*,\ldots\ldots,w_{N-1}^*;\mathbf{h},\mathbf{\Sigma}\right) & m=N\end{cases} \tag{2.27}$$

where $\mathbf{h}=(h_1,\ldots,h_{N-1})$, $g^*\left(\cdot\right)$ is the conditional joint normal p.d.f. of the first N-1 latent shares w_i^*, and

$$J\left(\mathbf{w}\right)=\left[1+\left(w_2/w_1\right)^2+\ldots+\left(w_m/w_1\right)^2\right]^{1/2}, \tag{2.28}$$

$$a_{m+1}=1-\sum_{i=1}^{m}w_1^*\frac{w_i}{w_1}, \tag{2.29}$$

$$a_l=a_{m+1}-\sum_{i=m+1}^{l-1}w_i^*,\qquad l=m+2,\ldots,N-1. \tag{2.30}$$

The term $J(\mathbf{w})$ is the Jacobian between the latent and the observed shares and can be ignored in the likelihood function as it does not depend on the parameters of the model (Wales and Woodland, 1983). Wales and Woodland (1983) provide an example of the application of this ML estimator in the case of the Linear Expenditure System (LES), for estimation of household demand for three types of meat. In demand systems of more than three commodities, however, estimation of the likelihood function is somewhat more involved as a simulation procedure is required for approximation of the probability/density mass functions comprising the model's likelihood function. For example, the Amemiya-Tobin model by wales and Woodland (1983) and the GHK simulator (Geweke, 1991; Hajivasiliou, McFadden and Ruud, 1996; Keane, 1994) have been employed by Dong, Gould, and Kaiser (2004), for the estimation of Mexican household demand for 12 food categories, and by Dong,

shares too will satisfy the budget constraint. The adding-up issue is dealt with by dropping one of the shares as redundant during estimation, but the resulting parameter estimates are not invariant to the equation dropped.

Kaiser, and Myrland (2007) for the assessment of the effects of advertising on Norwegian household demand for four fish and meat categories.

The second approach for the estimation of censored demand systems is the Kuhn-Tucker model proposed by Wales and Woodland (1983), and its dual model proposed by Lee and Pitt (1986). In these models, the source of zero purchases is assumed to be the consumer's inability to afford a commodity at given prices relative to his/her income. That is, the consumer is at a corner solution to his/her utility maximisation problem. In addition, both models assume that preferences are randomly distributed over the population, thus, allowing for differences in consumer tastes. As far as the Kuhn-Tucker model is concerned, its derivation starts with the maximisation of a random direct utility function, subject to budget and non-negativity constraints:

$$\max_{\mathbf{q}}\{U(\mathbf{q},\boldsymbol{\varepsilon}):\mathbf{p}\bullet\mathbf{q}\leq x,\mathbf{q}>0^N\}. \tag{2.31}$$

where $\mathbf{q}=(q_1,\ldots,q_N)$ is a vector of commodity quantities, $\boldsymbol{\varepsilon}=(\varepsilon_1,\ldots,\varepsilon_N)$ is a vector of random disturbances assumed to be jointly distributed as multivariate normal with mean vector zero and covariance matrix $\boldsymbol{\Sigma}=\{\sigma_{ij}\}$, $\mathbf{p}=(p_1,\ldots,p_N)$ is a vector of commodity prices, x denotes total expenditure, and 0^N is the N-dimensional zero vector.[12] $U(\mathbf{q},\boldsymbol{\varepsilon})$ is assumed to be a random utility function of the form $U(\mathbf{q},\boldsymbol{\varepsilon})\equiv\bar{U}(\mathbf{q})+\boldsymbol{\varepsilon}\bullet\mathbf{q}$, consisting, of a deterministic and a random component. The random component, $\boldsymbol{\varepsilon}$, of the utility function is the one that allows for differences in consumers' tastes. Thus, the difference between the Kuhn-Tucker model and the Amemiya-Tobin model by Wales and Woodland (1983) lies in the way the stochastic components are incorporated. In the former model, a random utility function is employed. The maximisation of this random utility function, with regard to non-

[12] Notation: $\mathbf{p}\bullet\mathbf{q}$ is the inner product (or, dot product) of the two (column) vectors **p** and **q**, that is, $\mathbf{p}\bullet\mathbf{q}=\sum_{i=1}^{N}p_iq_i$. Also, $\mathbf{q}\gg\mathbf{q}'$ means that $q_i>q_i'$ $\forall i$; $\mathbf{q}\geq\mathbf{q}'$ means that $q_i\geq q_i'$ $\forall i$; and $\mathbf{q}>\mathbf{q}'$ means that $q_i\geq q_i'$ $\forall i$ and for at least one element *j*, $q_j\neq q_j'$.

negativity constraints, yields the optimal consumption vector, which is allowed to occur at a corner. On the other hand, in the Amemiya-Tobin model preferences are non-random, *i.e.*, the consumers have the same utility function which they maximise subject to the budget constraint, and the utility maximising (the latent) expenditure shares add-up to unity whereas the observed expenditure shares do not. The observed expenditure shares differ from the utility-maximising ones, and random error terms are used to account for these differences.

The necessary and sufficient Kuhn-Tucker conditions for a solution to the problem in (2.31) are

$$\bar{U}_i(\mathbf{q}) + \varepsilon_i - \lambda\, p_i \le 0\,, \qquad i = 1,\dots,N\,, \tag{2.32.a}$$

$$\mathbf{p}\bullet\mathbf{q} - x = 0\,, \tag{2.32.b}$$

where $\bar{U}_i(\mathbf{q}) \equiv \partial\bar{U}(\mathbf{q})/\partial q_i$ is the marginal utility of the *i*-th commodity, and λ is the Lagrange multiplier. Let us assume that one commodity is purchased, say the first commodity. Then, at the optimum, $\bar{U}_1(\mathbf{q}) + \varepsilon_1 - \lambda\, p_1 = 0$. Solving this equality for λ, and substituting it into the remaining $N-1$ conditions in (2.32.a) yields

$$\left(p_1\varepsilon_i - p_i\varepsilon_1\right) + \left(p_1\bar{U}_i(\mathbf{q}) - p_i\bar{U}_1(\mathbf{q})\right) \le 0\,,\ i = 2,\dots,N\,, \tag{2.33.a}$$

$$\mathbf{p}\bullet\mathbf{q} - x = 0\,. \tag{2.33.b}$$

Taking into account that the budget constraint allows one commodity, say, q_1, to be written as a function of the remaining $N-1$ ones, Wales and Woodland (1983) re-write the Kuhn-Tucker conditions as

$$y_i - \bar{y}_i(\hat{\mathbf{q}}) \le 0\,, \quad i = 2,\dots,N\,, \tag{2.34}$$

where $\hat{\mathbf{q}} = \left(q_2,\dots,q_N\right)$, $y_i \equiv p_1\varepsilon_i - p_i\varepsilon_1$, and $\bar{y}_i(\hat{\mathbf{q}}) \equiv p_i\bar{U}_1(\hat{\mathbf{q}}) - p_1\bar{U}_i(\hat{\mathbf{q}})$. Since the ε_i random disturbances are assumed to be jointly distributed as multivariate normal with zero mean and constant covariance matrix $\mathbf{\Sigma}$, the y_i's – being linear functions of the

ε_i 's – are jointly distributed as multivariate normal with zero mean and non-constant covariance matrix $\boldsymbol{\Omega} = \{p_1^2\sigma_{ij} - p_i p_1 \sigma_{1i} - p_j p_1 \sigma_{1j} + p_i p_j \sigma_{11}\}$ (Pudney, 1989).

As Wales and Woodland (1983) note, the vector of random disturbances $\boldsymbol{\varepsilon}$ is known to the consumer and, to him/her, the utility function $U(\mathbf{q},\boldsymbol{\varepsilon})$ and the utility-maximising commodity vector $\mathbf{q}$ are non-stochastic. This is not the case for the researcher, though. The researcher treats the vector of random disturbances $\boldsymbol{\varepsilon}$, and, consequently, the utility-maximising commodity vector **q**, as random drawings from a population. Hence, specification of a distribution for the random disturbances as above allows, in turn, the derivation of a distribution for the consumption vector. In particular, for a purchase regime where the first m goods are purchased, while consumption for the remaining $N-m$ goods is zero, the observed demands $q_2,\ldots,q_N$ will have a mixed discrete-continuous joint distribution characterised by the following probability/density mass function[13]

$$f\left(q_2,\ldots,q_m,0,\ldots,0\right) = \begin{cases} \int_{-\infty}^{\bar{y}_N} \cdots \int_{-\infty}^{\bar{y}_{m+1}} g\left(\bar{y}_2,\ldots,\bar{y}_m,y_{m+1},\ldots,y_N;0^{N-1},\boldsymbol{\Omega}\right) \\ \qquad \times \operatorname{abs}\left|J\left(\hat{\mathbf{q}}\right)\right| dy_{m+1}\cdots y_N, & \text{if } 1 \le m < N, \\ g(\bar{y}_2,\ldots,\bar{y}_N;0^{N-1},\boldsymbol{\Omega}) \operatorname{abs}\left|J\left(\hat{\mathbf{q}}\right)\right|, & \text{if } m = N, \end{cases} \tag{2.35}$$

where $\bar{y}_i = \bar{y}_i(\hat{\mathbf{q}}) \equiv p_i \bar{U}_1(\hat{\mathbf{q}}) - p_1 \bar{U}_i(\hat{\mathbf{q}})$ and $y_i \equiv p_1\varepsilon_i - p_i\varepsilon_1$, and $J\left(\hat{\mathbf{q}}\right)$ is the Jacobian of the transformation from $\left(y_2,\ldots,y_m\right)$ to $\left(q_2,\ldots,q_m\right)$, *i.e.*,

$$J\left(\hat{\mathbf{q}}\right) = \begin{bmatrix} \dfrac{\partial \bar{y}_2(q_2,\ldots,q_m,0,\ldots,0)}{\partial q_2} & \cdots & \dfrac{\partial \bar{y}_2(q_2,\ldots,q_m,0,\ldots,0)}{\partial q_m} \\ \vdots & & \vdots \\ \dfrac{\partial \bar{y}_m(q_2,\ldots,q_m,0,\ldots,0)}{\partial q_2} & \cdots & \dfrac{\partial \bar{y}_m(q_2,\ldots,q_m,0,\ldots,0)}{\partial q_m} \end{bmatrix} \tag{2.36}$$

[13] Purchase, or demand, regime is the set of positively consumed goods.

The probability/density mass for purchase regimes where $q_1 = 0$ are obtained by re-ordering goods so that the ones with positive consumption occur first, and some other good is used in the place of good 1 for the derivation of the model. Thus, the complete probability/density mass function for $\hat{\mathbf{q}}$ is constituted upon 2^{N-1} probabilities of the type described in (2.35). The sample log-likelihood function is then the sum, over all households, of the log of the probabilities of the type in (2.35), where each household is associated with one of the purchase regime-specific probabilities. Maximisation of this likelihood function yields consistent, asymptotically efficient and normally distributed estimates of the parameters of the underlying random utility function and of the covariance matrix $\mathbf{\Sigma}$ (Wales and Woodland, 1983).

Wales and Woodland (1983) also provide an empirical application of the Kuhn-Tucker model, using the quadratic utility function, for the estimation of household demand for three types of meat, while an example of its use in the literature of recreation demand is provided by Phaneuf, Kling, and Herriges (2000). In particular, Phaneuf, Kling, and Herriges (2000), employed the Kuhn-Tucker model and a direct utility function which is a variant of the LES in order to estimate the demand for fishing in four sites of the Wisconsin Great Lakes region. The use of a direct utility function for representation of preferences, however, is a drawback of the Kuhn-Tuck model, since many widely used consumer demand models, such as the translog model, are based upon the specification of an indirect utility or a cost function.

An econometric framework for modelling corner solutions in price space has been proposed by Lee and Pitt (1986). Specifically, Lee and Pitt (1986) extended the Kuhn-Tucker approach to a dual form, taking the maximisation of the direct utility function without regard to non-negativity constraints as a starting point. That is, they start by defining the indirect utility function as the solution to the following maximisation problem,

$$V(\mathbf{r},\boldsymbol{\varepsilon}) = \max_{\mathbf{q}}\{U(\mathbf{q},\boldsymbol{\varepsilon}) : \mathbf{r}\cdot\mathbf{q} = 1\}, \qquad (2.37)$$

where $\mathbf{q} = (q_1,\ldots,q_N)$ is a vector of quantities of the N commodities, $\boldsymbol{\varepsilon} = (\varepsilon_1,\ldots,\varepsilon_N)$ is a vector of random error terms, $U(\mathbf{q},\boldsymbol{\varepsilon})$ is a strictly quasi-concave utility function, and $\mathbf{r} = (r_1, \ldots, r_N)$ is a vector of expenditure-normalised commodity prices. Application of Roy's identity yields the *notional demands*

$$q_i = \frac{\partial V(\mathbf{r}, \boldsymbol{\varepsilon}) / \partial r_i}{\sum_{j=1}^{N} r_j (\partial V(\mathbf{r}, \boldsymbol{\varepsilon}) / \partial r_j)}, \qquad i = 1, ..., N. \qquad (2.38)$$

The demands q_i in (2.38), are called by Lee and Pitt (1986) as notional because they are derived from a utility maximization problem which does not include non-negativity constraints and, thus, they may take on negative values. In this sense, the notional demands are rather latent variables corresponding to the non-negativity constrained (*i.e.*, observed) demands via virtual prices. The virtual prices are treated by Lee and Pitt (1986) as reservation, or shadow, prices which exactly support zero purchase of commodities. The derivation of the model of Lee and Pitt (1986) proceeds as follows. Assume that the demands for the first k commodities is zero, while the demands for the remaining $N-k$ ones are positive. Let $\overline{\mathbf{r}} = (r_{k+1}, \ldots, r_N)$ denote the vector of expenditure-normalised market prices of the $N-k$ positively consumed commodities, and let $\boldsymbol{\pi} = (\pi_1(\overline{\mathbf{r}}), \ldots, \pi_k(\overline{\mathbf{r}}))$ denote the vector of virtual prices that support zero consumption of the first k commodities. Then, for $i = 1, ..., k$ the denominator in Roy's identity is equal to zero, and the vector of virtual prices $\boldsymbol{\pi}$ can be solved from the equations

$$0 = \frac{\partial V(\pi_1(\overline{\mathbf{r}}), \ldots, \pi_k(\overline{\mathbf{r}}), \overline{\mathbf{r}}, \boldsymbol{\varepsilon})}{\partial r_i}, \quad i = 1, ..., k. \qquad (2.39)$$

Once the virtual prices for the non-consumed commodities are derived from (2.39), they can be substituted into the equations in (2.38). Specifically, the demands for the $N-k$ consumed commodities are given by

$$\overline{q}_i = \frac{\partial V(\pi_1(\overline{\mathbf{r}}), \ldots, \pi_k(\overline{\mathbf{r}}), \overline{\mathbf{r}}, \boldsymbol{\varepsilon}) / \partial r_i}{\sum_{j=1}^{N} r_j \left(\partial V(\pi_1(\overline{\mathbf{r}}), \ldots, \pi_k(\overline{\mathbf{r}}), \overline{\mathbf{r}}, \boldsymbol{\varepsilon}) / \partial r_j \right)}, \qquad i = k+1, ..., N. \qquad (2.40)$$

Comparison of virtual and market prices determines the consumption or non-consumption of commodities, and, thus, the purchase regimes. Thus, the purchase regime in which the first k commodities are not consumed is characterised by the inequalities

$$\pi_i(\bar{\mathbf{r}}) \leq r_i, \qquad i = 1, \ldots, k. \tag{2.41}$$

The regime switching conditions in (2.41) state that a commodity is not consumed unless the consumer's reservation price exceeds the commodity's market price. Moreover, they are shown by Lee and Pitt (1986) to be equivalent to the Kuhn-Tucker conditions when the latter are expressed in terms of virtual prices.

Lee and Pitt (1986) show how equations (2.39), (2.40), and (2.41) can be used for the derivation of the probability for each purchase regime and, hence, for the model's likelihood function. Specifically, Lee and Pitt (1986) employ the translog indirect utility function, and use Roy's identity in order to derive the notional expenditure shares. Then, they use equations (2.39) and (2.41) in order to obtain, for each of the k non-consumed commodities, an inequality relation between the i-th random error term and a function of the market prices of the i-th and of the $N-k$ positively consumed commodities. Moreover, they use relations (2.40) in order to obtain, for each of the $N-k-1$ positively consumed commodities[14], an equality relation between the i-th random error term and a function of the i-th budget share, of the market prices of the $N-k$ positively consumed commodities, and of the error terms associated with the k non-consumed commodities. Using these inequality and equality relations, and making appropriate assumptions about the joint distribution of the random error terms, they obtain the probability/density mass function for this regime. Since there are 2^{N-1} possible purchase regimes, the sample log-likelihood function is the sum, over all households, of the log of the purchase regime-specific probabilities, where each household is associated with one of the purchase regime-specific probabilities. The model is then estimated with the use of maximum likelihood. Examples of empirical applications of the dual approach of Lee and Pitt (1986) are provided in Phaneuf (1999), and Chakir, Bousquet, and Ladoux (2004).

[14] One of the positively consumed commodities is dropped as redundant since the budget constraint must be satisfied. Hence, $N-k-1$ equations in (2.40) are solved.

Specifically, Phaneuf (1999), employed a translog indirect utility function in order to estimate the demand for fishing in four sites of the Wisconsin Great Lakes region, while Chakir, Bousquet, and Ladoux (2004) used panel data and a translog cost function in order to estimate the industrial demand for electricity, gas and oil, in France.

An alternative approach to the one-step estimators presented above lies in the use of two-step estimators. Amongst the two-step procedures for estimation of censored equation systems, the two-step estimator proposed by Heien and Wessells (1990) for estimation of multivariate sample-selection models has been the most widely used in demand studies.[15,16] The Heien and Wessells (1990) procedure assumes the following system of limited dependent variables

$$w_i^* = h_i(\mathbf{x};\mathbf{b}_i) + \varepsilon_i, \tag{2.42}$$

$$d_i^* = \mathbf{z}'\mathbf{a}_i + \upsilon_i, \tag{2.43}$$

$$d_i = \begin{cases} 1 & \text{if } d_i^* > 0 \\ 0 & \text{if } d_i^* \le 0 \end{cases}, \tag{2.44}$$

$$w_i = d_i w_i^*, \tag{2.45}$$

where $i = 1, \ldots, N$ denotes commodities; w_i^* denotes the demand system's *i*-th latent expenditure share and w_i denotes the respective observed expenditure share; d_i and d_i^* are a binary indicator variable for the *i*-th commodity and its corresponding latent variable, respectively; $\mathbf{x}$ and $\mathbf{z}$ denote vectors of exogenous variables; and ε_i and υ_i are random error terms. The vector of random error terms $(\boldsymbol{\upsilon}, \boldsymbol{\varepsilon})' \equiv (\upsilon_1, \ldots, \upsilon_N, \varepsilon_1, \ldots, \varepsilon_N)'$ is assumed to be distributed as (2*N*)-variate normal with zero mean and variance-covariance matrix given by

[15] See, Shonkwiler and Yen (1999), and Chen and Yen (2005) for a review of empirical studies employing the Heien and Wessells (1990) estimator.

[16] The bivariate sample-selection model is attributed to Heckman (1979).

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}, \tag{2.46}$$

where Σ_{11} is the matrix of covariances between the υ_i's, Σ_{22} is the matrix of covariances between the ε_i's, and $\Sigma_{12} = \Sigma'_{21}$ is the matrix of covariances between the υ_i's and the ε_i's. Finally, the normalisation restriction $\sigma_\upsilon^2 = 1$ is assumed.

Equations (2.43) and (2.44) constitute a standard probit model, and provide the censoring mechanism, that is, they represent the mechanism which governs the consumer's consumption or non-consumption decision. Applying the probit technique to each equation *i* of the model in (2.43) and (2.44), consistent ML probit estimates $\hat{\mathbf{a}}_i$ are obtained. These parameter estimates are then used for computation of the inverse *Mills ratio*, which for purchased commodities, is

$$R_i = -\frac{\varphi(\mathbf{z}'\hat{\mathbf{a}}_i)}{\Phi(\mathbf{z}'\hat{\mathbf{a}}_i)}, \tag{2.47}$$

while for non-purchased commodities is given by

$$R_i = \frac{\varphi(\mathbf{z}'\hat{\mathbf{a}}_i)}{1-\Phi(\mathbf{z}'\hat{\mathbf{a}}_i)}, \tag{2.48}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the univariate standard normal probability density function and cumulative distribution function, respectively. In the second step, the inverse Mills ratio and the truncated sample (conditional on $d_i = 1$) are used in order to estimate the regression

$$w_i = h_i(\mathbf{x};\mathbf{b}_i) + \delta_i R_i + \varepsilon_i^*, \qquad i = 1,\ldots,N, \tag{2.49}$$

where δ_i is the covariance between the υ_i's and the ε_i's, and ε_i^* is a heteroscedastic error term. Since the expenditure shares in (2.49) must add-up to unity for the budget constraint to be satisfied, one of the expenditure shares must be dropped as redundant during estimation. The system of $N-1$ share equations is then estimated using SUR. However, as Heien and Wessells (1990) point out, the parameter estimates of the system in (2.49) are not invariant to the equation dropped. The adding-up property of the expenditure shares implies that $\sum_{i-1}^{N} \delta_i R_i = 0$. Yet, such a restriction is not possible in general, since the inverse Mills ratio can take on any value (Heien and Wessells, 1990).

The two-step estimator discussed above, along with more recent ones (e.g., the ones proposed by Shonkwiler and Yen (1999), and Perali and Chavas (2000)), offer simplified procedures for the estimation of systems of censored equations compared to the maximum likelihood estimators of Wales and Woodland (1983), and Lee and Pitt (1986). However, the resulting parameter estimates, although consistent, they lack in efficiency compared to the one-step estimators. Moreover, under these two-step estimators, the problem of adding-up of observed expenditure shares is inadequately dealt with, since the resulting parameter estimates are not invariant to the equation that is dropped during estimation. The problem of adding-up of observed shares is more adequately addressed in the Amemiya-Tobin model by Wales and Woodland (1983), as the budget constraint is accounted for in the mapping of latent to observed expenditure shares, as well as in the Kuhn-Tucker and its dual. The applicability of the latter models is quite limited, though, as they require that Kuhn-Tucker conditions or virtual price relationships (in the case of the dual model) be solved. In addition, application of the Kuhn-Tucker model and its dual requires that the researcher tackles the problem of incoherency of the demand model, which arises form violations of negativity of the Slutsky matrix. Specifically, van Soest, Kapteyn, and Kooreman (1993) show that if coherency is not imposed then the resulting ML parameter estimates may be inconsistent. As a solution to this problem, van Soest, Kapteyn, and Kooreman (1993) propose sufficient conditions for regularity which can be imposed during estimation, but not without a cost in terms of model flexibility. The Amemiya-Tobin model by Wales and Woodland (1983), on the other hand, can be applied to any demand system specification, and is free from the incoherency problem. The shortcoming of this model – as well as of the Kuhn-Tucker model and its dual – lies in the requirement for evaluation of multiple probability integrals in the likelihood function. The use of simulation procedures, though, such as the GHK simulator, can serve this aim even when there are many goods in the demand system.

2.3 THE MODEL AND EMPIRICAL FRAMEWORK

The demand system that is adopted for estimation of household demand for organic and non-organic milk & yoghurt and fruits & vegetables, in London, UK, is the IAIDS as represented by relations (2.11) and (2.12), which we re-write below:

$$w_i^* = \alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln Q,$$

$$\ln Q = \alpha_0 + \sum_j \alpha_j \ln q_j + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln q_i \ln q_j ,$$

where w_i^* denotes the i-th latent expenditure share. Now, relations (2.11) and (2.12) correspond to the functional form of the latent expenditure shares. As the Amemiya-Tobin model of Wales and Woodland (1983) requires, the observed expenditure shares are modelled in accordance with the mapping of latent to observed expenditure shares in relations (2.26.a) and (2.26.b), which we re-write below:

$$w_i = \frac{w_i^*}{\sum_{j \in \mathbf{M}} w_j^*} \qquad \text{if } w_i^* > 0,$$

$$w_i = 0 \qquad \text{if } w_i^* \le 0,$$

where w_i is the observed expenditure share of good i, respectively, and **M** is the set of positive latent expenditure shares.

In order to account for household heterogeneity, variables involving household characteristics must also be included in the IAIDS. A common way to include socio-demographic variables, which does not add extra non-linearity to the model, is by augmenting the shares' constant terms, α_i, so that

$$\alpha_i = \alpha_{i0} + \sum_k \zeta_{ik} Z_k , \qquad (2.50)$$

where Z_k denotes socio-demographic variables. Since our system of latent expenditure shares must add-up to unity, restrictions must be imposed on the coefficients of the socio-demographic variables as well. That is, the adding-up restriction $\sum_i \zeta_{i\kappa} = 0$ must supplement the adding-up, homogeneity and symmetry restrictions $\sum_i \alpha_i = 1$, $\sum_i \gamma_{ij} = 0$, $\sum_i \beta_i = 0$, $\sum_j \gamma_{ij} = 0$, and $\gamma_{ij} = \gamma_{ji}$.

As already discussed in Chapter 1, the presence of zero purchases in the sample not only gives rise to a censoring problem, but also prohibits the use of widely used inverse demand systems, such as the IAIDS, since the logarithm of zero consumption cannot be defined. One way to deal with this problem would be to estimate the IAIDS system only for the households that report positive purchases, but this may result in sample selection bias. Another way is full-sample estimation by assigning some small positive number or unity to the zero purchases. However, this approach has also serious drawbacks: it is not independent of the units of measurement of the respective explanatory variable(s), and if there is a large number of households in the sample that report zero purchases then the resulting parameter estimates may be biased (Battese, 1997). In order to overcome this problem, we employ the approach proposed by Battese (1997), and modify the IAIDS share equations to include 0-1 dummy variables indicating positive-zero consumption of commodities, and redefine the explanatory variables for the non-purchased commodities as the maximum between the actual value of the explanatory variable and the respective dummy.[17] For example, in a case of N commodities where the first N-1 commodities are purchased by all the households in the sample, whereas zero purchase of the N-th commodity is reported by some of the households, the IAIDS latent expenditure share equation for the i-th commodity can be re-written as

$$\begin{aligned} w_i^* &= \alpha_i + \delta_N D_N + \sum_{j=1}^{N-1} \gamma_{ij} \ln q_j + \gamma_{iN} \ln q_N^* \\ &+ \beta_i \begin{pmatrix} \alpha_0 + \sum_{j=1}^{N-1} \alpha_j \ln q_j + \alpha_N \ln q_N^* + \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \gamma_{ij} \ln q_i \ln q_j \\ + \frac{1}{2} \sum_{i=1}^{N-1} \gamma_{iN} \ln q_i \ln q_N^* + \frac{1}{2} \sum_{j=1}^{N-1} \gamma_{Nj} \ln q_N^* \ln q_j + \frac{1}{2} \gamma_{NN} \left(\ln q_N^*\right)^2 \end{pmatrix} \end{aligned} \tag{2.51}$$

[17] An example of the use of the Battese (1997) approach is also provided by Battese, Malik, and Gill (1996), who applied it the context of the stochastic production frontier estimation where zero input use is reported by farmers in the sample.

where D_N denotes the dummy variable for the *N*-th commodity, which takes on the value of zero for positive purchases of commodity *N* and the value of unity otherwise, and q_N^* is defined as

$$\text{if } q_N > 0\text{, then: } D_N = 0 \quad \text{and} \quad \ln q_N^* = \ln\left(\max\{q_N, D_N\}\right) = \ln q_N\,,$$

$$\text{if } q_N = 0\text{, then: } D_N = 1 \quad \text{and} \quad \ln q_N^* = \ln\left(\max\{q_N, D_N\}\right) = \ln(1) = 0\,.$$

Under this example, the model represented by equation by (2.51) can be estimated for all the households in the sample. In conclusion, in the case of an inverse demand system where the explanatory variables involving purchased quantities are expressed in logarithms, this technique is very convenient. It allows the same system of expenditure share equations to be estimated for all households, either reporting zero purchases or reporting positive purchases, without resulting in biased estimates. Moreover, as Battese (1997) points out, the additional constant parameter introduced by the dummy variable whenever zero purchases are reported acts only as a demand shifter, leaving the slope of the demand functions and the own- and cross-quantity flexibilities unaffected.

The IAIDS for the four commodity groups and three socio-demographic variables was modified so that household heterogeneity and the problem of the logarithm of explanatory variables taking on zero values are accounted for. As zero purchases have been reported for all the commodities in our system except for the non-organic fruits & vegetables category, three $0-1$ dummy variables are included in the model. Thus, the *i*-th equation of the final IAID system of latent expenditure share equations is given by:

$$w_i^* = \alpha_{i0} + \sum_{k=1}^{3}\zeta_{ik}Z_k + \sum_{j=1}^{3}\delta_{ij}D_j + \sum_{j=1}^{3}\gamma_{ij}\ln q_j^* + \gamma_{i4}\ln q_4 + \beta_i \ln Q + \varepsilon_i\,, \tag{2.52}$$

$$\begin{aligned} \ln Q = \alpha_0 &+ \sum_{j=1}^{3}\alpha_j \ln q_j^* + \alpha_4 \ln q_4 + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\gamma_{ij}\ln q_i^* \ln q_j^* \\ &+ \frac{1}{2}\sum_{i=1}^{3}\gamma_{i4}\ln q_i^* \ln q_4 + \frac{1}{2}\sum_{j=1}^{3}\gamma_{4j}\ln q_4 \ln q_j^* + \frac{1}{2}\gamma_{44}\left(\ln q_4\right)^2, \end{aligned} \tag{2.53}$$

where Z_k indicates variables involving household characteristics (age of the main shopper in the family, household's social class, and number of children), D_j indicates commodity-specific dummies taking the value of zero if $q_j > 0$ and the value of one if $q_j = 0$, $\ln q_j^* = \ln\left(\max\{q_j, D_j\}\right)$, and ε_i is the error term associated with the *i*-th share equation. The introduction of commodity-specific dummies and the budget constraint now require that the adding-up restriction $\sum_i \delta_{ij} = 0$ be imposed on the δ_{ij} parameters. Finally, in order to accommodate heteroscedasticity of the random errors, it is assumed that the diagonal elements of the variance-covariance matrix of the error terms depend additively on the social class and the number of children in the family. That is, for each household, the variance of the error term associated with the *i*-th commodity is given by

$$\sigma_i^2 = \theta_{0i} + \theta_{1i}\text{Class} + \theta_{2i}\text{Children}\,. \tag{2.54}$$

In order to estimate the model given by equations (2.52)-(2.54), (2.26.a), and (2.26.b), the probability/density mass function in (2.27), for purchase regimes where more than two commodities are not purchased, is evaluated via simulation. In the present study, we follow the suggestions by Tallis (1965) for reducing plane truncation of normal distributions to rectangular truncation, in order to transform the probability/density mass function for these regimes into rectangular normal probabilities. The latter are then approximated by the GHK simulator.[18]

Once the IAIDS has been estimated, the economic interpretation of the parameter estimates requires computation of the consumption scale, uncompensated, and compensated (*Antonelli*) flexibilities, based on the unconditional expectation of the observed shares. In order to derive the expected observed shares, the simulation procedure suggested by Dong, Gould, and Kaiser (2004) is followed: we start by

[18] In this study, application of the transformations suggested by Tallis (1965) is enough to allow the GHK simulation procedure to be used. For demand systems of more than four commodities, however, additional transformations are required. These transformations can be found in the appendix in Dong, Gould and Kaiser (2004), and also in the appendix in Dong, Kaiser and Myrland (2007).

simulating the $N-1$ error terms in (2.52), then substitute the simulated error terms in (2.52) to derive the simulated latent shares, and, finally, we use the simulated latent shares and the mapping rule of Wales and Woodland (1983) in order to compute the expected observed shares. Specifically, the procedure of Dong, Gould and Kaiser (2004) starts with the estimation of *R* replicates of the $N-1$ error terms of the latent share equations. Let us write the IAIDS *i*-th latent expenditure share, in a more compact form, as $w_i^* = h_i(\mathbf{x};\mathbf{b}_i) + \varepsilon_i$. The $N-1$ simulated latent shares are then calculated by

$$w_{ir}^* = h_i(\overline{\mathbf{x}};\hat{\mathbf{b}}_i) + \varepsilon_{ir}, \tag{2.55}$$

where $\overline{\mathbf{x}}$ is the vector of the exogenous variables evaluated at the sample means, $\hat{\mathbf{b}}_i$ denotes the vector of the maximum likelihood parameter estimates, and ε_{ir} is the *r*th replicate of the *i*-th error term. The *N*-th simulated latent share is computed via the adding-up property of the latent shares. The *r*th replicate of the *i*-th observed share is derived according to the mapping rule of Wales and Woodland (1983), *i.e.*,

$$w_{ir} = \frac{w_{ir}^*}{\sum_{j\in\mathbf{M}} w_{jr}^*} \qquad \text{if } w_{ir}^* > 0, \tag{2.56.a}$$

$$w_{ir} = 0 \qquad \text{if } w_{ir}^* \le 0. \tag{2.56.b}$$

Hence, the expected value of the *i*th observed share can be computed as the average of its *R* replicates, that is,

$$E(w_i) = \frac{1}{R}\sum_{r=1}^{R} w_{ir} \tag{2.57}$$

Dong, Gould, and Kaiser (2004) substitute the simulated expected observed shares in an arch elasticity formula in order to calculate the simulated uncompensated elasticities for the direct AIDS. Following their suggestion, we compute the change in the normalised price of good i that would induce the consumer to absorb a small increase in the quantity of good j with the use of the following formula:

$$f_{ij} = -\lambda_{ij} + \frac{\Delta E(w_i)}{\Delta q_j} \frac{q_j + \Delta q_j/2}{E(w_i) + \Delta E(w_i)/2} \tag{2.58}$$

where $\lambda_{ij} = 1$ for $i = j$ and $\lambda_{ij} = 0$ for $i \neq j$, and $\Delta E(w_i)$ is the change in the expected value of the *i*-th observed expenditure share due to a small change in the quantity of commodity *j*, Δq_j. Once the simulated uncompensated flexibilities, f_{ij}, have been computed, the simulated consumption scale flexibilities, f_i, and the simulated compensated flexibilities, f_{ij}^*, can be computed from the following relations,[19]

$$f_i = \sum_j f_{ij} \tag{2.59.a}$$

$$f_{ij}^* = f_{ij} - w_j f_i \tag{2.59.a}$$

The estimated parameters of the IAIDS can also be used for analysing the effects of changes in consumed quantities on households' welfare levels. In particular, Palmquist (1988), and Kim (1997) have shown how the distance function can be used for the computation of exact welfare measures for quantity changes. Using the estimated parameters of the IAIDS for recovering the underlying logarithmic distance function, the *compensating variation* (CV) and *equivalent variation* (EV) for a change in commodity quantities from $\mathbf{q}^0$ to $\mathbf{q}^1$ can be obtained as follows:

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[19] Scale flexibilities measure the change in the normalised price of good *i* in response to a scale expansion in the consumption bundle. Compensated flexibilities (or compensated quantity elasticities) are the analogues to compensated elasticities of direct demands, and they measure the change in the consumer's marginal valuation of good *i*, resulting from a marginal change in the consumption of good *j*, which is required for the consumer's utility level to remain unchanged. Relations (2.59.a) and (2.59.b) were derived by Anderson (1980). Relation (2.59.a) is the restriction of homogeneity of degree zero of inverse demands, expressed in terms of flexibilities, whereas relation (2.59.b) is the Antonelli equation, *i.e.*, the analogue to the Slutsky equation for direct demands.

$$CV = D(u^0, \mathbf{q}^1) - D(u^0, \mathbf{q}^0) = D(u^0, \mathbf{q}^1) - 1, \qquad (2.60)$$

$$EV = D(u^1, \mathbf{q}^1) - D(u^1, \mathbf{q}^0) = 1 - D(u^1, \mathbf{q}^0), \qquad (2.61)$$

where u^0 and u^1 denote the initial and new utility levels, respectively, and $\mathbf{q}^0$ and $\mathbf{q}^1$ denote the initial and new consumption vectors, respectively. The initial and new utility levels are obtained by setting the distance function $D(u^0, \mathbf{q}^0)$ and $D(u^1, \mathbf{q}^1)$, respectively, equal to 1 and solving for the utility level. CV is the amount of additional (normalised) expenditure necessary for a consumer facing the new commodity bundle $\mathbf{q}^1$ to maintain the initial utility level u^0. EV, on the other hand, is the amount of additional (normalised) expenditure necessary for a consumer to achieve the new utility level u^1 while facing the initial commodity bundle $\mathbf{q}^0$. For $\mathbf{q}^1 < (>) \mathbf{q}^0$, CV measures willingness to pay (accept), while EV measure willingness to accept (pay). Negative (positive) values of CV and EV imply that the consumer is better (worse) off under the new commodity vector $\mathbf{q}^1$. Finally, the CV and EV measures presented in eqs. (2.60) and (2.61) can be expressed as non-normalised measures of welfare change by their multiplication with total expenditure.

2.4 DATA DESCRIPTION AND EMPIRICAL RESULTS

2.4.1. Data Description

The data used in the empirical analysis are drawn from a British household survey conducted by the *TNS* market research institute. The survey provides information on weekly purchases of organic and non-organic milk, yoghurt, fruits, and vegetables. The surveyed households reported the volume of and expenditure on the aforementioned organic and non-organic commodities purchased at every shopping trip, as well as the shop that was visited and the time spend. The data base also includes information on the socio-demographic characteristics of the surveyed households, such as the number of adults and children in the household, the age of the main shopper in the family, social class, and the region of residence.

Table 2.1. Summary Statistics of the Data.

Variable	Mean	Min.	Max.	Standard Deviation
Expenditure shares				
Organic Milk & Yoghurt				
All households	0.04	0.00	0,79	0.10
Households consuming (T=390)	0.13	0.00	0,79	0.13
Non-Organic Milk & Yoghurt				
All households	0.26	0.00	0,95	0.19
Households consuming (T=1061)	0.28	0.01	0,95	0.18
Organic Fruits & Vegetables				
All households	0.03	0.00	0,70	0.08
Households consuming (T=401)	0.10	0.01	0,70	0.11
Non-Organic Fruits & Vegetables				
All households	0.66	0.02	1,00	0.19
Households consuming (T=1155)	0.66	0.02	1,00	0.19
Quantities purchased				
Organic Milk & Yoghurt				
All households	1.42	0.00	38.62	4.16
Households consuming (T=390)	4.20	0.13	38.62	6.30
Non-Organic Milk & Yoghurt				
All households	8.39	0.00	55.66	8.90
Households consuming (T=1061)	9.13	0.15	55.66	8.91
Organic Fruits & Vegetables				
All households	0.55	0.00	21.39	1.53
Households consuming (T=401)	1.59	0.15	21.39	2.26
Non-Organic Fruits & Vegetables				
All households	9.60	0.19	58.54	7.44
Households consuming (T=1155)	9.60	0.19	58.54	7.44
Household characteristics				
Age of main shopper	50.05	22	84	15.25
Social class	3.66	1	6	1.29
Number of children	0.60	0	6	0.96

Note: The sample consists of 1155 households. T denotes number of households. The explanatory variable for social class takes on the value of 1 for the highest social class and the value of 6 for the lowest one.

For the purposes of the present analysis, the region of London was selected in order to avoid problems associated with the consumption of home-grown agricultural products in rural areas. The final data set used for estimation of the IAIDS is a cross section of 1155 households in London, and contains household consumption and expenditure data on four commodity groups, namely, organic milk & yoghurt, non-organic milk & yoghurt, organic fruits & vegetables, and non-organic fruits & vegetables, for November 2006. Aggregation of quantities for the creation of the milk & yoghurt, and fruits & vegetables commodity groups was carried out with the use of *Divisia* indices, with expenditure shares serving as weights. The quantities of the fruits & vegetables group are measured in kilograms, while the quantities of milk and yoghurt, before aggregation, were measured in litres and kilos, respectively. Total expenditure is the sum of expenditures on all commodities, and is measured in Pound-Sterling.

The socio-demographic variables used in our analysis are the age of the main shopper in the household, the household's social class, and the number of children in the family. Social class is a classification of the households into six categories representing social grade, social status, and occupation. In particular, the classification is: Class A (upper middle class, higher managerial, administrative or professional), Class B (middle class, intermediate managerial, administrative or professional), Class C1 (lower middle class, supervisory or clerical, junior managerial, administrative or professional), Class C2 (skilled working class, skilled manual workers), Class D (working class, semi and unskilled manual workers), and Class E (households at lowest level of subsistence, state pensioners or widows (no other earner), casual or lowest grade workers).[20] The descriptive statistics for the household data are summarised in Table 2.1. As shown in Table 2.1, non-organic milk & yoghurt, and non-organic fruits & vegetables are associated with the highest average expenditure shares, as 1061 (92%) and 1155 households (100%) households, respectively, reported positive consumption for these commodity groups, whereas 390 (34%) and 401 households (35%) households reported positive consumption of organic milk & yoghurt, and organic fruits & vegetables, respectively.

2.4.2 Empirical Results

Estimates of the parameters of the IAIDS were obtained from maximum likelihood estimation of the model using the GAUSS software. The IAIDS was estimated with

[20] In estimation, the explanatory variable used for the households' social class takes on the value of 1 for the highest social class and the value of 6 for the lowest one.

the homogeneity and symmetry restrictions imposed, and the constant parameter α_0 set to zero.[21] Likelihood Ratio tests that were carried out indicated that the error terms in the share equations for organic milk & yoghurt and organic fruits & vegetables are heteroscedastic. Therefore, heteroscedastic rather than a homoscedastic error structure was employed for these commodity groups. The share equation that was dropped as redundant during estimation was the one for organic milk & yoghurt. Estimates of the parameters associated with this commodity group were computed via the adding-up, homogeneity, and symmetry restrictions, and their standard errors were approximated by the delta method (see, for example, Spanos (1999)).

The maximum likelihood parameter estimates, along with their standard errors, for the socio-demographic, quantity, total consumption, and dummy coefficients are presented in Table 2.2. As shown in Table 2.2, all the all the own- and cross-quantity coefficients, γ_{ij}, were statistically significant at the 1% level of significance. In addition, three out of the four total consumption coefficients, β_i, were found to be statistically significant. Regarding the parameter estimates of the socio-demographic coefficients, eight out of the twelve parameters were found to be statistically different from zero. In particular, the coefficient of the age of the main shopper in the household is statistically significant in all expenditure share equations except in the one for organic fruits & vegetables, and appears to have a positive effect on the expenditure shares for the organic and non-organic milk & yoghurt and a negative one on the expenditure share for non-organic fruits & vegetables. The coefficient of social class is statistically significant in all expenditure share equations except in the one for non-organic fruits & vegetables. In addition, results indicate that as the value of this socio-demographic variable increases (*i.e.*, as we are moving to lower social classes) the expenditure share for organic milk & yoghurt tends to decrease, and the share for organic fruits & vegetables tends to increase. Finally, the coefficient of the number of children in the family is statistically significant only in the expenditure share equations for the non-organic commodities, and appears to have a small and positive

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[21] Eales and Unnevehr (1994) report problems with the estimation of α_0. Identification of this parameter is problematic since it appears only in the IAIDS quantity index. The same problem arises in the AIDS as well (Deaton and Muellbauer, 1980*b*).

(negative) effect on the expenditure share associated with non-organic milk & yoghurt (non-organic fruits & vegetables).

Table 2.2. Parameter Estimates of the IAIDS for Organic and Non-Organic Foods in London, UK

	Organic Milk & Yoghurt	Non-Organic Milk & Yoghurt	Organic Fruits & Vegetables	Non-Organic Fruits & Vegetables
Intercept	0.0168	0.2609	0.0095	0.7128
	(0.0135)	(0.0083)*	(0.0146)	(0.0195)*
Household Characteristics				
Age	0.0605	0.0463	0.0066	-0.1134
	(0.0093)*	(0.0056)*	(0.0111)	(0.0144)*
Soc. Class	-0.0307	0.0088	0.0178	0.0041
	(0.0154)**	(0.0037)**	(0.0076)**	(0.0107)
No. of Children	0.0009	0.0073	0.0001	-0.0083
	(0.0022)	(0.0008)*	(0.0021)	(0.0028)*
Quantities				
Organic Milk & Yoghurt	0.1774			
	(0.0057)*			
Non-Organic Milk & Yoghurt	-0.04	0.1346		
	(0.0027)*	(0.0011)*		
Organic Fruits & Vegetables	-0.011	-0.0196	0.0741	
	(0.0034)*	(0.0020)*	(0.0031)*	
Non-Organic Fruits & Vegetables	-0.1264	-0.075	-0.0435	0.2449
	(0.0045)*	(0.0019)*	(0.0037)*	(0.0057)*
Total Consumption	0.129	-0.0065	0.0023	-0.1248
	(0.0071)*	(0.0038)***	(0.0049)	(0.0065)*
Commodity Specific Dummies				
Organic Milk & Yoghurt	-0.2311	0.0079	0.0247	0.1985
	(0.0098)*	(0.0069)	(0.0061)*	(0.0112)*
Non-Organic Milk & Yoghurt	0.0162	-0.2678	0.0587	0.1929
	(0.0077)**	(0.0171)*	(0.0108)*	(0.0203)*
Organic Fruits & Vegetables	0.0474	0.0105	-0.2199	0.162
	(0.0066)*	(0.0054)**	(0.0065)*	(0.0099)*
Log-Likelihood	-865.3029			

Notes: Asymptotic standard errors in parentheses. * (**, and ***) indicate significance at the 1% (5%, and 10%).

Interpretation of the γ_{ij} and β_i parameter estimates is provided by the (simulated) consumption scale, uncompensated and compensated flexibilities presented in Tables 2.3-2.5. The scale flexibilities, shown in Table 2.3, measure the change in the expenditure-normalised price of good *i* (i.e., the consumer's marginal valuation of good *i*) in response to a scale expansion in the consumption bundle. Scale flexibilities can be used to classify goods as luxuries $(f_i > -1)$ or necessities $(f_i < -1)$. In our case, organic milk & yoghurt are classified as luxuries, while the scale flexibility for the rest of the commodity groups are quite close to –1.

Table 2.3. Simulated Consumption Scale Flexibilities.

Commodities	Scale Flexibilities
Organic Milk & Yoghurt	-0.2292
Non-Organic Milk & Yoghurt	-0.9413
Organic Fruits & Vegetables	-0.9259
Non-Organic Fruits & Vegetables	-1.0767

The (simulated) compensated (*Antonelli*) flexibilities are reported in Table 2.5. Compensated flexibilities measure the change in the consumer's valuation for good *i*, required for him/her to remain on the initial indifference curve, in response to a change in the quantity of good *j*. As pointed out by Barten and Bettendorf (1989), however, compensated flexibilities are imperfect measures of the interrelationships between commodities because the negative-semidefiniteness of the *Antonelli* matrix and the homogeneity restriction lead to dominance of complementarity in the *Antonelli* matrix (*i.e.* dominance of positive cross-quantity effects).[22] Here, the majority of the cross-quantity compensated flexibilities are positive, implying *net q-complementarity* between the commodities in the demand system. In addition, as Dong, Gould, and Kaiser (2004) note, symmetry restrictions required by economic theory are applied to the latent shares. Thus, the use of observed shares and simulated

[22] Dominance of complementarity in the *Antonelli* matrix has been reported in several studies of consumer demand for goods that are *a priori* expected to be substitutes, such as different categories of meats (e.g., Eales and Unnevehr, 1994; Holt, 2002) and fish (e.g., Barten and Bettendorf, 1989; Fousekis and Karagiannis, 2001).

uncompensated flexibilities (elasticities in the paper of Dong, Gould, and Kaiser (2004)) for the computation of the simulated compensated flexibilities does not guarantee that the latter will be symmetric. For these reasons, we will not discuss the compensated flexibilities, and turn to the analysis of uncompensated flexibilities instead.

Table 2.4. Simulated Uncompensated Flexibilities.

Prices \ Quantities	Organic Milk & Yoghurt	Non-Organic Milk & Yoghurt	Organic Fruits & Vegetables	Non-Organic Fruits & Vegetables
Organic Milk & Yoghurt	-0.2219	-0.0614	0.0270	0.0271
Non-Organic Milk & Yoghurt	-0.0264	-0.5925	-0.0263	-0.2961
Organic Fruits & Vegetables	-0.0250	-0.1537	-0.3410	-0.4062
Non-Organic Fruits & Vegetables	-0.0684	-0.1291	-0.0325	-0.8467

Table 2.5. Simulated Compensated (Antonelli) Flexibilities.

Prices \ Quantities	Organic Milk & Yoghurt	Non-Organic Milk & Yoghurt	Organic Fruits & Vegetables	Non-Organic Fruits & Vegetables
Organic Milk & Yoghurt	-0.2119	-0.0023	0.0351	0.1792
Non-Organic Milk & Yoghurt	0.0147	-0.3498	0.0066	0.3285
Organic Fruits & Vegetables	0.0154	0.0851	-0.3086	0.2081
Non-Organic Fruits & Vegetables	-0.0214	0.1485	0.0051	-0.1323

As shown in Table 2.4, all own-quantity uncompensated flexibilities are negative, *i.e.*, each commodity is its own substitute, and are lower than one in absolute values. Small responses of normalised prices to own-quantity changes suggest that, in terms of a direct demand system, the goods in the consumption bundle are price elastic. A good reason for this is that organic and non-organic commodities are very good substitutes for one another, and, consequently, the demand for them is quite elastic. In particular, the organic foods in our demand system are less quantity elastic and, hence, more price elastic than their non-organic counterparts, which is to be expected since they are more expensive than the latter. Moreover, each of the cross-quantity flexibilities is lower, in absolute values, than the corresponding own-quantity flexibility, indicating that increases in the consumption of commodity *i* mostly affect the expenditure-normalised price (*i.e.*, consumer's valuation, or willingness to pay) of that commodity itself. Thus, it is the expenditure-normalised prices of the organic commodities themselves, and not those of their non-organic counterparts, that play the most important role in inducing consumers to increase their consumption of organic commodities.

As far as the substitution effects are concerned, the sign of the cross-quantity uncompensated flexibilities indicates that organic and non-organic milk & yoghurt, as well as organic and non-organic fruits & vegetables, are *gross q-substitutes* as expected. Our results also suggest that non-organic milk & yoghurt and the two fruits & vegetables groups are *gross q-substitutes*, while the type of the interrelationship between organic milk & yoghurt and the two fruits & vegetables groups cannot be inferred. A comparison of the cross-quantity flexibilities by pairs of organic and non-organic commodities shows that the response of the expenditure-normalised prices of the non-organic commodities to changes in the quantities of the organic ones is smaller (in absolute values) than the response of the expenditure-normalised price of the organic commodities to changes in the quantities of the non-organic ones. That is, it takes a comparatively higher decrease in consumer's valuation of organic commodities to induce consumers to increase the consumption of non-organic ones. Put another way, despite the higher prices of organic commodities, it is relatively difficult to induce consumers habitually buying organic commodities to "revert" to non-organic ones. This situation is even more striking in the case of organic and non-organic fruits & vegetables. An explanation for this could be that organic fruits and vegetables have made their appearance in the market well before organic milk and yoghurt did, and as a result, there are more varieties of organic fruits and vegetables for the consumers to choose amongst and their price premiums are not as high as the price premiums of organic milk and yoghurt.[23] In short, the values of the cross-quantity flexibilities, along with organic commodities' being more price-elastic than their non-organic counterparts, imply that small decreases in the prices of the organic

[23] For the sample used in the present study, the percentage difference in unit values (*i.e.*, the price of the *i*-th commodity group computed from expenditure and quantity data) between organic and non-organic milk & yoghurt is larger than the difference in unit values between organic and non-organic fruits & vegetables.

commodities and/or small increases in the prices of the non-organic commodities can increase the demand for the organic commodities substantially.

Table 2.6. Compensating and Equivalent Variation for Households Reporting Positive Consumption of Non-Organic Milk & Yoghurt and Zero Consumption of Organic Milk & Yoghurt.

Quantity Changes	Household Group	CV	EV
Substitution of 5% of non-organic milk & yoghurt by organic ones	Higher social classes	3.81	1.90
	Lower social classes	2.82	1.50
		(2.1134)*	(1.9983)*
	Without children	3.42	1.72
	With children	3.31	1.75
		(0.2397)	(-0.1062)
	All households	3.38	1.73
Substitution of 10% of non-organic milk & yoghurt by organic ones	Higher social classes	1.40	0.81
	Lower social classes	0.96	0.62
		(1.5393)	(0.9539)
	Without children	1.22	0.70
	With children	1.19	0.77
		(0.1100)	(-0.3092)
	All households	1.21	0.73
Substitution of 15% of non-organic milk & yoghurt by organic ones	Higher social classes	0.96	0.67
	Lower social classes	0.72	0.57
		(0.7972)	(0.3315)
	Without children	0.86	0.60
	With children	0.85	0.67
		(0.0269)	(-0.2181)
	All households	0.85	0.63

Notes: Average CVs and EVs are measured in Pound-Sterling. This sub-sample consists of 723 households (full sample size is 1155). The group of households in higher (lower) social classes consists of 411 (312) households in social classes A, B, and C1 (C2, D, and E). The group of households without children consists of 476 households. The group of households with children consists of 247 households with 1 to 5 children. *t*-statistics for the differences in CV and EV between household groups are in parenthesis, and * indicates significance at the 5% level.

Substitution of the estimated parameters of the demand system back into the IAIDS logarithmic distance function in (2.10) allows derivation of the CV and EV measures of welfare change due to changes in consumed quantities. For the purposes of the present analysis, it is of interest to examine the households in our sample which reported positive consumption of non-organic commodities and zero consumption of organic ones, and to analyse these households' welfare changes due to substitution of a portion of the non-organic commodities by organic ones. Table 2.6 provides the average CV and EV measures of welfare change (measured in Pound-Sterling) due to substitution of a 5%, 10% and 15% of the consumed non-organic milk & yoghurt by organic ones. As shown in Table 2.6, the average CVs and EVs for all the 723 households in the sub-sample are positive, indicating that the households in this sub-sample become worse off under the substitution of a portion of non-organic milk & yoghurt by their organic counterparts. However, as the portion of non-organic milk & yoghurt which is substituted by their organic counterparts increases, the average CVs and EVs diminish, indicating that the households become less and less worse off. The results are also broken down according to household groups. Specifically, we computed the average CVs and EVs for the following groups: households in higher social classes (social classes A, B, and C1), households in lower social classes (classes C2, D, and E), households without children, and households with children. The hypothesis that the difference between the average CVs (EVs) for the households in higher and lower social classes is zero was tested, and it was rejected only for the case of substitution of 5% of non-organic milk & yoghurt by organic ones.[24] Thus, in the case of substitution of 5% of non-organic milk & yoghurt by organic ones, households in different social classes are affected differently by these quantity changes, with households in higher social classes becoming more worse off than the households in lower ones. Similar tests which were also carried out for the household groups with and without children indicate that the substitution of a portion of non-organic milk & yoghurt by organic ones does not affect the average CVs and EVs of

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[24] The hypothesis that the difference between the average CVs for households in different groups is zero was tested using the statistic

$$\left(\overline{CV}_1 - \overline{CV}_2\right)\Big/\sqrt{s_1^2/T_1 + s_2^2/T_2}\,,$$

where $\overline{CV}_1$ and $\overline{CV}_2$ are the average CVs for household groups 1 and 2, and s_1 and s_2 are the standard deviations of the CVs for household groups 1 and 2. This statistic follows the *t* distribution with $\nu = T_1 + T_2 - 2$ degrees of freedom, where T_1 and T_2 denote the number of households in groups 1 and 2, respectively. Likewise for the EVs.

these household groups differently. A more detailed analysis of the results, according to smaller household groups, indicates that households in higher and lower social classes, *without children*, are affected differently by the substitution of 5% , or 10%, of non-organic milk & yoghurt by organic ones, while this difference vanishes for the 15% substitution case (see Table A.1 in the Appendix).

Table 2.7. Compensating and Equivalent Variation for Households Reporting Positive Consumption of Fruits & Vegetables and Zero Consumption of Organic Fruits & Vegetables.

Quantity Changes	Household Group	CV	EV
Substitution of 5% of	Higher social classes	1.57	1.22
non-organic	Lower social classes	1.96	1.56
fruits & vegetables		(-1.7534)*	(-1.6777)*
by organic ones	Without children	1.75	1.38
	With children	1.77	1.39
		(-0.0820)	(-0.0443)
	All households	1.75	1.38
Substitution of 10% of	Higher social classes	-2.61	-3.15
non-organic	Lower social classes	-2.08	-2.54
fruits & vegetables		(-1.6848)*	(-1.6297)
by organic ones	Without children	-2.38	-2.89
	With children	-2.32	-2.82
		(-0.1834)	(-0.1624)
	All households	-2.36	-2.87
Substitution of 15% of	Higher social classes	-4.78	-6.07
non-organic	Lower social classes	-4.17	-5.26
fruits & vegetables		(-1.5755)	(-1.5534)
by organic ones	Without children	-4.52	-5.73
	With children	-4.44	-5.62
		(-0.2009)	(-0.1898)
	All households	-4.50	-5.69

Notes: Average CVs and EVs are measured in Pound-Sterling. This sub-sample consists of 754 households (full sample size is 1155). The group of households in higher (lower) social classes consists of 406 (348) households in social classes A, B, and C1 (C2, D, and E). The group of households without children consists of 517 households. The group of households with children consists of 237 households with 1 to 6 children. *t*-statistics for the differences in CV and EV between household groups are in parenthesis, and * indicates significance at the 10% level.

A similar analysis is carried out for fruits & vegetables. Table 2.7 presents the average CVs and EVs for the households reporting positive consumption of non-organic fruits & vegetables and zero consumption of organic fruits & vegetables (754 households). The average CVs and EVs for the households in this sub-sample are positive for the case of substitution of a 5% of the non-organic fruits & vegetables by their organic counterparts, indicating that the households become worse off under this change in quantities. They do become better off, though, under the 10% substitution case, and even more favoured under the 15% substitution case. The hypothesis that the difference between the average CVs (EVs) for the households in higher and lower social classes is zero was tested, and it was rejected for the cases of substitution of 5%, and 10% (CVs only), of non-organic fruits & vegetables by organic ones. Similar tests for the households groups with and without children indicate that the substitution of a portion of non-organic fruits & vegetables by their organic counterparts does not affect the CVs and EVs of these household groups differently. Finally, a more detailed analysis of the results, according to smaller household groups, indicates that it is the households in higher and lower social classes, *without children*, that are affected differently by the substitution of a 5%, 10% or 15% of non-organic fruits & vegetables by organic ones (see Table A.2 in the Appendix).

3. Measuring Efficiency in Consumption: A Theoretical Model

3.1. INTRODUCTION

The study presented in the previous chapter employed standard theoretical approaches for the analysis of consumer demand for organic and non-organic commodities; standard, in the sense that it is implicitly assumed that consumers behave optimally and, thus, efficiently. However, as stressed out in Chapter 1, optimality is a restrictive assumption to make for consumers' actual behaviour. The study presented in the present chapter moves away from this restrictive assumption, and develops a theoretical model for the analysis of consumer's inefficiency in price-quantity space. In particular, it contributes to the literature of measuring efficiency in consumption by proposing indices of commodity-oriented efficiency and utility-oriented efficiency for measuring consumer's efficiency. The commodity-oriented efficiency measures which are developed allow consumer's efficiency to be studied not only in terms of budget that is wasted (*i.e.*, as in the models developed by Afriat and Varian), but also in terms of quantities that are wasted. Also, the proposed utility-oriented efficiency measures allow consumer's inefficiency to be studied in terms of utility that that the consumer could have, but has not, attained. As already stated, this side of consumer's non-optimal behaviour has not been explicitly dealt with in the literature of consumer's efficiency. Specifically, an index which resembles our index of utility overall efficiency can be found in Russell (1998). However, the issue of consumer's efficiency is not studied by Russell (1998); it is only implied that consumer's efficiency could be studied in terms of utility that is wasted. Finally, the construction of the proposed utility-oriented efficiency measures is based on the use of the *output distance function*. The output distance function is well-established in the producer theory context, but it has never been used before in the consumer theory context.

The structure of this chapter is as follows. The approaches proposed so far for measuring efficiency in consumption are discussed in Section 3.2. In particular, this section provides a presentation of Afriat's *cost efficiency index* and Varian's money-metric goodness-of-fit measure for measuring consumer's efficiency in price-quantity space, and of the recent developments in measuring consumer's efficiency in price-quality space. The same section also provides a critique of the aforementioned approaches. The proposed model for measuring inefficiency in consumption is presented analytically in Section 3.3. In particular, Section 3.3.1 provides a detailed presentation of the consumer's expenditure-minimisation problem, and derives some interesting results with straightforward application in empirical work. The notion of commodity efficiency and a proposed measure of it are described in Section 3.3.2, whereas the notions of and measures for expenditure (or overall) efficiency and allocative efficiency are the subject-matter of Section 3.3.3. In the same section, a decomposition of expenditure efficiency into commodity and allocative efficiency is also proposed and illustrated. Finally, in Section 3.3.4, the notion and a measure of utility efficiency are provided, and its relation to the measure of commodity efficiency is explained.[25]

3.2. HISTORICAL BACKGROUND

3.2.1 Afriat's Cost Efficiency Index

The first attempt to measure consumption efficiency in price-quantity space is attributed to Afriat (1967), who employed revealed preference theory in order to construct an non-parametric index for measuring consumption efficiency in a price-quantity space, where consumption inefficiency is represented as a measure of wasted expenditure. A brief description of Afriat's index can also be found in Afriat (1988) and Varian (1990). For describing the index in question, we will start by letting **q** and **p** denote a vector of commodity quantities and the vector of their associated market prices, respectively. Assuming that a *utility order R* (*i.e.*, a preference ordering) underlies a demand (**q**,**p**) (*i.e.*, a set of quantity and price observations), Afriat defines a demand (**q**,**p**) to be efficient if the following conditions are satisfied:

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[25] The study in this chapter was presented at the 5^{th} *North American Productivity Workshop*, Stern Business School, New York, USA, June 24-27, 2008, under the title "Measurement of Consumption Efficiency in Price-Quantity Space: A Distance Function Approach" (with M. Genius, P. Midmore, and V. Tzouvelekas).

$$H1 \equiv \mathbf{p} \bullet \mathbf{y} \leq \mathbf{p} \bullet \mathbf{q} \Rightarrow \mathbf{q} R \mathbf{y}\,, \tag{3.1}$$

$$H2 \equiv \mathbf{y} R \mathbf{q} \Rightarrow \mathbf{p} \bullet \mathbf{y} \geq \mathbf{p} \bullet \mathbf{q}\,. \tag{3.2}$$

That is, the utility provided by **q** is required to be at least as great as the utility provided by any other bundle **y** which is affordable with that expenditure (condition *H*1), and if **y** provides at least the same utility as **q** then it should be at least as expensive (condition *H*2).[26,27] Afriat defines a demand (**q**,**p**) to be *compatible* with a utility order *R*, if it satisfies condition *H*, which is the conjunction of conditions *H*1 and *H*2. Condition *H* gives Afriat's model for exact efficiency. In order to define a model that allows for inefficiency, Afriat introduces a scalar *e*, which takes on values between 0 and 1 and which he refers to as a *level of cost-efficiency*. Using *e*, conditions *H*1 and *H*2 are redefined as follows:

$$H1(e) \equiv \mathbf{p} \bullet \mathbf{y} \leq e(\mathbf{p} \bullet \mathbf{q}) \Rightarrow \mathbf{q} R \mathbf{y}\,, \tag{3.3}$$

$$H2(e) \equiv \mathbf{y} R \mathbf{q} \Rightarrow \mathbf{p} \bullet \mathbf{y} \geq e(\mathbf{p} \bullet \mathbf{q})\,. \tag{3.4}$$

That is, the utility provided by **q** is required to be at least as great as the utility provided by any other bundle **y** which is affordable with a fraction *e* of the expenditure $\mathbf{p} \bullet \mathbf{q}$ (condition $H1(e)$), and the utility provided by **q** should not be attainable with expenditure less than $e(\mathbf{p} \bullet \mathbf{q})$ (condition $H2(e)$). In effect, what Afriat does for modelling consumption inefficiency is to represent inefficiency as a measure of budget wastage: an amount $[(\mathbf{p} \bullet \mathbf{q}) - e(\mathbf{p} \bullet \mathbf{q})]$ is allowed to be wasted, since by better programming at least the same utility could have been obtained with an expenditure not exceeding $e(\mathbf{p} \bullet \mathbf{q})$ (Afriat, 1988, pp. 255-6). Hence, the closer *e* is

[26] Put another way, if the amount of money spent on **q** is at least as great as that spent on any other bundle **y**, then **q** is "at –least-as-good-as" **y** (condition *H*1), while, if a bundle **q** is "no-better-than" any other bundle **y**, then the amount of money spent on **q** should not exceed that spent on **y** (condition *H*2).

[27] Afriat (1998) notes that, if *R* is representable by a continuous non-decreasing utility function (*i.e.*, if preferences satisfy the axioms of reflexivity, completeness, transitivity, continuity and non-satiation), the conditions *H*1 and *H*2 become equivalent.

to 1, the smaller the portion of the budget that the consumers waste. Thus, e can be thought of as measuring the "overall efficiency" of consumers' choice behaviour. Formally, Afriat's model for inefficiency is given by the condition $H(e)$, which is the conjunction of conditions $H1(e)$ and $H2(e)$ for any e between 0 and 1, and a demand (**q**,**p**) is defined to be *compatible* with utility order *R at the level of cost efficiency e*, if it satisfies condition $H(e)$.

Evidently, if $e=1$, that is, if no amount of the budget is allowed to be wasted, then condition $H(e)$ becomes equivalent to the model of full efficiency given by the compatibility condition *H*. On the other hand, if $e=0$, that is, if the entire budget is allowed to be wasted, then the compatibility condition $H(e)$ is vacuous in the sense that we cannot find any bundles **y** that **q** is at-least-as-good-as.[28] In general, if a demand (**q**,**p**) is compatible with utility order *R* at a level of cost efficiency *e*, then it is also compatible with *R* at any level of cost efficiency e' such that $e' \leq e$. Hence, as *e* varies from 1 to 0, the number of bundles **y** that **q** is at-least-as-good-as decreases.

Finally, Afriat (1998) also provides as alternative representation of the model for inefficiency. In particular, he employs a *utility cost function*, $c(\mathbf{p},\mathbf{q})$, which is defined in terms of a utility order *R* by

$$c(\mathbf{p},\mathbf{q}) = \inf\{\mathbf{p} \cdot \mathbf{y} : \mathbf{y} R \mathbf{q}\}. \tag{3.5}$$

The function given by (3.5) is actually a money-metric utility function defined in terms of a preference ordering, and allows the condition $H2(e)$ in (4.4) to be restated as

$$c(\mathbf{p},\mathbf{q}) \geq e(\mathbf{p} \cdot \mathbf{q}). \tag{3.6}$$

If preferences satisfy certain axioms of choice so that conditions $H1(e)$ and $H2(e)$ are equivalent, the condition (3.6) alone can be used in order to express the compatibility condition $H(e)$. In particular, rearranging terms in relation (3.6) yields

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[28] Using Afriat's words, "$H(0)$ holds unconditionally; that is, every demand is compatible with every utility at a level of cost efficiency 0" (Afriat, 1988, p. 256.)

$$e^* \equiv \frac{c(\mathbf{p},\mathbf{q})}{\mathbf{p}\cdot\mathbf{q}} \geq e. \qquad (3.7)$$

The compatibility condition $H(e)$, which gives Afriat's model for consumption inefficiency is now given by $e^* \geq e$, where Afriat defines e^* to be the *cost efficiency* of the demand $(\mathbf{q},\mathbf{p})$ as determined by the utility order *R*.

3.2.2 Varian's Goodness-of-Fit Measure

Afriat's cost efficiency index measures the overall consistency of a set of data with optimising behaviour (*i.e.*, the overall efficiency of the data). That is, a single index *e* is used that applies to all observations. Following Afriat (1967), Varian (1990) shows how to construct a parametric goodness-of-fit measure for violations in optimising behaviour, which serves as a parametric consumption efficiency index and can be applied to each observation in a sample. Drawing on Varian (1990), we will start from the presentation of the efficiency index in its non-parametric form and continue on to the description and use of it as a parametric efficiency index.[29] First, let **q** and **p** denote vectors of commodity quantities and their associated prices, and let $\mathbf{q}^t$ and $\mathbf{p}^t$ denote the *t*-th observation in **q** and **p**, $t = 1,\ldots,T$. Then, for all pairs of observations *t* and *s*, define the direct revealed preference relation R^0 by

$$\mathbf{q}^t R^0 \mathbf{q}^s \text{ if and only if } \mathbf{p}^t\mathbf{q}^t \geq \mathbf{p}^t\mathbf{q}^s, \qquad (3.8)$$

that is, $\mathbf{q}^t$ is directly revealed preferred to $\mathbf{q}^s$ if $\mathbf{q}^t$ is purchased when $\mathbf{q}^s$ was affordable. Varian (1990) defines the following extension to the direct revealed preference relation:

$$\mathbf{q}^t R_e^0 \mathbf{q}^s \text{ if and only if } e^t\mathbf{p}^t\mathbf{q}^t \geq \mathbf{p}^t\mathbf{q}^s, \qquad (3.9)$$

[29] An extensive non-parametric analysis of optimising behaviour, including the measurement of violations in optimising behaviour can be found in Varian (1982, 1983, 1985). For the computation of goodness-of-fit measures and Afriat's efficiency index, see, for example, Varian (1996).

where e^t denotes numbers such that $0 \leq e^t \leq 1$. This extension of the direct revealed preference relation says that observation *t* is directly revealed preferred to observation *s* at efficiency level e^t if, and only if, $e^t p^t x^t \geq p^t x^s$ (Varian, 1996). Just as was the case in Afriat's models for exact efficiency (condition *H*) and inefficiency (condition $H(e)$), Varian's revealed preference relations (3.8) and (3.9) are identical for $e^t = 1$, whereas, for $e^t = 0$, (3.9) is vacuous. Hence, as e^t decreases from 1, the number of observations revealed preferred to other observations monotonically decreases (Varian, 1990, p. 130).

In turn, letting the revealed preference relation R_e be the transitive closure of the relation R_e^0, Varian (1990) defines the Generalised Axiom of Revealed Preference at efficiency level *e*, $GARP_e$, by[30]

$$\mathbf{q}^s R_e \mathbf{q}^t \quad \text{implies} \quad e^t \mathbf{p}^t \mathbf{q}^t \leq \mathbf{p}^t \mathbf{q}^s. \tag{3.10}$$

That is, some data $(\mathbf{p}^t, \mathbf{q}^t, e^t)$ are consistent with the utility maximisation model if, and only if, they satisfy the $GARP_e$. Obviously, if $e^t = 1$ for all *t*, then the definition of $GARP_e$ becomes identical with the standard definition of *GARP*. Varian (1990) restates the definition of $GARP_e$ as follows: if some data $(\mathbf{p}^t, \mathbf{q}^t, e^t)$ satisfy $GARP_e$, then

$$\text{for all } \mathbf{q}^s R_e \mathbf{q}^t \quad \text{we have} \quad e^t \mathbf{p}^t \mathbf{q}^t \leq \mathbf{p}^t \mathbf{q}^s, \tag{3.11}$$

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[30] Definitions: let the standard direct revealed preference relation R^0 be defined by: $\mathbf{q}^t R^0 \mathbf{q}$ if and only if $\mathbf{p}^t \mathbf{q}^t \geq \mathbf{p}^t \mathbf{q}$. Then the transitive closure of the relation R^0, denoted by R, is defined as: $\mathbf{q}^t R \mathbf{q}$ if and only if there is some chain of observations $(\mathbf{q}^s, \mathbf{q}^r, \ldots, \mathbf{q}^v)$ such that $\mathbf{q}^t R^0 \mathbf{q}^s$, $\mathbf{q}^s R^0 \mathbf{q}^r, \ldots, \mathbf{q}^v R^0 \mathbf{q}$. Finally some observed choices $(\mathbf{p}^t, \mathbf{q}^t)$ are consistent with the utility maximisation model if, and only if, they satisfy the Generalised Axiom of Revealed Preference (GARP), which is defined as: $\mathbf{q}^t R \mathbf{q}^s$ implies not $\mathbf{q}^s R \mathbf{q}^t$.

Relation (3.11) can now be written as

$$e^t \leq \mathbf{p}^t\mathbf{q}^s \big/ \left(\mathbf{p}^t\mathbf{q}^t\right) \quad \text{for all} \quad \mathbf{q}^s R_e \mathbf{q}^t, \tag{3.12}$$

For observations $\mathbf{q}^s$ such that $\mathbf{q}^s R_e \mathbf{q}^t$ and $e^t\mathbf{p}^t\mathbf{q}^t = \mathbf{p}^t\mathbf{q}^s$, e^t can be defined by the ratio $e^t = \mathbf{p}^t\mathbf{q}^s \big/ \left(\mathbf{p}^t\mathbf{q}^t\right)$. Hence, for relation (3.12) to hold as an equality, $\mathbf{q}^s$ must be such the expenditure $\mathbf{p}^t\mathbf{q}^s$ are the minimal ones. That is, e^t can be defined by

$$e^t = \min_{\mathbf{q}^s R \mathbf{q}^t} \{\mathbf{p}^t\mathbf{q}^s / \mathbf{p}^t\mathbf{q}^t\}. \tag{3.13}$$

Varian (1990) also shows that, by employing a money metric utility function, his goodness-of-fit measure can be used for the construction of a parametric consumption efficiency index. His analysis starts with the assumption that an observed commodity vector $\mathbf{q}^0$, is the result of the maximization of a utility function $U(\mathbf{q}^0;\boldsymbol{\beta})$, where $\boldsymbol{\beta}$ denotes a vector of parameters. The money-metric utility function , or *direct (income) compensation function*, $M(\mathbf{p},\mathbf{q}^0;\boldsymbol{\beta})$ is then defined as the solution to the problem of minimising the total expenditure required for attaining at least the same utility as that generated by the reference (*i.e.*, the observed) commodity vector $\mathbf{q}^0$, at market prices $\mathbf{p}$. That is,

$$M(\mathbf{p},\mathbf{q}^0;\boldsymbol{\beta}) = \min_{\mathbf{q}} \{\mathbf{p} \cdot \mathbf{q} : U(\mathbf{q};\boldsymbol{\beta}) \geq U(\mathbf{q}^0;\boldsymbol{\beta})\} \tag{3.14}$$

or,

$$M(\mathbf{p},\mathbf{q}^0;\boldsymbol{\beta}) = C(U(\mathbf{q}^0;\boldsymbol{\beta}),\mathbf{p}), \tag{3.15}$$

where $C(\cdot)$ is the consumer's expenditure function. Notice that the difference between the minimisation problem in (3.14) and the traditional functional representation of the consumer's expenditure minimisation problem is that the target indifference surface in the former problem is specified in terms of a reference commodity vector that lies on the surface, while the target indifference surface in the latter problem is specified in terms of a reference utility level. It is obvious that the money-metric utility function can be regarded as an expenditure function: for fixed $\mathbf{q}^0$, $M(\mathbf{p},\mathbf{q}^0;\boldsymbol{\beta})$ is a function of **p** and shares similar properties with the expenditure function, *i.e.*, it is an increasing, linearly homogeneous, and concave function of prices. Moreover, holding **p** fixed and regarding $M(\mathbf{p},\mathbf{q}^0;\boldsymbol{\beta})$ as a function of the reference bundle $\mathbf{q}^0$, the money-metric utility function can be thought of as a utility function. Specifically, since the preference ordering represented by a utility function is an *ordinal* property, *i.e.*, it is preserved under any strictly increasing transformation of the utility function, and since the expenditure function is increasing in utility, the money-metric utility function as defined in relation (3.15) is an increasing monotonic transformation of a utility function and, hence, is itself a utility function.

Using the money-metric utility function, Varian (1990) defines the *money-metric goodness-of-fit measure* for violations in optimising behaviour by

$$i^t = M(\mathbf{p}^t,\mathbf{q}^{t0};\boldsymbol{\beta})\big/(\mathbf{p}^t \cdot \mathbf{q}^{t0}), \tag{3.16}$$

where t denotes the t-th observation. Thus, Varian's (1990) index of the degree of violation of utility-maximising behaviour is given by the ratio of the minimum expenditure required for the consumer to be as well off as he would by consuming the reference bundle $\mathbf{q}^{t0}$, to the actual expenditure for the bundle $\mathbf{q}^{t0}$.

Varian (1990) illustrated the use of the *money metric goodness-of-fit measure* using U.S. data for aggregate consumption of durables, nondurables, and services from 1947 to 1987. Specifically, he employed a Cobb-Douglas utility function $U(q_1,q_2,q_3) = q_1^{\alpha_1} q_2^{\alpha_2} q_3^{\alpha_3}$, with the restriction that $\alpha_1+\alpha_2+\alpha_3=1$, and estimated it using three different techniques: (1) by taking the average expenditure share of each commodity; (2) by obtaining SUR estimates of the Cobb-Douglas system of uncompensated demand functions, $q_i = \alpha_i x/p_i \quad i=1,2,3$, where x denotes the total (*i.e*, the observed) expenditure on the consumption bundle; (3) by using logs in relation (3.16) to specify the log of the goodness-of-fit measure as the difference between the log of the value of the money-metric utility function and the log of actual expenditure, and determining the values of the parameters of the Cobb-Douglas utility function which maximised the goodness-of-fit. In order to describe the third method, Varian (1990) starts by specifying the Cobb-Douglas uncompensated demand functions as $q_i = \alpha_i m / p_i$, where *m* denotes the value of the money-metric utility function, *i.e.*, the minimum amount of money required, at prices (p_1, p_2, p_3), for attaining the same utility level as that generated by the reference (observed) commodity vector (q_1, q_2, q_3). These demand functions are then substituted into the associated direct utility function to yield the following Cobb-Douglas indirect utility function

$$q_1^{\alpha_1} q_2^{\alpha_2} q_3^{\alpha_3} = (\alpha_1 m / p_1)^{\alpha_1} (\alpha_2 m / p_2)^{\alpha_2} (\alpha_3 m / p_3)^{\alpha_3} . \qquad (3.17)$$

Solving the indirect utility function for *m* and taking logs yields:

$$\begin{aligned} M(\mathbf{p}, \mathbf{q}) &= \alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2} \alpha_3^{-\alpha_3} (p_1 q_1)^{\alpha_1} (p_2 q_2)^{\alpha_2} (p_3 q_3)^{\alpha_3} \Rightarrow \\ \ln M(\mathbf{p}, \mathbf{q}) &= -\alpha_1 \ln \alpha_1 - \alpha_2 \ln \alpha_2 - \alpha_3 \ln \alpha_3 + \alpha_1 \ln(p_1 q_1) \\ &\quad + \alpha_2 \ln(p_2 q_2) + \alpha_3 \ln(p_3 q_3) , \end{aligned} \qquad (3.18)$$

where $M(\mathbf{p}, \mathbf{q})$ now denotes that we have expressed *m* as a function of prices and the reference (observed) commodity vector. At this point, Varian (1990) makes the assumption that the log of the actual expenditure in period *t*, $\ln x^t$,can be defined as the sum of the log of the expenditure minimising amount, $\ln M(\mathbf{p}^t, \mathbf{q}^t)$, and an error term, ε^t, which represents the optimisation error. In order to explain what this error term is, let us rearrange terms in relation (3.16) and take logs to obtain

$$\ln(1/i^t) = \ln x^t - \ln M(\mathbf{p}^t, \mathbf{q}^t) . \qquad (3.19)$$

It is obvious from relation (3.19) that the error term representing the optimisation error is the log of the inverse of the efficiency index i^t. Under this assumption, and using equation (3.18), Varian (1990) derived, and estimated using nonlinear least squares, the following equation:

$$\begin{aligned} \ln x^t &= \ln M(\mathbf{p}^t, \mathbf{q}^t) + \varepsilon^t \Rightarrow \\ \ln x^t &= -\alpha_1 \ln \alpha_1 - \alpha_2 \ln \alpha_2 - \alpha_3 \ln \alpha_3 \end{aligned}$$

$$+\alpha_1 \ln(p_1^t q_1^t) + \alpha_2 \ln(p_2^t q_2^t) + \alpha_3 \ln(p_3^t q_3^t) + \varepsilon^t \,. \tag{3.20}$$

3.2.3 Consumption Efficiency Measurement in Price-Quality Space

Measurement of consumption efficiency in price-quality space is based upon the theoretical framework for consumer demand analysis developed by Lancaster (1966). Lancaster (1966) defines consumer preferences in terms of the characteristics that goods possess, rather than in terms of the quantities of the goods themselves: preference orderings and, consequently, utility, are assumed to rank characteristic vectors, while commodity vectors are ranked indirectly through the characteristic vectors they give rise to. The consumer's budget and the relative commodity prices (*i.e.*, the budget constraint on the goods) determine the set of commodity vectors that are feasible for the consumer to choose, and hence the set of characteristic vectors that are attainable by linear combinations of the commodities. The boundary of the set of characteristic vectors, which are attainable by linear combinations of the commodities, is Lancaster's *consumer efficiency frontier*, which changes with relative commodity prices. However, as Hendler (1975) shows, restrictive assumptions should be made for Lancaster's (1996) theory to be valid. In particular, Hendler (1975) points out that Lancaster's model is based on the restrictive assumption that the marginal utility of characteristics is always non-negative (*i.e.*, the utility that a characteristic conveys can never turn from positive to negative). This assumption, if accepted, restricts the applicability of the model, whereas, if relaxed, changes the shape of the efficiency frontier. Moreover, according to Lancaster's linear combination model, if a linear combination of commodities yields larger amount of total characteristics than that which is derived from a single commodity, then it is preferred to the single commodity. Hendler (1975), however, indicates that the consumer's objective is the maximisation of utility, and not of the amount of characteristics. Consequently, in order to say that a linear combination of commodities is preferred to a single commodity the assumption should be made that (a) the total utility of one characteristic, although dependent on the total amount of other characteristics, should be independent of the ratio of characteristics per unit of consumption, or (b) the mix of goods containing different ratios of characteristics can yield the ratio of characteristics as another good per unit of consumption (Hendler, 1975, p. 197).

Despite its drawbacks, Lancaster's (1966) theoretical framework for the analysis of consumer behaviour in terms of the characteristics of the commodities provided the initial basis for researchers interested in efficiency analysis to examine efficiency in the price-quality space. The latest and more complete advancement in this field is found in the paper of Lee, Hwang, and Kim (2005), who developed a theoretical and empirical framework for measuring the degree of consumption efficiency of multi-attribute products in price-quality space, where consumption inefficiency is assumed to arise from various sources, such as product complexity, asymmetric information due to search-costs, bounded rationality of consumers, imperfect markets, etc. The theoretical framework developed by Lee, Hwang, and Kim (2005) tackles the problem of indivisibility of quality attributes and, hence, accommodates fully the discreteness of consumption choice. Specifically, Lee, Hwang, and Kim (2005) start their theoretical analysis by defining the *feasible consumption set*, B, as the set of *observed* price-quality combinations $(p,\mathbf{z}') \in \mathbb{R}^{N+1}$, where $\mathbf{z}' \in \mathbb{R}^{N}$ is the vector of quality attributes and p is the scalar price. Notice that the feasible consumption set, B, contains only those price-quality vectors which are observed (*i.e.*, exist in the market) at a certain point in time. Any other price-quality vector that is not observed in the market does not exist and it is not feasible for the consumer to choose, that is, it can not be consumed. This means that the feasible consumption set is not continuous, but discrete instead, and it is not convex (if two price-quality combinations belong to B then a linear combination of them will also belong to B only if his new price-quality combination exists in the market).

Lee, Hwang, and Kim (2005) go on to present a definition of *efficient price-quality vectors* which is similar to Koopman's (1951) definition of technically efficient input-output vectors. Specifically, they define an observed price-quality vector $(p,\mathbf{z}') \in B$ to be efficient in consumption if, and only if, $(\tilde{p},\tilde{\mathbf{z}}') \notin B$ for any $(-\tilde{p},\tilde{\mathbf{z}}') > (-p,\mathbf{z}')$. Based on this definition, they define a *consumption frontier*, F, as the set of efficient price-quality vectors. In addition, just as the measure of technical efficiency in production employs the distance function in order to measure the distance between observed and technically efficient input-output combinations, the measure of consumption efficiency that Lee, Hwang, and Kim (2005) propose employs, what they call, "a kind of distance function" in order to measure the distance between observed and efficient price-quality combinations. In particular, they define the slack $(S^-, S^+) \in \mathbb{R}^{1+N}$, where $S^- \in \mathbb{R}$ and $S^+ \in \mathbb{R}^N$ denote the distance between an observed consumption choice and an efficient price-quality combination (*i.e.*, a point on the consumption frontier) in terms of price and in terms of quality attributes, respectively. Then, they define a function $D(S^-, S^+)$, which is strictly increasing in (S^-, S^+) and $D(0,0) = 1$. Given a reference price-quality combination $(p, \mathbf{z}') \in B$, Lee, Hwang, and Kim (2005) define a *measure of consumption efficiency* as a function $\gamma : B \to \mathbb{R}$,

$$\gamma(p, \mathbf{z}') = \min D(S^-, S+) \text{ s.t. } (p - S^-, \mathbf{z}' + S^+) \in F \text{ and } S^-, S^+ \geq 0. \qquad (3.21)$$

Let (S^{*-}, S^{*+}) denote the solution to the minimisation problem written above. Then, it is against the reference point $(p - S^{*-}, \mathbf{z}' + S^{*+}) \in F$ that the efficiency of the observed price-quality combination $(p, \mathbf{z}') \in B$ is measured. Specifically, since the reference point $(p - S^{*-}, \mathbf{z}' + S^{*+}) \in F$ – associated with the observed choice $(p, \mathbf{z}') \in B$ – is itself a member of the consumption frontier, *F*, this reference point represents the best price-quality combination for a consumer who has chosen $(p, \mathbf{z}') \in B$.

In order to measure consumption efficiency empirically, Lee, Hwang, and Kim (2005) proposed a Discrete Range-Adjusted Model, which is a combination of Range-Adjusted Models and Free Disposal Hull models. Their empirical model accommodates the discrete nature of consumption choice and allows the consumption efficiency measure to satisfy the properties of efficiency indices. Their model was applied to the Korean mobile phone market, and their empirical analysis was focused on the relationship between consumption efficiency and market share. The empirical findings suggested that the market share of firms with inefficient products had decreased. Moreover, Lee, Hwang, and Kim (2005) included in their theoretical model a parameter taking on values in the [0,1] interval, which reflects the choice ability of the consumers, and they showed that an inefficient product is more likely to be selected by consumers who are unable to recognise the efficient product due to low choice ability. Hence, by incorporating a choice ability parameter in the model, an explanation can be provided of why producers selling inefficient products (i.e., products with high price and low quality) can survive in the market. However, the choice ability parameter was not included in the empirical model of Lee, Hwang, and Kim (2005); it is merely a theoretical discussion on the role of a choice ability parameter and its relationship with the consumption efficiency measure that is provided.

3.2.4 Summary and Critique

Both Varian's (1990) and Afriat's (1967) indices summarise how close the observed choices are to maximising choices. Varian's (1990) measure can be considered as a generalisation of Afriat's (1967) efficiency index in that it involves computation of a set of efficiency indices, one for each observation in the data set. Afriat's (1967) approach involves computation of a single efficiency index that applies to all observations and measures the overall efficiency of a set of consumption choices. However, computation of a single e that applies to all observations in the data set is much more easier. One must start with $e = 1$ and test for violations of the strong axiom of revealed preference. If the data violate the strong axiom of revealed preference, then a lower value must be chosen for the index, say $e = 1/2$. If the data do (do not) violate the strong axiom of revealed preference for $e = 1/2$, then a lower (higher) index must be chosen, and so on. On the other hand, as Varian (1990) states, computation of his index is more difficult since a set of efficiency indices that are as close as possible to 1 in some norm must be computed. For example, if a quadratic norm is chosen, then one would have to solve a minimisation problem such as $E = \min_{(e^t)} \sum_{t=1}^{T} (e^t - 1)^2$, subject to the constraint that the revealed preference relation R_e, satisfies the Generalised Axiom of Revealed Preference (Varian, 1990). Moreover, Varian (1990) shows how his index can be parameterised with the use of a money-metric utility function, and notes that this money-metric measure can also be used as a criterion to estimate the parameters of an optimising model. A Bayesian implementation of the use of Varian's (1990) money-metric measure as a criterion for estimating a demand system is provided by Ley and Steel (1996). In particular, they used a Cobb-Douglas utility function in order to derive the relevant money-metric utility function and the same data as the ones in the example provided by Varian (1990), so their results are comparable to those of Varian (1990). Nevertheless, a problem with empirical applications of Varian's (1990) money-metric measure is that it is not easy to specify the money metric utility function for utility functions that are of a more complex form than the Cobb-Douglas utility function.

In addition, what is implied by the measures of Afriat (1967) and Varian (1990) is that inefficiency occurs because a portion of the consumers' budget is wasted, and not a portion of the purchased quantities. However, it is this latter assumption that allows us to define the measure of *commodity efficiency*, and, in particular, in its simplest form with no need to take into account commodity prices or impose a behavioural objective on consumers. Moreover, money-metric utility functions pick out a commodity vector that makes the consumer as well off as he/she would be consuming a reference vector, *i.e.*, they pick out a commodity vector that lies on the same utility curve as the reference vector. Thus, by comparing optimal expenditure (the value of the money-metric utility function) to actual expenditure, Varian's (1990) index measures the overall efficiency of what we define below as *commodity efficient* commodity vectors. By leaving no room for commodity inefficiency to arise, and hence for a decomposition of overall efficiency into commodity and allocative efficiency, Varian's (1990) measure may assign to consumers a higher efficiency score than it should. In short, it is more proper to use the measures of Afriat and Varian for explaining the goodness-of-fit of employed demand models and attributing any departure from optimising behaviour to the functional specification of the model, rather than use them in order to measure consumers' inefficiency.

Regarding the consumption efficiency measure proposed by Lee, Hwang, and Kim (2005), it is a measure that employs theoretical tools found in consumer demand theory (*i.e.*, consumption analysis in price-quality space), in order to analyse the efficiency of products. Indeed, the name that Lee, Hwang, and Kim (2005) have assigned to their efficiency measure is misleading in that it does not concern an efficiency analysis of consumer's behaviour itself. Instead, the latter is used as a tool to construct an efficiency measure for measuring product efficiency and firm market performance. Lee, Hwang, and Kim (2005) themselves suggest that their consumption efficiency measure can be considered as an extension of the traditional framework of production efficiency analysis. Moreover, just as was the case with Varians' (1990) money-metric measure, their consumption efficiency index measures only the "overall efficiency" of a product. In addition, the applicability of the model proposed by Lee, Hwang, and Kim (2005) is restricted by the paucity of data on quality attributes on commodities. Finally, it seems that the consumption efficiency measure defined in price-quality space is appropriate when we examine different varieties of the same commodity (variants of a differentiated product).

3.3. A MODEL FOR MEASURING EFFICIENCY IN CONSUMPTION

3.3.1 Mathematical Background: The Expenditure Minimisation Problem Subject to Inequality and Non-Negativity Constraints and Some Further Results

A consumer's *expenditure function* $C(u,\mathbf{p})$ is defined as the solution to the problem of minimising the total expenditure required to attain a fixed utility level *u*, given that the consumer faces a vector of strictly positive commodity prices, $\mathbf{p} \in \mathbb{R}^N_{++}$.[31] Following Blackorby, Primont, and Russell (1978), the expenditure function, $C : R(U) \times \mathbb{R}^N_{++} \to \mathbb{R}^1_{++}$ is defined by

$$C(u,\mathbf{p}) = \min_{\mathbf{q}}\{\mathbf{p}\bullet\mathbf{q} : U(\mathbf{q}) \geq u, \mathbf{q} \in Q^N\} = \mathbf{p}\bullet\mathbf{q}(u,\mathbf{p}), \qquad (3.22)$$

where $R(U)$ is the range of U with its infimum value excluded, $\mathrm{Q}^N = \{\mathbf{q} \in \mathbb{R}^N : \mathbf{q} > 0^N\}$ is the *choice set* (the set of the quantities of goods over which consumer's preferences are defined), and $\mathbf{q}(u,\mathbf{p})$ is the solution vector, if unique, to the minimisation problem.[32] Note that $\mathbf{q}(u,\mathbf{p})$ is the vector of Hicksian demand

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[31] $\mathbb{R}^N_{++}$ denotes the positive Euclidian *N*-orthant, while $\mathbb{R}^N_{+}$ denotes the non-negative Euclidian *N*-orthant.

[32] The infimum value of utility is excluded from the expenditure function's domain. The reason is that, since the choice set is assumed not to include its origin, then if the infimum of the range of *U is* an element of the range of *U*, the cost minimisation problem as defined by (3.22) will not have a solution when the level of utility is at its infimum value (Blackorby, Primont, and Russell, 1978). Moreover, if preferences are convex (instead of strictly convex), the solution vector for the minimisation problem in (3.22) may not be unique. In this case, an arbitrary element of the set of solution

functions or compensated demand functions (or correspondences, if the solution vector is not unique). Given that the utility function is a continuous, non-decreasing and quasi-concave real-valued function, the expenditure function is jointly continuous in $(u,\mathbf{p})$, increasing in *u*, and non-decreasing, concave, and positively linearly homogeneous in **p**.

The distance function representation of consumer preferences, which is used in standard consumer theory, corresponds to the *input distance function* representation of production technologies. From this point onwards we will refer to the distance function presented so far as *input distance function*, in order to distinguish between this function and the *output distance function* which will be introduced in the sections that follow. The duality between the input distance function and the expenditure function allows us to provide an alternative definition of the expenditure function. Given a fixed utility level *u* and a reference commodity vector **q**, the distance function $D^I : R(U)\times Q_+^N \to \mathbb{R}_{++}^1$ is defined by

$$D^I(u,\mathbf{q}) = \min_{\mathbf{r}}\left\{\mathbf{r}\cdot\mathbf{q} : \overset{*}{C}(u,\mathbf{r}) \geq 1, \mathbf{r}\in\mathbb{R}_+^N\right\} = \mathbf{r}(u,\mathbf{q})\cdot\mathbf{q}\,, \tag{3.23}$$

where $\mathrm{Q}_+^N = \{\mathbf{q}\in\mathbb{R}^N : \mathbf{q} \gg 0^N\}$, $\mathbf{r} = \mathbf{p}/x$ is the vector of minimum-expenditure normalised commodity prices, $\overset{*}{C}(\cdot)$ is an extension of $C(\cdot)$ to $R(U)\times\mathbb{R}_+^N$ by continuity from above, and $\mathbf{r}(u,\mathbf{q})$ is the solution vector to the minimisation problem in (3.23), referred to as the vector of compensated inverse demand functions or correspondences (or minimum-expenditure deflated shadow prices, or marginal willingness to pay) for the goods in question.[33] Relation (3.23) defines the input distance function as the solution to the problem of finding that vector of minimum-expenditure deflated prices that will minimise the value of a reference consumption bundle so that a fixed utility level is achieved. Given the properties of the expenditure

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values can be used in the inner product in (3.22) (Blackorby, Primont, and Russell, 1978).

[33] If the solution vector for the minimisation problem in (3.23) is not unique, then an arbitrary element of the set of solution values can be used in the inner product in (3.23) (Blackorby, Primont, and Russell, 1978).

function, the input distance function is jointly continuous in $(u, \mathbf{x})$, decreasing in u, and non-decreasing, concave, and positively linearly homogeneous in **q**.

The expenditure function can now be defined by

$$C(u,\mathbf{p}) = \min_{\mathbf{q}} \left\{ \mathbf{p} \bullet \mathbf{q} : \overset{*}{D}{}^{I}(u,\mathbf{q}) \geq 1, \mathbf{q} \in Q^{N} \right\}, \tag{3.24}$$

where $\overset{*}{D}{}^{I}(\cdot)$ is an extension of $D^{I}(\cdot)$ to $R(U) \times Q^{N}$ by continuity from above.[34]

In order to take into account that consumers may choose to purchase only a fraction of the goods available, that is, expenditure-minimisation may occur at a corner, we will also include a non-negativity constraint, $\mathbf{q} > 0^{N}$, in the minimisation problem expressed in (3.24). The Lagrangian associated with the above minimisation problem is defined as

$$L = \mathbf{p} \bullet \mathbf{q} + \lambda \left[1 - \overset{*}{D}{}^{I}(u,\mathbf{q}) \right]. \tag{3.25}$$

For the problem at hand, the Kuhn-Tucker (necessary) conditions for a minimum subject to inequality and non-negativity constraints say that if $\mathbf{q}^{*} = \mathbf{q}(u,\mathbf{p})$ is a solution to the expenditure minimisation problem (3.24), then there exists a Lagrange multiplier $\lambda^{*} = \lambda(u,\mathbf{p})$ such that for all $i = 1,\ldots,N$:

$$\nabla_{\mathbf{q}} L \geq 0 \Rightarrow \mathbf{p} \geq \lambda^{*} \nabla_{\mathbf{q}} \overset{*}{D}{}^{I}(u,\mathbf{q}^{*}), \tag{3.26.a}$$

$$\mathbf{q}^{*} \bullet \nabla_{\mathbf{q}} L = 0 \Rightarrow \mathbf{q}^{*} \bullet \left[\mathbf{p} - \lambda^{*} \nabla_{\mathbf{q}} \overset{*}{D}{}^{I}(u,\mathbf{q}^{*}) \right] = 0, \tag{3.26.b}$$

$$\mathbf{q}^{*} > 0^{N}, \tag{3.26.c}$$

[34] These extensions of the domains of the expenditure and input distance functions are necessary for the original expenditure function to be derived from the original input distance function and *vice versa*. For details on this subject, see, for example, Blackorby, Primont, and Russell (1978), Diewert (1982), and Weymark (1980).

$$\frac{\partial L}{\partial \lambda} \le 0 \Rightarrow 1 - \overset{*}{D}^I(u,\mathbf{q}^*) \le 0\,, \tag{3.26.d}$$

$$\lambda^* \frac{\partial L}{\partial \lambda} = 0 \Rightarrow \lambda^* \left[1 - \overset{*}{D}^I(u,\mathbf{q}^*) \right] = 0\,, \tag{3.26.e}$$

$$\lambda^* \ge 0\,. \tag{3.26.f}$$

If $p_i > \lambda^* \partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_i$, then $q_i^* = 0$. That is, if the minimum of expenditure is observed to occur when $p_i > \lambda^* \partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_i$, then the *i*th commodity is not purchased (if $q_i^* > 0$, *then* condition (3.26.a) holds with equality). In addition, if $\overset{*}{D}^I(u,\mathbf{q}^*) > 1$, then $\lambda^* = 0$ (if $\lambda^* > 0$, *then* condition (3.26.d) holds with equality).

Since the constraint in (3.24) is generally binding, *i.e.*, the value of the input distance function at the optimum is equal to unity so that the first-order conditions require that $\partial L/\partial \lambda = 0$ and $\lambda^* > 0$, let's see what happens when $\lambda^* > 0$ but one or more commodities are not purchased. For ease of exposition, we will explore the two-commodities case and assume that at the optimum point, $p_1 = \lambda^* \partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_1$ and $p_2 > \lambda^* \partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_2$ so that $q_1^* > 0$ and $q_2^* = 0$. Then, at the expenditure-minimising point,

$$\lambda^* = \frac{p_1}{\partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_1} < \frac{p_2}{\partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_2}\,.$$

Rearranging terms yields

$$\frac{\partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_1}{\partial \overset{*}{D}^I(u,\mathbf{q}^*) \big/ \partial q_2} > \frac{p_1}{p_2}\,. \tag{3.27}$$

The left-hand side of relation (3.27) is an input-distance function representation of the marginal rate of substitution of good 1 for good 2 at $\mathbf{q}^*$, MRS_{12}, that is, (the negative) of the slope of the indifference curve generating utility level u at $\mathbf{q}^*$.

Interpretation of relation (3.27) is made easier with the use of the Shephard-Hanoch lemma. First, we will define the input distance function by $\overset{*}{D}{}^I(u,\mathbf{q}) = \mathbf{r}^* \bullet \mathbf{q}$, where $\mathbf{r}^* = \mathbf{r}(u,\mathbf{q})$ is the solution vector to the minimisation problem in (3.23). Recall that the Shephard-Hanoch lemma states that, if the input distance function is continuously differentiable, the vector of shadow prices can be readily obtained from partial differentiation of the distance function with respect to quantities, *i.e.*, $\partial \overset{*}{D}{}^I(u,\mathbf{q}) \big/ \partial q_1 = r_i(u,\mathbf{q})$.[35] Using this property, relation (3.27) can now be written as

$$MRS_{12} = \frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*) \big/ \partial q_1}{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*) \big/ \partial q_2} = \frac{r_1(u,\mathbf{q}^*)}{r_2(u,\mathbf{q}^*)} > \frac{p_1}{p_2}, \tag{3.28}$$

The situation described by relation (3.28) is depicted in Figure 3.1. For a given utility level u^*, the set of feasible consumption bundles in problem (3.24) is $L(u^*)$.

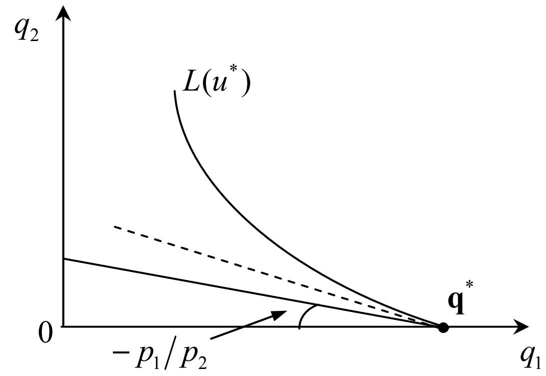

Figure 3.1 Minimisation of Expenditure at a Corner

[35] Note that this derivative is evaluated at $\overset{*}{D}{}^I(u,\mathbf{q}) = 1$. Moreover, as Russell (1998, p. 61) points out, if the reference commodity vector in the definition of the input distance function is not expenditure-minimising at $\mathbf{r}^* = \mathbf{r}(u,\mathbf{q})$ and utility u, then $\mathbf{r}(u,\mathbf{q})$ must be interpreted as the shadow price vector deflated by shadow expenditures $C(u,\mathbf{r}(u,\mathbf{q}))$.

Given u^* and commodity prices **p**, expenditure minimisation occurs at the point $\mathbf{q}^*$, and the minimum expenditure is given by $C(u,\mathbf{p}) = p_1 q_1^* + p_2 q_2^*$. In the Figure, the slope $-p_1/p_2$ of the iso-expenditure line with value $C(u,\mathbf{p})$ is less steep (in absolute terms) than the slope $r_1(u,\mathbf{q}^*)/r_2(u,\mathbf{q}^*)$ of the indifference curve $I(u)$ at $q_1^* > 0$, $q_2^* = 0$. In other words, if the consumer's subjective marginal evaluation of q_2 (in terms of the amount of q_1 the consumer would willingly forgo in order to consume an extra unit of q_2) is less than the market's evaluation of q_2, then the consumer will choose to consume no q_2 at all at the minimum-expenditure point.

Deaton (1981) provided a definition of the marginal rate of substitution between two goods, as the one expressed in relation (3.28), for the case where all goods are purchased. For empirical applications, however, we propose the use of a *normalised marginal rate of substitution*. In our two-commodities example, strictly convex indifference curves exhibit a diminishing MRS, that is, their slope $r_1(u,\mathbf{q})/r_2(u,\mathbf{q})$ decreases (in absolute value) as consumption of q_1 increases. Put another way, the consumer's subjective marginal evaluation of q_1, in terms of the amount of q_2 the consumer is willing to give up in order to acquire an extra unit of q_1, is falling as the ratio q_2/q_1 falls. Hence, the change in $r_1(u,\mathbf{q})/r_2(u,\mathbf{q})$ depends upon how relative quantities (the quantity mix) changes. For this reason, when measuring empirically the slope of an indifference curve at an observed point, the relative shadow prices in (3.28) should be normalised by the observed commodity vector so that the MRS, when all goods are purchased, is written as

$$Sub_{ij} = \frac{r_i(u,\mathbf{q})/r_j(u,\mathbf{q})}{q_j/q_i}. \tag{3.29}$$

This normalised MRS provides an indication of the ease of substitution between two goods. Specifically, values of this normalised MRS greater (less) than unity reflect relative difficulty (ease) in substitution between goods *i* and *j*.

Let us go back to the expenditure minimisation problem in (3.24) and derive some further interesting results which have straightforward application in empirical work. To start with, substitution of the expenditure minimising solution, $\mathbf{q}^* = \mathbf{q}(u,\mathbf{p})$, and the optimal value of the Lagrangian multiplier, $\lambda^* = \lambda(u,\mathbf{p})$, into (3.25) and application of the envelope theorem yields the following relations:

$$\frac{\partial C(u,\mathbf{p})}{\partial u} = -\lambda^* \frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*)}{\partial u}, \tag{3.30}$$

$$\nabla_{\mathbf{p}} C(u,\mathbf{p}) = \mathbf{q}^* . \tag{3.31}$$

Equation (3.30) is the (unobservable) *marginal cost of utility*, while the *N* equations in (3.31) are the derivative property of the expenditure function called as Shephard's lemma.[36]

It is common in applied demand analyses with time-series data to estimate a demand system where a deterministic time trend is also included as an explanatory variable. This variable is usually used to capture changes in consumers' tastes or changes in the demographic composition of the population through time. In our example, if both the expenditure function and the input distance function in (3.24) are also functions of time, then application of the envelope theorem will yield

$$\frac{\partial C(u,\mathbf{p},t)}{\partial t} = -\lambda^* \frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*,t)}{\partial t}, \tag{3.32}$$

where, now, $\mathbf{q}^* = \mathbf{q}(u,\mathbf{p},t)$ and $\lambda^* = \lambda(u,\mathbf{p},t)$. This partial derivative may be thought of as reflecting the rate of change in tastes.

An interesting result provided by Shephard (1970) is that the optimal value of the Lagrangian multiplier associated with a producer's cost minimisation problem is equal to the value of his cost function. Jacobsen (1972) provides a proof of this result, which makes use of the envelope theorem and the positive linear homogeneity of the

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[36] It can be shown that Shephard's lemma is not just an application of the envelope theorem. Färe and Primont (1995) provide an alternative proof of Shephard's lemma, in the context of production theory, which does not make use of the envelope theorem.

input distance function in input quantities.[37] In the context of consumption theory, it is easy to show that the value of the Lagrangian multiplier associated with (3.25) is equal to the value of the consumer's expenditure function. Following the proof by Jacobsen (1972), we start by assuming that the optimal commodity vector for the problem in (3.24) is $\mathbf{q}^*$, so that $C(u,\mathbf{p}) = \mathbf{p}\bullet\mathbf{q}^*$ and $\overset{*}{D}{}^I(u,\mathbf{q}^*) = 1$. By the positive linear homogeneity of the input distance function in commodity quantities it follows that, for $\alpha > 0$, $\alpha \overset{*}{D}{}^I(u,\mathbf{q}^*) = \alpha \Rightarrow \overset{*}{D}{}^I(u,\alpha\mathbf{q}^*) = \alpha$. Let $C(u,\mathbf{p};\alpha)$ denote the cost of the commodity vector $\alpha\mathbf{q}^*$. Then, $C(u,\mathbf{p};\alpha) = \mathbf{p}\bullet\alpha\mathbf{q}^* = \alpha(\mathbf{p}\bullet\mathbf{q}^*) = \alpha C(u,\mathbf{p})$. By the envelope theorem, the optimal Lagrange multiplier for the problem $C(u,\mathbf{p};\alpha) = \min_{\mathbf{q}}\left\{\mathbf{p}\bullet\mathbf{q} : \overset{*}{D}{}^I(u,\mathbf{q}) \geq \alpha, \mathbf{q} \in Q^N\right\}$ is $\lambda(u,\mathbf{p};\alpha) = \partial C(u,\mathbf{p};\alpha)/\partial\alpha =$ $\partial[\alpha C(u,\mathbf{p})]/\partial\alpha = C(u,\mathbf{p})$. Hence, at $\alpha = 1$, it follows that

$$\lambda(u,\mathbf{p};1) = \lambda(u,\mathbf{p}) = C(u,\mathbf{p}). \tag{3.33}$$

Suppose that the inequality constraint in (3.24) is binding, *i.e.*, at the optimum, $\overset{*}{D}{}^I(u,\mathbf{q}^*) = 1$. Then, using (3.33) we can re-express relation (3.32) as follows:

$$\frac{\partial C(u,\mathbf{p},t)}{\partial t} = -C(u,\mathbf{p},t)\frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*,t)}{\partial t} \Rightarrow \frac{\partial C(u,\mathbf{p},t)}{\partial t}\frac{1}{C(u,\mathbf{p},t)} = -\frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*,t)}{\partial t}$$

$$\Rightarrow \frac{\partial \ln C(u,\mathbf{p},t)}{\partial t} = -\frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*,t)}{\partial t} \Rightarrow \frac{\partial \ln C(u,\mathbf{p},t)}{\partial t} = -\frac{\partial \overset{*}{D}{}^I(u,\mathbf{q}^*,t)}{\partial t}\frac{1}{\overset{*}{D}{}^I(u,\mathbf{q}^*,t)}$$

$$\Rightarrow \frac{\partial \ln C(u,\mathbf{p},t)}{\partial t} = -\frac{\partial \ln \overset{*}{D}{}^I(u,\mathbf{q}^*,t)}{\partial t} \tag{3.34}$$

[37] An alternative proof of this result is provided by Färe and Primont (1995, p. 52). Their proof makes use of the first order conditions for the producer's cost minmisation problem and the linear homogeneity of the input distance function in input quantities.

Equation (3.34) now allows the rate of change in tastes to be computed either in price (dual) space or in quantity (primal) space.

3.3.2 Commodity Efficiency

Standard consumer theory assumes that consumer's preferences satisfy a number a number of properties, known as the *axioms of choice*. Specifically, consumer's preferences are assumed to be reflexive, complete, transitive, continuous, non-satiated (or, (strongly) monotone), and (strictly) convex. However, before proceeding to the analysis of measurement of consumer inefficiency, expressions for the non-satiation axiom of choice – alternative to the ones commonly used in consumer theory – are needed. In particular, Russell (1998) uses the terms *strong* and *weak non-satiation* to define

(i) *Strong Non-Satiation*: for every $\mathbf{q}, \mathbf{q}' \in \mathrm{Q}^N$, if $\mathbf{q} \geq \mathbf{q}'$ and $\mathbf{q}' \in L(u)$, then $\mathbf{q} \in L(u)$,

(ii) *Weak Non-Satiation*: for all $\mathbf{q} \in \mathrm{Q}^N$, if $\mathbf{q} \in L(u)$, then $\lambda \mathbf{q} \in L(u)$ for $\lambda \geq 1$,

where $L(u)$ is the consumption (requirement) set defined as $L(u) = \{\mathbf{q} \in \mathrm{Q}^N : U(\mathbf{q}) \geq u\}$. Defined in terms of the consumption (requirement) set, these definitions state explicitly what is implicit in the usual definitions of strong and weak monotonicity: if a vector **q** can generate utility *u*, then so can a vector with more of at least one commodity (or more of all commodities) than **q**. Put this way, strong and weak non-satiation imply that the consumer can dispose of or eliminate the extra amount of commodities at no cost.[38] As a result, room is left for consumer inefficiency to be defined.

Having provided alternative representations of the non-satiation axiom of choice, we may proceed to the analysis of consumer inefficiency. Firstly, let us assume that the consumer's objective is to choose a feasible commodity vector in

[38] In fact, this is exactly the reason why, in production theory, the strong and weak monotonicity properties are usually called strong (or free) and weak disposability.

order to achieve a target utility level u. If we also make the assumption that any unwanted quantities of the purchased commodities can be disposed of, then the quantities of the purchased commodities may well be higher than the ones required to just attain u, and the consumer may well have chosen an inefficient way of attaining u. We will use the term *commodity efficiency* in order to describe the consumer's ability to avoid wasting any quantities of the purchased commodities, by minimising quantity purchases in the achievement of a target utility level. The definition of commodity efficiency of consumption bundles which is proposed here is in accordance with the input-oriented case of Koopman's (1951) definition of technical efficiency in production. In particular, we define a commodity vector, which is feasible for a given utility level u, to be commodity efficient if, and only if, a reduction in any of the purchased quantities renders the commodity vector infeasible. Formally,

Definition 3.1: a commodity vector $\mathbf{q} \in L(u)$ is commodity efficient if, and only if, $\mathbf{q}' \notin L(u)$ for $\mathbf{q}' < \mathbf{q}$.

Following this definition of commodity efficiency, we propose a Debreu-Farrell-like (Debreu, 1951; Farrell, 1957) *measure of commodity efficiency* in consumption, defined as

$$CE(u,\mathbf{q}) = \min\left\{\zeta : \zeta\mathbf{q} \in L(u)\right\}. \tag{3.35}$$

The proposed measure of commodity efficiency calls a reference commodity vector commodity efficient if, when radially contracted, it no longer attains the given utility level u. Let the indifference curve associated with utility level u be defined as $I(u) = \left\{\mathbf{q} \in L(u) : \lambda\mathbf{q} \notin L(u)\ \forall\ \lambda < 1\right\}$, that is, the set of commodity vectors which can generate utility u but which, when radially contracted, they no longer generate the utility level u. It is obvious then that the measure of commodity efficiency calls a reference commodity vector commodity efficient if it is an element of the indifference curve associated with the utility level u. Thus, just as input isoquants provide a standard against which input-oriented technical efficiency of the producer can be measured, indifference curves provide a standard against which *commodity efficiency* of commodity vectors can be measured. However, when preferences are weakly monotone, this measure of efficiency may assign the same efficiency score to different commodity inefficient commodity vectors.[39] In order to provide a stricter standard for measuring commodity efficiency, we will introduce a notion similar to that of input efficient subsets in production theory, the *commodity efficient subsets*, which we define as

$$Eff(u) = \{\mathbf{q} \in L(u) : \mathbf{q}' < \mathbf{q} \Rightarrow \mathbf{q}' \notin L(u)\}. \qquad (3.36)$$

Thus, the commodity efficient subsets are those sets of commodity vectors which can generate utility u but which, when decreased in any dimension, can no longer generate utility u. It is obvious from Definition 3.1 that a commodity vector $\mathbf{q} \in L(u)$ is commodity efficient if, and only if, $\mathbf{q} \in Eff(u)$. Moreover, a comparison of the definitions of indifference curves and of commodity efficient subsets reveals that commodity efficient subsets are subsets of the indifference curves. As such, commodity efficient subsets represent stricter standards for measuring commodity efficiency in that, if a feasible commodity vector is commodity efficient against $Eff(u)$, then it is also commodity efficient against $I(u)$ but not *vice versa*. Consequently, it is only when preferences satisfy strong non-satiation, that $Eff(u) = I(u)$ and a commodity vector called commodity efficient by the proposed measure (3.35) is also commodity efficient on the basis of Definition 3.1.

The measure of commodity efficiency can also be defined in terms of the input distance function, defined by $D^I(u,\mathbf{q}) = \max_\lambda\{\lambda > 0 : \mathbf{q}/\lambda \in L(u)\}$.[40] Given this definition of the input distance function, the consumption requirement set can also be defined in terms of the input distance function as[41]

$$L(u) = \{\mathbf{q} \in \mathrm{Q}^N : D^I(u,\mathbf{q}) \geq 1\}. \qquad (3.37)$$

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[39] See the example provided by Russell (1998, p. 29).

[40] As before, the input distance function $D^I(\cdot)$ can be extended to $\overset{*}{D}{}^I(\cdot)$, with domain $R(U) \times Q^N$, by continuity from above.

[41] For a proof of this in the context of production theory, see Färe and Primont (1995).

The assumption of weak non-satiation is required for (3.37) to be equivalent to the definition of $L(u)$ as $L(u) = \{\mathbf{q} \in Q^N : U(\mathbf{q}) \geq u\}$, that is, for the input distance function to be able to completely characterise the consumption requirement set. The input distance function can also completely characterise the indifference curves, without the requirement that preferences are weakly monotone. Formally, $\mathbf{q} \in I(u)$ if and only if $D^I(u,\mathbf{q}) = 1$. Hence, indifference curves can be defined as $I(u) = \{\mathbf{q} \in Q^N : D^I(u,\mathbf{q}) = 1\}$, that is, the sets of commodity bundles having an input-distance function value of unity. For all other commodity bundles, the value of the input distance function is greater than unity because the analysis is made under the expenditure-minimisation framework and consumption bundles in the region below and to the left of $L(u)$ are not feasible.

Having defined the consumption (requirement) set in terms of the input distance function, the measure of commodity efficiency can now be given by

$$CE(u,\mathbf{q}) = \min\left\{\zeta : D^I(u,\zeta\mathbf{q}) \geq 1\right\}. \qquad (3.38)$$

This definition of commodity efficiency shows that there is a close relation between the measure of commodity efficiency and the input distance function. In fact, the measure of commodity efficiency is the reciprocal of the input distance function. To see this, note that the reciprocal of $D^I(\cdot)$ is given by $(1/D^I(u,\mathbf{q})) =$ $\min\{\lambda : \lambda\mathbf{q} \in L(u)\}$. Then, by (3.35),

$$CE(u,\mathbf{q}) = 1/D^I(u,\mathbf{q}). \qquad (3.39)$$

There is a number of properties of efficiency indices that our proposed measure of commodity efficiency satisfies.[42] To begin with, the measure of commodity efficiency takes on values in the (0,1] interval. This is easy to check, considering, for example, the definition of the input distance function. Given a reference commodity vector $\mathbf{q} \in L(u)$, the input distance function seeks that scalar λ that will scale $\mathbf{q}$ down

[42] For a discussion on the properties that efficiency indices must satisfy, see, for example, Färe and Lovell (1978) and Russell (1998).

to that commodity vector that just attains u. Assuming that the solution to this problem is λ^*, $D^I(u,\mathbf{q}) = \lambda^* \geq 1$, with $D^I(u,\mathbf{q}) = 1$ if, and only if, $\mathbf{q} \in I(u)$. Since $CE(u,\mathbf{q})$ is the inverse of the input distance function, it follows that it takes on values in the (0,1] interval. In addition, the properties of non-decreasingness and homogeneity of degree one in **q** of the input distance function imply that $CE(u,\mathbf{q})$ satisfies the properties of weak monotonocity and homogeneity of efficiency indices: $CE(u,\mathbf{q})$ is non-increasing and homogeneous of degree minus one in **q**. Moreover, just like its Debereu-Farrell counterpart in production theory, as long as $CE(u,\mathbf{q})$ is assumed to satisfy the weak monotonicity property of efficiency indices instead of the strong monotonicity property, it also satisfies the property of commensurability, *i.e.*, it is independent of the units in which **q** is measured. The properties of $CE(u,\mathbf{q})$ can now be stated formally as follows: for any $\mathbf{q} \in L(u)$,

(i) $0 < CE(u,\mathbf{q}) \leq 1$,

(ii) $CE(u,\mathbf{q}) = 1 \Leftrightarrow \mathbf{q} \in I(u)$,

(iii) $CE(u,\mathbf{q})$ is non-increasing in **q**,

(iv) $CE(u,\mathbf{q})$ is homogeneous of degree -1 in **q**,

(v) $CE(u,\mathbf{q})$ is independent of the units in which **q** is measured.

Property (ii) follows from relation (3.35), according to which a commodity vector is commodity efficient if it is an element of the indifference curve $I(u)$. However, as already mentioned, when preferences satisfy strong non-satiation a commodity vector defined by $CE(u,\mathbf{q})$ as commodity efficient is also commodity efficient on the basis of Definition 3.1.

Figure 3.2 illustrates the measure of commodity efficiency, for the case of two commodities. Suppose that a consumer chooses a commodity bundle, say, $\mathbf{q}^0$, such that $\mathbf{q}^0 \in L(u^*)$ and $\mathbf{q}^0 \notin I(u^*)$, with a view to achieve a target utility level u^*. The consumer in our example is commodity inefficient in that the same utility level u^* could be attained with proportionally less of all commodities. In particular, he/she could have chosen the commodity vector $\mathbf{q}^0 / D^I(u^*,\mathbf{q}^0)$, which contains a fraction 0*S*/0*R* of the quantities (q_1^0, q_2^0) and it just generates the target utility level u^*. It is obvious that the ratio 0*S*/0*R*, which is the inverse of $D(u^*,\mathbf{q}^0)$, can serve as a measure of the consumer's commodity inefficiency. That is, $CE(u^*,\mathbf{q}^0) = 0S/0R = \|\mathbf{q}^0/D(u^*,\mathbf{q}^0)\|/\|\mathbf{q}^0\| = 1/D^I(u^*,\mathbf{q}^0) < 1$.

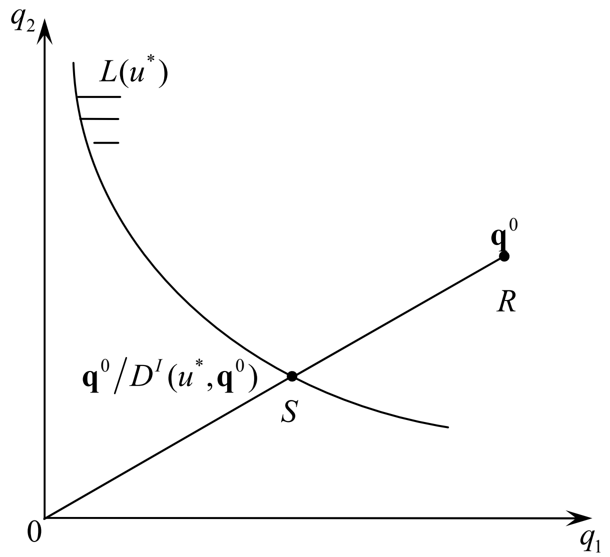

Figure 3.2 Commodity Efficiency

3.3.3 Expenditure Efficiency and Allocative Efficiency

Assume that consumers face strictly positive commodity prices, and that their objective is to attain a target utility level, *u*, from the consumption of a feasible vector of quantities **q**. Consumers behave optimally as long as the expenditure for the chosen commodity vector is the minimum one required for the attainment of *u*. However, there is no reason why one should assume that consumers always behave optimally; as long as any unwanted quantities of the purchased commodities can be disposed of, consumers' actual (observed) expenditure may be higher than the expenditure required for the achievement of *u*. The consumer's ability to avoid wasting expenditures, by minimising the cost of purchased commodities in the achievement of a target utility level is what is described in this section as *expenditure efficiency* or *overall efficiency*. The proposed measure of this type of efficiency is defined as the ratio of minimum to actual expenditure. Formally, assuming that consumers face strictly positive commodity prices, $\mathbf{p} \in R_{++}^{N}$, and that their objective is to choose that feasible vector of quantities $\mathbf{q}$ that will minimise the level of total expenditure $\mathbf{p} \cdot \mathbf{q}$ required to attain a fixed utility level *u*, a *measure of expenditure efficiency* (or *overall efficiency*) is given by the ratio of the minimum expenditure required for the achievement of *u* to the actual expenditure, that is,

$$EE(u, \mathbf{q}, \mathbf{p}) = C(u, \mathbf{p}) / (\mathbf{p} \cdot \mathbf{q}), \tag{3.40}$$

where $C(u, \mathbf{p})$ is the expenditure function defined as $C(u, \mathbf{p}) = \min_{\mathbf{q}} \{\mathbf{p} \cdot \mathbf{q} : U(\mathbf{q}) \geq u\}$, or, equivalently, $C(u, \mathbf{p}) = \min_{\mathbf{q}} \{\mathbf{p} \cdot \mathbf{q} : D(u, \mathbf{q}) \geq 1\}$.

Not all expenditure inefficiency can be attributed to commodity inefficiency. This is because even if consumers behave at 100% commodity efficiency they could choose a wrong combination of commodity quantities, given the market prices of the commodities. This type of efficiency, which is concerned with how close a chosen commodity vector $\mathbf{q} \in I(u)$ is to the expenditure-minimising commodity vector on $I(u)$, is defined here as *allocative efficiency*. Allocative efficiency can be measured as the ratio of expenditure efficiency to commodity efficiency, that is,

$$AE(u, \mathbf{q}, \mathbf{p}) = EE(u, \mathbf{q}, \mathbf{p}) / CE(u, \mathbf{q}). \tag{3.41}$$

A comparison of the proposed efficiency measures to Varian's (1990) money-metric measure is suitable at this point. As shown in relation (3.16), Varian's measure is given by the ratio of the minimum expenditure required for the consumer to be as well off as he/she would by consuming a reference commodity vector, to the actual expenditure for that reference vector. Hence, Varian's money metric measure resembles the expenditure efficiency measure proposed here. However, since the money-metric utility function picks out a commodity vector that makes the consumer as well off as he/she would be consuming a reference vector, it picks out a commodity vector that lies on the same indifference curve as the reference vector. As a result, Varian's measure can be thought of as measuring the overall inefficiency of commodity efficient commodity vectors. However, by assuming that observed commodity vectors are commodity efficient, and hence, by leaving no room for a decomposition of expenditure efficiency such as the one developed here, Varian's (1990) measure may assign to consumers a higher efficiency score than it should.

The proposed measure of allocative efficiency can also be defined as a cost ratio. Recall that $CE(u,\mathbf{q}) = 1/D^I(u,\mathbf{q})$. Multiply and divide $CE(u,\mathbf{q})$ by actual expenditure, $\mathbf{p}\cdot\mathbf{q}$, and then rearrange terms to write $CE(u,\mathbf{q})$ as

$$CE(u,\mathbf{q}) = \frac{\mathbf{p}\cdot\mathbf{q}/D^I(u,\mathbf{q})}{\mathbf{p}\cdot\mathbf{q}}, \tag{3.42}$$

that is, the ratio of the expenditure for the commodity efficient commodity vector to the expenditure for the actual commodity vector. Using (3.40) and (3.42), the measure of allocative efficiency can be written as

$$AE(u,\mathbf{q},\mathbf{p}) = \frac{C(u,\mathbf{p})}{\mathbf{p}\cdot\mathbf{q}/D^I(u,\mathbf{q})}. \tag{3.43}$$

Hence, the measure of allocative efficiency is simply the ratio of the minimum expenditure required for attaining u to the expenditure for the commodity efficient commodity vector.

Both the measure of expenditure efficiency and the measure of allocative efficiency take on values in the (0,1] interval. A consumer is said to be expenditure efficient (*i.e.*, $EE(u,\mathbf{q},\mathbf{p}) = 1$) if he/she chooses the expenditure-minimising commodity vector, whereas he/she is said to be allocatively efficient (*i.e.*, $AE(u,\mathbf{q},\mathbf{p}) = 1$) if he/she chooses a feasible commodity vector which is proportional to the expenditure-minimising vector. In addition, $EE(u,\mathbf{q},\mathbf{p})$ is homogeneous of degree zero in **p**, since, for any $\lambda > 0$, $EE(u,\mathbf{q},\lambda\mathbf{p}) = C(u,\lambda\mathbf{p})/(\lambda\mathbf{p}\cdot\mathbf{q}) = \lambda C(u,\mathbf{p})/(\lambda(\mathbf{p}\cdot\mathbf{q})) = EE(u,\mathbf{q},\mathbf{p})$.[43] In the same manner, it can also be shown that $EE(u,\mathbf{q},\mathbf{p})$ is homogeneous of degree minus one in **q**, and $AE(u,\mathbf{q},\mathbf{p})$ is homogeneous of degree zero in **p** and **q**. Finally, the property of non-decreasingness

[43] The second equality follows from the property of the expenditure function of homogeneity of degree one in **p**.

in u of the expenditure function implies that $EE(u,\mathbf{q},\mathbf{p})$ is non-decreasing in u. Formally, for any for any $\mathbf{q} \in L(u)$, the properties of $EE(u,\mathbf{q},\mathbf{p})$ can be stated as

(i) $0 < EE(u,\mathbf{q},\mathbf{p}) \leq 1$,

(ii) $EE(u,\mathbf{q},\mathbf{p}) = 1 \Leftrightarrow \mathbf{q} = \mathbf{q}(u,\mathbf{p})$ so that $\mathbf{p} \bullet \mathbf{q} = C(u,\mathbf{p})$,

(iii) $EE(u,\lambda\mathbf{q},\mathbf{p}) = \lambda^{-1} EE(u,\mathbf{q},\mathbf{p})$ for $\lambda > 0$.

(iv) $EE(u,\mathbf{q},\lambda\mathbf{p}) = EE(u,\mathbf{q},\mathbf{p})$ for $\lambda > 0$,

(v) $EE(\lambda u,\mathbf{q},\mathbf{p}) \geq EE(u,\mathbf{q},\mathbf{p})$ for $\lambda \geq 1$,

while the properties of $AE(u,\mathbf{q},\mathbf{p})$ can be stated as

(i) $0 < AE(u,\mathbf{q},\mathbf{p}) \leq 1$,

(ii) $AE(u,\mathbf{q},\mathbf{p}) = 1 \Leftrightarrow \lambda\mathbf{q} = \mathbf{q}(u,\mathbf{q})$ for $\lambda \leq 1$,

(iii) $AE(u,\lambda\mathbf{q},\mathbf{p}) = AE(u,\mathbf{q},\mathbf{p})$ for $\lambda > 0$,

(iv) $AE(u,\mathbf{q},\lambda\mathbf{p}) = AE(u,\mathbf{q},\mathbf{p})$ for $\lambda > 0$.

It is obvious from definition (3.41) that expenditure efficiency can be decomposed into commodity and allocative efficiency. This decomposition of expenditure efficiency is illustrated in Figure 3.3 for the case of two commodities. Given commodity prices $\mathbf{p}^0$, let $\mathbf{q}^*$ and $C(u^*,\mathbf{p}^0) = \mathbf{p}^0 \bullet \mathbf{q}^*$ denote the expenditure-minimising commodity vector and the minimum expenditure required for attaining a target utility level u^*, respectively. Suppose now that a consumer facing commodity prices $\mathbf{p}^0$ chooses a commodity bundle which is more than enough to generate u^*. That, is he/she chooses $\mathbf{q}^0$, such that $\mathbf{q}^0 \in L(u^*)$ and $\mathbf{q}^0 \notin I(u^*)$. Suppose also that the consumer does not use the purchased commodities at hand as efficiently as he/she could, so that the actual utility-quantity combination is $(u^*,\mathbf{q}^0)$. As shown in Section 3.3.2, this consumer is commodity inefficient since the same utility level u^* could be attained with the commodity vector $\mathbf{q}^0 / D^I(u^*,\mathbf{q}^0)$ which is a radial contraction of $\mathbf{q}^0$. Hence, the degree of the consumer's commodity efficiency is given by $CE(u^*,\mathbf{q}^0) = 0S/0R = [\mathbf{p}^0 \bullet \mathbf{q}^0 / D^I(u^*,\mathbf{q}^0)] / \mathbf{p}^0 \bullet \mathbf{q}^0 = 1/D^I(u^*,\mathbf{q}^0)$. However, in the situation depicted in Figure 3.3, the commodity efficient commodity vector $\mathbf{q}^0/D^I(u^*,\mathbf{q}^0)$ does not coincide with the expenditure-minimising vector $\mathbf{q}^*$. Hence, even if the consumer in our example had chosen the commodity efficient commodity vector, he/she would still not be expenditure (or overall) efficient: given relative commodity prices as they are reflected in the slope of the tangent at *A*, the commodity vector $\mathbf{q}^0/D^I(u^*,\mathbf{q}^0)$ contains the wrong mix of commodity quantities.

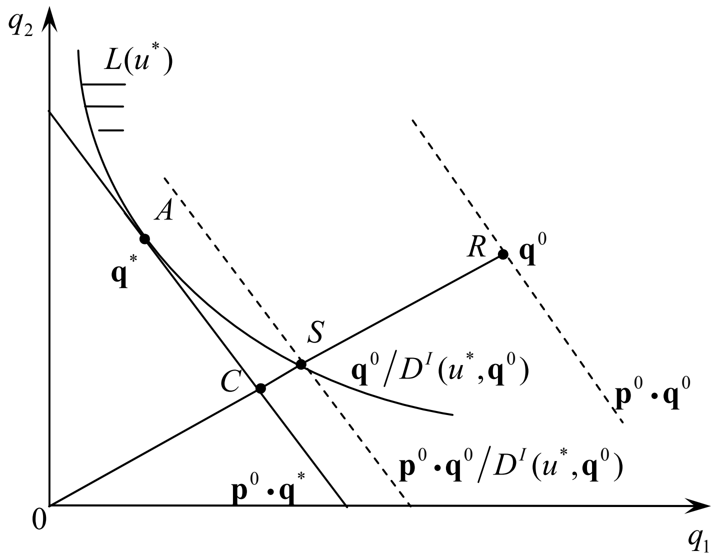

Figure 3.3 The Decomposition of Expenditure Efficiency

This remaining portion of expenditure inefficiency is measured by allocative efficiency, which is given by $AE(u^*,\mathbf{q}^0,\mathbf{p}^0) = 0C/0S = C(u^*,\mathbf{p}^0)/[\mathbf{p}^0\cdot\mathbf{q}^0/D^I(u^*,\mathbf{q}^0)]$. Finally, if the observed utility-commodity combination is $(u^*,\mathbf{q}^0)$, the expenditure efficiency of the consumer can be measured by the ratio ratio 0*C*/0*R*, which is the product of the measures of commodity and allocative efficiency. That is, $EE(u^*,\mathbf{q}^0,\mathbf{p}^0) = 0C/0R = C(u^*,\mathbf{p}^0)/(\mathbf{p}^0\cdot\mathbf{q}^0)$. The decomposition of expenditure efficiency into commodity efficiency and allocative efficiency can now be summarised as follows:

$$EE(u^*,\mathbf{q}^0,\mathbf{p}^0) = CE(u^*,\mathbf{q}^0)\times AE(u^*,\mathbf{q}^0,\mathbf{p}^0)$$

$$\Rightarrow \frac{0C}{0R} = \frac{0S}{0R}\frac{0C}{0S} \Rightarrow \frac{\mathbf{p}^0 \bullet \mathbf{q}^*}{\mathbf{p}^0 \bullet \mathbf{q}^0} = \frac{\mathbf{p}^0 \bullet \mathbf{q}^0 / D^I(u^*,\mathbf{q}^0)}{\mathbf{p}^0 \bullet \mathbf{q}^0} \frac{\mathbf{p}^0 \bullet \mathbf{q}^*}{\mathbf{p}^0 \bullet \mathbf{q}^0 / D^I(u^*,\mathbf{q}^0)}$$

$$\Rightarrow \frac{C(u^*,\mathbf{p}^0)}{\mathbf{p}^0 \bullet \mathbf{q}^0} = \frac{1}{D^I(u^*,\mathbf{q}^0)} \frac{C(u^*,\mathbf{p}^0)}{\mathbf{p}^0 \bullet \mathbf{q}^0 / D^I(u^*,\mathbf{q}^0)}.$$

An alternative definition of the measures of commodity and allocative efficiency can be given with the use of the shadow prices. To this aim, we will present a relation known as *Mahler's Inequality* and the notions of *direct* and *indirect conjugacy*. Following Blackorby, Primont, and Russell (1978), let $\mathbf{q}^*$ denote the expenditure-minimising commodity vector for the mimisation problem in (3.22). Also, let λ^* denote the solution to the maximisation problem $D^I(u,\mathbf{q}) = \max_\lambda \{\lambda > 0 : \mathbf{q}/\lambda \in L(u)\}$, so that $(\mathbf{q}/\lambda^*) \in L(u)$, and write the product of the input distance function and the expenditure function as follows:

$$D^I(u,\mathbf{q})C(u,\mathbf{p}) = \lambda^* \mathbf{p} \bullet \mathbf{q}^* \le \lambda^* \mathbf{p} \bullet \frac{\mathbf{q}}{\lambda^*} = \mathbf{p} \bullet \mathbf{q} \Rightarrow$$

$$D^I(u,\mathbf{q})C(u,\mathbf{p}) \le \mathbf{p} \bullet \mathbf{q}, \qquad \forall (\mathbf{p},\mathbf{q}) \in Q_+^{2N}. \tag{3.44}$$

Relation (3.44) is known as *Mahler's inequality*. Blackorby, Primont, and Russell (1978, p. 28) point out that the commodity vector $\mathbf{q}/D^I(u,\mathbf{q})$ may not be optimal at prices $\mathbf{p}$, and define a pair $(\overline{\mathbf{p}},\overline{\mathbf{q}})$ to be *direct conjugates* at utility level *u* if the *u*-level set is supported at $\overline{\mathbf{q}}/D^I(u,\overline{\mathbf{q}})$ by a hyperplane with normal $\overline{\mathbf{p}}$ where $\overset{*}{C}(u,\overline{\mathbf{p}}) = 1$. That is, if the following equality holds

$$D^I(u,\overline{\mathbf{q}})\overset{*}{C}(u,\overline{\mathbf{p}}) = \overline{\mathbf{p}} \bullet \overline{\mathbf{q}} \tag{3.45}$$

where $\overset{*}{C}(\cdot)$ is an extension of $C(\cdot)$ to $R(U) \times \mathbb{R}_+^N$ by continuity from above.

Balk (1998) uses a different approach for the derivation of relation (3.44). Following his example, we will provide an alternative representation of Mahler's inequality. For ease of exposition, we will assume that the choice set is Q^N and that the constraint in the minimisation problem in (3.23) is defined for unit level expenditure, *i.e.*, the constraint becomes $C(u,\mathbf{p}) \geq 1$ and minimisation is now over $\mathbf{r} = \mathbf{p}$. The positive linear homogeneity of the expenditure function in **p** implies that for all utility-price combinations $C(u,\mathbf{p}/C(u,\mathbf{p})) = 1$. Thus, the price vector $\mathbf{p}/C(u,\mathbf{p})$ satisfies the constraint $C(u,\mathbf{p}) \geq 1$ in the minimisation problem in (3.23). Consequently,

$$D^I(u,\mathbf{q}) = \mathbf{p}^* \cdot \mathbf{q} \leq \frac{\mathbf{p}}{C(u,\mathbf{p})} \cdot \mathbf{q} \Rightarrow$$

$$D^I(u,\mathbf{q})C(u,\mathbf{p}) \leq \mathbf{p} \cdot \mathbf{q}, \qquad \forall (\mathbf{p},\mathbf{q}) \in Q_+^{2N}. \tag{3.46}$$

where $\mathbf{p}^* = \mathbf{p}(u,\mathbf{q})$ is now the vector of optimal shadow prices. Blackorby, Primont, and Russell (1978, p. 28) point out that the (imputed) price vector $\mathbf{p}/C(u,\mathbf{p})$ may not be optimal at quantities **q**, and define a pair $(\overline{\mathbf{p}},\overline{\mathbf{q}})$ to be *indirect conjugates* at utility level u if the u-level set is supported at $\overline{\mathbf{p}}/C(u,\overline{\mathbf{p}})$ by a hyperplane with normal $\overline{\mathbf{q}}$ where $\overset{*}{D}{}^I(u,\overline{\mathbf{q}}) = 1$. That is, if the following equality holds

$$\overset{*}{D}{}^I(u,\overline{\mathbf{q}})C(u,\overline{\mathbf{p}}) = \overline{\mathbf{p}} \cdot \overline{\mathbf{q}} \tag{3.47}$$

where $\overset{*}{D}{}^I(\cdot)$ is an extension of $D^I(\cdot)$ to $R(U) \times Q^N$ by continuity from above.

Using relations (3.45) and (3.47), Blackorby, Primont, and Russell (1978) describe an optimal pair $(\overline{\mathbf{p}},\overline{\mathbf{q}})$ at $\overline{u}$ as a pair which is simultaneously directly and indirectly conjugate at $\overline{u}$, that is,

$$\overline{\mathbf{p}} \cdot \frac{\overline{\mathbf{q}}}{\overset{*}{D}{}^I(\overline{u},\overline{\mathbf{q}})} = \frac{\overline{\mathbf{p}}}{\overset{*}{C}(\overline{u},\overline{\mathbf{p}})} \cdot \overline{\mathbf{q}},$$

which implies that

$$\overset{*}{D}{}^I(\overline{u},\overline{\mathbf{q}}) = \overset{*}{C}(\overline{u},\overline{\mathbf{p}}) = 1 \text{ and } U(\overline{\mathbf{q}}) = \overline{u} = \overset{*}{V}(\overline{\mathbf{p}}) = 1,$$

where $\overset{*}{V}(\cdot)$ is the extension of the consumer's indirect utility function $V(\cdot)$ from $\mathbb{R}^N_{++}$ to $\mathbb{R}^N_{+}$ by continuity from below.[44] It follows therefore that $\overline{\mathbf{q}} = \mathbf{q}(\overline{u}, \overline{\mathbf{p}})$ and $\overline{\mathbf{p}} = \mathbf{p}(\overline{u}, \overline{\mathbf{q}})$ (Blackorby, Primont, and Russell, 1978, pp. 31-2).

Let us provide now an alternative interpretation of the measures of commodity and allocative efficiency. Relation (3.47) says that $\overline{\mathbf{p}}/C(u,\overline{\mathbf{p}})$ is the vector of deflated shadow prices that minimises the value of the commodity vector $\overline{\mathbf{q}}$, given *u*. If we rearrange terms in relation (3.47) to obtain

$$C(u,\overline{\mathbf{p}}) = \overline{\mathbf{p}} \cdot \frac{\overline{\mathbf{q}}}{\overset{*}{D}{}^I(u,\overline{\mathbf{q}})}, \tag{3.48}$$

then $\overline{\mathbf{p}}$ can be interpreted as the vector of shadow prices that make $\overline{\mathbf{q}} \big/ \overset{*}{D}{}^I(u,\overline{\mathbf{q}})$ the least expenditure solution for attaining *u*. Recall that the measure of commodity efficiency can be defined as the ratio of the cost of the commodity efficient commodity vector to the cost of the actual commodity vector, say, $\mathbf{p} \cdot \overline{\mathbf{q}}$. Recall also that the commodity efficient commodity vector may not be expenditure–minimising given market prices. If the latter is the case, then the commodity efficient commodity vector will be optimal at shadow prices, say, $\overline{\mathbf{p}}$, and the shadow expenditure for this commodity vector is given from relation (3.48). However, as Balk (1998) notes, since, the expenditure function is homogeneous of degree one in prices, if $\overline{\mathbf{p}}$ satisfies (3.48), then $\lambda\overline{\mathbf{p}}$ also satisfies (3.48) for $\lambda > 0$. Since the shadow prices are determined up to a multiplicative factor, we can choose a normalisation such that $\overline{\mathbf{p}} \cdot \overline{\mathbf{q}} = \mathbf{p} \cdot \overline{\mathbf{q}}$, where $\mathbf{p}$ is the vector of market prices (Balk, 1998, p. 29). Hence, substituting this equality into (3.48), and using the definition of commodity efficiency (3.42), we obtain

$$CE(u,\overline{\mathbf{q}}) = \frac{C(u,\overline{\mathbf{p}})}{\mathbf{p} \cdot \overline{\mathbf{q}}}. \tag{3.49}$$

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[44] See, for example, Blackorby, Primont, and Russell (1978), Diewert (1982), and Weymark (1980).

That is, the measure of commodity efficiency can be interpreted as the ratio of the minimum expenditure of attaining u under shadow prices $\overline{\mathbf{p}}$ to the actual expenditure. Finally, using (3.41), (3.40) and (3.49), we can define the measure of allocative efficiency as

$$AE(u,\overline{\mathbf{q}},\mathbf{p}) = \frac{C(u,\mathbf{p})}{C(u,\overline{\mathbf{p}})}, \tag{3.50}$$

that is, the ratio of the minimum expenditure of attaining u under market prices $\mathbf{p}$ to the minimum expenditure of attaining u under shadow prices $\overline{\mathbf{p}}$.

3.3.4 Utility Efficiency, Budget Allocative Efficiency and Utility Overall Efficiency

In Section 3.3.2, we assumed that the consumer's objective is to choose a feasible commodity vector in order to achieve a target utility level. We also proposed a measure of commodity efficiency, where commodity efficiency was described as the consumer's ability to avoid wasting any quantities of the purchased commodities, by minimising quantity purchases in the achievement of a target utility level. But, if the consumer is wasting some of the quantities of the purchased commodities, then he/she is inefficient, not only in terms of the commodity quantities that were wasted, but also in terms of utility that could have been, but was not, attained at the end of the day. This observation gives rise to a different type of efficiency in consumption.

In this section, it will be assumed that the consumer's objective is to obtain maximum utility from a given commodity vector. If the consumer does not achieve this objective, *i.e.*, if he/she does not use the purchased commodities as efficiently as he/she could, then the utility attained may well be lower than the maximum attainable one. We will use the notion of *utility efficiency*, in order to describe consumer's ability to avoid wasting utility, by obtaining maximum utility from given purchased commodities. This type of efficiency applies not only to commodities the quantities of which the consumer may dispose off. Since the starting point is the consumer's objective of obtaining maximum utility from a given commodity vector, the notion of utility efficiency can be used to study nonfoods as well. For example, it could be lack of information about the commodities' characteristics, the way(s) they could and/or should be used, etc., that may lead the consumer to obtain a utility level that is lower than the one the commodities could potentially allow him/her to. Particularly, in the case of commodities that are complementary, inefficient use of one commodity will also lead to inefficient use of the other.

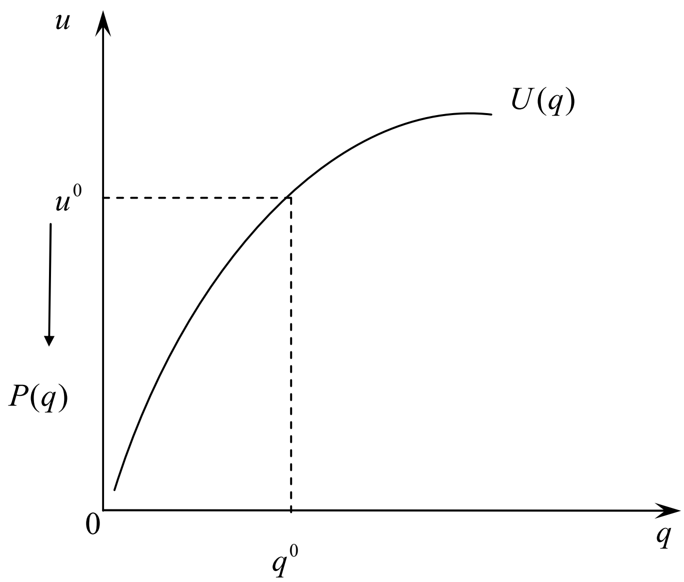

Figure 3.4 The Utility Consumption (Requirement) Set in the One-good Case

Before examining this new type of inefficiency in consumption some necessary definitions need to be presented. Firstly, we will define the *utility requirement set* as the set of utility levels that, for each commodity vector $\mathbf{q} \in Q^N$, are feasible for the consumer to attain, that is,

$$P(\mathbf{q}) = \{u \in \mathbb{R}^1_{++} : u \text{ can be attained with } \mathbf{q}\}, \qquad (3.51)$$

It is assumed that $u \in \mathbb{R}^1_{++}$, *i.e.*, $u > 0$, since the point $\mathbf{q} = (0,\ldots,0)$ is excluded from the choice set. The utility (requirement) set is closed, bounded from above (we have $u \le U(\mathbf{q})$ for all $\mathbf{q} \in Q^N$), and convex. Moreover, it satisfies a monotonicity property which we define as *utility disposability*:

Utility Disposability: for every $\mathbf{q} \in Q^N$, if $u > u'$ and $u \in P(\mathbf{q})$, then $u' \in P(\mathbf{q})$ (or, alternatively, if $u \in P(\mathbf{q})$ and $u > u'$, then $L(u) \subset L(u')$).

The utility (requirement) set is depicted in Figure 3.4, for the case of one commodity. In this figure, $U(q)$ denotes the direct utility function for one commodity, q denotes quantities of the commodity in question, and u denotes utility levels. The maximum utility level that can be generated by the quantity q^0 is u^0. However, a consumer purchasing q^0 quantities of the commodity can obtain any utility level which is less than, or equal to, u^0. In this case, the utility (requirement) set is given by $P(q) = \{u \in \mathbb{R}^1_{++} : u \leq u^0\}$.

The definition of utility efficiency which is proposed in this section is in accordance with the output-oriented case of Koopman's (1951) definition of output-oriented technical efficiency in production. Specifically, we define a utility level, which is feasible for a given commodity vector **q**, to be utility efficient, if, and only if, no increase in utility is feasible. Formally,

Definition 3.2: a utility level $u \in P(\mathbf{q})$ is utility efficient if, and only if, $u' \notin P(\mathbf{q})$ for $u' > u$.

Following this definition of utility efficiency, we propose a Debreu-Farrell-like (Debreu, 1951; Farrell, 1957) *measure of utility efficiency* in consumption, defined as

$$UE(\mathbf{q},u) = \left[\max\{\psi : \psi u \in P(\mathbf{q})\}\right]^{-1} = \left[\max\{\psi : \mathbf{q} \in L(\psi u)\}\right]^{-1}. \tag{3.52}$$

The proposed measure calls a reference utility level utility efficient if, when increased, it can no longer be generated by the given commodity vector **q**. That is, a utility level is utility efficient if it is associated with the indifference curve on which the given commodity vector **q** lies. Since indifference curves can also be thought of as representing the maximum utility that can be attained by a given commodity vector, they provide a standard against which utility efficiency of utility levels can be measured.

The measure of utility efficiency can also be defined in terms of the *output distance function*. Although it has never been used before in the consumer theory context, the output distance function is well-established in producer theory. In the producer theory context, this function is defined as $D^O(\mathbf{x},\mathbf{y}) = \min_\mu\{\mu : \mathbf{y}/\mu \in P(\mathbf{x})\}$, where **x** denotes a vector of inputs, **y** denotes a vector of feasible outputs, and $P(\mathbf{x})$ is the output requirement set, *i.e.*, the set of output vectors that are feasible for each input vector. The output distance function is lower semi-continuous, non-increasing in inputs, and homogeneous of degree one, non-decreasing and convex in outputs. Let us define the output distance function in the consumer theory context as

$$D^O(\mathbf{q},u) = \min_\psi\{\psi : u/\psi \in P(\mathbf{q})\}, \tag{3.53}$$

and assume that is a lower semi-continuous function, non-increasing in quantities, and non-decreasing and convex in utility. The output distance function holds a target commodity vector **q** fixed, and seeks that scalar that will scale up a reference utility level $u \in P(\mathbf{q})$ to the boundary of $P(\mathbf{q})$. Hence, for $u \in P(\mathbf{q})$, the output distance function takes on values lower than or equal to unity. In analogy with the description of the output requirement set in terms of the output distance function, we will re-define the utility requirement set as[45]

$$P(\mathbf{q}) = \{u \in \mathbb{R}_+ : D^O(\mathbf{q},u) \le 1\}. \tag{3.54}$$

Now, we can define the measure of utility efficiency in terms of the output distance function, as

$$UE(\mathbf{q},u) = \left[\max\left\{\psi : D^O(\mathbf{q},\psi u) \le 1\right\}\right]^{-1}. \tag{3.55}$$

[45] In producer theory, the assumption of *weak output disposability* is required for the output distance function to completely characterise the output requirement set (see, Färe and Primont, 1995). In the consumer theory context, the assumption of utility disposability should be made for the utility requirement set to be defined in terms of the output distance function.

The measure of utility efficiency coincides with the output distance function, since, by by (4.52), $\left[\max\{\psi : \psi u \in P(\mathbf{q})\}\right]^{-1} = \min\{\psi : u/\psi \in P(\mathbf{q})\}$, and consequently,

$$UE(\mathbf{q},u) = D^O(\mathbf{q},u). \qquad (3.56)$$

The properties of the utility efficiency index proposed here can be inferred form the properties of the output distance function. Firstly, the measure of utility efficiency takes on values in the (0,1] interval, and is equal to unity if, and only if, the reference utility level u is associated with the boundary of $P(\mathbf{q})$. Moreover, $UE(\mathbf{q},u)$ is non-decreasing in utility, and it is independent of the units in which $\mathbf{q}$ is measured. Formally, for $u \in P(\mathbf{q})$,

(i) $0 < UE(\mathbf{q},u) \le 1$,

(ii) $UE(\mathbf{q},u) = 1 \Leftrightarrow u$ is a member of the boundary of $P(\mathbf{q})$,

(iii) $UE(\mathbf{q},u)$ is non-decreasing in u,

(iv) $UE(\mathbf{q},u)$ is independent of the units in which $\mathbf{q}$ is measured.

Figure 3.5 illustrates the measure of utility efficiency, for the case of two commodities. Suppose that a consumer chooses a commodity bundle, say, $\mathbf{q}^0$, lying on the indifference curve associated with utility level u^0, *i.e.*, $\mathbf{q}^0 \in I(u^0)$. The commodity vector $\mathbf{q}^0$ is capable of generating utility less than or equal to u^0. Suppose also the consumer does not use the purchased commodities as efficiently as he/she could, thus attaining utility a lower utility level $\hat{u}$. The consumer in this example is utility inefficient in that the achieved utility level is less than the maximum one that the purchased commodity vector can generate. In particular, if the commodities were used efficiently, the consumer could have achieved utility $u^0 = \hat{u}/D^O(\mathbf{q}^0,\hat{u})$. The measure of the consumer's utility inefficiency is then given by $UE(\mathbf{q}^0,\hat{u}) = \hat{u}/u^0 = \hat{u}/\left[\hat{u}/D^O(\mathbf{q}^0,\hat{u})\right] = D^O(\mathbf{q}^0,\hat{u}) < 1$.

Recall that in the diagrammatical example of Section 3.3.2 the consumer chooses the commodity bundle $\mathbf{q}^0$ with a view to achieve a target utility level $\hat{u}$, and that this consumer is commodity inefficient in that the same utility level $\hat{u}$ could be attained with proportionally less of all commodities. The consumer in that example could have chosen the commodity vector $\mathbf{q}^0 / D^I(\hat{u}, \mathbf{q}^0)$, which contains a fraction 0*S*/0*R* of the quantities (q_1^0, q_2^0) and it just generates the target utility level $\hat{u}$.

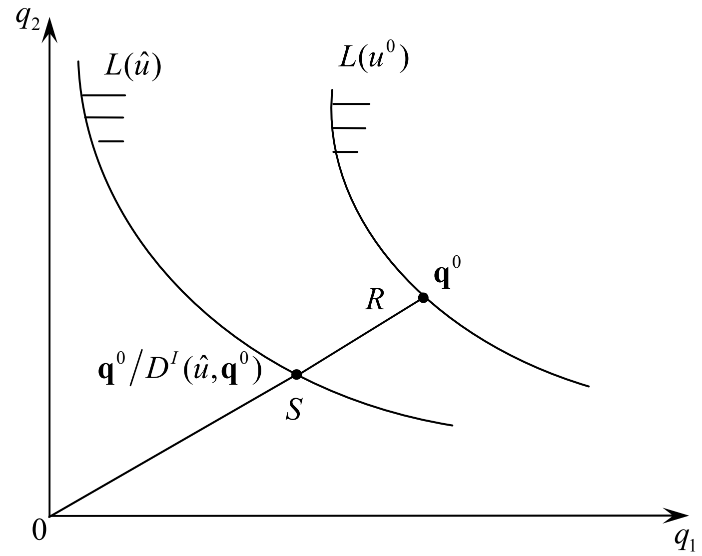

Figure 3.5 Utility Efficiency

Also, we showed that his/her commodity inefficiency is $CE(\hat{u}, \mathbf{q}^0) = 0S/0R = \| \mathbf{q}^0 / D^I(\hat{u}, \mathbf{q}^0) \| / \| \mathbf{q}^0 \| = 1/D^I(\hat{u}, \mathbf{q}^0) < 1$. If the utility function of that consumer was assumed to be homogeneous of degree one in commodity quantities, we could reverse the aforementioned argument and say that the consumption vector (q_1^0, q_2^0) can, if used efficiently, generate 0*R*/0*S* times as much utility as the level $\hat{u}$.[46] In fact, under

[46] A function is said to be homothetic if it is the positive monotonic transformation of a homogenous function. Due to the ordinality property of the direct utility function, any positive monotonic transformation of the direct utility function represents the same preference ordering as the original function does. Hence, we can choose a monotonic increasing transformation of the direct utility function as our representation of preferences. Then, the assumption that the direct utility function is

this restrictive assumption about consumer preferences, the functions $D^I(u,\mathbf{q})$ and $D^O(\mathbf{q},u)$ assign reciprocal values to each utility-commodity combination. That is, if consumer's preferences in our diagrammatical example were homothetic, then $D^O(\mathbf{q}^0,\hat{u}) = 1/D^I(\hat{u},\mathbf{q}^0) \Rightarrow UE(\mathbf{q}^0,\hat{u}) = CE(\hat{u},\mathbf{q}^0)$.[47] Hence, the consumer's utility efficiency score is readily obtained from knowledge of his/her commodity efficiency score.

Utility inefficiency represents one type of consumer's inefficiency in his/her attempt to obtain maximum utility level from a given consumption bundle. However, if the consumption bundle in question is not the utility-maximising one, given the prices the consumer faces and his/her budget, then two more types of efficiency arise: *budget allocative efficiency* and *utility overall efficiency*. In order to describe the notion of *budget allocative inefficiency*, let us assume that the consumer's objective is to choose a feasible vector of quantities $\mathbf{q}$ that will maximise his/her utility level, given strictly positive prices $\mathbf{p} \in R^N_{++}$ and expenditure $\mathbf{p} \cdot \mathbf{q} = x$. Consumers behave optimally as long as the commodity bundle they spend their budget x on is the utility-maximising one. However, due to lack of information about the commodities' characteristics, the way(s) the commodities could and/or should be used, whims, accidents, or other factors that may affect consumers' decisions, consumers may spend their budget on a commodity mix which is not the utility-maximising one. It is the consumers' ability to choose a utility-maximising commodity vector, given budget $x > 0$ and commodity prices $\mathbf{p} \in R^N_{++}$, that we will define as *budget allocative efficiency*. A *measure of budget allocative efficiency* can be defined as the ratio of the utility efficient utility level (*i.e.*, the highest utility level that the observed commodity vector is capable of generating) to the maximum utility that can be achieved given prices $\mathbf{p}$ and budget x (*i.e.*, the highest utility level that can be generated by the utility-maximising commodity vector). Recalling that the indirect utility function, $V(x,\mathbf{p})$, is

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homogeneous of degree one in commodity quantities implies homotheticity of consumer preferences. Note also that the property of homotheticity is more general than the property of homogeneity. That that is, a homogeneous function is also homothetic, but not *vice versa*.

[47] For a proof of this result in the context of production theory, see, for example, Färe and Primont (1995).

defined as the solution to the consumer's utility maximisation problem, and that its value is the maximum utility level attained, the measure of budget allocative efficiency is given by

$$BAE(\mathbf{q},u,x,\mathbf{p}) = \left(u/D^{O}(\mathbf{q},u)\right)\big/V(x,\mathbf{p}). \tag{3.57}$$

Just as commodity efficiency and allocative efficiency constitute two parts of the consumer's expenditure efficiency, utility efficiency and budget allocative efficiency constitute two parts of the consumer's *utility overall efficiency*. A *measure of utility overall efficiency* is given by the ratio of the actual (*i.e.*, the utility inefficient) utility level to the maximum utility that can be achieved, given prices **p** and budget *x*, that is,

$$UOE(\mathbf{q},u,x,\mathbf{p}) = u/V(x,\mathbf{p}), \tag{3.58}$$

whereas, the decomposition of utility overall efficiency into utility efficiency and budget allocative efficiency is given by the relation

$$UOE(\mathbf{q},u,x,\mathbf{p}) = UE(\mathbf{q},u)\times BAE(\mathbf{q},u,x,\mathbf{p}). \tag{3.59}$$

Both the measure of utility overall efficiency and the measure of budget allocative efficiency take on values in the (0,1] interval. A consumer is said to be utility overall efficient (*i.e.*, $UOE(\mathbf{q},u,x,\mathbf{p})=1$) if he/she attains the highest utility level that can be generated by the utility-maximising commodity vector. On the other hand, a consumer is said to be budget allocatively efficient (*i.e.*, $BAE(\mathbf{q},u,x,\mathbf{p})=1$) if he/she chooses the utility-maximising commodity vector but attains a utility level that is lower than the one that the optimal commodity vector can generate. In addition, since the indirect utility function is homogeneous of degree zero in **p** and *x*, $UOE(\mathbf{q},u,x,\mathbf{p})$ and $BAE(\mathbf{q},u,x,\mathbf{p})$ are homogeneous of degree zero in **p** and *x*. That is, for any $\lambda>0$, $UOE(\mathbf{q},u,\lambda x,\lambda\mathbf{p}) = u/V(\lambda x,\lambda\mathbf{p}) = u/V(x,\mathbf{p}) = UOE(\mathbf{q},u,x,\mathbf{p})$, and $BAE(\mathbf{q},u,\lambda x,\lambda\mathbf{p}) = \left(u/D_{O}(\mathbf{q}^{0},u)\right)\big/V(\lambda x,\lambda\mathbf{p}) = BAE(\mathbf{q},u,x,\mathbf{p})$. Moreover, it is obvious from the definition of $BAE(\mathbf{q},u,x,\mathbf{p})$ that this measure is unaffected by changes in consumer's actual utility level, u. On the other hand, by the non-decreasingness of $UE(\mathbf{q},u)$ in u and relation (3.59), $UOE(\mathbf{q},u,x,\mathbf{p})$ is non-decreasing in u. Formally, for $u \in P(\mathbf{q})$,

(i) $0 < UOE(\mathbf{q},u,x,\mathbf{p}) \le 1$,

(ii) $UOE(\mathbf{q},u,x,\mathbf{p}) = 1 \Leftrightarrow \mathbf{q} = \mathbf{q}(x,\mathbf{p})$ and $u = V(x,\mathbf{p})$,

(iii) $UOE(\mathbf{q},u',x,\mathbf{p}) > UOE(\mathbf{q},u,x,\mathbf{p})$ for $u' > u$,

(iv) $UOE(\mathbf{q},u,\lambda x,\lambda \mathbf{p}) = UOE(\mathbf{q},u,x,\mathbf{p})$ for $\lambda > 0$,

and

(i) $0 < BAE(\mathbf{q},u,x,\mathbf{p}) \le 1$,

(ii) $BAE(\mathbf{q},u,x,\mathbf{p}) = 1 \Leftrightarrow \mathbf{q} = \mathbf{q}(x,\mathbf{p})$ and $\lambda u = V(x,\mathbf{p})$ for $\lambda \ge 1$,

(iii) $BAE(\mathbf{q},u',x,\mathbf{p}) = BAE(\mathbf{q},u,x,\mathbf{p})$ for $u' > u$,

(iv) $BAE(\mathbf{q},u,\lambda x,\lambda \mathbf{p}) = BAE(\mathbf{q},u,x,\mathbf{p})$ for $\lambda > 0$.

The decomposition of utility overall efficiency into utility efficiency and budget allocative efficiency is illustrated in Figure 3.6 for the case of two commodities. Suppose that a consumer facing commodity prices $\mathbf{p}^0$ spends his/her budget, x, on the purchase of a commodity vector $\mathbf{q}^0$, which can generate at most utility level u^0. Suppose also that the consumer does not use the purchased commodities as efficiently as he/she could, so that the actual utility level attained is $\hat{u}$. This consumer is utility inefficient in that he/she has attained a utility level which is lower than the one that can be generated by the purchased commodity vector. Hence, the degree of the consumer's utility inefficiency is $UE(\mathbf{q}^0,\hat{u}) = \hat{u}/u^0 = \hat{u}\big/\left[\hat{u}/D^O(\mathbf{q}^0,\hat{u})\right] = D^O(\mathbf{q}^0,\hat{u})$. Suppose now that the purchased commodity vector $\mathbf{q}^0$ does not coincide with the utility-maximising commodity vector $\mathbf{q}^*$. The utility-maximising commodity vector $\mathbf{q}^*$ is capable of generating utility level $u^* \equiv V(\mathbf{p}^0,x)$, which is higher than the one that can be generated by $\mathbf{q}^0$. As a result, the consumer in our example is also budget allocative inefficient since, given prices $\mathbf{p}^0$ and budget x, he/she has made a wrong allocation his/her budget, has purchased a commodity vector that contains a non-optimal mix of commodity quantities, and has not attained the highest utility level possible. The degree of the consumer's budget allocative inefficiency is given by $BAE(\mathbf{q}^0,\hat{u},x,\mathbf{p}^0) = u^0/u^* = \left(\hat{u}/D^O(\mathbf{q}^0,\hat{u})\right)/V(x,\mathbf{p}^0)$. Finally, the utility overall efficiency of the consumer and its decomposition into utility efficiency and budget allocative efficiency is

$$UOE(\mathbf{q}^0,\hat{u},x,\mathbf{p}^0) = UE(\mathbf{q}^0,\hat{u}) \times BAE(\mathbf{q}^0,\hat{u},x,\mathbf{p}^0)$$

$$\Rightarrow \frac{\hat{u}}{u^*} = \frac{\hat{u}}{u^0}\frac{u^0}{u^*}$$

$$\Rightarrow \frac{\hat{u}}{V(x,\mathbf{p}^0)} = \frac{\hat{u}}{\left[\hat{u}/D^O(\mathbf{q}^0,\hat{u})\right]}\frac{\left[\hat{u}/D^O(\mathbf{q}^0,\hat{u})\right]}{V(x,\mathbf{p}^0)}$$

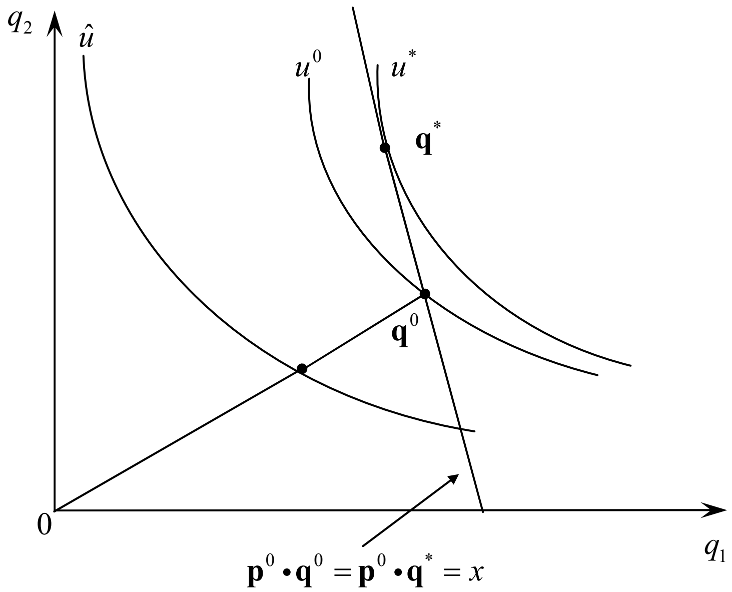

Figure 3.6 Decomposition of Utility Overall Efficiency

4. Measuring Efficiency in Consumption: Empirical Modelling

4.1 INTRODUCTION

Chapter 3 proposed a theoretical model for measuring efficiency in consumption, in price-quantity space. Econometric estimation of this theoretical model calls for establishment of an appropriate empirical framework which will accommodate consumers' non-optimal behaviour. In this context, the aim of the present chapter is to suggest an econometric framework for the aforementioned theoretical model and illustrate its empirical application. In particular, the focus is on the empirical measurement of commodity efficiency, allocative efficiency and expenditure efficiency, whereas the empirical measurement of utility efficiency, budget allocative efficiency and utility overall efficiency is left for future research.

The analysis starts with the computation of the commodity efficiency index which requires estimation of an input distance function. The translog input distance function is employed here and its estimation is carried out using two different approaches. The first approach concerns the use of a proxy for consumer's unobserved utility level (Lewbel and Pendakur, 2006; Färe, Grosskopf, Hayes, and Margaritis, 2008). The second approach concerns the treatment and estimation of utility as a random error term, and represents the contribution of this chapter to the literature of measurement of efficiency in consumption. As already stated, this proposed empirical approach yields an empirical model for consumer's efficiency which resembles, but is not similar to, the two-tiered frontier framework by Polachek and Yoon (1987, 1996). However, the estimators which are proposed in the present chapter for the one-sided error terms are distinctively different from the ones proposed by Polachek and Yoon (1987, 1996).

The structure of this chapter is as follows. The two empirical approaches that are employed for the empirical estimation of consumer's efficiency are presented in detail in Section 4.2, along with the methodology for the computation of the measures of commodity, allocative, and expenditure efficiency. The model is applied to a panel of British household data on purchases of three groups of highly perishable foods, namely, milk & yoghurt, fruits, and vegetables. The last section provides a description of the data and the interpretation of the empirical results.[48]

4.2. EMPIRICAL FRAMEWORK

As mentioned in the introductory section, a translog input distance function is estimated in order to compute the commodity efficiency index. The translog input distance function adopted is given by[49]

$$\begin{aligned}\ln D_{it}^{I}(u,\mathbf{q}) &= \alpha_0 + \sum_j \alpha_j \ln q_{jit} + \frac{1}{2}\sum_j \sum_k \gamma_{jk} \ln q_{jit} \ln q_{kit} + \sum_j \delta_j \ln q_{jit} \ln t + \delta_t \ln t \\ &\quad + \frac{1}{2}\delta_{tt}(\ln t)^2 + \left(\beta_0 + \sum_j \beta_j \ln q_{jit} + \delta_0 \ln t\right)\ln u_{it} + \frac{1}{2}\zeta(\ln u_{it})^2 + \varepsilon_{it} \\ &= f(\ln \mathbf{q}_{it}, t; \mathbf{b}_1) + g(\ln \mathbf{q}_{it}, t; \mathbf{b}_2)\ln u_{it} + \frac{1}{2}\zeta(\ln u_{it})^2 + \varepsilon_{it}, \qquad (4.1)\end{aligned}$$

where $j,k = 1,\ldots,N$ is the number of commodities, $i = 1,\ldots,I$ is the number of households, $t = 1,\ldots,T$ is the number of time-periods, $\mathbf{q}$ denotes the commodity vector, t denotes a time-trend capturing autonomous changes in consumer's preferences over time, u_{it} denotes the period-*t* utility level of the *i*-th consumer, ε_{it} is a random error term which is assumed to be distributed as iid normal with zero mean and variance σ_{ε}^2, and $\alpha_0, \alpha_j, \gamma_{jk}, \delta_j, \delta_t, \delta_{tt}, \beta_0, \beta_j, \delta_0$ and ζ are parameters to be estimated. As consumer theory suggests, the input distance function should satisfy the restrictions of symmetry and homogeneity, which imply the following restrictions on

---

[48] The study in this chapter was presented at the *5th North American Productivity Workshop*, Stern Business School, New York, USA, June 24-27, 2008, under the title "Measurement of Consumption Efficiency in Price-Quantity Space: A Distance Function Approach" (with M. Genius, P. Midmore, and V. Tzouvelekas).

[49] This general translog input distance function can be found in Diewert (1993, pp. 212-3).

the parameters of the distance function given by equation (5.1): $\gamma_{jk} = \gamma_{kj}$, $\sum_{j=1}^{N} \alpha_j = 1$, $\sum_{k=1}^{N} \gamma_{jk} = 0$ for $j = 1, \ldots, N$, $\sum_{j=1}^{N} \beta_j = 0$, and $\sum_{j=1}^{N} \delta_j = 0$.

Since the input distance function is homogeneous of degree 1 in commodity quantities, imposition of homogeneity through division of commodity quantities by, say, q_1, yields:

$$
\begin{aligned}
-\ln q_{1it} &= \alpha_0 + \sum_j \alpha_j \ln\left(q_{jit}/q_{1it}\right) + \frac{1}{2}\sum_j \sum_k \gamma_{jk} \ln\left(q_{jit}/q_{1it}\right)\ln\left(q_{kit}/q_{1it}\right) \\
&\quad + \sum_j \delta_j \ln\left(q_{jit}/q_{1it}\right)\ln t + \delta_t \ln t + \frac{1}{2}\delta_{tt}(\ln t)^2 \\
&\quad + \left(\beta_0 + \sum_j \beta_j \ln\left(q_{jit}/q_{1it}\right) + \delta_0 \ln t\right)\ln u_{it} + \frac{1}{2}\zeta(\ln u_{it})^2 - \ln D_{it}^I(u,\mathbf{q}) + \varepsilon_{it} \\
&= f\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_1\right) + g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_2\right)\ln u_{it} + \frac{1}{2}\zeta(\ln u_{it})^2 \\
&\quad - \ln D_{it}^I(u,\mathbf{q}) + \varepsilon_{it}. \qquad (4.2)
\end{aligned}
$$

As mentioned in Chapter 1, the difficulty in estimating an input distance function representation of consumer preferences lies on the fact that it is a function not only of purchased quantities, but also of consumer's unobserved utility level. The first methodology that is adopted for dealing with this problem concerns the use of a proxy for utility. As already mentioned in Chapter 1, Lewbel and Pendakur (2006) invented Implicit Marshallian Demand systems, which are systems of Hicksian demands where utility u is substituted by *implicit utility*, a simple function of observables. Following Lewbel and Pendakur (2006), Färe, Grosskopf, Hayes, and Margaritis (2008) proxy utility with household annual income in order to estimate and assess systems of demand equations which are derived from expenditure and benefit functions. In the present study, we use the log of total expenditures as a proxy for utility, in order to estimate the model given by equation (4.2). We adopted the Battese and Coelli (1995) specification for panel-data maximum likelihood estimation, which assumes that the ε_{it}'s are random variables distributed as iid normal with zero mean and variance σ_{ε}^2, and that $v_{it} \equiv \ln D_{it}$ are non-negative random variables, independent of the ε_{it}, and independently distributed as truncations at zero of the normal distribution with constant variance σ_v^2 and non-constant mean μ_{it}. In particular, μ_{it} is given by

$$\mu_{it} = \sum_h \mu_h H_{hit}, \tag{4.3}$$

where μ_h is a vector of parameters to be estimated, and H_{it} are variables that may have an effect on consumer's efficiency levels (e.g., socio-demographic variables). The composite error term is given by $s_{it} \equiv -\ln D_{it}^I(u,\mathbf{q}) + \varepsilon_{it} \equiv -v_{it} + \varepsilon_{it}$, and the estimator for commodity efficiency is defined as $E[(D_{it}^I)^{-1} \mid s_{it}] = E[e^{-v_{it}} \mid s_{it}]$. Battese and Coelli (1993) provide the following expressions for the marginal density of the composite error term and the estimator for commodity efficiency:

$$f(s_{it}) = \frac{\exp\left\{-\frac{1}{2}\left(s_{it} + \mu_{it}\right)^2 \Big/ \left(\sigma_\varepsilon^2 + \sigma_v^2\right)\right\}}{\sqrt{2\pi}\left(\sigma_\varepsilon^2 + \sigma_v^2\right)^{1/2}\left[\Phi\left(\mu_{it}/\sigma_v^2\right)\Big/\Phi\left(\mu_{it}^*/\sigma^*\right)\right]}, \tag{4.4}$$

and

$$\begin{aligned} CE_{it}(u,\mathbf{q}) &= E[(D_{it}^I)^{-1} \mid s_{it}] = E[e^{-v_{it}} \mid s_{it}] \\ &= \left\{\exp\left[-\mu_{it}^* + \frac{1}{2}(\sigma^*)^2\right]\right\}\left\{\frac{\Phi\left[\left(\mu_{it}^*/\sigma^*\right) - \sigma^*\right]}{\Phi\left(\mu_{it}^*/\sigma^*\right)}\right\}, \end{aligned} \tag{4.5}$$

where

$$(\sigma^*)^2 = \frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}, \quad \mu_{it}^* = \frac{\sigma_\varepsilon^2\mu_{it} - \sigma_v^2 s_{it}}{\sigma_\varepsilon^2 + \sigma_v^2},$$

and $\Phi(\cdot)$ is the standard normal cumulative distribution function.

The second methodology that is adopted is to treat and estimate consumer's unobserved utility level as a one-sided positive error term. Under such a treatment, however, the translog term for utility-squared will not be included in the model since this would complicate the model considerably because the composite error term would be the sum of four different error terms. Since the two one-sided error terms in equation (4.2), *i.e.*, utility and the distance, must be preceded by opposite signs in order to be discernible in estimation, certain assumptions must be made on the utility and distance terms. Firstly, our theoretical model requires, by construction, that the value of the input distance function be greater than or equal to unity, i.e., $D_{it}^{I} \geq 1$, so that $\ln D_{it}^{I} \geq 0$. Secondly, as consumer theory suggests, the input distance function must be non-decreasing in quantities and decreasing in utility. Thus, monotonicity of the input distance function with respect to utility requires that

$$\frac{\partial D_{it}^{I}}{\partial u_{it}} = \frac{\partial \ln D_{it}^{I}}{\partial \ln u_{it}} \frac{D_{it}^{I}}{u_{it}} = g(\ln q_{jit}, t; \mathbf{b}_2) \frac{D_{it}^{I}}{u_{it}} < 0, \tag{4.6}$$

that is, the function $g(\cdot)$ must be negative. Moreover, since $u_{it} > 0$, $\ln u_{it}$ can take on any values in the interval $(-\infty, +\infty)$. However, since consumer's preferences are ordinal, no harm is done by normalising utility so that $u_{it} \in (0,1)$ and $\ln u_{it} < 0$. Under these assumptions, we can re-write the model to be estimated as

$$\begin{aligned} -\ln q_{1it} &= f\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_1\right) + \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_2\right)\right]\left(-\ln u_{it}\right) - \ln D_{it}^{I}(u, \mathbf{q}) + \varepsilon_{it} \\ &= f\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_1\right) + s_{it}, \end{aligned} \tag{4.7}$$

where $s_{it} \equiv \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_2\right)\right]\left(-\ln u_{it}\right) - \ln D_{it}^{I}(u, \mathbf{q}) + \varepsilon_{it}$. Hence, assuming that $D_{it}^{I} \geq 1$ so that $\ln D_{it}^{I} \geq 0$, and assuming that $u_{it} \in (0,1)$ so that $\ln u_{it} < 0$, we can treat the variables $\ln D_{it}^{I}$ and $(-\ln u_{it})$ as iid exponentially distributed error terms. In addition, under the assumption that $g(\ln \mathbf{q}_{it}; t, \mathbf{b}_2) < 0$, the terms $\left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_2\right)\right]\left(-\ln u_{it}\right)$ and $\ln D_{it}^{I}(u, \mathbf{q})$ are preceded by opposite signs and can be discerned in estimation. Another reason for assigning a specific sign to the function $g(\cdot)$ is that it was necessary for the construction of the density of the composite error term s_{it} to decide on the sign of the function $g(\cdot)$, and since consumer theory suggests that it should be negative for monotonicity of the distance function with respect to utility to hold, we couldn't have chosen otherwise.

In summary, the distributional assumptions we have made for the three random error terms are as follows:

$$z_{it} \equiv (-\ln u_{it}) \sim \text{ iid exponential}(\sigma_z, \sigma_z^2),$$

$$v_{it} \equiv \ln D_{it}^I \sim \text{ iid exponential}(\sigma_v, \sigma_v^2),$$

$$\varepsilon_{it} \sim \text{ iid} N(0, \sigma_\varepsilon^2).$$

Assuming that the z_{it}, v_{it} and ε_{it} are independent with respect to one another, the marginal density of the composite error term is given by

$$f(s_{it}) = \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s_{it}}{\sigma_v}\right) \Phi\left(-\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v}\right)$$
$$+ \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s_{it}}{\sigma_w}\right) \Phi\left(\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w}\right), \quad (4.8)$$

which is the same as the density of the composite error term in the two-tiered frontier model of Polachek and Yoon (1987, 1996) with the exception that σ_w is not a parameter to be estimated, but it is defined, instead, as

$$\sigma_w \equiv \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t; \mathbf{b}_2\right)\right]\sigma_z. \quad (4.9)$$

We propose the following estimators for commodity efficiency and consumer's utility level

$$CE_{it}(u, \mathbf{q}) = E[(D_{it}^I)^{-1} \mid s_{it}] = E[e^{-v_{it}} \mid s_{it}]$$
$$= \frac{\sigma^*}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s_{it}}{\sigma_w}\right) \frac{1}{f_s(s_{it})}$$
$$\times \left\{\Phi\left(\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w}\right) + \exp\left[\frac{\sigma_\varepsilon^2}{2\sigma^{*2}} + \frac{1}{\sigma^*}\left(s_{it} - \frac{\sigma_\varepsilon^2}{\sigma_w}\right)\right] \Phi\left(-\frac{s_{it}}{\sigma_\varepsilon} + \frac{\sigma_\varepsilon}{\sigma_w} - \frac{\sigma_\varepsilon}{\sigma^*}\right)\right\}, \quad (4.10)$$

and

$$E[u_{it} \mid s_{it}] = \frac{\mu_{it}^*}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s_{it}}{\sigma_v} \right) \frac{1}{f_s(s)}$$

$$\times \left\{ \Phi\left(-\frac{s_{it}}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v} \right) + \exp\left[\frac{\sigma_\varepsilon^2}{2\mu_{it}^{*2}} - \frac{1}{\mu_{it}^*}\left(s_{it} + \frac{\sigma_\varepsilon^2}{\sigma_v} \right) \right] \Phi\left(\frac{s_{it}}{\sigma_\varepsilon} + \frac{\sigma_\varepsilon}{\sigma_v} - \frac{\sigma_\varepsilon}{\mu_{it}^*} \right) \right\}, \quad (4.11)$$

where

$$\sigma^* = \frac{\sigma_v \sigma_w}{\sigma_v + \sigma_w + \sigma_v \sigma_w}, \quad \mu_{it}^* = \frac{[-g(\cdot)]\sigma_v \sigma_w}{[-g(\cdot)]\sigma_v + [-g(\cdot)]\sigma_w + \sigma_v \sigma_w},$$

and $\Phi(\cdot)$ is the standard normal cumulative distribution function.[50]

Once the translog input distance function has been estimated and the commodity efficiency index has been computed, the value of the expenditure or overall efficiency measure can be computed from relation (3.41) which gives the decomposition of expenditure efficiency into commodity and allocative efficiency. Calculation of the value of the measure of allocative efficiency is more problematic since it requires knowledge of either the expenditure function or the expenditure-minimising commodity vector. The notion of virtual prices as defined by Grosskopf, Hayes, and Hirschberg (1995) and the procedure developed by Karagiannis, Midmore, and Tzouvelekas (2004) for derivation of optimal input vectors can also be employed here. In particular, suppose the expenditure function is a linear function of the commodity market prices, that is,

$$C(u, \mathbf{p}) = \sum_{j=1}^{N} p_j q_j^*, \quad (4.12)$$

Where $\mathbf{p}$ denotes the vector of market prices, and q_j^* denotes the expenditure-minimising quantity of commodity *j*. Dividing relation (4.12) through by a commodity, say, q_1^*, yields

[50] The derivation of the density in eq. (4.8), and of the estimators for commodity efficiency and consumer's utility level in eqs. (4.10) and (4.11) are presented in Appendix B.

$$\frac{C(u,\mathbf{p})}{q_1^*} = p_1 + p_2\left(\frac{q_2^*}{q_1^*}\right) + \ldots + p_N\left(\frac{q_N^*}{q_1^*}\right). \tag{4.13}$$

Relations (4.13) and (3.43) can be used to compute the index of allocative efficiency as follows:

$$AE(u,\mathbf{q},\mathbf{p}) = \frac{D^I(u,\mathbf{q})C(u,\mathbf{p})}{(\mathbf{p}\cdot\mathbf{q})} = D^I(u,\mathbf{q})\frac{C(u,\mathbf{p})/q_1}{(\mathbf{p}\cdot\mathbf{q})/q_1}, \tag{4.14}$$

given that the observed (actual) and the optimal quantity of commodity 1 coincide, that is, $q_1 = q_1^*$. Recall that the Shephard-Hanoch lemma allows the vector of shadow prices to be derived from partial differentiation of the input distance function with respect to quantities. Recall also that in Section 3.3.1 we noted that if the reference quantity vector in the definition of the input distance function is not the expenditure-minimising one, then the vector of these prices is interpreted as the vector of shadow prices deflated by shadow expenditure (*i.e.*, virtual prices). On the other hand, if the reference quantity vector in the definition of the input distance function is the expenditure-minimising one, then the vector of shadow prices coincides with the expenditure-normalised vector of market (*i.e.*, observed) prices. Hence, at the expenditure-minimising commodity vector, $\mathbf{q}^*$, we have: $\partial \ln D^I(u,\mathbf{q}^*)/\partial \ln q_j^* = \left(\partial D^I(u,\mathbf{q}^*)/\partial q_j^*\right)\left(q_j^*/D^I(u,\mathbf{q}^*)\right) = \left(p_j/C(u,\mathbf{p})\right)\left(q_j^*/D^I(u,\mathbf{q}^*)\right)$. Using this result, we can derive the following system of N-1 equations

$$\frac{\partial \ln D^I(u,\mathbf{q}^*)/\partial \ln q_j^*}{\partial \ln D^I(u,\mathbf{q}^*)/\partial \ln q_1} = \frac{\left(p_j/C(u,\mathbf{p})\right)\left(q_j^*/D^I(u,\mathbf{q}^*)\right)}{\left(p_1/C(u,\mathbf{p})\right)\left(q_1/D^I(u,\mathbf{q}^*)\right)} = \frac{p_j q_j^*}{p_1 q_1}, \quad j = 2,\ldots,N. \tag{4.15}$$

In the case of the translog input distance function given by relation (4.1), this system becomes

$$\left(\frac{p_j}{p_1}\right)\left(\frac{q_j^*}{q_1}\right) = \frac{\alpha_j + \sum_{k=2}^{N}\gamma_{jk}\ln\left(q_{kit}^*/q_{1it}\right) + \delta_j \ln t_t + \beta_j \ln u_{it}}{\alpha_1 + \sum_{k=2}^{N}\gamma_{1k}\ln\left(q_{kit}^*/q_{1it}\right) + \delta_1 \ln t_t + \beta_1 \ln u_{it}}, \quad j = 2,\ldots,N, \tag{4.16}$$

where the restrictions of homogeneity and symmetry have been imposed, and the assumption that $q_1 = q_1^*$ has been made. This system can be solved to obtain the ratios of expenditure-minimising quantities in terms of the observed market prices, the estimated expected value of utility (in the case of the two-tiered frontier empirical specification), and the estimated parameters of the distance function. These expenditure-minimising commodity ratios can then be substituted into (4.13) to derive estimates of $AE(u,\mathbf{q},\mathbf{p})$ from relation (4.14).

Finally, knowledge of the expenditure minimising quantity ratios also allows the derivation of the optimal expenditure shares. The latter can be computed from the following relation:

$$w_i(u,\mathbf{p}) = \frac{p_i q_i(u,\mathbf{p})}{C(u,\mathbf{p})} = \frac{p_i q_i^*}{C(u,\mathbf{p})} = \frac{p_i\left(q_i^*/q_1^*\right)}{\left(C(u,\mathbf{p})/q_1^*\right)}, \tag{4.17}$$

The optimal expenditure share for the first commodity can be computed residually, using the adding-up restriction.

4.3 DATA DESCRIPTION AND EMPIRICAL RESULTS

4.3.1 Data Description

The data used in the empirical analysis are drawn from a panel of British household data provided by the *TNS* market research institute. The panel provides information on weekly purchases of organic and non-organic eggs, milk, yoghurt, fruits, and vegetables, from December 2004 to November 2006. The surveyed households reported the volume of and expenditure on the aforementioned organic and non-organic products purchased at every shopping trip, as well as the shop that was visited and the time spend. The data base also includes information on the socio-demographic characteristics of the surveyed households, such as number of adults and children in the household, age of the main shopper in the family, social class, and region of residence.

For the purposes of the present analysis, the region of London was selected in order to avoid problems associated with the consumption of home-grown agricultural products in rural areas. Since accounting for censoring would add extra complexity to the adopted empirical models, and since our aim is to provide an illustration of the econometric estimation of the proposed model for consumer efficiency, the data on quantities and expenditure were aggregated to monthly figures, and the households selected were the ones that reported positive consumption of all the following three commodity groups: milk & yoghurt, fruits, and vegetables. In particular, the selected sample consists of 884 households in London, which reported positive consumption of all three commodities for a period of 12 months, from July 2005 το June 2006.

Table 4.1. Summary Statistics of the Data

	Time Period	Quantities			Total Expenditures	Social Class	Age of Family Head	Family Size
		Milk & Yoghurt	Fruits	Vegetables				
Mean	Jul. 2005	11.69	9.96	12.30	46.74			
	Aug. 2005	11.38	9.58	11.54	43.22			
	Sep. 2005	11.20	8.71	11.36	40.12			
	Oct. 2005	11.41	8.48	12.17	39.38			
	Nov. 2005	11.68	8.21	12.43	38.42			
	Dec. 2005	13.02	8.83	13.16	40.60			
	Jan. 2006	11.76	7.70	12.63	37.65			
	Feb. 2006	11.77	8.30	12.06	39.21			
	Mar. 2006	13.10	9.51	13.44	44.36			
	Apr. 2006	11.81	8.78	12.29	40.57			
	May 2006	11.81	9.06	13.09	42.20			
	Jun. 2006	11.50	9.02	12.48	44.03			
Jul. 2005 - Jun. 2006								
	Mean	11.84	8.85	12.41	41.38	3.83	54.67	2.60
	Min.	0.15	0.10	0.12	2.37	1	22	1
	Max.	109.30	80.01	89.36	303.71	6	91	8
	St.Dev.	10.52	6.62	8.88	24.96	1.35	15.73	1.29

Note: Social class takes on the value of 1 for the highest social class and the value of 6 for the lowest one.

The variables included in the empirical analysis are the quantities of milk & yoghurt, fruits, and vegetables, total expenditures on all the three commodities, and three socio-demographic variables, namely, the household's social class, the age of the main shopper in the household, and family size. Aggregation of the quantities for the creation of the milk & yoghurt commodity group was carried out with the use of a *Divisia* index with expenditure shares serving as weights. The quantities of fruits and vegetables are measured in kilograms, while the quantities of milk and yoghurt, before aggregation, were measured in litres and kilos, respectively. Total expenditure is the sum of expenditures on milk & yoghurt, fruits and vegetables, and are measured in Pound-Sterling. Social class is a classification of the households in the panel into six categories representing social grade, social status, and occupation.[51] In particular, the classification is: Class A (upper middle class, higher managerial, administrative or professional), Class B (middle class, intermediate managerial, administrative or professional), Class C1 (lower middle class, supervisory or clerical, junior managerial, administrative or professional), Class C2 (skilled working class, skilled manual workers), Class D (working class, semi and unskilled manual workers), and Class E (households at lowest level of subsistence, state pensioners or widows (no other earner), casual or lowest grade workers). Finally, family size is the sum of the number of adults and the number of children in the household. The descriptive statistics for the household data are summarized in Table 4.1. As shown in Table 4.1, the average consumption of the three commodities is rather constant during the period of the 12 months, which is to be expected for commodities such as foods.

4.3.2 Empirical Results

The parameter estimates for the proxy model were obtained from maximum likelihood estimation of the model in equation (4.2) using the FRONTIER 4.1 software. As already mentioned, it is assumed that the distance follows a truncated-normal distribution with constant variance and non-constant mean. In particular, the mean of the distance was specified as a polynomial function of the following variables: social class, family size, age of the main shopper in the family, time, and time-squared. As far as the parameter estimates for the two-tiered model are

[51] In estimation, the explanatory variable used for the households' social class takes on the value of 1 for the highest social class and the value of 6 for the lowest one.

Table 4.2. Parameter Estimates of the Translog Distance Function

	Proxy Model		Two-Tiered Model	
Parameter	Estimate	Std. Err.	Estimate	Std. Err.
α_0	1.7634	(0.0411)*		
α_{MY}	0.2814	(0.0334)*	0.2684	(0.0094)*
α_F	0.5572	(0.0371)*	0.3626	(0.0106)*
α_V	0.1614	(0.0376)*	0.3690	(0.0114)*
γ_{MY}	-0.0814	(0.1001)	0.1397	(0.0216)*
γ_F	-0.4603	(0.0423)*	0.2139	(0.0103)*
γ_V	-0.2996	(0.0464)*	0.2952	(0.0106)*
γ_{MYF}	0.1210	(0.0289)*	-0.0292	(0.0074)*
γ_{MYV}	-0.0397	(0.0376)	-0.1105	(0.0079)*
γ_{FV}	0.3393	(0.0384)*	-0.1847	(0.0087)*
δ_{MY}	0.2793	(0.0460)*	0.0065	(0.0083)
δ_F	-0.2099	(0.0551)*	0.0198	(0.0096)**
δ_V	-0.0693	(0.0569)	-0.0263	(0.0102)*
δ_T	0.6266	(0.0729)*	-0.0628	(0.0161)*
δ_{TT}	1.6591	(0.1145)*	-0.1051	(0.0200)*
β_0	-0.9686	(0.0560)*		
β_{MY}	-0.1812	(0.0525)*	0.0687	(0.0218)*
β_F	-0.2281	(0.0576)*	0.0397	(0.0245)***
β_V	0.4094	(0.0561)*	-0.1084	(0.0263)*
δ_0	0.9767	(0.0675)*	0.0124	(0.0281)
ζ	0.0780	(0.0986)		
μ_0	-65.7178	(1.3730)*		
μ_{Class}	6.2678	(0.3518)*		
$\mu_{FamSize}$	-2.5783	(0.3469)*		
μ_{Age}	31.5932	(0.7132)*		
μ_{Time}	40.7733	(1.6117)*		
$\mu_{TimeSquared}$	-19.5422	(0.8010)*		
σ_z			0.3747	(0.0108)*
σ_v	6.9563	(0.0653)*	0.1893	(0.0110)*
σ_ε	7.3956	(0.0578)*	0.4212	(0.0110)*
Log-likelihood	-20751.8640		-9363.6816	

Notes: MY refers to milk & yoghurt, *F* to fruits, and *V* to vegetables. Asymptotic standard errors in parentheses. * (**, and ***) indicate significance level at the 1 (5, and 10) percent.

concerned, they were obtained from pooled-data maximum likelihood estimation of the model in equation (4.7), with the use of the GAUSS software. The model was estimated without the translog constant term. In addition, as is obvious in relation (4.9), there is a problem with identification of the standard deviation σ_z of the random error term which is associated with utility. In order to be able to estimate the parameter σ_z, we normalised β_0 to unity.

The maximum likelihood parameter estimates of the two models, along with standard errors, are displayed in Table 4.2. Both models were estimated with homogeneity and symmetry imposed, where homogeneity was imposed by division of all quantities by the quantity of milk & yoghurt. Parameters a_{MY}, γ_{MY}, γ_{MYF}, γ_{MYV}, β_{MY}, and δ_{MY} were computed via the homogeneity and symmetry restrictions, and, their standard errors were approximated by the delta method (see, for example Spanos, 1999). In the case of the proxy model, the standard deviations for the distance and the normal error term, σ_v and σ_ε, respectively, were computed from the FRONTIER 4.1 estimates for *sigma-squared* $(\sigma^2 = \sigma_v^2 + \sigma_\varepsilon^2)$ and *gamma* $(\gamma = \sigma_v^2 / \sigma^2)$. The standard errors for the parameter estimates for σ_v and σ_ε were also approximated by the delta method.

As shown in Table 4.2, 25 out of 29 parameters of the proxy model, and 19 out of 21 parameters of the two-tiered frontier model were statistically significant. In the case of the proxy model, the variance of the one-sided error term associated with the distance is found to be σ_v^2 = 48.3902, and that of the normal error term is found to be σ_ε^2 = 54.6952. In addition, all the parameters in equation (4.3) for the mean of the one-sided error tem in the proxy model were found to be statistically significant at the 1% level, indicating that households' socio-demographic characteristics and time may affect the pattern of inefficiency. Under the two-tiered frontier empirical specification, the variance of the one-sided error term associated with household's utility is found to be σ_z^2 = 0.1404, while the variance of the one-sided error term associated with the distance is found to be σ_v^2 = 0.0358, and that of the normal error term is found to be σ_ε^2 = 0.1774.

The estimated efficiency indices for the two empirical specifications are presented in Tables 4.3 and 4.4. Under the proxy model, the estimated mean commodity, allocative, and expenditure efficiency scores were found to be 76.10%, 77.97%, and 58.78%, respectively, during the period July 2005 - June 2006.

Table 4.3. Frequency Distribution of Commodity, Allocative, and Expenditure Efficiency for the Proxy Model

	2005						2006					
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
Commodity Efficiency												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	1	1	2	8	2	1	2	0	2	2	1	0
0.5-0.6	9	15	22	48	39	42	35	32	22	18	25	27
0.6-0.7	31	126	211	295	246	229	251	226	159	145	117	116
0.7-0.8	93	288	405	428	458	482	470	490	504	445	422	358
0.8-0.9	225	313	202	97	131	127	126	136	195	270	314	368
>0.9	525	141	42	8	8	3	0	0	2	4	5	15
Mean	0.89	0.8	0.75	0.72	0.73	0.73	0.73	0.73	0.75	0.76	0.77	0.78
Allocative Efficiency												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	0	0	0	0	0	0	0	0	0	0	0	0
0.5-0.6	9	14	8	13	8	8	7	11	9	9	5	8
0.6-0.7	137	130	125	140	137	134	134	130	125	122	115	144
0.7-0.8	383	391	358	391	428	397	394	421	429	438	390	417
0.8-0.9	266	272	308	266	250	267	271	256	258	246	298	244
>0.9	89	77	85	74	61	78	78	66	63	69	76	71
Mean	0.78	0.78	0.79	0.78	0.77	0.78	0.78	0.78	0.78	0.78	0.79	0.78
Expenditure Efficiency												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	50	54	60	19	30	30	35	31	25	43	39	34
0.5-0.6	428	503	456	484	518	464	492	530	476	392	473	465
0.6-0.7	403	327	368	381	336	390	357	323	383	449	372	385
0.7-0.8	3	0	0	0	0	0	0	0	0	0	0	0
0.8-0.9	0	0	0	0	0	0	0	0	0	0	0	0
>0.9	0	0	0	0	0	0	0	0	0	0	0	0
Mean	0.59	0.58	0.58	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59

Specifically, 70% of the households in the sample achieved scores of commodity efficiency between 70 and 80%, 80% of the households achieved scores of allocative efficiency between 70 and 80%, and 83% of the households achieved scores of expenditure efficiency between 50 and 60%. Regarding the estimates of the efficiency indices for the two-tiered frontier empirical specification, they were found to be higher than the ones under the proxy model: the estimated mean commodity, allocative, and expenditure efficiency scores were found to be 84.06%, 80.14%, and 67.22%, respectively, during the during the time-span of the panel. In particular, the majority of the households in the sample (87%) achieved scores of commodity efficiency between 80 and 90%, 45% of the households achieved scores of allocative efficiency between 70 and 80% and the remaining 55% of the households achieved scores of allocative efficiency between 80 and 90%, and, finally, almost all the households (99%) achieved scores of expenditure efficiency between 60 and 70%.

Table 4.4. Frequency Distribution of Commodity, Allocative, and Expenditure Efficiency for the Two-Tiered Frontier Model

	2005						2006					
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
Commodity Efficiency												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	0	1	0	0	0	1	0	0	1	0	1	0
0.5-0.6	2	6	1	4	1	0	1	2	2	2	1	2
0.6-0.7	9	15	15	14	11	20	9	7	27	7	5	9
0.7-0.8	112	124	119	136	128	147	116	102	148	111	120	113
0.8-0.9	761	738	749	730	744	716	758	773	706	764	757	760
>0.9	0	0	0	0	0	0	0	0	0	0	0	0
Mean	0.84	0.84	0.84	0.84	0.84	0.83	0.84	0.84	0.83	0.84	0.84	0.84
Allocative Efficiency												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	0	0	0	0	0	0	0	0	0	0	0	0
0.5-0.6	25	0	2	2	2	4	7	3	11	2	1	1
0.6-0.7	99	0	40	22	33	69	51	18	25	9	98	120
0.7-0.8	221	362	315	402	434	406	430	416	329	269	307	375
0.8-0.9	402	469	460	416	393	362	370	427	469	539	423	350
>0.9	137	53	67	42	22	43	26	20	50	65	55	38
Mean	0.81	0.81	0.81	0.80	0.79	0.79	0.79	0.80	0.81	0.82	0.80	0.78
Expenditure Efficiency												
<0.4	0	0	0	0	0	0	0	0	0	0	0	0
0.4-0.5	19	0	0	1	0	13	8	3	22	7	0	0
0.5-0.6	142	9	75	43	47	122	83	27	27	5	170	162
0.6-0.7	345	605	526	562	627	528	556	597	562	499	452	484
0.7-0.8	351	270	283	278	210	221	237	257	273	373	262	238
0.8-0.9	27	0	0	0	0	0	0	0	0	0	0	0
>0.9	0	0	0	0	0	0	0	0	0	0	0	0
Mean	0.68	0.68	0.67	0.68	0.67	0.66	0.66	0.68	0.68	0.69	0.66	0.66

An interpretation of the findings with regard to efficiency scores is suitable here. Since the empirical specifications we have adopted for the econometric estimation of the theoretical model and, hence, the derived results, are not comparable, we will focus on the interpretation of the results derived from of the two-tier frontier empirical specification only. The central assumption we have made for the development of the proposed efficiency measures is that any unwanted quantities of the purchased commodities can be disposed of. This means that the quantities of the purchased commodities, and hence, actual (observed) expenditures, may well be higher than the ones required to just attain a target utility level. Using relation (3.42) which provides a definition of commodity efficiency as a cost ratio, the finding that the estimated mean commodity efficiency was 84.06% during the period July 2005 - June 2006 indicates that, on average, a 15.94% of the households' budget was wasted, or that households, say, by better planning, could have decreased their total expenditure by 15.94% and could still have achieved the same utility level with a portion of the purchased commodities. Recall the definition (3.43) of the measure of allocative efficiency as the ratio of the minimum expenditure required for attaining a target utility level to the expenditure for the commodity efficient commodity vector. Under this definition, the finding that mean allocative efficiency was 80.14% during the time period covered by the panel indicates that, given some target utility level and the commodity prices that households face, a 19.86% decrease in total expenditures could be made feasible if households had chosen a different combination of commodity quantities. Moreover, since expenditure efficiency is defined as the ratio of minimum expenditure required for the achievement of u to the actual expenditure, a mean expenditure efficiency score of 67.22% indicates that, on average, a 32.78% of the households' budget was wasted due to the presence of commodity and allocative inefficiency.

With regard to households' behaviour over time, it cannot be concluded from the empirical results that the inefficiency scores are diminishing over time. As shown in Tables 4.3 and 4.4, the mean inefficiency scores are rather steady over time. This is to be expected, considering the type of commodities under analysis combined with the short time-span of our panel (12 months): the commodities under analysis are foods that have an important role to play in any average household's diet, and, in addition, the time-span of our panel is too short to allow for changes in households' socio-demographic characteristics which would affect consumption habits and hence efficiency scores. Finally, an analysis of the findings in terms of household classification according to social class (Table 4.5) indicates that, as expected, the higher the social class, the higher the expenditure savings that can be obtained by elimination of overall efficiency.

Table 4.5. Mean expenditure Savings per Social Class (in £, Jul. 2005 – Jun. 2006)

Social Class	No. of Households	Proxy Model			Two-Tier Frontier Model		
		CE	AE	EE	CE	AE	EE
A	10	12.70	11.11	23.81	9.27	10.71	19.97
B	106	11.88	10.33	22.21	7.45	9.26	16.71
C1	338	10.10	9.36	19.46	6.75	8.35	15.10
C2	179	9.71	8.86	18.57	6.47	8.18	14.65
D	73	9.12	8.60	17.72	6.26	7.88	14.14
E	178	8.73	8.28	17.01	5.93	7.38	13.31
Mean		10.37	9.42	19.79	7.02	8.63	15.65

5. Summary and Conclusions

This Ph.D. thesis provides an insight on consumer demand for organic and non-organic commodities and on the effects of changes in consumed quantities on consumer welfare, by employing techniques which are used in applied production analysis in order to make possible the econometric estimation of well-known consumer demand systems. Furthermore, it deviates from the standard consumer demand analysis which assumes *a priori* that consumers behave optimally, develops a model for measuring efficiency in consumption, and suggests two different empirical frameworks for the empirical application of the theoretical model.

The majority of studies on consumer demand for organic commodities focus on consumers' attitudes, and consumers' motivation for purchasing organic commodities willingness to pay for them, as well as on socio-economic, demographic, psychological and ethical factors which may have an effect on consumer's acceptance of organic commodities. There is only a small number of empirical studies that analyse consumer demand for organic products together with the demand for non-organic ones, and explore the interrelationships between the two commodity types by providing estimates of cross-price elasticities. In this context, this Ph.D. thesis contributes to the literature of empirical studies of consumer demand for organic products by providing empirical evidence on the interrelationships between organic and non-organic commodities and on the changes which occur on households' welfare levels due to changes in consumed quantities. This task was accomplished by employing cross-sectional data and the IAIDS model of Eales and Unnevehr (1994), and Moschini and Vissa (1992) for the empirical analysis of household demand for organic milk & yoghurt, non-organic milk & yoghurt, organic fruits & vegetables, and non-organic fruits & vegetables, in London, UK. The occurrence of zero purchases in the sample was accounted for by using the Amemiya-Tobin model by Wales and Woodland (1983). Moreover, we accounted, in a theoretically consistent way, for the problem introduced in inverse demand systems, in which consumed quantities are expressed in logarithms, when zero consumption is reported. Specifically, our censored inverse demand system was modified by using an approach proposed by

Battese (1997), which has been applied in the empirical production analysis literature and allows full-sample estimation and derivation of efficient and unbiased parameter estimates.

After estimation of the censored IAIDS, expected observed shares were computed via simulations and used for the computation of flexibilities. Scale flexibilities suggest that organic milk & yoghurt are luxuries. The scale flexibility for organic fruits & vegetables, on the other hand, was found to be close to –1 and close to the scale flexibility of their non-organic counterparts. Hence, it is not clear that these commodities are luxuries, a situation that could be explained by the lower price premium that is paid for organic fruits & vegetables compared to the price premium paid for the organic milk & yoghurt commodity group in our demand system. In addition, the organic commodities in our system were found to be more own-quantity inflexible, and, hence, more own-price elastic than their non-organic counterparts. Moreover, the magnitudes of the own- and cross-quantity flexibilities suggest that it is the own-prices of the organic commodities that play the most important role in inducing consumers to increase their consumption of organic commodities. Hence, the results suggest that expansion of the organic farming, which could lead to decreases in the price premium of the organic commodities due to increased supply of them in the market, will, in turn, increase the demand for the organic commodities significantly. Finally, estimates of the CV and EV measures associated with quantities changes indicate that, for the households in our sample reporting positive consumption of non-organic commodities and zero consumption of organic ones, substitution of a portion of non-organic fruits & vegetables by/with organic ones is likely to make them better off, whereas a similar substitution in the case of milk & yoghurt is likely to make them worse off. The latter result could be due to the relatively higher discrepancy in unit values of the organic and non-organic milk & yoghurt commodity groups.

This Ph.D. thesis also contributes to the literature of measurement of consumer efficiency by proposing a theoretical and empirical framework for analysing consumer's efficiency in price-quantity space. In this context, a measure of consumer's expenditure inefficiency was proposed, which can be decomposed into two associated measures of efficiency in consumption, namely, commodity and allocative efficiency, in a manner similar to the one met in production efficiency analysis. The empirical application of the theoretical framework that has been developed was required to tackle the problem of econometric estimation of a distance function which is a function of not only purchased commodity quantities, but also of consumer's unobserved utility level. Two different empirical frameworks were adopted in order to illustrate the empirical application of the proposed theoretical model: the first empirical model is based on the use of observable variables as proxies for utility, while the second one allows consumer's utility to be treated and estimated as an unobserved random error term. The model was then applied to a panel data set of British household purchases of highly perishable foods. Although it seems restrictive to employ highly perishable commodities in order to accommodate the assumption that consumers are free to dispose of any unwanted quantities of the purchased commodities, studying consumer's inefficiency with respect to such type of commodities is important since a significant portion of the consumer's budget is allocated to them.

The conclusions derived from the analysis of the empirical results in studies of consumer demand usually direct attention to market and production implications. Our proposed measure of expenditure or overall efficiency can also serve such a goal, but the measures of commodity and allocative efficiency in consumption cannot; they can, however, provide a deeper insight into how expenditure inefficiency arises. None of the three measures can explain in full why consumers are inefficient. For, even if socio-demographic and economic characteristics of the consumers are accounted for in empirical analysis, there are still many characteristics of them, e.g. psychological, that cannot be observed. For example, it could be lack of information, awareness and responsibility from the part of consumers with respect to the full social costs of their consumption decisions that lead to excess purchases and spending, and consumption inefficiency. Nonetheless, the importance of studying inefficiency in consumption lies not only on the fact that optimal behaviour is a restrictive assumption to make for consumers actual behaviour, or on that changes in consumption efficiency levels may have an effect on products' prices in a competitive market. It also lies on the fact that consumer's non-optimal behaviour has a negative impact on welfare levels. In particular, it has a negative impact on consumer's welfare levels in terms of budget that was wasted and which could have been allocated to the satisfaction of other wants. In addition, since over-consumption leads to increased and more industrialised production, which itself fuels over-consumption, through, say, advertising, this circle implies excessive use and misuse of natural resources, and a negative impact on social welfare. Reduction of consumer's inefficiency and mitigation of its negative impact on welfare levels could be accomplished though, say, advertising. If advertising plays an important role in creating and/or sustaining consumer's non-optimal behaviour, then advertising could perhaps be used as a means of awareness raising and initiation of changes in consumer's shopping, purchasing and consumption patterns.

Appendix A. Tables for Households' Welfare Changes

Table A.1. Compensating and Equivalent Variation for Households Reporting Positive Consumption of Non-Organic Milk & Yoghurt and Zero Consumption of Organic Milk & Yoghurt.

Quantity Changes	Household Group	CV	EV
Substitution of 5% of non-organic milk & yoghurt by organic ones	Higher social classes, without children (group 1; T_1=263)	3.96	1.95
	Higher social classes, with children (group 2; T_2=148)	3.54	1.82
	Lower social classes, without children (group 3; T_3=213)	2.75	1.44
	Lower social classes, with children (group 4; T_4=99)	2.95	1.63
	t-stat. (groups 1 & 2)	(0.58)	(0.42)
	t-stat. (groups 3 & 4)	(-0.34)	(-0.67)
	t-stat. (groups 1 & 3)	(1.92)**	(1.99)*
	t-stat. (groups 2 & 4)	(0.86)	(0.57)
	Higher social classes (group 5; T_5=411)	3.81	1.90
	Lower social classes (group 6; T_6=312)	2.82	1.50
	t-stat. (groups 5 & 6)	(2.11)*	(2.00)*
	Without children (group 7; T_7=476)	3.42	1.72
	With children (group 8; T_8=247)	3.31	1.75
	t-stat. (groups 7 & 8)	(0.24)	(-0.10)
	All households in the sub-sample (T=723)	3.38	1.73
Substitution of 10%of non-organic milk & yoghurt by organic ones	Higher social classes, without children (group 1; T_1=263)	1.51	0.85
	Higher social classes, with children (group 2; T_2=148)	1.21	0.73
	Lower social classes, without children (group 3; T_3=213)	0.87	0.52
	Lower social classes, with children (group 4; T_4=99)	1.17	0.83
	t-stat. (groups 1 & 2)	(0.70	(0.40)
	t-stat. (groups 3 & 4)	(-0.81)	(-1.02)
	t-stat. (groups 1 & 3)	(1.70)**	(1.33)
	t-stat. (groups 2 & 4)	(0.09)	(-0.27)
	Higher social classes (group 5; T_5=411)	1.40	0.81
	Lower social classes (group 6; T_6=312)	0.96	0.62
	t-stat. (groups 5 & 6)	(1.54)	(0.95)
	Without children (group 7; T_7=476)	1.22	0.70
	With children (group 8; T_8=247)	1.19	0.77
	t-stat. (groups 7 & 8)	(0.11)	(-0.31)
	All households in the sub-sample (T=723)	1.21	0.73

Table A.1. (*continued*).

Quantity Changes	Household Group	CV	EV
Substitution of 15% of non-organic milk & yoghurt by organic ones	Higher social classes, without children (group 1; T_1=263)	1.07	0.72
	Higher social classes, with children (group 2; T_2=148)	0.76	0.57
	Lower social classes, without children (group 3; T_3=213)	0.60	0.46
	Lower social classes, with children (group 4; T_4=99)	0.97	0.83
	t-stat. (groups 1 & 2)	(0.66)	(0.37)
	t-stat. (groups 3 & 4)	(-0.85)	(-0.84)
	t-stat. (groups 1 & 3)	(1.23)	(0.79)
	t-stat. (groups 2 & 4)	(-0.41)	(-0.51)
	Higher social classes (group 5; T_5=411)	0.96	0.67
	Lower social classes (group 6; T_6=312)	0.72	0.57
	t-stat. (groups 5 & 6)	(0.80)	(0.33)
	Without children (group 7; T_7=476)	0.86	0.60
	With children (group 8; T_8=247)	0.85	0.67
	t-stat. (groups 7 & 8)	(0.03)	(-0.22)
	All households in the sub-sample (T=723)	0.85	0.63

Notes: Average CVs and EVs are measured in Pound-Sterling. This sub-sample consists of 723 households (full sample size is 1155), and T_g, g=1,…,8, denotes the number of households in each household group. The group of households in higher (lower) social classes consists of households in social classes A, B, and C1 (C2, D, and E). The group of households with children (group 8) consists of households with 1 to 5 children. *t*-statistics for the differences in CV and EV between household groups are in parenthesis, and * (**) indicates significance at the 5% (10%) level.

Table A.2. Compensating and Equivalent Variation for Households Reporting Positive Consumption of Fruits & Vegetables and Zero Consumption of Organic Fruits & Vegetables.

Quantity Changes	Household Group	CV	EV
Substitution of 5% of non-organic fruits & vegetables by organic ones	Higher social classes, without children (group 1; T_1=267)	1.54	1.18
	Higher social classes, with children (group 2; T_2=139)	1.65	1.29
	Lower social classes, without children (group 3; T_3=250)	1.97	1.58
	Lower social classes, with children (group 4; T_4=98)	1.94	1.52
	t-stat. (groups 1 & 2)	(-0.34)	(-0.35)
	t-stat. (groups 3 & 4)	(0.09)	(0.19)
	t-stat. (groups 1 & 3)	(-1.65)*	(-1.61)
	t-stat. (groups 2 & 4)	(-0.70)	(-0.59)
	Higher social classes (group 5; T_5=406)	1.57	1.22
	Lower social classes (group 6; T_6=348)	1.96	1.56
	t-stat. (groups 5 & 6)	(-1.75)*	(-1.68)*
	Without children (group 7; T_7=517)	1.75	1.38
	With children (group 8; T_8=237)	1.77	1.39
	t-stat. (groups 7 & 8)	(-0.08)	(-0.04)
	All households in the sub-sample (T=754)	1.75	1.38
Substitution of 10% of non-organic fruits & vegetables by organic ones	Higher social classes, without children (group 1; T_1=267)	-2.70	-3.27
	Higher social classes, with children (group 2; T_2=139)	-2.42	-2.93
	Lower social classes, without children (group 3; T_3=250)	-2.04	-2.48
	Lower social classes, with children (group 4; T_4=98)	-2.18	-2.68
	t-stat. (groups 1 & 2)	(-0.69)	(-0.61)
	t-stat. (groups 3 & 4)	(0.28)	(0.33)
	t-stat. (groups 1 & 3)	(-1.74)*	(-1.71)*
	t-stat. (groups 2 & 4)	(-0.43)	(-0.37)
	Higher social classes (group 5; T_5=406)	-2.61	-3.15
	Lower social classes (group 6; T_6=348)	-2.08	-2.54
	t-stat. (groups 5 & 6)	(-1.68)*	(-1.63)
	Without children (group 7; T_7=517)	-2.38	-2.89
	With children (group 8; T_8=237)	-2.32	-2.82
	t-stat. (groups 7 & 8)	(-0.18)	(-0.16)
	All households in the sub-sample (T=754)	-2.36	-2.87

Table A.2. (*continued*).

Quantity Changes	Household Group	CV	EV
Substitution of 15% of non-organic fruits & vegetables by organic ones	Higher social classes, without children (group 1; T_1=267)	-4.90	-6.24
	Higher social classes, with children (group 2; T_2=139)	-4.54	-5.74
	Lower social classes, without children (group 3; T_3=250)	-4.11	-5.18
	Lower social classes, with children (group 4; T_4=98)	-4.30	-5.45
	t-stat. (groups 1 & 2)	(-0.63)	(-0.64)
	t-stat. (groups 3 & 4)	(0.30)	(0.33)
	t-stat. (groups 1 & 3)	(-1.66)*	(-1.65)*
	t-stat. (groups 2 & 4)	(-0.35)	(-0.31)
	Higher social classes (group 5; T_5=406)	-4.78	-6.07
	Lower social classes (group 6; T_6=348)	-4.17	-5.26
	t-stat. (groups 5 & 6)	(-1.58)	(-1.55)
	Without children (group 7; T_7=517)	-4.52	-5.73
	With children (group 8; T_8=237)	-4.44	-5.62
	t-stat. (groups 7 & 8)	(-0.20)	(-0.19)
	All households in the sub-sample (T=754)	-4.50	-5.69

Notes: Average CVs and EVs are measured in Pound-Sterling. This sub-sample consists of 754 households (full sample size is 1155), and T_g, g=1,…,8, denotes the number of households in each household group. The group of households in higher (lower) social classes consists of households in social classes A, B, and C1 (C2, D, and E). The group of households with children (group 8) consists of households with 1 to 6 children. *t*-statistics for the differences in CV and EV between household groups are in parenthesis, and * indicates significance at the 10% level.

Appendix B. Treatment of Utility as a Random Error Term: Econometric Specification of the Consumer Efficiency Model

B.1. DERIVATION OF THE DENSITY OF THE COMPOSITE ERROR TERM IN THE "TWO-TIERED FRONTIER" EMPIRICAL SPECIFICATION

Under the treatment of the consumer's unobserved utility level as a one-sided positive error term, the term of the translog input distance function which associated with utility-squared is not included in the model. Thus, the homogeneous translog input distance function which is estimated in order to compute the commodity efficiency index is given by:

$$\begin{aligned}
-\ln q_{1it} &= \alpha_0 + \sum_j \alpha_j \ln\left(q_{jit}/q_{1it}\right) + \frac{1}{2}\sum_j\sum_k \gamma_{jk} \ln\left(q_{jit}/q_{1it}\right)\ln\left(q_{kit}/q_{1it}\right) \\
&\quad + \sum_j \delta_j \ln\left(q_{jit}/q_{1it}\right)\ln t_t + \delta_t \ln t_t + \frac{1}{2}\delta_{tt}(\ln t_t)^2 \\
&\quad + \left(\beta_0 + \sum_j \beta_j \ln\left(q_{jit}/q_{1it}\right) + \delta_0 \ln t_t\right)\ln u_{it} - \ln D^I_{it}(u,\mathbf{q}) + \varepsilon_{it} \\
&= f\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t_t; \mathbf{b}_1\right) + g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right), t_t; \mathbf{b}_2\right)\ln u_{it} \\
&\quad - \ln D^I_{it}(u,\mathbf{q}) + \varepsilon_{it}. \qquad \text{(B.1)}
\end{aligned}$$

where $j,k = 1,\ldots,N$ is the number of commodities, $i = 1,\ldots,I$ is the number of households, $t = 1,\ldots,T$ is the number of time-periods, $\mathbf{q}$ denotes the commodity vector, t_t denotes a time-trend capturing autonomous changes of consumer's preferences over time, u_{it} denotes the period-*t* utility level of the *i*-th consumer, ε_{it} is a random error term which is assumed to be distributed as iid normal with zero mean and variance σ^2_ε, and $\alpha_0, \alpha_j, \gamma_{jk}, \delta_j, \delta_t, \delta_{tt}, \beta_0, \beta_j$ and δ_0 are parameters to be estimated.

Since the two one-sided error terms in equation (B.1), *i.e.*, utility and the distance, must be preceded by opposite signs in order to be discernible in estimation, certain assumptions must be made on the utility and distance terms. Firstly, our theoretical model requires, by construction, that the value of the input distance function be greater than unity, i.e., $D_{it}^{I} \geq 1$, so that $\ln D_{it}^{I} \geq 0$. Secondly, as consumer theory suggests, the input distance function must be non-decreasing in quantities and decreasing in utility. Thus, monotonicity of the input distance function with respect to utility requires that the function $g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)$ be negative. Moreover, since $u_{it} > 0$, $\ln u_{it}$ can take on any values in the interval $(-\infty,+\infty)$. However, since consumer's preferences are ordinal, no harm is done by normalising utility so that $u_{it} \in (0,1)$ and $\ln u_{it} < 0$. Under these assumptions, we can re-write the model to be estimated as

$$\begin{aligned} -\ln q_{1it} &= f\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_1\right) + \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)\right]\left(-\ln u_{it}\right) - \ln D_{it}^{I}(u,\mathbf{q}) + \varepsilon_{it} \\ &= f\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_1\right) + s_{it}, \end{aligned} \tag{B.2}$$

where $s_{it} \equiv \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)\right]\left(-\ln u_{it}\right) - \ln D_{it}^{I}(u,\mathbf{q}) + \varepsilon_{it}$ is the composite error term. Hence, assuming that $D_{it}^{I} \geq 1$ so that $\ln D_{it}^{I} \geq 0$, and assuming that $u_{it} \in (0,1)$ so that $\ln u_{it} < 0$, the variables $\ln D_{it}^{I}$ and $(-\ln u_{it})$ can be treated as iid exponentially distributed error terms. In addition, under the assumption that $g(\ln \mathbf{q}_{it};t,\mathbf{b}_2) < 0$, the terms $\left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)\right]\left(-\ln u_{it}\right)$ and $\ln D_{it}^{I}(u,\mathbf{q})$ are preceded by opposite signs and can be discerned in estimation. Assuming also that the term $\left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)\right]\left(-\ln u_{it}\right)$ is iid exponentially distributed, the distributional assumptions we have made for our model can be summarised as follows:

$$z_{it} \equiv (-\ln u_{it}) \sim \text{ iid exponential}(\sigma_z,\sigma_z^2),$$

$$w_{it} \equiv \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)\right]\left(-\ln u_{it}\right) \sim \text{ iid exponential}(\sigma_w,\sigma_w^2)$$

$$v_{it} \equiv \ln D_{it}^{I} \sim \text{ iid exponential}(\sigma_v,\sigma_v^2),$$

$$\varepsilon_{it} \sim \text{ iid}N(0,\sigma_\varepsilon^2).$$

where $\sigma_w \equiv \left[-g\left(\ln\left(\mathbf{q}_{it}/q_{1it}\right),t_t;\mathbf{b}_2\right)\right]\sigma_z$, $z_{it} \in [0,+\infty)$, $w_{it} \in [0,+\infty)$, $v_{it} \in [0,+\infty)$, $\varepsilon_{it} \in (-\infty,+\infty)$, and the densities of the random variables z_{it}, w_{it}, v_{it}, and ε_{it} are given by

$$f_z(z_{it}) = \frac{1}{\sigma_z}\exp\left(-z_{it}/\sigma_z\right), \tag{B.3}$$

$$f_w(w_{it}) = \frac{1}{\sigma_w}\exp\left(-w_{it}/\sigma_w\right), \tag{B.4}$$

$$f_v(v_{it}) = \frac{1}{\sigma_v}\exp\left(-v_{it}/\sigma_v\right), \tag{B.5}$$

$$f_\varepsilon(\varepsilon_{it}) = \frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left(-\varepsilon_{it}^2/(2\sigma_\varepsilon^2)\right). \tag{B.6}$$

Let's drop subscripts and write the composite error term, *s*, as

$$\begin{aligned} s &\equiv \left[-g\left(\ln\left(\mathbf{q}/q_1\right),t;\mathbf{b}_2\right)\right]\left(-\ln u\right) - \ln D^I(u,\mathbf{q}) + \varepsilon \\ &\equiv (-g)z - v + \varepsilon \\ &\equiv w - v + \varepsilon, \end{aligned} \tag{B.7}$$

where $g \equiv g\left(\ln\left(\mathbf{q}/q_1\right),t;\mathbf{b}_2\right)$. In order to derive the density of the composite error term, *s*, we will first derive the density of $k \equiv w - v$, and then the density of $s \equiv k + \varepsilon$. Starting with the derivation of the density of *k*, we assume that the random variables *w* and *v* are independent. Then, the joint density of *w* and *v* is

$$f_{w,v}(w,v) = f_w(w)f_v(v) = \frac{1}{\sigma_w\sigma_v}\exp\left(-\frac{w}{\sigma_w} - \frac{v}{\sigma_v}\right). \tag{B.8}$$

We define the transformation :

$$\begin{pmatrix} w \\ v \end{pmatrix} \rightarrow \begin{pmatrix} w \\ w - v \end{pmatrix} = \begin{pmatrix} w \\ k \end{pmatrix}. \tag{B.9}$$

The inverse transformation and its Jacobian are,

$$\begin{cases} w = w \\ v = w - k \end{cases}, \quad \text{and} \quad |J| = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1, \tag{B.10}$$

respectively. The joint density of the random variables *k* and *w* is given by

$$f_{k,w}(k,w) = f_{w,v}(w, w-k)\text{abs}(|J|) = \frac{1}{\sigma_w \sigma_v}\exp\left(-\frac{w}{\sigma_w} - \frac{w-k}{\sigma_v}\right)$$
$$= \frac{1}{\sigma_w \sigma_v}\exp\left(\frac{k}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_w} - \frac{w}{\sigma_v}\right). \tag{B.11}$$

In order to derive the marginal density of *k*, the joint density in eq. (B.11) must be integrated with respect to *w*. However, care must be taken with regard to the limits of integration, that is, with regard to the interval that the random variable *w* takes values on. Since *w* is exponentially distributed, we know that $w \geq 0$. But, the random variable *v* is also exponentially distributed, $v \geq 0$, and, hence, the inverse transformation in (B.10) requires that: $v \geq 0 \Rightarrow w - k \geq 0 \Rightarrow w \geq k$. In addition, since $w \geq 0$ and $-v \leq 0$, the random variable $k = w - v$ takes on values in the interval $(-\infty, +\infty)$. Consequently, if $k \geq 0$, we must choose $w \in [k, +\infty)$ as the limits of integration, whereas, if $k < 0$, we must choose $w \in [0, +\infty)$. As a result, the marginal density of *k* has two different functional form representations, depending on whether $k \geq 0$ (*i.e.*, $w - v \geq 0 \Rightarrow w \geq v$) or $k < 0$ (*i.e.*, $w - v < 0 \Rightarrow w < v$). Thus, the marginal density of *k* is given by:

$$f_k(k) = \begin{cases} \int_k^{+\infty} f_{k,w}(k,w)dw \\ \int_0^{+\infty} f_{k,w}(k,w)dw \end{cases} = \begin{cases} \int_k^{+\infty} \frac{1}{\sigma_w \sigma_v}\exp\left(\frac{k}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_w} - \frac{w}{\sigma_v}\right)dw \\ \int_0^{+\infty} \frac{1}{\sigma_w \sigma_v}\exp\left(\frac{k}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_w} - \frac{w}{\sigma_v}\right)dw \end{cases}$$

$$= \ldots = \begin{cases} \dfrac{1}{\sigma_v + \sigma_w} \exp\left(-\dfrac{k}{\sigma_w} \right), & \text{if } k \geq 0 \\ \dfrac{1}{\sigma_v + \sigma_w} \exp\left(\dfrac{k}{\sigma_v} \right), & \text{if } k \leq 0 \end{cases} \tag{B.12}$$

Having derived the marginal density of *k*, we can now derive the density for the composite error term, $s \equiv k + \varepsilon$. Assuming that the random variables *k* and ε are independent, their joint density is the product of their densities, that is,

$$f_{k,\varepsilon}(k,\varepsilon) = f_k(k) f_\varepsilon(\varepsilon) = \begin{cases} \dfrac{1}{\sigma_v + \sigma_w} \exp\left(-\dfrac{k}{\sigma_w} \right) \dfrac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\dfrac{\varepsilon^2}{2\sigma_\varepsilon^2} \right), & \text{if } k \geq 0 \\ \dfrac{1}{\sigma_v + \sigma_w} \exp\left(\dfrac{k}{\sigma_v} \right) \dfrac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\dfrac{\varepsilon^2}{2\sigma_\varepsilon^2} \right), & \text{if } k \leq 0 \end{cases} \tag{B.13}$$

We define the transformation

$$\begin{pmatrix} k \\ \varepsilon \end{pmatrix} \to \begin{pmatrix} k \\ k + \varepsilon \end{pmatrix} = \begin{pmatrix} k \\ s \end{pmatrix}. \tag{B.14}$$

The inverse transformation and its Jacobian are

$$\begin{cases} k = k \\ \varepsilon = s - k \end{cases}, \quad \text{and} \quad |J| = \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = -1, \tag{B.15}$$

respectively. Thus, the joint density of *k* and *s* is given by

$$f_{s,k}(s,k) = f_{k,\varepsilon}(k, s-k)\,\text{abs}(|J|)$$
$$= \begin{cases} \dfrac{1}{\sigma_v + \sigma_w} \exp\left(-\dfrac{k}{\sigma_w} \right) \dfrac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\dfrac{(s-k)^2}{2\sigma_\varepsilon^2} \right) \\ \dfrac{1}{\sigma_v + \sigma_w} \exp\left(\dfrac{k}{\sigma_v} \right) \dfrac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\dfrac{(s-k)^2}{2\sigma_\varepsilon^2} \right) \end{cases} = \ldots$$

$$= \begin{cases} \dfrac{1}{\sigma_v + \sigma_w} \exp\left(\dfrac{\sigma_\varepsilon^2}{2\sigma_w^2} - \dfrac{s}{\sigma_w} \right) \dfrac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left[\left(-\dfrac{1}{2} \right) \left(\dfrac{k - s + \sigma_\varepsilon^2 / \sigma_w}{\sigma_\varepsilon} \right)^2 \right], & \text{if } k \geq 0 \\ \dfrac{1}{\sigma_v + \sigma_w} \exp\left(\dfrac{\sigma_\varepsilon^2}{2\sigma_v^2} + \dfrac{s}{\sigma_v} \right) \dfrac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left[\left(-\dfrac{1}{2} \right) \left(\dfrac{k - s - \sigma_\varepsilon^2 / \sigma_v}{\sigma_\varepsilon} \right)^2 \right], & \text{if } k \leq 0 \end{cases}$$

(B.16)

In order to find the marginal density of *s*, the joint density $f_{s,k}(s,k)$ in eq. (B.16) must be integrated with respect to the random variable *k*. Since $k \in (-\infty, +\infty)$, the integral in question can be broken down into two different ones:

$$f_s(s) = \int_{-\infty}^{+\infty} f_{s,k}(s,k)dk = \int_{-\infty}^{0} f_{s,k}(s,k)dk + \int_{0}^{+\infty} f_{s,k}(s,k)dk =$$

$$= \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s}{\sigma_v} \right) \int_{-\infty}^{0} \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left[\left(-\frac{1}{2} \right) \left(\frac{k - s - \sigma_\varepsilon^2 / \sigma_v}{\sigma_\varepsilon} \right)^2 \right] dk$$

$$+ \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s}{\sigma_w} \right) \int_{0}^{+\infty} \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left[\left(-\frac{1}{2} \right) \left(\frac{k - s + \sigma_\varepsilon^2 / \sigma_w}{\sigma_\varepsilon} \right)^2 \right] dk$$

$$= \ldots = \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s}{\sigma_v} \right) \Phi\left(-\frac{s}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v} \right)$$

$$+ \frac{1}{\sigma_v + \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s}{\sigma_w} \right) \Phi\left(\frac{s}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w} \right) \qquad \text{(B.17)}$$

where $\Phi(\cdot)$ denotes standard normal cumulative distribution function.

B.2. DERIVATION OF THE ESTIMATOR FOR COMMODITY EFFICIENCY IN THE "TWO-TIERED FRONTIER" EMPIRICAL SPECIFICATION

Recall that the measure of commodity efficiency is the inverse of the value of the input distance function. Since the distance is treated as a random variable (an error term to be estimated), we need to compute its expected value conditional on the composite error term, *s*. Recall also that the random variable *v* denotes the logarithm of the value of the distance function, that is, $v = \ln D^I(u,\mathbf{q})$, so that, $D^I(u,\mathbf{q}) = e^v$. The expected value of *v* conditional on the composite error term, *s*, is given by:

$$E[v \mid s] = \int_0^{+\infty} v f(v \mid s) dv = \int_0^{+\infty} v \frac{f_{v,s}(v,s)}{f_s(s)} dv, \tag{B.18}$$

where $f_{v,s}(v,s)$ is the joint density of the random variables *v* and *s*, and the limits of integration follow from the assumption that $v \geq 0$. Hence, the measure of commodity efficiency is given by

$$CE(u,\mathbf{q}) = E[(D^I(u,\mathbf{q}))^{-1} \mid s] = E[e^{-v} \mid s] = \int_0^{+\infty} e^{-v} f(v \mid s) dv = \int_0^{+\infty} e^{-v} \frac{f_{v,s}(v,s)}{f_s(s)} dv. \tag{B.19}$$

The density of the composite error term, $f_s(s)$, is given by equation (B.17). But, we still need to find the joint density of the random variables *v* and *s*, *i.e.*, $f_{v,s}(v,s)$, so that we can derive the functional form of $CE(u,\mathbf{q})$. Recall that the random variable *s* is a function of the random variable *v*. Hence, *s* and *v* are not independent, *i.e.*, $f_{v,s}(v,s) \neq f_v(v) f_s(s)$. In order to derive the joint density $f_{v,s}(v,s)$, we will start by assuming that the random variables *w*, *v*, and ε are independent, so that their joint density is given by the product of their densities, that is,

$$f_{w,v,\varepsilon}(w,v,\varepsilon) = f_w(w) f_v(v) f_\varepsilon(\varepsilon) = \frac{1}{\sigma_w \sigma_v} \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\frac{w}{\sigma_w} - \frac{v}{\sigma_v} - \frac{\varepsilon^2}{2\sigma_\varepsilon^2} \right) \tag{B.20}$$

We define the transformation

$$\begin{pmatrix} w \\ v \\ \varepsilon \end{pmatrix} \rightarrow \begin{pmatrix} w - v + \varepsilon \\ v \\ \varepsilon \end{pmatrix} = \begin{pmatrix} s \\ v \\ \varepsilon \end{pmatrix}. \tag{B.21}$$

The inverse system and its Jacobian are,

$$\begin{cases} w = s + v - \varepsilon \\ v = v \\ \varepsilon = \varepsilon \end{cases}, \quad \text{and} \quad |J| = \begin{vmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1, \tag{B.22}$$

respectively. Then, the joint density of *s*, *v*, and ε is given by:

$$\begin{aligned} f_{s,v,\varepsilon}(s,v,\varepsilon) &= f_{w,v,\varepsilon}(s+v-\varepsilon,v,\varepsilon)\text{abs}(|J|) = \ldots \\ &= \frac{1}{\sigma_v\sigma_w}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2}-\frac{s}{\sigma_w}\right)\exp\left(-\frac{v}{\sigma_v}-\frac{v}{\sigma_w}\right) \\ &\quad\times\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left[\left(-\frac{1}{2}\right)\left(\frac{\varepsilon-\sigma_\varepsilon^2/\sigma_w}{\sigma_\varepsilon}\right)^2\right]. \end{aligned} \tag{B.23}$$

The joint density $f_{v,s}(v,s)$ is found by integrating the joint density in (B.23) with respect to the random variable ε. However, care must be taken with regard to the limits of integration, that is, with regard to the interval that the random variable ε takes values on. Since ε is normally distributed, $\varepsilon \in (-\infty,+\infty)$. But, the random variable *w* is exponentially distributed, so $w \geq 0$, and, hence, the transformation in (B.22) requires that: $w \geq 0 \Rightarrow s+v-\varepsilon \geq 0 \Rightarrow \varepsilon \leq s+v$. Consequently, integration of $f_{s,v,\varepsilon}(s,v,\varepsilon)$ with respect to ε will be carried out in the interval $(-\infty, s+v)$:

$$\begin{aligned} f_{v,s}(v,s) &= \int_{-\infty}^{s+v} f_{s,v,\varepsilon}(s,v,\varepsilon)d\varepsilon \\ &= \frac{1}{\sigma_v\sigma_w}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2}-\frac{s}{\sigma_w}\right)\exp\left(-\frac{v}{\sigma_v}-\frac{v}{\sigma_w}\right) \\ &\quad\times\int_{-\infty}^{s+v}\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left[\left(-\frac{1}{2}\right)\left(\frac{\varepsilon-\sigma_\varepsilon^2/\sigma_w}{\sigma_\varepsilon}\right)^2\right]d\varepsilon \\ &= \frac{1}{\sigma_v\sigma_w}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2}-\frac{s}{\sigma_w}\right)\exp\left(-\frac{v}{\sigma_v}-\frac{v}{\sigma_w}\right)\Phi\left(\frac{s+v}{\sigma_\varepsilon}-\frac{\sigma_\varepsilon}{\sigma_w}\right), \end{aligned} \tag{B.24}$$

where $\Phi(\cdot)$ denotes standard normal cumulative distribution function. Thus, the conditional expected value of $1/D(u,\mathbf{q}) = e^{-v}$ is given by

$$CE(u,\mathbf{q}) = E[(D^I(u,\mathbf{q}))^{-1} \mid s] = E[e^{-v} \mid s] = \int_0^{+\infty} e^{-v} \frac{f(v,s)}{f(s)} dv$$
$$= \frac{1}{f(s)} \int_0^{+\infty} e^{-v} f(v,s) dv = \frac{1}{f(s)} \frac{1}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s}{\sigma_w} \right)$$
$$\times \int_0^{+\infty} e^{-v} \exp\left(-\frac{v}{\sigma_v} - \frac{v}{\sigma_w} \right) \Phi\left(\frac{s+v}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w} \right) dv. \tag{B.25}$$

Using integration by parts in order to compute the integral in eq. (B.25), the expression for the conditional expected value of $1/D^I(u,\mathbf{q}) = e^{-v}$ becomes:

$$CE(u,\mathbf{q}) = E[(D^I(u,\mathbf{q}))^{-1} \mid s] = E[e^{-v} \mid s]$$
$$= \frac{1}{f(s)} \frac{\sigma^*}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_w^2} - \frac{s}{\sigma_w} \right)$$
$$\times \left\{ \Phi\left(\frac{s}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_w} \right) + \exp\left[\frac{\sigma_\varepsilon^2}{2\sigma^{*2}} + \left(s - \frac{\sigma_\varepsilon^2}{\sigma_w} \right) \frac{1}{\sigma^*} \right] \Phi\left(-\frac{s}{\sigma_\varepsilon} + \frac{\sigma_\varepsilon}{\sigma_w} - \frac{\sigma_\varepsilon}{\sigma^*} \right) \right\}, \tag{B.26}$$

where

$$\sigma^* = \frac{\sigma_v \sigma_w}{\sigma_v + \sigma_w + \sigma_v \sigma_w}, \tag{B.27}$$

and $\Phi(\cdot)$ denotes standard normal cumulative distribution function.

B.3. DERIVATION OF THE ESTIMATOR FOR UTILITY IN THE "TWO-TIERED FRONTIER" EMPIRICAL SPECIFICATION

Recall that the random variable w was defined to be $w \equiv \left[-g\left(\ln\left(\mathbf{q}/q_1 \right), t; \mathbf{b}_2 \right) \right] (-\ln u) \equiv (-g)(-\ln u)$, so that, $u = e^{w/g}$, where u is the utility level. The expected value of w conditional on the composite error term, s, is given by:

$$E[w \mid s] = \int_{0}^{+\infty} w f(w \mid s) dw = \int_{0}^{+\infty} w \frac{f_{w,s}(w,s)}{f_s(s)} dw, \tag{B.28}$$

where $f_{w,s}(w,s)$ is the joint density of the random variables *w* and *s*, and the limits of integration follow from the assumption that $w \geq 0$. Hence, the estimator for the consumer's utility level is given by

$$E[u \mid s] = E[e^{-w/(-g)} \mid s] = \int_{0}^{+\infty} e^{-w/(-g)} f(w \mid s) dw = \int_{0}^{+\infty} e^{-w/(-g)} \frac{f_{w,s}(w,s)}{f_s(s)} dw. \tag{B.29}$$

The density of the composite error term, $f_s(s)$, is given by equation (B.17). But, we still need to find the joint density of the random variables *w* and *s*, $f_{w,s}(w,s)$, so that we can derive the functional form of $E[u \mid s]$. Recall that the random variable *s* is a function of the random variable *w*. Hence, *s* and *w* are not independent, *i.e.*, $f_{w,s}(w,s) \neq f_w(w) f_s(s)$. In order to derive the joint density $f_{w,s}(w,s)$, we will start by assuming that the random variables *w*, *v*, and ε are independent, so that their joint density is given by the product of their densities, that is,

$$f_{w,v,\varepsilon}(w,v,\varepsilon) = f_w(w) f_v(v) f_\varepsilon(\varepsilon) = \frac{1}{\sigma_w \sigma_v} \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\frac{w}{\sigma_w} - \frac{v}{\sigma_v} - \frac{\varepsilon^2}{2\sigma_\varepsilon^2} \right) \tag{B.30}$$

We define the transformation

$$\begin{pmatrix} w \\ v \\ \varepsilon \end{pmatrix} \rightarrow \begin{pmatrix} w \\ w - v + \varepsilon \\ \varepsilon \end{pmatrix} = \begin{pmatrix} w \\ s \\ \varepsilon \end{pmatrix}. \tag{B.31}$$

The inverse system and its Jacobian are,

$$\begin{cases} w = w \\ v = w + \varepsilon - s \\ \varepsilon = \varepsilon \end{cases}, \quad \text{and,} \quad |J| = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \end{vmatrix} = 1, \tag{B.32}$$

respectively. Then, the joint density of *s*, *w*, and ε is given by:

$$\begin{aligned} f_{s,w,\varepsilon}(s,w,\varepsilon) &= f_{w,v,\varepsilon}(w, w-v+\varepsilon, \varepsilon)\text{abs}(|J|) \\ &= \frac{1}{\sigma_v\sigma_w}\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left(-\frac{w}{\sigma_w}-\frac{(w-\varepsilon+s)}{\sigma_v}-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}\right) \\ &= \ldots = \frac{1}{\sigma_w\sigma_v}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2}+\frac{s}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_v}-\frac{w}{\sigma_w}\right)\frac{1}{\sigma_\varepsilon\sqrt{2\pi}} \\ &\quad \times\exp\left[\left(-\frac{1}{2}\right)\left(\frac{\varepsilon+\sigma_\varepsilon^2/\sigma_v}{\sigma_\varepsilon}\right)^2\right] \end{aligned} \tag{B.33}$$

The joint density $f_{w,s}(w,s)$ is found by integrating the joint density in (B.33) with respect to the normal random variable ε. However, care must be taken with regard to the limits of integration, that is, with regard to the interval that the random variable ε takes values on. Since ε is normally distributed, $\varepsilon \in (-\infty, +\infty)$. But, the random variable v is exponentially distributed, so $v \geq 0$, and, hence, the inverse transformation in (B.32) requires that: $v \geq 0 \Rightarrow w+\varepsilon-s \geq 0 \Rightarrow \varepsilon \geq s-w$. Consequently, integration of $f_{s,w,\varepsilon}(s,w,\varepsilon)$ with respect to ε will be carried out in the interval $\varepsilon \in (s-w, +\infty)$;

$$\begin{aligned} f_{w,s}(w,s) &= \int_{s-w}^{+\infty} f_{s,w,\varepsilon}(s,w,\varepsilon)d\varepsilon \\ &= \frac{1}{\sigma_v\sigma_w}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2}+\frac{s}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_v}-\frac{w}{\sigma_w}\right) \\ &\quad \times\int_{s-w}^{+\infty}\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left[\left(-\frac{1}{2}\right)\left(\frac{\varepsilon+\sigma_\varepsilon^2/\sigma_v}{\sigma_\varepsilon}\right)^2\right]d\varepsilon \\ &= \frac{1}{\sigma_v\sigma_w}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2}+\frac{s}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_v}-\frac{w}{\sigma_w}\right)\left[1-\Phi\left(\frac{s-w}{\sigma_\varepsilon}+\frac{\sigma_\varepsilon}{\sigma_v}\right)\right] \\ &= \frac{1}{\sigma_v\sigma_w}\exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2}+\frac{s}{\sigma_v}\right)\exp\left(-\frac{w}{\sigma_v}-\frac{w}{\sigma_w}\right)\Phi\left(-\frac{(s-w)}{\sigma_\varepsilon}-\frac{\sigma_\varepsilon}{\sigma_v}\right), \end{aligned} \tag{B.34}$$

where $\Phi(\cdot)$ denotes standard normal cumulative distribution function. Thus, the conditional expected value of $u = e^{-w/(-g)}$ is given by

$$E[u \mid s] = E[e^{-w/(-g)} \mid s] = \int_0^{+\infty} e^{-w/(-g)} \frac{f(w,s)}{f(s)} dw = \frac{1}{f(s)} \int_0^{+\infty} e^{-w/(-g)} f(w,s) dw$$
$$= \frac{1}{f(s)} \frac{1}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s}{\sigma_v} \right)$$
$$\times \int_0^{+\infty} e^{-w/(-g)} \exp\left(-\frac{w}{\sigma_v} - \frac{w}{\sigma_w} \right) \Phi\left(-\frac{(s-w)}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v} \right) dw \qquad \text{(B.35)}$$

Using integration by parts in order to compute the integral in eq. (B.35), the expression for the conditional expected value of $u = e^{-w/(-g)}$ becomes:

$$E[u \mid s] = E[e^{-w/(-g)} \mid s]$$
$$= \frac{1}{f(s)} \frac{\sigma^*}{\sigma_v \sigma_w} \exp\left(\frac{\sigma_\varepsilon^2}{2\sigma_v^2} + \frac{s}{\sigma_v} \right)$$
$$\times \left\{ \Phi\left(-\frac{s}{\sigma_\varepsilon} - \frac{\sigma_\varepsilon}{\sigma_v} \right) + \exp\left[\frac{\sigma_\varepsilon^2}{2\sigma^{*2}} - \left(s + \frac{\sigma_\varepsilon^2}{\sigma_v} \right) \frac{1}{\sigma^*} \right] \Phi\left(\frac{s}{\sigma_\varepsilon} + \frac{\sigma_\varepsilon}{\sigma_v} - \frac{\sigma_\varepsilon}{\sigma^*} \right) \right\}, \qquad \text{(B.36)}$$

where

$$\sigma^* = \frac{(-\alpha)\sigma_v \sigma_w}{\sigma_v \sigma_w + (-\alpha)\sigma_w + (-\alpha)\sigma_v}, \qquad \text{(B.37)}$$

and $\Phi(\cdot)$ denotes standard normal cumulative distribution function.

Appendix C. Greek Summary

Η παρούσα διδακτορική διατριβή αποτελεί μία σύνθεση μελετών πάνω σε διαφορετικά ζητήματα σχετικά με τη συμπεριφορά καταναλωτή: παρέχει μια ανάλυση της ζήτησης καταναλωτή για βιολογικά και μη-βιολογικά αγαθά και διαπραγματεύεται το ζήτημα της αναποτελεσματικότητας του καταναλωτή. Ειδικότερα, (α) παρέχει εμπειρικές ενδείξεις σχετικά με τις αλληλοσυσχετίσεις μεταξύ βιολογικών και μη-βιολογικών αγαθών με τη μορφή σταυροειδών αντίστροφων ελαστικοτήτων ζήτησης (cross-quantity flexibilities), (β) αναλύει τις επιπτώσεις που έχει η υποκατάσταση μη-βιολογικών με βιολογικά αγαθά πάνω στο επίπεδο ευημερίας του καταναλωτή, (γ) αναπτύσσει ένα θεωρητικό υπόδειγμα για τη μέτρηση της αποτελεσματικότητας στην κατανάλωση και (δ) προτείνει ένα οικονομετρικό πλαίσιο για την εμπειρική μέτρηση της αναποτελεσματικότητας του καταναλωτή.

Τα πρώτα δύο ζητήματα τα οποία μελετώνται αφορούν στη ζήτηση καταναλωτή για βιολογικά και μη-βιολογικά αγαθά και στις μεταβολές στην ευημερία του καταναλωτή οι οποίες προκύπτουν από την υποκατάσταση μη-βιολογικών με βιολογικά αγαθά. Η ζήτηση καταναλωτή για βιολογικά αγαθά επιλέχθηκε ως αντικείμενο μελέτης λόγω της σημασίας που έχει η βιολογική γεωργία ως ένα σύστημα παραγωγής το οποίο βασίζεται σε ανανεώσιμες εισροές, οι οποίες παράγονται τοπικά ή στην φάρμα και το οποίο στοχεύει στην προώθηση και στη βελτίωση της υγείας του οικοσυστήματος. Στην Ευρωπαϊκή Ένωση (ΕΕ), η σημασία της βιολογικής γεωργίας αντικατοπτρίζεται στις πρόσφατες μεταρρυθμίσεις της Κοινής Αγροτικής Πολιτικής (ΚΑΠ), καθώς και σε σχετικούς κανονισμούς. Με τη μεταρρύθμιση της ΚΑΠ το 1992 η βιολογική γεωργία απέκτησε έναν σημαντικό ρόλο στην προώθηση της προστασίας του περιβάλλοντος σε όλη την ΕΕ, ενώ οι Ευρωπαϊκοί Κανονισμοί 2078/92 και 2092/91 παρείχαν συγκεκριμένα κίνητρα για την μετατροπή των αγροκτημάτων σε βιολογικά και τη διατήρηση της βιολογικής γεωργίας και καθιέρωσαν τα βιολογικά προϊόντα ως διαφορετικά των μη-βιολογικών (παροχή προτύπων και πιστοποίησης). Όμως, οι υψηλότερες τιμές των βιολογικών αγαθών σε σχέση με τα μη-βιολογικά αποτελούν το σημαντικότερο κίνητρο προς τους αγρότες για μετατροπή των μονάδων τους σε βιολογικές (Burton, Rigby, και Young, 1999, 2003, O'Riordan και Cobb, 2001). Οι αγρότες λαμβάνουν υψηλότερες τιμές για τα βιολογικά προϊόντα όταν οι καταναλωτές πιστεύουν ότι υπάρχει ένα ασφάλιστρο ποιότητας (quality premium) στα χαρακτηριστικά των βιολογικών προϊόντων (Loureiro, McCluskey, και Mittelhammer, 2001, Boland και Schroeder, 2002). Έτσι, η αποδοχή των βιολογικών προϊόντων εκ μέρους των καταναλωτών είναι ζωτικής σημασίας για την ανάπτυξη του τομέα της βιολογικής γεωργίας.

Οι περισσότερες μελέτες των συνθηκών ζήτησης για βιολογικά προϊόντα εξετάζουν τη στάση των καταναλωτών ως προς τα βιολογικά προϊόντα, προσδιορίζουν τα κίνητρα των καταναλωτών για αγορά βιολογικών προϊόντων και εξάγουν την προθυμία των καταναλωτών να πληρώσουν γι' αυτά (willingness to pay) (βλ., για παράδειγμα, Yiridoe, Bonti-Ankomah, και Martin (2005) για μία ανασκόπηση των μελετών που διερευνούν αυτά τα ζητήματα, και Gracia και de Magistris (2008) για μία ανασκόπηση των πρόσφατων εμπειρικών μελετών πάνω στους οικονομικούς και δημογραφικούς παράγοντες που επηρεάζουν τη ζήτηση του καταναλωτή για βιολογικά προϊόντα). Τέτοιες μελέτες λαμβάνουν υπόψη κοινωνικο-οικονομικούς, δημογραφικούς, ψυχολογικούς και ηθικούς παράγοντες, οι οποίοι μπορεί να επηρεάζουν την αποδοχή των βιολογικών προϊόντων εκ μέρους των καταναλωτών, αλλά αμελούν σχεδόν παντελώς την παρουσία στην αγορά προϊόντων ανταγωνιστικών των βιολογικών, όπως τα μη-βιολογικά. Ειδικότερα, υπάρχουν πολύ λίγες εμπειρικές μελέτες της ζήτησης καταναλωτή που χρησιμοποιούν πραγματικά δεδομένα τιμών και ποσοτήτων για να διερευνήσουν τις αλληλοσυσχετίσεις μεταξύ βιολογικών και μη-βιολογικών αγαθών με τη μορφή σταυροειδών ελαστικοτήτων ζήτησης (ενδεικτικά αναφέρουμε τις μελέτες των Glaser και Thompson (1998, 2000) και των Wier, Hansen, και Smed (2001)). Ο περιορισμένος αριθμός εμπειρικών μελετών που εξάγουν σταυροειδείς ελαστικότητες ζήτησης μεταξύ βιολογικών και μη-βιολογικών αγαθών οφείλεται στο ότι τα βιολογικά προϊόντα είναι καινούργια αγαθά σε σχέση με τα μη-βιολογικά και, ως εκ τούτου, στην σπανιότητα επαρκών ιστορικών δεδομένων πάνω σε λιανικές τιμές και καταναλισκόμενες ποσότητες.

Σε αυτά τα πλαίσια, ο στόχος του Κεφαλαίου 3 είναι να παράσχει εμπειρικές ενδείξεις σχετικά με τις αλληλοσυσχετίσεις μεταξύ βιολογικών και μη-βιολογικών αγαθών και τις μεταβολές στην ευημερία του καταναλωτή, οι οποίες προκύπτουν από υποκατάσταση μη-βιολογικών με βιολογικά αγαθά. Προκειμένου να επιτευχθεί ο στόχος αυτός, χρησιμοποιείται ένα σύστημα εξισώσεων ζήτησης και για τους δύο τύπους αγαθών. Χρησιμοποιούνται διαστρωματικά στατιστικά δεδομένα και το αντίστροφο υπόδειγμα AIDS (Inverse Almost Ideal Demand System – IAIDS) των Eales και Unnevehr (1994), και Moschini και Vissa (1992), προκειμένου να γίνει εμπειρική ανάλυση της ζήτησης νοικοκυριών για δύο ομάδες βιολογικών και μη-βιολογικών αγαθών (γάλα & γιαούρτι, φρούτα & λαχανικά). Αντίστροφα συστήματα ζήτησης, δηλαδή, συστήματα ζήτησης στα οποία οι καταναλισκόμενες ποσότητες θεωρούνται ως προκαθορισμένες, μπορούν να χρησιμοποιηθούν όταν οι τιμές είτε δεν υπάρχουν είτε είναι πλασματικά διαστρεβλωμένες (Deaton και Muellbauer, 1980*a*), ή στην περίπτωση ευπαθών τροφίμων και ευπαθών προϊόντων γεωργίας και αλιείας που οι τιμές τους δεν μπορούν να προσαρμοστούν βραχυχρόνια (Barten και Bettendorf, 1989). Στη δεύτερη περίπτωση, η βασική υπόθεση που γίνεται είναι ότι αφού η προσφορά τέτοιων αγαθών μπορεί να είναι αρκετά ανελαστική κατά τη διάρκεια μικρών χρονικών διαστημάτων, οι τιμές πρέπει να προσαρμοστούν ώστε η διαθέσιμη ποσότητα να καταναλωθεί. Στην μελέτη που παρουσιάζεται στο Κεφάλαιο 3, τα υπό εξέταση αγαθά είναι ευπαθή και δεν μπορούν να αποθηκευθούν. Επιπλέον, τα στατιστικά δεδομένα που χρησιμοποιούνται παρέχουν πληροφόρηση σχετικά με τις ποσότητες των αγαθών που αγοράζονται και την δαπάνη που καταβάλλουν τα νοικοκυριά, αλλά δεν παρέχουν πληροφόρηση σχετικά με τις τιμές. Ως εκ τούτου, ένα αντίστροφο σύστημα ζήτησης είναι καταλληλότερο για την παρούσα μελέτη. Ένα άλλο ζήτημα που εγείρεται από την υιοθέτηση ενός αντίστροφου συστήματος ζήτησης αφορά στην ενδογένεια των ποσοτήτων των αγαθών. Από την άλλη πλευρά, οι εμπειρικές μελέτες ζήτησης καταναλωτή που χρησιμοποιούν διαστρωματικά δεδομένα και άμεσα συστήματα ζήτησης (direct demand systems), υπολογίζουν τις τιμές των αγαθών από τα δεδομένα για τις ποσότητες και την δαπάνη. Όμως, αυτές οι υπολογισμένες τιμές (unit values) αντανακλούν, όχι μόνον την τιμή των αγαθών, αλλά και την ποιότητά τους, και είναι ενδογενείς με την έννοια ότι καθορίζονται από τις αποφάσεις των νοικοκυριών (Deaton, 1990, Nelson, 1991).

Η χρήση διαστρωματικών στοιχείων στην ανάλυσή μας, όμως, δεν γίνεται χωρίς δυσκολίες. Είναι σύνηθες σε μελέτες ζήτησης καταναλωτή που χρησιμοποιούν διαστρωματικά στατιστικά δεδομένα, να απαντώνται πολλά νοικοκυριά τα οποία δηλώνουν μηδενική κατανάλωση για ορισμένα αγαθά. Η παρουσία αυτών των παρατηρήσεων στο δείγμα εγείρει δύο προβλήματα. Πρώτον, στην περίπτωση ενός αντίστροφου συστήματος ζήτησης, όπως το IAIDS, όπου τα μερίδια δαπάνης είναι συναρτήσεις του λογάριθμου των ποσοτήτων των αγαθών, ο λογάριθμος των μηδενικών ποσοτήτων δεν μπορεί να οριστεί. Προκειμένου να αντιμετωπίσομε το πρόβλημα αυτό, χρησιμοποιούμε την προσέγγιση η οποία προτάθηκε από τον Battese (1997) στο πλαίσιο εκτίμησης στοχαστικών συνόρων παραγωγής (stochastic production frontiers), και η οποία επιτρέπει οικονομετρική εκτίμηση με τη χρήση όλου του δείγματος και δίδει αποτελεσματικές και αμερόληπτες εκτιμήσεις. Το δεύτερο πρόβλημα το οποία σχετίζεται με την παρουσία μηδενικών καταναλισκόμενων ποσοτήτων στο δείγμα είναι ότι οι συνήθεις μέθοδοι εκτίμησης συστημάτων εξισώσεων, π.χ. η μέθοδος των φαινομενικά ασυσχέτιστων παλινδρομήσεων (seemingly unrelated regressions) ή η μέθοδος μεγίστης πιθανοφάνειας (maximum likelihood), οδηγούν σε μεροληπτικές εκτιμήσεις των παραμέτρων. Το Amemiya-Tobin μοντέλο των Wales και Woodland (1983) είναι αυτό το οποίο χρησιμοποιείται στην παρούσα μελέτη προκειμένου να αντιμετωπιστεί το πρόβλημα της παρουσίας μηδενικών ποσοτήτων στο δείγμα μας. Τα πλεονεκτήματα αυτής της προσέγγισης για την εκτίμηση συστημάτων λογοκριμένων (censored) εξισώσεων, σε σχέση με άλλες προσεγγίσεις που χρησιμοποιούνται ευρέως στη βιβλιογραφία, αφορούν στο ότι (α) μπορεί να εφαρμοστεί σε συστήματα ζήτησης οποιασδήποτε συναρτησιακής μορφής, (β) αντιμετωπίζει με ικανοποιητικό τρόπο το πρόβλημα της ικανοποίησης του εισοδηματικού περιορισμού εκ μέρους των παρατηρούμενων μεριδίων δαπάνης, και (γ) δίνει αποτελεσματικές εκτιμήσεις των παραμέτρων του συστήματος. Το Amemiya-Tobin μοντέλο των Wales και Woodland έχει επίσης χρησιμοποιηθεί από τους Dong, Gould, και Kaiser (2004), για την εκτίμησης της ζήτησης των Μεξικάνικων νοικοκυριών για 12 κατηγορίες τροφίμων, και από τους Dong, Kaiser, και Myrland (2007) για την αποτίμηση των επιδράσεων που έχει η διαφήμιση πάνω στη ζήτηση των νοικοκυριών της Νορβηγίας για τέσσερις κατηγορίες κρέατος και ψαριών.

Χρησιμοποιώντας το υπόδειγμα IAIDS, την τεχνική του Battese (1997) για την επίλυση του προβλήματος που προκύπτει από ερμηνευτικές μεταβλητές που λαμβάνουν και μηδενικές τιμές, και το Amemiya-Tobin μοντέλο των Wales και Woodland (1983), το λογοκριμένο υπόδειγμα IAIDS εκτιμήθηκε με διαστρωματικά στοιχεία για δαπάνες και καταναλισκόμενες ποσότητες αγαθών για 1155 νοικοκυριά του Λονδίνου, για τον Νοέμβριο του έτος 2006. Στις ερμηνευτικές μεταβλητές του υποδείγματος συμπεριλήφθησαν, επίσης, και κοινωνικο-δημογραφικές μεταβλητές, προκειμένου να ληφθεί υπόψη η ετερογένεια των νοικοκυριών του δείγματος. Ειδικότερα, οι σταθεροί όροι των εξισώσεων δαπάνης του IAIDS επαυξήθηκαν έτσι ώστε να έχουν τη μορφή πολυωνυμικών συναρτήσεων της ηλικίας του βασικού αγοραστή στο νοικοκυριό, της κοινωνικής τάξης του νοικοκυριού και του αριθμού των παιδιών σε αυτό. Επιπλέον, προκειμένου να αντιμετωπίσουμε το πρόβλημα της ετεροσκεδαστικότητας των διαταρακτικών όρων των εξισώσεων μεριδίων δαπάνης του IAIDS, υποθέσαμε ότι τα διαγώνια στοιχεία της μήτρας διακύμανσης-συνδιακύμανσης των διαταρακτικών όρων εξαρτώνται αθροιστικά από την κοινωνική τάξη του νοικοκυριού και από τον αριθμό των παιδιών σε αυτό. Τέλος, αφού επιβλήθηκαν οι περιορισμοί της ομογένειας και της συμμετρίας, το τελικό υπόδειγμα εκτιμήθηκε με τη μέθοδο της μεγίστης πιθανοφάνειας και με τη βοήθεια του υπολογιστικού πακέτου GAUSS.

Έλεγχοι λόγου πιθανοφάνειας (likelihood ratio tests) που έγιναν κατόπιν της εκτίμησης του υποδείγματος δείχνουν ότι οι διαταρακτικοί όροι των εξισώσεων μεριδίων δαπάνης για βιολογικό γάλα & γιαούρτι και βιολογικά φρούτα & λαχανικά είναι ετεροσκεδαστικοί. Για τον λόγο αυτό, προτιμήθηκε μία ετεροσκεδαστική διάρθρωση για τους διαταρακτικούς όρους των εξισώσεων αυτών. Λόγω του περιορισμού της αθροιστικότητας (adding-up) των εξισώσεων μεριδίων δαπάνης σε συστήματα ζήτησης, η μήτρα διακύμανσης-συνδιακύμανσης των διαταρακτικών όρων είναι ιδιάζουσα. Ως εκ τούτου, μία από τις εξισώσεις του συστήματος ζήτησης πρέπει να αφεθεί έξω από την εκτίμηση του συστήματος ως περιττή. Στην περίπτωσή μας, η εξίσωση μεριδίου δαπάνης η οποία αφέθηκε έξω από την εκτίμηση ήταν εκείνη του βιολογικού γάλακτος & γιαουρτιού. Οι εκτιμήσεις των παραμέτρων της εν λόγω εξίσωσης υπολογίστηκαν μετά την εκτίμηση του υποδείγματος από τους περιορισμούς της αθροιστικότητας, της ομογένειας και της συμμετρίας, ενώ τα τυπικά σφάλματά τους υπολογίστηκαν προσεγγιστικά με τη *μέθοδο δέλτα* (*delta method*) (βλ., για παράδειγμα, Spanos (1999)).

Οι εκτιμήσεις των παραμέτρων και τα τυπικά σφάλματά τους παρουσιάζονται στον Πίνακα 1. Όπως φαίνεται στον πίνακα αυτό, οι συντελεστές των ιδίων και σταυροειδών ποσοτήτων είναι στατιστικά σημαντικοί σε επίπεδο σημαντικότητας 1%. Επιπλέον, τρεις από τους τέσσερις συντελεστές της συνολικής κατανάλωσης, οκτώ από τους δώδεκα συντελεστές των δημογραφικών μεταβλητών και έντεκα από τους δώδεκα συντελεστές των ψευδομεταβλητών που χρησιμοποιήθηκαν για την εφαρμογή της τεχνικής του Battese (1997) είναι στατιστικά σημαντικοί.

Πίνακας 1. Εκτιμήσεις Παραμέτρων του IAIDS Βιολογικά και Μη-Βιολογικά Τρόφιμα στο Λονδίνο (Ηνωμένο Βασίλειο)

	Βιολογικό Γάλα & Γιαούρτι	Μη-Βιολογικό Γάλα & Γιαούρτι	Βιολογικά Φρούτα & Λαχανικά	Μη-Βιολογικά Φρούτα & Λαχανικά
Σταθερά	0,0168	0,2609	0,0095	0,7128
	(0,0135)	(0,0083)*	(0,0146)	(0,0195)*
Χαρακτηριστικά των Νοικοκυριών				
Ηλικία	0,0605	0,0463	0,0066	-0,1134
	(0,0093)*	(0,0056)*	(0,0111)	(0,0144)*
Κοινωνική Τάξη	-0,0307	0,0088	0,0178	0,0041
	(0,0154)**	(0,0037)**	(0,0076)**	(0,0107)
Αριθμός Παιδιών	0,0009	0,0073	0,0001	-0,0083
	(0,0022)	(0,0008)*	(0,0021)	(0,0028)*
Ποσότητες				
Βιολογικό Γάλα & Γιαούρτι	0,1774			
	(0,0057)*			
Μη-Βιολογικό Γάλα & Γιαούρτι	-0,04	0,1346		
	(0,0027)*	(0,0011)*		
Βιολογικά Φρούτα & Λαχανικά	-0,011	-0,0196	0,0741	
	(0,0034)*	(0,0020)*	(0,0031)*	
Μη-Βιολογικά Φρούτα & Λαχανικά	-0,1264	-0,075	-0,0435	0,2449
	(0,0045)*	(0,0019)*	(0,0037)*	(0,0057)*
Συνολική Κατανάλωση	0,129	-0,0065	0,0023	-0,1248
	(0,0071)*	(0,0038)***	(0,0049)	(0,0065)*
Ψευδομεταβλητές για τα Αγαθά				
Βιολογικό Γάλα & Γιαούρτι	-0,2311	0.0079	0.0247	0.1985
	(0,0098)*	(0.0069)	(0.0061)*	(0.0112)*
Μη-Βιολογικό Γάλα & Γιαούρτι	0,0162	-0.2678	0.0587	0.1929
	(0,0077)**	(0.0171)*	(0.0108)*	(0.0203)*
Βιολογικά Φρούτα & Λαχανικά	0,0474	0.0105	-0.2199	0.162
	(0,0066)*	(0.0054)**	(0.0065)*	(0.0099)*
Log-Likelihood		-865,3029		

Σημειώσεις: Τα ασυμπτωτικά τυπικά σφάλματα βρίσκονται σε παρενθέσεις. Τα * (**, και ***) δηλώνουν στατιστική σημαντικότητα σε 1% (5%, και 10%) επίπεδο σημαντικότητας.

Η ερμηνεία των συντελεστών ποσοτήτων και συνολικής κατανάλωσης παρέχεται μέσω των αντίστροφων ελαστικοτήτων κλίμακας κατανάλωσης (consumption scale flexibilities) και των μη-σταθμισμένων και σταθμισμένων αντίστροφων ελαστικοτήτων ζήτησης (uncompensated flexibilities και compensated flexibilities).

Στην παρούσα μελέτη, χρησιμοποιήθηκε η διαδικασία που προτάθηκε από τους Dong, Gould και Kaiser (2004) για την προσομοίωση αυτών των αντίστροφων ελαστικοτήτων και τα αποτελέσματα παρουσιάζονται στους Πίνακες 2, 3 και 4. Οι αντίστροφες ελαστικότητες κλίμακας κατανάλωσης μετρούν τη μεταβολή στην ομαλοποιημένη τιμή ενός αγαθού (δηλαδή, στην οριακή αξιολόγηση ενός αγαθού από τον καταναλωτή), η οποία οφείλεται σε μία ισο-ποσοστιαία αύξηση των ποσοτήτων του διανύσματος αγαθών. Ανάλογα με το αν παίρνουν τιμές μεγαλύτερες ή μικρότερες του –1, οι αντίστροφες ελαστικότητες κλίμακας κατανάλωσης χρησιμοποιούνται για την ταξινόμηση των αγαθών ως αγαθά πολυτελείας ή ως αναγκαία αγαθά, αντίστοιχα. Όπως φαίνεται στον Πίνακα 2, το βιολογικό γάλα & γιαούρτι μπορούν να ταξινομηθούν ως αγαθά πολυτελείας, ενώ οι προσομοιωμένες αντίστροφες ελαστικότητες κλίμακας κατανάλωσης για τις υπόλοιπες ομάδες αγαθών είναι πολύ κοντά στο –1.

Πίνακας 2. Προσομοιωμένες Αντίστροφες Ελαστικοτήτες Κλίμακας Κατανάλωσης.

Αγαθά	Ελαστικοτήτες Κλίμακας Κατανάλωσης
Βιολογικό Γάλα & Γιαούρτι	-0,2292
Μη-Βιολογικό Γάλα & Γιαούρτι	-0,9413
Βιολογικά Φρούτα & Λαχανικά	-0,9259
Μη-Βιολογικά Φρούτα & Λαχανικά	-1,0767

Οι προσομοιωμένες σταθμισμένες (*Antonelli*) αντίστροφες ελαστικότητες ζήτησης παρουσιάζονται στον Πίνακα 4. Οι σταθμισμένες αντίστροφες ελαστικότητες ζήτησης μετρούν τη μεταβολή στην αξιολόγηση ενός αγαθού *i* από τον καταναλωτή έπειτα από μια μεταβολή στην ποσότητα του αγαθού *j*, η οποία απαιτείται προκειμένου αυτός να παραμείνει στην αρχική καμπύλη αδιαφορίας. Όμως, όπως τονίζουν οι Barten και Bettendorf (1989), οι σταθμισμένες αντίστροφες ελαστικότητες ζήτησης αποτελούν ατελή μέτρα των αλληλοσυσχετίσεων μεταξύ των αγαθών καθώς το γεγονός ότι η μήτρα *Antonelli* πρέπει να είναι αρνητικά ημι-ορισμένη, σε συνδυασμό με τον περιορισμό της ομογένειας, οδηγούν σε κυριαρχία της συμπληρωματικότητας των αγαθών στη μήτρα *Antonelli* (δηλαδή, κυριαρχία των θετικών σταυροειδών αποτελεσμάτων). Στην παρούσα μελέτη, οι σταυροειδείς σταθμισμένες αντίστροφες ελαστικότητες ζήτησης είναι στην πλειοψηφία τους θετικές, γεγονός που υποδεικνύει ότι τα αγαθά στο υπόδειγμά μας είναι *καθαρά συμπληρωματικά* (*net q-complements*). Για τον λόγο αυτό, δεν θα εξετάσουμε τις σταθμισμένες αντίστροφες ελαστικότητες ζήτησης, αλλά θα προχωρήσουμε στην ανάλυση των μη-σταθμισμένων αντίστροφων ελαστικοτήτων ζήτησης.

Πίνακας 3. Προσομοιωμένες Μη-Σταθμισμένες Αντίστροφες Ελαστικοτήτες Ζήτησης.

Τιμές \ Ποσότητες	Βιολογικό Γάλα & Γιαούρτι	Μη-Βιολογικό Γάλα & Γιαούρτι	Βιολογικά Φρούτα & Λαχανικά	Μη-Βιολογικά Φρούτα & Λαχανικά
Βιολογικό Γάλα & Γιαούρτι	-0,2219	-0,0614	0,0270	0,0271
Μη-Βιολογικό Γάλα & Γιαούρτι	-0,0264	-0,5925	-0,0263	-0,2961
Βιολογικά Φρούτα & Λαχανικά	-0,0250	-0,1537	-0,3410	-0,4062
Μη-Βιολογικά Φρούτα & Λαχανικά	-0,0684	-0,1291	-0,0325	-0,8467

Πίνακας 4. Προσομοιωμένες Σταθμισμένες Αντίστροφες Ελαστικοτήτες Ζήτησης.

Τιμές \ Ποσότητες	Βιολογικό Γάλα & Γιαούρτι	Μη-Βιολογικό Γάλα & Γιαούρτι	Βιολογικά Φρούτα & Λαχανικά	Μη-Βιολογικά Φρούτα & Λαχανικά
Βιολογικό Γάλα & Γιαούρτι	-0,2119	-0,0023	0,0351	0,1792
Μη-Βιολογικό Γάλα & Γιαούρτι	0,0147	-0,3498	0,0066	0,3285
Βιολογικά Φρούτα & Λαχανικά	0,0154	0,0851	-0,3086	0,2081
Μη-Βιολογικά Φρούτα & Λαχανικά	-0,0214	0,1485	0,0051	-0,1323

Όπως φαίνεται στον Πίνακα 3, όλες οι ίδιες μη-σταθμισμένες αντίστροφες ελαστικότητες ζήτησης είναι αρνητικές, δηλαδή κάθε αγαθό είναι υποκατάστατο του εαυτού του, και μικρότερες της μονάδας σε απόλυτες τιμές. Μικρές αντιδράσεις της ομαλοποιημένης τιμής ενός αγαθού σε μεταβολές της ποσότητας αυτού υποδηλώνουν ότι, σε όρους ενός συνήθους συστήματος ζήτησης (direct demand system), τα αγαθά στο διάνυσμα κατανάλωσης είναι ελαστικά ως προς τις τιμές. Μία καλή αιτία για το εύρημα αυτό είναι ότι τα βιολογικά και μη-βιολογικά αγαθά αποτελούν πολύ καλά υποκατάστατα του ενός για το άλλο και, συνεπώς, η ζήτηση γι' αυτά είναι αρκετά ελαστική. Ειδικότερα, τα βιολογικά αγαθά στο σύστημά μας είναι λιγότερο ελαστικά ως προς τις ποσότητες και, άρα, περισσότερο ελαστικά ως προς τις τιμές από τα μη-βιολογικά αντίστοιχά τους. Αυτό ήταν αναμενόμενο, αφού τα πρώτα είναι περισσότερο ακριβά από τα τελευταία. Επιπλέον, κάθε μία από τις σταυροειδείς μη-σταθμισμένες αντίστροφες ελαστικότητες ζήτησης είναι μικρότερη (σε απόλυτες τιμές) από την αντίστοιχη ίδια σταθμισμένη αντίστροφη ελαστικότητα ζήτησης. Αυτό υποδηλώνει ότι αυξήσεις στην κατανάλωση του αγαθού i επηρεάζουν κυρίως την ομαλοποιημένη τιμή (δηλαδή, την αξιολόγηση του καταναλωτή) του ίδιου του αγαθού i. Έτσι, οι ομαλοποιημένες τιμές των βιολογικών αγαθών, και όχι εκείνες των μη-βιολογικών αντίστοιχών τους, είναι εκείνες που διαδραματίζουν το σημαντικότερο ρόλο στο να δώσουν κίνητρο στους καταναλωτές να αυξήσουν την κατανάλωση των βιολογικών αγαθών. Όσον αφορά στα αποτελέσματα υποκατάστασης, το πρόσημο των σταυροειδών μη-σταθμισμένων αντίστροφων ελαστικοτήτων δείχνει ότι το βιολογικό και μη-βιολογικό γάλα & γιαούρτι, καθώς και τα βιολογικά και μη-βιολογικά φρούτα & λαχανικά είναι *ακαθάριστα υποκατάστατα αγαθά* (*gross q-substitutes*), όπως ήταν αναμενόμενο. Τέλος, τα αποτελέσματά μας δείχνουν ότι το μη-βιολογικό γάλα & γιαούρτι και οι δύο κατηγορίες φρούτων & λαχανικών είναι ακαθάριστα υποκατάστατα αγαθά, ενώ η σχέση που διέπει το βιολογικό γάλα & γιαούρτι και τις δύο κατηγορίες φρούτων & λαχανικών είναι ακαθόριστη.

Ακολουθώντας τους Palmquist (1988) και Kim (1997), μπορούμε να αντικαταστήσουμε τις εκτιμημένες παραμέτρους του υποδείγματος στη συνάρτηση Ευκλείδειας απόστασης από την οποία προκύπτει το IAIDS και να μπορέσουμε να υπολογίσουμε ακριβή μέτρα των μεταβολών στην ευημερία που οφείλονται σε μεταβολές των καταναλισκόμενων ποσοτήτων. Για τους σκοπούς της παρούσας ανάλυσης, είναι περισσότερο ενδιαφέρον να εξεταστεί η συμπεριφορά των νοικοκυριών του δείγματος που έχουν δηλώσει θετική κατανάλωση για τα μη-βιολογικά αγαθά και μηδενική κατανάλωση για τα βιολογικά και να αναλυθούν οι μεταβολές στην ευημερία αυτών των νοικοκυριών όταν γίνεται υποκατάσταση ενός μέρους των ποσοτήτων των μη-βιολογικών αγαθών με βιολογικά. Στον Πίνακα 5 παρουσιάζονται οι μέσες τιμές της αποζημιωμένης μεταβολής (compensating variation – CV) και της ισοδύναμης μεταβολής (equivalent variation – EV) έπειτα από υποκατάσταση του 5%, 10% και 15% του καταναλισκόμενου μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό. Σύμφωνα με τον πίνακα αυτό, οι μέσες τιμές των CV και EV για τα 723 νοικοκυριά του υπό-δείγματος (sub-sample) είναι θετικές, δείχνοντας ότι η θέση των νοικοκυριών αυτών χειροτερεύει μετά από υποκατάσταση ενός μέρους του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό.

Πίνακας 5. Αποζημιωμένη Μεταβολή (CV) και Ισοδύναμη Μεταβολή (EV) για Νοικοκυριά που Δηλώνουν Θετική Κατανάλωση Μη-Βιολογικού Γάλακτος & Γιαουρτιού και Μηδενική Κατανάλωση Βιολογικού Γάλακτος & Γιαουρτιού.

Μεταβολές Ποσοτήτων	Ομάδες Νοικοκυριών	CV	EV
Υποκατάσταση του 5% του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό	Ανώτερες Κ. Τάξεις	3,81	1,90
	Κατώτερες Κ. Τάξεις	2,82	1,50
		(2,1134)*	(1,9983)*
	Χωρίς παιδιά	3,42	1,72
	Με παιδιά	3,31	1,75
		(0,2397)	(-0,1062)
	Όλα τα νοικοκυριά	3,38	1,73
Υποκατάσταση του 10% του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό	Ανώτερες Κ. Τάξεις	1,40	0,81
	Κατώτερες Κ. Τάξεις	0,96	0,62
		(1,5393)	(0,9539)
	Χωρίς παιδιά	1,22	0,70
	Με παιδιά	1,19	0,77
		(0,1100)	(-0,3092)
	Όλα τα νοικοκυριά	1,21	0,73
Υποκατάσταση του 15% του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό	Ανώτερες Κ. Τάξεις	0,96	0,67
	Κατώτερες Κ. Τάξεις	0,72	0,57
		(0,7972)	(0,3315)
	Χωρίς παιδιά	0,86	0,60
	Με παιδιά	0,85	0,67
		(0,0269)	(-0,2181)
	Όλα τα νοικοκυριά	0,85	0,63

Σημειώσεις: Οι μέσες τιμές των CV και EV μετρώνται σε Βρετανικές λίρες. Αυτό το υπό-δείγμα αποτελείται από 723 νοικοκυριά (το πλήρες δείγμα αποτελείται από 1155 νοικοκυριά). Η ομάδα των νοικοκυριών σε ανώτερες (κατώτερες) κοινωνικές τάξεις αποτελείται από 411 (312) νοικοκυριά. Η ομάδα των νοικοκυριών χωρίς παιδιά (με 1 έως 5 παιδιά) αποτελείται από 476 (247) νοικοκυριά. Οι στατιστικές-*t* για τη διαφορά στα CV και EV μεταξύ ομάδων νοικοκυριών βρίσκονται σε παρενθέσεις και ο αστερίσκος (*) δηλώνει στατιστική σημαντικότητα σε επίπεδο σημαντικότητας 5%.

Όμως, καθώς αυξάνεται το ποσοστό του μη-βιολογικού γάλακτος & γιαουρτιού που υποκαθίσταται με βιολογικό, οι μέσες τιμές των CV και EV μειώνονται, γεγονός που υποδεικνύει ότι η θέση των νοικοκυριών γίνεται ολοένα και λιγότερο κακή. Τα αποτελέσματα στον Πίνακα 5 είναι επίσης χωρισμένα σύμφωνα με ομάδες νοικοκυριών: νοικοκυριά σε ανώτερες κοινωνικές τάξεις, νοικοκυριά σε κατώτερες κοινωνικές τάξεις, νοικοκυριά χωρίς παιδιά, και νοικοκυριά με παιδιά. Η υπόθεση ότι η διαφορά μεταξύ των μέσων τιμών των CV (EV) για τα νοικοκυριά σε ανώτερες και κατώτερες κοινωνικές τάξεις ελέγχθηκε, και δεν απορρίφθηκε στην περίπτωση της υποκατάστασης του 5% του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό.[52] Έτσι, στην περίπτωση της υποκατάστασης του 5% του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό, τα νοικοκυριά που ανήκουν σε διαφορετικές κοινωνικές τάξεις επηρεάζονται με διαφορετικό τρόπο από αυτές τις μεταβολές στις καταναλισκόμενες ποσότητες, με τα νοικοκυριά στις ανώτερες κοινωνικές τάξεις να χειροτερεύουν τη θέση τους περισσότερο από ότι τα νοικοκυριά στις κατώτερες κοινωνικές θέσεις. Παρόμοιοι έλεγχοι διεξήχθησαν και για τις ομάδες νοικοκυριών με και χωρίς παιδιά. Οι έλεγχοι αυτοί δίνουν ενδείξεις ότι η υποκατάσταση ενός μέρους του μη-βιολογικού γάλακτος & γιαουρτιού με βιολογικό δεν επηρεάζει με διαφορετικό τρόπο τις μέσες τιμές των CV και EV για τα νοικοκυριά αυτά.

Παρόμοια ανάλυση έγινε και για τη συμπεριφορά των νοικοκυριών ως προς τα φρούτα & λαχανικά. Στον Πίνακα 6 παρουσιάζονται οι μέσες τιμές των CV και EV για τα νοικοκυριά που δήλωσαν θετική κατανάλωση μη-βιολογικών φρούτων & λαχανικών και μηδενική κατανάλωση βιολογικών φρούτων & λαχανικών (517 νοικοκυριά). Οι μέσες τιμές των CV και EV για τα νοικοκυριά αυτού του υπό-

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[52] Η υπόθεση ότι η διαφορά μεταξύ των μέσων τιμών των CV για τα νοικοκυριά σε διαφορετικές ομάδες είναι μηδενική, ελέγχθηκε με τη χρήση της στατιστικής

$$\left(\overline{CV}_1 - \overline{CV}_2\right) \Big/ \sqrt{s_1^2/T_1 + s_2^2/T_2}\ ,$$

όπου $\overline{CV}_1$ και $\overline{CV}_2$ είναι τα μέσα CV για τις ομάδες νοικοκυριών 1 και 2, και s_1 και s_2 είναι οι τυπικές αποκλίσεις των CV των ομάδων νοικοκυριών 1 και 2. Η στατιστική αυτή ακολουθεί την *t*-κατανομή με $v = T_1 + T_2 - 2$ βαθμούς ελευθερίας, όπου T_1 και T_2 είναι ο αριθμός των νοικοκυριών στις ομάδες 1 και 2, αντίστοιχα. Με τον ίδιο τρόπο γίνεται ο έλεγχος για τα EV.

δείγματος είναι θετικές για την περίπτωση της υποκατάστασης του 5% της ποσότητας των μη-βιολογικών φρούτων & λαχανικών με βιολογικά, υποδεικνύοντας ότι η μεταβολή αυτή στις καταναλισκόμενες ποσότητες χειροτερεύει τη θέση των

Πίνακας 6. Αποζημιωμένη Μεταβολή (CV) και Ισοδύναμη Μεταβολή (EV) για Νοικοκυριά που Δηλώνουν Θετική Κατανάλωση Μη-Βιολογικών Φρούτων & Λαχανικών και Μηδενική Κατανάλωση Βιολογικών Φρούτων & Λαχανικών.

Μεταβολές Ποσοτήτων	Ομάδες Νοικοκυριών	CV	EV
Υποκατάσταση του 5%	Ανώτερες Κ. Τάξεις	1,57	1,22
των μη-βιολογικών	Κατώτερες Κ. Τάξεις	1,96	1,56
φρούτων & λαχανικών		(-1,7534)*	(-1,6777)*
με βιολογικά	Χωρίς παιδιά	1,75	1,38
	Με παιδιά	1,77	1,39
		(-0,0820)	(-0,0443)
	Όλα τα νοικοκυριά	1,75	1,38
Υποκατάσταση του 5%	Ανώτερες Κ. Τάξεις	-2,61	-3,15
των μη-βιολογικών	Κατώτερες Κ. Τάξεις	-2,08	-2,54
φρούτων & λαχανικών		(-1,6848)*	(-1,6297)
με βιολογικά	Χωρίς παιδιά	-2,38	-2,89
	Με παιδιά	-2,32	-2,82
		(-0,1834)	(-0,1624)
	Όλα τα νοικοκυριά	-2,36	-2,87
Υποκατάσταση του 5%	Ανώτερες Κ. Τάξεις	-4,78	-6,07
των μη-βιολογικών	Κατώτερες Κ. Τάξεις	-4,17	-5,26
φρούτων & λαχανικών		(-1,5755)	(-1,5534)
με βιολογικά	Χωρίς παιδιά	-4,52	-5,73
	Με παιδιά	-4,44	-5,62
		(-0,2009)	(-0,1898)
	Όλα τα νοικοκυριά	-4,50	-5,69

Σημειώσεις: Οι μέσες τιμές των CV και EV μετρώνται σε Βρετανικές λίρες. Αυτό το υπόδείγμα αποτελείται από 754 νοικοκυριά (το πλήρες δείγμα αποτελείται από 1155 νοικοκυριά). Η ομάδα των νοικοκυριών σε ανώτερες (κατώτερες) κοινωνικές τάξεις αποτελείται από 406 (348) νοικοκυριά. Η ομάδα των νοικοκυριών χωρίς παιδιά (με 1 έως 6 παιδιά) αποτελείται από 517 (237) νοικοκυριά. Οι στατιστικές-t για τη διαφορά στα CV και EV μεταξύ ομάδων νοικοκυριών βρίσκονται σε παρενθέσεις και ο αστερίσκος (*) δηλώνει στατιστική σημαντικότητα σε επίπεδο σημαντικότητας 10%.

νοικοκυριών. Όμως, η θέση των νοικοκυριών γίνεται καλύτερη στις περιπτώσεις υποκατάστασης του 10% και 15% της ποσότητας των μη-βιολογικών φρούτων & λαχανικών με βιολογικά. Η υπόθεση της μηδενικής διαφοράς στις μέσες τιμές των

CV (EV) για τα νοικοκυριά σε ανώτερες και κατώτερες κοινωνικές τάξεις ελέγχθηκε και απορρίφθηκε για τις περιπτώσεις υποκατάστασης του 5% και 10% των μη-βιολογικών φρούτων & λαχανικών με βιολογικά. Παρόμοιοι έλεγχοι για τις ομάδες νοικοκυριών με και χωρίς παιδιά υποδεικνύουν ότι η υποκατάσταση ενός μέρους της ποσότητας των μη-βιολογικών φρούτων & λαχανικών με βιολογικά δεν επηρεάζει με διαφορετικό τρόπο τα CV και EV αυτών των ομάδων νοικοκυριών.

Το τρίτο ζήτημα το οποίο διαπραγματεύεται η παρούσα διδακτορική διατριβή αφορά στην ανάπτυξη ενός θεωρητικού υποδείγματος για την αποτελεσματικότητα στην κατανάλωση, καθώς και στην εμπειρική μέτρηση αυτής. Η συνήθης ανάλυση ζήτησης καταναλωτή υποθέτει εκ των προτέρων ότι οι καταναλωτές συμπεριφέρονται βέλτιστα, δηλαδή, ότι επιτυγχάνουν να επιλέξουν ένα διάνυσμα αγαθών που θα τους δώσει τη μέγιστη χρησιμότητα, ή ότι επιτυγχάνουν στο να επιλέξουν τις ελάχιστες ποσότητες που απαιτούνται για την επίτευξη ενός δεδομένου επιπέδου χρησιμότητας. Όμως, η υπόθεση της βέλτιστης συμπεριφοράς είναι περιοριστική συγκρινόμενη με την πραγματική συμπεριφορά των καταναλωτών. Όπως τονίζει ο Afriat (1988, σελ. 252), «[η] συνήθης θεωρία του καταναλωτή βασίζεται στην χρησιμότητα – και στη μη-αμφισβητούμενη αποτελεσματικότητα. Ακόμα και όταν η χρησιμότητα είναι δεδομένη, η τέλεια αποτελεσματικότητα μοιάζει να είναι μία υπερβολική απαίτηση. Οι γνωστές αστάθειες των πραγματικών καταναλωτών κάνουν αυτή την έλλειψη ανοχής ακατάλληλη.» Είναι λοιπόν περισσότερο λογικό το να υποθέσουμε ότι οι καταναλωτές μπορεί να μην συμπεριφέρονται βέλτιστα και να υιοθετήσουμε θεωρητικά και εμπειρικά υποδείγματα τα οποία μπορούν να λάβουν υπόψη οποιαδήποτε απομάκρυνση από το βέλτιστο, δηλαδή, την αναποτελεσματικότητα, και να επιτρέψουν την μέτρησή της.

Η σημασία της μελέτης της αναποτελεσματικότητας στην κατανάλωση έγκειται όχι μόνο στο ότι η βέλτιστη συμπεριφορά, και άρα η αποτελεσματικότητα, είναι μία περιοριστική υπόθεση για την πραγματική συμπεριφορά των καταναλωτών. Έγκειται, επίσης, και στο ότι η μη-βέλτιστη συμπεριφορά των καταναλωτών έχει έναν αρνητικό αντίκτυπο στα επίπεδα ευημερίας. Συγκεκριμένα, έχει έναν αρνητικό αντίκτυπο στα επίπεδα ευημερίας των ίδιων των καταναλωτών σε όρους χρηματικών πόρων που σπαταλήθηκαν ενώ θα μπορούσαν να είχαν χρησιμοποιηθεί για την ικανοποίηση άλλων αναγκών. Επιπρόσθετα, η υπερκατανάλωση οδηγεί σε αυξημένη παραγωγή, η οποία με τη σειρά της παρακινεί την υπερκατανάλωση μέσω, π.χ., της διαφήμισης. Αυτός ο κύκλος συνεπάγεται υπερβολική χρήση των φυσικών πόρων και/ή εσφαλμένη κατανομή αυτών στην παραγωγή των αγαθών, αυξημένες ποσότητες αποβλήτων τόσο από την κατανάλωση όσο και από την παραγωγή, και έναν αρνητικό αντίκτυπο στην κοινωνική ευημερία.

Η υπόθεση της μη-βέλτιστης συμπεριφοράς του καταναλωτή μπορεί να υποστηριχτεί στην περίπτωση αγαθών όπως τα ευπαθή τρόφιμα και τα ευπαθή προϊόντα κτηνοτροφίας, αλιείας και γεωργίας. Σε αυτές τις περιπτώσεις, οι καταναλωτές μπορεί να είναι αναποτελεσματικοί επειδή κάνουν ανακριβείς εκτιμήσεις του όγκου των ποσοτήτων των αγαθών και του συνδυασμού των ποσοτήτων που τους είναι επαρκείς για την επίτευξη ενός επιθυμητού επιπέδου χρησιμότητας: όταν οι καταναλωτές επιλέγουν ένα διάνυσμα αγαθών, το επιλέγουν βάσει των εκτιμήσεών τους για το ποιος συνδυασμός ποσοτήτων των αγαθών είναι ο κατάλληλος για τις ανάγκες τους. Επίσης, οι καταναλωτές μπορεί να είναι αναποτελεσματικοί επειδή δεν μπορούν να προβλέψουν το μέλλον επακριβώς: αφού η καθημερινή ζωή των ατόμων δεν μπορεί να προγραμματιστεί έως την τελευταία λεπτομέρεια, είναι αναμενόμενο ένα μέρος των αγορασθέντων αγαθών να μην καταναλωθεί και – ιδίως στην περίπτωση ευπαθών τροφίμων τα οποία δεν μπορούν να αποθηκευθούν – να πεταχτεί. Ακόμη, οι καταναλωτές μπορεί να συνεχίζουν να παίρνουν μη-βέλτιστες αποφάσεις και να είναι αναποτελεσματικοί επειδή μπορεί να έχουν ένα πρότυπο για τις αγορές και την κατανάλωση, το οποίο να ακολουθούν χωρίς να το επανεξετάζουν. Ή θα μπορούσε να είναι η έλλειψη πληροφόρησης, συναίσθησης και υπευθυνότητας εκ μέρους των καταναλωτών σχετικά με το πλήρες κοινωνικό κόστος των αποφάσεών τους για κατανάλωση, που οδηγεί σε υπερβάλλουσες αγορές και δαπάνες και σε αναποτελεσματικότητα στην κατανάλωση. Έτσι, οι καταναλωτές μπορεί να αγοράσουν ένα διάνυσμα αγαθών το οποίο αργότερα θα αποδειχθεί μη-βέλτιστο: θα μπορούσαν να είχαν αγοράσει λιγότερες ποσότητες από όλα τα αγαθά (*αναποτελεσματικότητα ως προς τα αγαθά*) μειώνοντας έτσι τις δαπάνες τους, και/ή θα μπορούσαν να είχαν ανακατανείμει τις δαπάνες τους επιλέγοντας ένα διαφορετικό συνδυασμό ποσοτήτων των αγαθών (*διανεμητική αναποτελεσματικότητα*) μειώνοντας έτσι τις δαπάνες τους ακόμα περισσότερο. Η μείωση της αναποτελεσματικότητας του καταναλωτή και ο μετριασμός του αρνητικού αντίκτυπού της πάνω στα επίπεδα ευημερίας θα μπορούσε, για παράδειγμα, να επιτευχθεί μέσω της διαφήμισης. Αν η διαφήμιση παίζει έναν σημαντικό ρόλο στη δημιουργία και/ή στη διατήρηση της μη-βέλτιστης συμπεριφοράς του καταναλωτή, τότε η διαφήμιση ίσως να μπορούσε να χρησιμοποιηθεί ως ένα μέσο ενημέρωσης και έναρξης αλλαγών στα πρότυπα αγορών και κατανάλωσης.

Στα πλαίσια αυτά, ο στόχος του Κεφαλαίου 4 είναι να προτείνει ένα θεωρητικό πλαίσιο για την ανάλυση της αποτελεσματικότητας του καταναλωτή στο χώρο τιμών-ποσοτήτων. Το θεωρητικό υπόδειγμα το οποίο αναπτύσσεται βασίζεται στην απλή παρατήρηση ότι οι προτιμήσεις του καταναλωτή είθισται να ορίζονται σε σχέση με τα επίπεδα κατανάλωσης και ότι δεν γίνεται διάκριση μεταξύ των ποσοτήτων των αγαθών που αγοράζονται και αυτών που καταναλίσκονται. Δηλαδή, υπονοείται η υπόθεση ότι οι ποσότητες που αγοράζονται και οι ποσότητες που καταναλίσκονται είναι οι ίδιες. Όμως, αν κάνουμε την υπόθεση ότι οι καταναλωτές μπορούν να πετάξουν οποιεσδήποτε μη-επιθυμητές ποσότητες των αγαθών που έχουν αγοράσει, τότε μπορούμε να ορίσουμε ένα μέτρο της αποτελεσματικότητας των καταναλωτών στην προσπάθειά τους να ελαχιστοποιήσουν τη δαπάνη τους για τα αγαθά. Οι ως τώρα προσπάθειες μελέτης της αποτελεσματικότητας του καταναλωτή στον χώρο τιμών-ποσοτήτων έχουν βασιστεί σε επιχειρήματα αποκαλυπτόμενων προτιμήσεων ή σε *money-metric* συναρτήσεις χρησιμότητας για να κατασκευάσουν μη-παραμετρικούς ή παραμετρικούς δείκτες αποτελεσματικότητας (Afriat, 1967 και 1988, Varian 1982, 1983, 1985 και 1990). Το σημείο εστίασης αυτών των μελετών, όμως, είναι η εξέταση του κατά πόσο τα υποδείγματα βελτιστοποίησης ταιριάζουν στα πραγματικά στατιστικά δεδομένα (goodness-of-fit), μέσω της μέτρησης της απομάκρυνσης από τη βελτιστοποίηση. Επιπλέον, αυτό που υπονοείται από αυτά τα υποδείγματα είναι ότι η αναποτελεσματικότητα συμβαίνει επειδή ένα μέρος του προϋπολογισμού του καταναλωτή είναι που σπαταλιέται, και όχι ένα μέρος των αγορασθέντων ποσοτήτων. Όμως, η υπόθεση ότι οι καταναλωτές μπορεί να σπαταλήσουν/πετάξουν ένα μέρος των ποσοτήτων των αγορασθέντων αγαθών είναι εκείνη που επιτρέπει την κατασκευή του μέτρου της *αναποτελεσματικότητας ως προς τα αγαθά*. Επιπρόσθετα, αφού τα υποδείγματα αυτά δεν επιτρέπουν τη δυνατότητα σε ένα παρατηρούμενο διάνυσμα αγαθών να είναι και *αναποτελεσματικό ως προς τα αγαθά*, δεν γίνεται καμία διάκριση μεταξύ αυτών που ορίζονται στην παρούσα μελέτη ως *διανεμητική αποτελεσματικότητα* και *αποτελεσματικότητα ως προς τη δαπάνη* ή *συνολική αποτελεσματικότητα*. Ως αποτέλεσμα, τα υπάρχοντα υποδείγματα μελέτης της αποτελεσματικότητας στην κατανάλωση μπορεί να αποδώσουν στους καταναλωτές ένα υψηλότερο επίπεδο αποτελεσματικότητας από αυτό που θα έπρεπε.

Η ανάλυση μάς γίνεται υπό το πρίσμα της ελαχιστοποίησης της δαπάνης του καταναλωτή, και το σημείο εκκίνησης είναι η υπόθεση ότι ο στόχος του καταναλωτή είναι να επιλέξει ένα εφικτό διάνυσμα αγαθών για να πετύχει τουλάχιστον ένα δεδομένο επίπεδο χρησιμότητας. Επίσης, υποθέτοντας ότι ο καταναλωτής δεν χρειάζεται να κάνει χρήση όλων των αγορασθέντων ποσοτήτων των αγαθών και ότι μπορεί να πετάξει οποιεσδήποτε μη-επιθυμητές ποσότητές τους, οι αγορασθείσες ποσότητες των αγαθών μπορούν κάλλιστα να είναι υψηλότερες από αυτές που χρειάζονται για την επίτευξη του δεδομένου επιπέδου χρησιμότητας, και ο καταναλωτής μπορεί κάλλιστα να έχει επιλέξει έναν αναποτελεσματικό τρόπο για να επιτύχει αυτό το δεδομένο επίπεδο χρησιμότητας. Αυτός ο τύπος αποτελεσματικότητας είναι αυτό που ορίζουμε εδώ ως αποτελεσματικότητα ως προς τα αγαθά. Μία άλλη μορφή αναποτελεσματικότητας είναι αυτό που ορίζουμε ως αποτελεσματικότητα ως προς τη δαπάνη, ή συνολική αποτελεσματικότητα, και την οποία περιγράφουμε ως την ικανότητα του καταναλωτή να αποφεύγει την σπατάλη χρηματικών πόρων με το να ελαχιστοποιεί το κόστος των αγορασθέντων αγαθών κατά την επίτευξη ενός επιπέδου χρησιμότητας. Μία τρίτη μορφή αποτελεσματικότητας είναι η διανεμητική αποτελεσματικότητα: υποθέτοντας ότι το διάνυσμα αγαθών το οποίο ελαχιστοποιεί τη δαπάνη και το παρατηρούμενο διάνυσμα αγαθών βρίσκονται πάνω στην ίδια καμπύλη αδιαφορίας, η διανεμητική αποτελεσματικότητα αφορά στο πόσο κοντά είναι μεταξύ τους τα δύο διανύσματα. Τέλος, δείχνουμε τη σχέση που υπάρχει μεταξύ των τριών μορφών αποτελεσματικότητας, δηλαδή, τη σχέση που περιγράφει τη διάσπαση της αποτελεσματικότητας ως προς τη δαπάνη στην αποτελεσματικότητα ως προς τα αγαθά και στη διανεμητική αποτελεσματικότητα.

Το θεωρητικό υπόδειγμα που μόλις περιγράφτηκε βασίζεται στην υπόθεση ότι οι καταναλωτές μπορούν να πετάξουν ένα μέρος των ποσοτήτων των αγορασθέντων αγαθών. Όμως, αυτή η υπόθεση δίνει τη δυνατότητα ορισμού της αναποτελεσματικότητας του καταναλωτή, όχι μόνο σε όρους ποσοτήτων και χρηματικών πόρων που σπαταλήθηκαν, αλλά και σε όρους χρησιμότητας που θα μπορούσε να επιτευχθεί αλλά εν τέλει δεν επιτεύχθηκε. Ειδικότερα, ας υποθέσουμε ότι ο στόχος του καταναλωτή είναι να πάρει την υψηλότερη χρησιμότητα που ένα διάνυσμα αγαθών μπορεί να του δώσει. Αν ο καταναλωτής δεν πετύχει αυτόν τον στόχο, δηλαδή, δεν χρησιμοποιήσει τα αγαθά τόσο αποτελεσματικά όσο θα μπορούσε, τότε η χρησιμότητα την οποία λαμβάνει μπορεί κάλλιστα να είναι μικρότερη από την δυνητικά μέγιστη. Θα χρησιμοποιήσουμε τον όρο *αποτελεσματικότητα ως προς τη χρησιμότητα* για να περιγράψουμε τη δυνατότητα του καταναλωτή να αποφεύγει τη σπατάλη χρησιμότητας με το να λαμβάνει το υψηλότερο επίπεδο χρησιμότητας που ένα διάνυσμα αγορασθέντων αγαθών μπορεί να του αποδώσει. Αυτή η μορφή αποτελεσματικότητας μπορεί να μελετηθεί όχι μόνο στην περίπτωση αγαθών των οποίων τις ποσότητες ο καταναλωτής μπορεί να πετάξει. Αφού το σημείο εκκίνησης είναι ο στόχος του καταναλωτή να λάβει τη μέγιστη χρησιμότητα που ένα διάνυσμα αγαθών μπορεί να του αποδώσει, η έννοια της αποτελεσματικότητας ως προς τη χρησιμότητα μπορεί να χρησιμοποιηθεί για τη μελέτη και άλλων αγαθών πέρα από τα τρόφιμα. Για παράδειγμα, θα μπορούσε η έλλειψη πληροφόρησης για τα χαρακτηριστικά των αγαθών, για τον τρόπο με τον οποίο θα μπορούσαν και/ή θα έπρεπε να χρησιμοποιηθούν, κτλ., να είναι εκείνη που οδηγεί τον καταναλωτή να λάβει από ένα διάνυσμα αγαθών ένα επίπεδο χρησιμότητας το οποίο είναι μικρότερο από αυτό που τα αγαθά θα του επέτρεπαν. Παραδείγματα αποτελούν τα αυτοκίνητα και η έλλειψη πληροφόρησης για την «πράσινη οδήγηση» (χρήση αυτοκινήτου και συμπεριφορά στην οδήγηση που μειώνουν τις αρνητικές εξωτερικότητες που παράγει η χρήση αυτοκινήτων), τα τρόφιμα και η έλλειψη πληροφόρησης σχετικά με την προετοιμασία υγιεινών γευμάτων, κτλ. Επιπρόσθετα, ένας καταναλωτής ο οποίος αντιμετωπίζει εξωγενείς τιμές για τα αγαθά και έχει ένα δεδομένο προϋπολογισμό για την αγορά αγαθών συμπεριφέρεται βέλτιστα εφόσον το διάνυσμα των αγαθών το οποίο αγοράζει είναι εκείνο το οποίο μεγιστοποιεί τη χρησιμότητά του. Όμως, λόγω έλλειψης πληροφόρησης για τα χαρακτηριστικά των αγαθών ή για τον τρόπο με τον οποίο θα μπορούσαν και/ή θα έπρεπε να χρησιμοποιηθούν, λόγω ξαφνικών επιθυμιών, τυχαίων γεγονότων, ή άλλων παραγόντων που μπορούν να επηρεάσουν τις αποφάσεις του καταναλωτή, ο τελευταίος μπορεί να δαπανήσει τους χρηματικούς πόρους του σε ένα μη-βέλτιστο διάνυσμα αγαθών. Αυτή η μορφή αποτελεσματικότητας είναι αυτό που ορίζουμε ως *διανεμητική αποτελεσματικότητα ως προς τη δαπάνη*, ενώ η έννοια της *συνολικής αποτελεσματικότητας ως προς τη χρησιμότητα* θα χρησιμοποιηθεί για την περιγραφή της ικανότητας του καταναλωτή να λαμβάνει το υψηλότερο επίπεδο χρησιμότητας που το βέλτιστο διάνυσμα αγαθών είναι ικανό να του αποδώσει. Τέλος, όπως και στην περίπτωση της αποτελεσματικότητας ως προς τη δαπάνη, η συνολική αποτελεσματικότητα ως προς τη χρησιμότητα μπορεί να διασπαστεί στις συνιστώσες της, δηλαδή, την αποτελεσματικότητα ως προς τη χρησιμότητα και τη διανεμητική αποτελεσματικότητα ως προς τη δαπάνη.

Το τέταρτο ζήτημα το οποίο διαπραγματεύεται η παρούσα διδακτορική διατριβή είναι εκείνο της εμπειρικής μέτρησης της αποτελεσματικότητας καταναλωτή. Η εμπειρική μας ανάλυση για την αποτελεσματικότητα του καταναλωτή επικεντρώνεται στην εμπειρική μέτρηση της αποτελεσματικότητας ως προς τα αγαθά, τη διανεμητική αποτελεσματικότητα και την αποτελεσματικότητα ως προς τη δαπάνη, ενώ η εμπειρική μέτρηση της αποτελεσματικότητας ως προς τη χρησιμότητα, της διανεμητικής αποτελεσματικότητας ως προς τη δαπάνη και της συνολικής αποτελεσματικότητας ως προς τη χρησιμότητα αφήνεται για μελλοντική έρευνα. Η εμπειρική μεθοδολογία που χρησιμοποιείται για την οικονομετρική εκτίμηση της πρώτης ομάδας δεικτών αποτελεσματικότητας έγκειται στη χρήση προσεγγίσεων που έχουν ακολουθηθεί όχι μόνο στην ανάλυση ζήτησης καταναλωτή, αλλά και σε άλλα πεδία. Ειδικότερα, ο δείκτης που προτείνουμε στο Κεφάλαιο 4 για τη μέτρηση της αποτελεσματικότητας ως προς τα αγαθά βασίζεται σε μια συνάρτηση Ευκλείδειας απόστασης η οποία αναπαριστά προτιμήσεις καταναλωτών. Επομένως, ο υπολογισμός του δείκτη αποτελεσματικότητας ως προς τα αγαθά απαιτεί γνώση της τιμής της συνάρτησης Ευκλείδειας απόστασης, η οποία μπορεί να αποκτηθεί με οικονομετρική εκτίμηση της τελευταίας. Όμως, η δυσκολία στην εκτίμηση μιας συνάρτησης Ευκλείδειας απόστασης η οποία αναπαριστά προτιμήσεις καταναλωτών έγκειται στο ότι είναι μία συνάρτηση, όχι μόνο των παρατηρούμενων ποσοτήτων των αγαθών, αλλά και του μη-παρατηρούμενου επιπέδου χρησιμότητας του καταναλωτή. Στο Κεφάλαιο 5 δείχνουμε πώς το πρόβλημα αυτό μπορεί να αντιμετωπιστεί, με το να εκτιμήσουμε μία υπερλογαριθμική συνάρτηση Ευκλείδειας απόστασης με διαστρωματικά στοιχεία χρονολογικών σειρών (panel) για τη ζήτηση νοικοκυριών για γάλα & γιαούρτι, φρούτα, και λαχανικά, και με τη χρήση δύο διαφορετικών προσεγγίσεων: με τη χρήση μιας παρατηρούμενης μεταβλητής για την προσέγγιση της χρησιμότητας (proxy), και με την αντιμετώπιση της χρησιμότητας του καταναλωτή ως ένα τυχαίο σφάλμα. Οι Lewbel και Pendakur (2006) δημιούργησαν τα Implicit Marshallian Demand συστήματα εξισώσεων, στα οποία η χρησιμότητα υποκαθίσταται από την *implicit χρησιμότητα*, η οποία είναι μία συνάρτηση παρατηρούμενων μεταβλητών. Ακολουθώντας τους Lewbel και Pendakur (2006), οι Färe, Grosskopf, Hayes, και Margaritis (2008) χρησιμοποίησαν το ετήσιο εισόδημα των νοικοκυριών για να προσεγγίσουν τη χρησιμότητα και να εκτιμήσουν και αποτιμήσουν συστήματα εξισώσεων ζήτησης τα οποία εξάγονται από συναρτήσεις δαπάνης και συναρτήσεις ωφέλειας (benefit functions). Το πλεονέκτημα αυτής της τεχνικής είναι ότι, από τη στιγμή που χρησιμοποιούνται παρατηρούμενες μεταβλητές για την προσέγγιση της χρησιμότητας, η συνάρτηση Ευκλείδειας απόστασης μπορεί να εκτιμηθεί με τη χρήση καθιερωμένων τεχνικών εκτίμησης συνόρου (frontier estimation) που χρησιμοποιούνται ευρέως στην ανάλυση αποτελεσματικότητας παραγωγού (βλ., για παράδειγμα, Kumbhakar και Lovell, 2000). Το μειονέκτημα αυτής της τεχνικής, όμως, είναι ότι εφόσον γίνεται η υπόθεση ότι το επίπεδο χρησιμότητας του καταναλωτή επηρεάζεται από ένα συνδυασμό παρατηρούμενων μεταβλητών, αμελούνται οποιοιδήποτε άλλοι παράγοντες που μπορεί να επηρεάζουν τις προτιμήσεις του καταναλωτή. Η δεύτερη προσέγγιση που χρησιμοποιούμε για την εκτίμηση της υπερλογαριθμικής συνάρτησης Ευκλείδειας απόστασης έγκειται στην αντιμετώπιση της μη-παρατηρούμενης χρησιμότητας του καταναλωτή ως ένα τυχαίο σφάλμα. Ειδικότερα, οι όροι της υπερλογαριθμικής συνάρτησης Ευκλείδειας απόστασης που αφορούν στο επίπεδο χρησιμότητας και την Ευκλείδεια απόσταση αντιμετωπίζονται ως μονόπλευρα και θετικά τυχαία σφάλματα. Αυτές οι υποθέσεις οδηγούν σε μία συνάρτηση πυκνότητας πιθανότητας για τον σύνθετο διαταρακτικό όρο, η οποία μοιάζει με το *two-tiered frontier* πλαίσιο οικονομετρικής εκτίμησης των Polachek και Yoon (1987, 1996). Η εκτιμημένη συνάρτηση Ευκλείδειας απόστασης μπορεί κατόπιν να χρησιμοποιηθεί ως δείκτης για τη μέτρηση της αναποτελεσματικότητας του καταναλωτή ως προς τα αγαθά. Όσον αφορά στον υπολογισμό του μέτρου για τη διανεμητική αποτελεσματικότητα, απαιτείται γνώση είτε της συνάρτησης δαπάνης, ή του διανύσματος αγαθών το οποίο ελαχιστοποιεί τη δαπάνη. Για τον σκοπό αυτό, ακολουθούνται τεχνικές οι οποίες χρησιμοποιούνται για τον υπολογισμό της διανεμητικής αποτελεσματικότητας του παραγωγού ως προς τις εισροές. Τέλος, το μέτρο της αποτελεσματικότητας ως προς τη δαπάνη υπολογίζεται με τη χρήση της σχέσης που προτείνουμε για τη διάσπαση της αποτελεσματικότητας ως προς τη δαπάνη σε αποτελεσματικότητα ως προς τα αγαθά και διανεμητική αποτελεσματικότητα.

Όπως παρουσιάζεται στο Κεφάλαιο 5, η εμπειρική μελέτη της αποτελεσματικότητας του καταναλωτή ως προς τα αγαθά έγινε με την εκτίμηση μιας υπερλογαριθμικής συνάρτησης Ευκλείδειας απόστασης με διαστρωματικά δεδομένα χρονολογικών σειρών. Τα στατιστικά δεδομένα αφορούν σε καταναλισκόμενες ποσότητες και δαπάνες 884 νοικοκυριών που διαμένουν Λονδίνο, από τον Ιούλιο του

2005 έως και τον Ιούνιο του 2006. Στα πλαίσια της πρώτης μεθόδου εκτίμησης της υπερλογαριθμικής συνάρτησης Ευκλείδειας απόστασης (εφεξής, proxy υπόδειγμα), χρησιμοποιήσαμε τη συνολική δαπάνη των νοικοκυριών για να προσεγγίσουμε το επίπεδο χρησιμότητας των νοικοκυριών. Στη συνέχεια, υιοθετήθηκε το υπόδειγμα των Battese και Coelli (1995), το οποίο αφορά σε οικονομετρική εκτίμηση με τη μέθοδο της μεγίστης πιθανοφάνειας με τη χρήση διαστρωματικών στοιχείων χρονολογικών σειρών, προκειμένου να εκτιμηθεί το proxy υπόδειγμα στο υπολογιστικό πακέτο FRONTIER 4.1. Στα πλαίσια της δεύτερης μεθόδου εκτίμησης της υπερλογαριθμικής συνάρτησης Ευκλείδειας απόστασης (εφεξής, two-tiered frontier υπόδειγμα), έγινε η υπόθεση ότι ο διαταρακτικός όρος κατανέμεται κανονικά, ενώ οι όροι που αφορούν στην αποτελεσματικότητα και στο επίπεδο χρησιμότητας είναι τυχαίες μεταβλητές που ακολουθούν την εκθετική κατανομή. Μετά την εξαγωγή της συνάρτησης πυκνότητας πιθανότητας αυτού του σύνθετου διαταρακτικού όρου, το two-tiered frontier υπόδειγμα εκτιμήθηκε στο υπολογιστικό πακέτο GAUSS με τη μέθοδο της μεγίστης πιθανοφάνειας. Και τα δύο υποδείγματα εκτιμήθηκαν με τους περιορισμούς της ομογένειας και της συμμετρίας, και στην πλειοψηφία τους οι παράμετροι και στα δύο υποδείγματα ήταν στατιστικά σημαντικές (25 από τις 29 παραμέτρους του proxy υποδείγματος και 19 από τις 21 παραμέτρους του two-tiered frontier υποδείγματος).

Οι εκτιμημένοι δείκτες αποτελεσματικότητας για τα δύο υποδείγματα παρουσιάζονται στους Πίνακες 7 και 8. Για το proxy υπόδειγμα, οι μέσες τιμές των δεικτών αποτελεσματικότητας ως προς τα αγαθά, διανεμητικής αποτελεσματικότητας και αποτελεσματικότητας ως προς την δαπάνη είναι 76,10%, 77,97%, και 58,78%, αντίστοιχα, κατά την περίοδο Ιούλιος 2005 – Ιούνιος 2006. Συγκεκριμένα, το 70% των νοικοκυριών του δείγματος έχει βαθμούς αποτελεσματικότητας ως προς τα αγαθά μεταξύ 70% και 80%, το 80% των νοικοκυριών έχει βαθμούς διανεμητικής αποτελεσματικότητας μεταξύ 70% και 80% και το 83% των νοικοκυριών έχει βαθμούς αποτελεσματικότητας ως προς τη δαπάνη μεταξύ 50% και 60%. Όσον αφορά στις εκτιμήσεις των δεικτών αποτελεσματικότητας για το two-tiered frontier υπόδειγμα, αυτές βρέθηκαν να έχουν υψηλότερες τιμές από ότι εκείνες του proxy υποδείγματος: οι μέσες τιμές των εκτιμημένων δεικτών αποτελεσματικότητας ως προς τα αγαθά, διανεμητικής αποτελεσματικότητας και αποτελεσματικότητας ως προς τη δαπάνη είναι 84,06%, 80,14%, και 67,22%, αντίστοιχα, για την περίοδο που καλύπτει το δείγμα μας. Ειδικότερα, η πλειοψηφία των νοικοκυριών στο δείγμα

(87%) πέτυχε βαθμούς αποτελεσματικότητας ως προς τα αγαθά μεταξύ 80% και 90%, 45% των νοικοκυριών πέτυχε βαθμούς διανεμητικής αποτελεσματικότητας μεταξύ 70% και 80% και το υπόλοιπο 55% των νοικοκυριών πέτυχε βαθμούς διανεμητικής αποτελεσματικότητας μεταξύ 80% και 90%. Τέλος, σχεδόν όλα τα νοικοκυριά του δείγματος (99%) πέτυχαν βαθμούς αποτελεσματικότητας ως προς τη δαπάνη μεταξύ 60% και 70%.

Τα εμπειρικά υποδείγματα που έχουμε χρησιμοποιήσει για την οικονομετρική εκτίμηση του θεωρητικού υποδείγματος για την αποτελεσματικότητα του καταναλωτή και, άρα, τα αποτελέσματα που εξήχθηκαν από αυτά, δεν είναι συγκρίσιμα. Για το λόγο αυτό, θα επικεντρωθούμε στην ερμηνεία των αποτελεσμάτων που εξήχθηκαν από το two-tier frontier υπόδειγμα μόνον. Η κύρια υπόθεση που έχουμε κάνει για την ανάπτυξη των δεικτών αποτελεσματικότητας που εκτιμήθηκαν είναι ότι οι καταναλωτές μπορούν να πετάξουν τις όποιες μη-επιθυμητές ποσότητες των αγαθών που αγόρασαν. Αυτό σημαίνει ότι οι αγορασθείσες ποσότητες των αγαθών και, ως εκ τούτου, η πραγματική (παρατηρούμενη) δαπάνη, μπορούν κάλλιστα να είναι υψηλότερες από αυτές που μόλις χρειάζονται για την επίτευξη ενός δεδομένου επιπέδου χρησιμότητας. Χρησιμοποιώντας τη σχέση (4.42) του Κεφαλαίου 4, η οποία παρέχει έναν ορισμό της αποτελεσματικότητας ως προς τα αγαθά ως έναν λόγο κόστους (cost ratio), το εύρημα ότι η εκτιμημένη μέση αποτελεσματικότητα ως προς τα αγαθά ήταν 84,06% κατά τη διάρκεια της περιόδου Ιούλιος 2005 – Ιούνιος 2006 υποδεικνύει ότι, κατά μέσο όρο, το 15,94% του προϋπολογισμού των νοικοκυριών σπαταλήθηκε, ή ότι, π.χ., μέσω καλύτερου προγραμματισμού των δαπανών τους, τα νοικοκυριά θα μπορούσαν να είχαν μειώσει τη συνολική τους δαπάνη κατά 15,94% και ταυτόχρονα να πετύχουν το ίδιο επίπεδο χρησιμότητας με ένα μέρος των αγορασμένων αγαθών. Η σχέση (4.43) του Κεφαλαίου 4 ορίζει το μέτρο της διανεμητικής αποτελεσματικότητας ως τον λόγο της ελάχιστης δαπάνης που απαιτείται για την επίτευξη ενός δεδομένου επιπέδου χρησιμότητας προς τη δαπάνη για το διάνυσμα αγαθών που είναι αποτελεσματικό ως προς τα αγαθά. Υπό αυτόν τον ορισμό, το εύρημα ότι η μέση διανεμητική αποτελεσματικότητα ήταν 80,14%, κατά τη χρονική διάρκεια που καλύπτει το δείγμα μας, υποδεικνύει ότι, δεδομένου ενός επιπέδου χρησιμότητας και των τιμών των αγαθών, τα νοικοκυριά θα μπορούσαν να είχαν μειώσει τη συνολική τους δαπάνη κατά 19,86% αν επέλεγαν ένα διαφορετικό συνδυασμό ποσοτήτων των αγαθών.

Πίνακας 7. Κατανομή Συχνότητας για την Αποτελεσματικότητα ως προς τα Αγαθά, τη Διανεμητική Αποτελεσματικότητα και την Αποτελεσματικότητα ως προς τη Δαπάνη, για το Proxy Υπόδειγμα.

	2005						2006					
	Ιούλ.	Αύγ.	Σεπ.	Οκτ.	Νοέ.	Δεκ.	Ιαν.	Φεβ.	Μαρ.	Απρ.	Μαϊ.	Ιούν.
Αποτελεσματικότητα ως προς τα Αγαθά												
<0,4	0	0	0	0	0	0	0	0	0	0	0	0
0,4-0,5	1	1	2	8	2	1	2	0	2	2	1	0
0,5-0,6	9	15	22	48	39	42	35	32	22	18	25	27
0,6-0,7	31	126	211	295	246	229	251	226	159	145	117	116
0,7-0,8	93	288	405	428	458	482	470	490	504	445	422	358
0,8-0,9	225	313	202	97	131	127	126	136	195	270	314	368
>0,9	525	141	42	8	8	3	0	0	2	4	5	15
Μέσος	0,89	0,8	0,75	0,72	0,73	0,73	0,73	0,73	0,75	0,76	0,77	0,78
Διανεμητική Αποτελεσματικότητα												
<0,4	0	0	0	0	0	0	0	0	0	0	0	0
0,4-0,5	0	0	0	0	0	0	0	0	0	0	0	0
0,5-0,6	9	14	8	13	8	8	7	11	9	9	5	8
0,6-0,7	137	130	125	140	137	134	134	130	125	122	115	144
0,7-0,8	383	391	358	391	428	397	394	421	429	438	390	417
0,8-0,9	266	272	308	266	250	267	271	256	258	246	298	244
>0,9	89	77	85	74	61	78	78	66	63	69	76	71
Μέσος	0,78	0,78	0,79	0,78	0,77	0,78	0,78	0,78	0,78	0,78	0,79	0,78
Αποτελεσματικότητα ως προς τη Δαπάνη												
<0,4	0	0	0	0	0	0	0	0	0	0	0	0
0,4-0,5	50	54	60	19	30	30	35	31	25	43	39	34
0,5-0,6	428	503	456	484	518	464	492	530	476	392	473	465
0,6-0,7	403	327	368	381	336	390	357	323	383	449	372	385
0,7-0,8	3	0	0	0	0	0	0	0	0	0	0	0
0,8-0,9	0	0	0	0	0	0	0	0	0	0	0	0
>0,9	0	0	0	0	0	0	0	0	0	0	0	0
Μέσος	0,59	0,58	0,58	0,59	0,59	0,59	0,59	0,59	0,59	0,59	0,59	0,59

Επιπλέον, αφού η αποτελεσματικότητα ως προς τη δαπάνη ορίζεται ως ο λόγος της ελάχιστης δαπάνης που απαιτείται για την επίτευξη ενός δεδομένου επιπέδου χρησιμότητας προς την πραγματική (παρατηρούμενη) δαπάνη, ένας μέσος βαθμός αποτελεσματικότητας ως προς τη δαπάνη ύψους 67,22% υποδεικνύει ότι, κατά μέσο όρο, το 32,78% του προϋπολογισμού των νοικοκυριών σπαταλήθηκε λόγω της παρουσίας της αναποτελεσματικότητας ως προς τα αγαθά και της διανεμητικής αναποτελεσματικότητας.

Πίνακας 8. Κατανομή Συχνότητας για την Αποτελεσματικότητα ως προς τα Αγαθά, τη Διανεμητική Αποτελεσματικότητα και την Αποτελεσματικότητα ως προς τη Δαπάνη, για το Two-Tiered Frontier Υπόδειγμα.

	2005						2006					
	Ιούλ.	Αύγ.	Σεπ.	Οκτ.	Νοέ.	Δεκ.	Ιαν.	Φεβ.	Μαρ.	Απρ.	Μαϊ.	Ιούν.
Αποτελεσματικότητα ως προς τα Αγαθά												
<0,4	0	0	0	0	0	0	0	0	0	0	0	0
0,4-0,5	0	1	0	0	0	1	0	0	1	0	1	0
0,5-0,6	2	6	1	4	1	0	1	2	2	2	1	2
0,6-0,7	9	15	15	14	11	20	9	7	27	7	5	9
0,7-0,8	112	124	119	136	128	147	116	102	148	111	120	113
0,8-0,9	761	738	749	730	744	716	758	773	706	764	757	760
>0,9	0	0	0	0	0	0	0	0	0	0	0	0
Μέσος	0,84	0,84	0,84	0,84	0,84	0,83	0,84	0,84	0,83	0,84	0,84	0,84
Διανεμητική Αποτελεσματικότητα												
<0,4	0	0	0	0	0	0	0	0	0	0	0	0
0,4-0,5	0	0	0	0	0	0	0	0	0	0	0	0
0,5-0,6	25	0	2	2	2	4	7	3	11	2	1	1
0,6-0,7	99	0	40	22	33	69	51	18	25	9	98	120
0,7-0,8	221	362	315	402	434	406	430	416	329	269	307	375
0,8-0,9	402	469	460	416	393	362	370	427	469	539	423	350
>0,9	137	53	67	42	22	43	26	20	50	65	55	38
Μέσος	0,81	0,81	0,81	0,80	0,79	0,79	0,79	0,80	0,81	0,82	0,80	0,78
Αποτελεσματικότητα ως προς τη Δαπάνη												
<0,4	0	0	0	0	0	0	0	0	0	0	0	0
0,4-0,5	19	0	0	1	0	13	8	3	22	7	0	0
0,5-0,6	142	9	75	43	47	122	83	27	27	5	170	162
0,6-0,7	345	605	526	562	627	528	556	597	562	499	452	484
0,7-0,8	351	270	283	278	210	221	237	257	273	373	262	238
0,8-0,9	27	0	0	0	0	0	0	0	0	0	0	0
>0,9	0	0	0	0	0	0	0	0	0	0	0	0
Μέσος	0,68	0,68	0,67	0,68	0,67	0,66	0,66	0,68	0,68	0,69	0,66	0,66

Τέλος, όσον αφορά στη συμπεριφορά των νοικοκυριών κατά τη διάρκεια του χρόνου, δεν μπορούμε να συμπεράνουμε από τα εμπειρικά αποτελέσματα ότι οι βαθμοί αναποτελεσματικότητας μειώνονται χρονικά. Όπως φαίνεται στους Πίνακες 7 και 8, οι μέσοι βαθμοί αποτελεσματικότητας είναι σταθεροί κατά τη διάρκεια του χρόνου. Αυτό είναι αναμενόμενο, αν λάβουμε υπόψη τον τύπο των υπό μελέτη αγαθών και τη μικρή χρονική διάρκεια που καλύπτει το δείγμα μας (12 μήνες). Πιο συγκεκριμένα, τα υπό εξέταση αγαθά παίζουν έναν σημαντικό ρόλο στη διατροφή του μέσου νοικοκυριού και, επιπλέον, η χρονική διάρκεια που καλύπτει το δείγμα μας είναι πολύ μικρή για να επιτρέψει να φανούν αλλαγές στα κοινωνικό-δημογραφικά χαρακτηριστικά τα οποία θα μπορούσαν να επηρεάσουν τις καταναλωτικές συνήθειες και, άρα, τους βαθμούς αποτελεσματικότητας.

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