Political Polarization and Economic Inequality in the European Union

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Abstract

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In the last 5 decades political polarization has been constantly rising. After going through a short period of consensus for a couple of decades after the end of WWII, it seems that politicians have opted for a strategy of hatred in order to increase partisanship and maximize voter turnout. This has led to a high level of political fractionalization today both in parliaments as well as among citizens. While this subject has been vastly studied in the USA, little can be found regarding the European Union, and even more so on a cross country level. In this thesis, we study the evolution of political polarization in 10 countries in the European Union for a thirty-years period, namely from 1989 until 2019. In order to do so, we use an index which takes under account both the heterogeneity between groups as well as the homogeneity within groups. The procedure of estimating the polarization index in each country is based on the ideal point estimation of the self-placement perception of voters. Utilizing individual level survey data on the self-placement of voters as well as their placement of their countries' parties, we estimate, non-parametrically, with the aid of a Gaussian kernel, and the application of a probabilistic scaling method, the probability density function of the distribution of public political opinion. We apply on these results the index of political polarization. The index is based on the ≪Identification-Alienation≫ framework. According to this, political polarization increases with the increase of heterogeneity among groups (the further away clusters move from each other) as well as the increase in homogeneity within groups (the stronger the identification sentiment between the members of each cluster). Additionally, we test the interrelationship between political polarization and two economic variables, namely income inequality and economic growth. Regressions are performed using the Fixed Effects method as well as the Random Effects Method. In both methods, individuals effects are also allowed for. As expected, results indicate that political polarization is positively affected by income inequality and negatively affected by economic growth.

Εκτεταμένη Περίληψη

Την τελευταία πενταετία η πολιτική πόλωση παρουσιάζει συνεχή αυξητική τάση. Υστέρα από μια σύντομη περίοδο όπου υπήρχε διάθεση συναίνεσης και συνεργασίας μεταξύ των πολιτικά αντίθετων παρατάξεων, φαίνεται ότι πλέον περνάμε σε μια φάση όπου κυριαρχεί η επιλογή της στρατηγικής και ρητορικής μίσους προκειμένου να αυξηθεί η προσέλκυση ψηφοφόρων καθώς και να μεγιστοποιηθεί η προσέλευση στις κάλπες. Αυτό σήμερα έχει οδηγήσει σε αυξημένο κατακερματισμό, τόσο εντός των κοινοβουλίων όσο και ανάμεσα στον λαό. Παρά το γεγονός ότι το ζήτημα αυτό έχει μελετηθεί εκτενώς στις Ηνωμένες Πολιτείες, δεν υπάρχουν πολλές μελέτες πάνω στο φαινόμενο αυτό για τις χώρες της Ευρώπαικης Ένωσης, πολλώ δε μάλλον συγκριτικές μελέτες μεταξύ των χωρών εντός αυτής. Στην παρούσα διατριβή, μελετήθηκε η εξέλιξη της πολιτικής πόλωσης σε 10 χώρες της Ευρωπαϊκής Ένωσης κατά την διάρκεια τριάντα ετών, από το 1989 έως και το 2019. Για την υλοποίηση της συγκριτικής αυτής μελέτης χρησιμοποιήθηκε ένας δείκτης ο οποίος λαμβάνει υπόψη του την ετερογένεια μεταξύ των ομάδων καθώς και την ομοιογένεια εντός των ομάδων. Η διαδικασία εκτίμησης του δείκτη πολιτικής πόλωσης εντός κάθε χώρας έχει βασιστεί στον υπολογισμό της εκτίμησης της ιδανικής θέσης (ideal point estimate) των ερωτώμενων αναφορικά με την αυτοτοποθέτησή τους στην πολιτική κλίμακα. Για τον υπολογισμό της ιδανικής θέσης χρησιμοποιήθηκε μια πιθανοθεωρητική μέθοδος αναπροσαρμογής μεγέθους. Χρησιμοποιώντας τα αποτελέσματα αυτά, σε συνδυασμό με τις απαντήσεις για την αντίληψη των ερωτηθέντων περί της θέσεως των κομμάτων των χωρών τους, εκτιμούμε την συνάρτηση πυκνότητας πιθανότητας της κατανομής της κοινής πολιτικής γνώμης. Η εκτίμηση αυτή γίνεται μη παραμετρικά, με τη βοήθεια μιας συνάρτησης πυρήνα της κανονικής κατανομής Gauss . Χρησιμοποιώντας τα αποτελέσματα που προκύπτουνε από την παραπάνω διαδικασία, εκτιμούμε τον δείκτη της πολιτικής πόλωσης. Ο δείκτης βασίζεται στην ιδέα ενός πλαισίου «Ταύτισης – Αποξένωσης» μεταξύ ατόμων της ίδιας ομάδας όσο και των ατόμων μεταξύ των ομάδων. Σύμφωνα με το πλαίσιο αυτό, η πολιτική πόλωση αυξάνεται με την αύξηση της ετερογένειας ανάμεσα στις ομάδες (δηλαδή με την αύξηση της αποξένωσης μεταξύ των ομάδων) καθώς και με την αύξηση της ομοιογένειας εντός των ομάδων (δηλαδή με την αύξηση του αισθήματος ταύτισης μεταξύ των μελών κάθε ομάδας). Επιπλέον, ελέγχεται η εξάρτηση του επιπέδου της πολιτικής πόλωσης από δύο ανεξάρτητες οικονομικές μεταβλητές, και συγκεκριμένα την εισοδηματική ανισότητα και την οικονομική μεγέθυνση. Οι παλινδρομήσεις πραγματοποιούνται χρησιμοποιώντας τη μέθοδο Σταθερών Επιδράσεων καθώς και τη Μέθοδο Τυχαίων Επιδράσεων. Και τα δύο μοντέλα εκτιμώνται και επιτρέποντας για ατομικές επιδράσεις (Individual effects). Όπως ήταν αναμενόμενο, τα αποτελέσματα δείχνουν ότι η πολιτική πόλωση επηρεάζεται θετικά από την εισοδηματική ανισότητα και αρνητικά από την οικονομική μεγέθυνση.

Dedication

To my husband and daughter

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Εκτεταμένη Περίληψη

ΕΙΣΑΓΩΓΗ

Η παρούσα διατριβή αναλύει την εξέλιξη της πολιτικής πόλωσης μετά το τέλος του Β΄ Παγκοσμίου Πολέμου, αναλύει θέματα σχετικά με τη μέτρησή της και προτείνει έναν κατάλληλο δείκτη για την παρακολούθησή της. Τέλος εξετάζει τη σχέση της πολιτικής πόλωσης με την οικονομική ανάπτυξη και την οικονομική ανισότητα. Στο πρώτο κεφάλαιο προσφέρεται μια εκτενής βιβλιογραφική ανασκόπηση, η οποία παρέχει μια εις βάθος ανάλυση του φαινομένου τα τελευταία εβδομήντα χρόνια. Στόχος της ανασκόπησης αυτής είναι η παράθεση και κατανόηση της υπάρχουσας έρευνας. Παράλληλα στοχεύει και σε μια ενδελεχή ανάλυση του θέματος από διάφορες οπτικές, αναφέροντας διάφορα θέματα που έχουν εγερθεί από τους μελετητές του αντικειμένου (όπως έλλειψη ενός σαφούς πλαισίου μελέτης σε συνάρτηση με έναν ξεκάθαρο ορισμό) . Παράλληλα με την ανασκόπηση της βιβλιογραφίας, παρατίθενται πραγματικά παραδείγματα, σε μια απόπειρα ανάδειξης της σημαντικότητας του φαινομένου και του άμεσου αντικτύπου που έχει στην ζωή όλων. Η ιστορική ανασκόπηση ξεκινάει από την λήξη του Β΄ Παγκοσμίου Πολέμου μέχρι και τις τελευταίες εκλογές σε κράτη-μέλη της ΕΕ καθώς και στις ΗΠΑ. Αναλύονται μέχρι και τα πιο πρόσφατα γεγονότα σχετικά με την αντιμετώπιση της υγειονομικής κρίσης που προκλήθηκε από το ξέσπασμα της πανδημίας της COVID-19. Αναφορικά με το ζήτημα αυτό, αναλύεται η ανταπόκριση των κυβερνήσεων έναντι στην πανδημία της COVID-19, καθώς και η αντίδραση των εκάστοτε κομμάτων της αντιπολίτευσης και κατά πόσο αυτές επηρεάζονται από ιδεοληψίες και ψηφοθηρία. Όλα τα παραπάνω προσφέρουν μια ξεκάθαρη εικόνα της σοβαρότητας της κατάστασης καθώς και της σημαντικότητας του προβλήματος στην σημερινή εποχή. Τα προβλήματα αυτά είναι απόρροια του αυξημένου κατακερματισμού των ομάδων μέσα στα κοινοβούλια καθώς και την αύξηση του χάσματος μεταξύ αριστεράς και δεξιάς, που μοιάζει να αυξάνεται συν τω χρόνω. Παράλληλα παρουσιάζεται, μέσα από βιλβιογραφικές αναφορές και στατιστικά δεδομένα, η ολοένα και αυξανόμενη οικονομική ανισότητα και τίθεται το ερώτημα του κατά πόσο υπάρχει αιτιώδης σύνδεση μεταξύ των δύο φαινομένων.

Το δεύτερο κεφάλαιο ξεκινάει με την απόπειρα απόδοσης ενός σαφούς και συγκεκριμένου ορισμού της πολιτικής πόλωσης. Παρά την πρόσφατη δημοτικότητα του θέματος, η βιβλιογραφία στερείται ενός ξεκάθαρου ορισμού της πολιτικής πόλωσης καθώς και ενός συγκεκριμένου πλαισίου γύρω από το οποίο θα μπορούσαν να γίνουν συγκεκριμένες μελέτες και συγκρίσεις αποτελεσμάτων. Μετά την παρουσίαση των επικρατέστερων θεωριών, παρουσιάζονται και αναλύονται οι δύο κύριοι παράγοντες από τους οποίους εξαρτάται η πολιτική πόλωση. Συνοπτικά, οι δύο παράγοντες αυτοί είναι η ομοιογένεια εντός των ομάδων (το συναίσθημα ταύτισης μεταξύ των μελών των ίδιων ομάδων) και η ετερογένεια μεταξύ των ομάδων (το συναίσθημα αποξένωσης μεταξύ των μελών των διαφορετικών ομάδων). Στην συνέχεια γίνεται αναλυτική παρουσίαση του δείκτη που εφαρμόστηκε σε αυτή τη μελέτη για την εκτίμηση του επιπέδου της πόλωσης, παραθέτοντας τις γενικές παραδοχές του υποδείγματος. Γίνεται επίσης και μια αναλυτική αξιωματική προσέγγιση του δείκτη που εξηγεί την εκτιμώμενη συμπεριφορά, ανάλογα με τις μεταβολές στην κατανομή της κοινής γνώμης. Στο δεύτερο κεφάλαιο γίνεται επίσης παρουσίαση των δεδομένων, σε ατομικό επίπεδο, που χρησιμοποιήθηκαν για την διεξαγωγή της μελέτης αυτής. Τα δεδομένα αφορούν 10 κράτη-μέλη της ευρωπαϊκής ένωσης για μια περίοδο τριάντα ετών, συγκεκριμένα από το 1989 έως το 2019. Τα δεδομένα που χρησιμοποιούνται αφορούν απαντήσεις σε ερωτήσεις σχετικά με την αντίληψη των ψηφοφόρων τόσο για την δική τους τοποθέτηση στο πολιτικό φάσμα όσο και για την τοποθέτηση των κομμάτων της χώρας τους. Τα δεδομένα αυτού του τύπου, που αφορούν σε θέματα υποκειμενικής αντίληψης χαρακτηρίζονται από δυο ειδών μεροληψίες. Από την μία είναι η αντίληψη του ερωτώμενου για την πολιτική σφαίρα και που θεωρεί ο ίδιος ότι ανήκει και παράλληλα η αντίληψη του ερωτώμενου για την κλίμακα του ερωτηματολογίου. Γίνεται εκτενής ανάλυση της μεθόδου για την αντιμετώπιση των δυο αυτών μεροληψιών.

Στο τρίτο κεφάλαιο γίνεται αρχικά μια θεωρητική υποστήριξη της υπόθεσης που έχει τεθεί στο πρώτο κεφάλαιο, δηλαδή ότι η πόλωση επηρεάζεται από οικονομικά μεγέθη όπως η οικονομική ανισότητα και η οικονομική μεγέθυνση. Από την ανάλυση της πόλωσης στο δευτερο κεφάλαιο έχει προκύψει ένα πάνελ δεδομένων για 10 χώρες για 7 πενταετίες. Με την βοήθεια οικονομικών δεδομένων που ανακτήθηκαν από την βάση δεδομένων World Inequality Database, κατασκευάζεται το τελικό σύνολο διαμηκών διαστρωματικών δεδομένων (panel data) το οποίο χρησιμοποιείται για τις παλινδρομήσεις που διεξάγονται με σκοπό να αποδειχτεί εμπειρικά η συσχέτιση που έχει υποτεθεί. Η αρχική υπόθεση επιβεβαιώνεται από τα αποτελέσματα της παλινδρόμησης, δηλαδή η οικονομική ανισότητα αυξάνει την πολιτική πόλωση και, αντίθετα, η οικονομική ανάπτυξη εμποδίζει την πολιτική πόλωση.

ΑΝΑΛΥΤΙΚΗ ΔΟΜΗ

Η παρούσα διπλωματική εργασία αποτελείται από τρία ξεχωριστά αλλά αλληλένδετα κεφάλαια. Τα τρία κεφάλαια συνδυαστικά προσφέρουν μια ενδελεχή κατανόηση της πολιτικής πόλωσης. Προσφέρουν ένα ξεκάθαρο τρόπο διαχείρισης (ορισμού, μέτρησης και σύγκρισης) μιας μεταβλητής, όπως η πολιτική πόλωση, η οποία είναι ποιοτική, συνεχής και αφορά σε υποκειμενική αντίληψη. Χάρη στην μεθοδολογία που χρησιμοποιείται, τα αποτελέσματα μπορούν να συγκριθούν είτε διαχρονικά για μια χώρα είτε για μια δεδομένη χρονική στιγμή για πολλές χώρες. Επίσης επιτρέπει την συσχέτισή της, αρνητικά ή θετικά, με άλλα οικονομικά μεγέθη όπως η οικονομική ανάπτυξη και η οικονομική ανισότητα.

Πιο αναλυτικά, το κάθε κεφάλαιο είναι δομημένο ως ακολούθως:

ΚΕΦΑΛΑΙΟ 1

Στο πρώτο κεφάλαιο προσφέρεται μια αναλυτική βιβλιογραφική ανασκόπηση σχετικά με το θέμα της πολιτικής πόλωσης και το θέμα της αυξανόμενης οικονομικής ανισότητας καθώς και την ενδεχόμενη μεταξύ τους σχέση. Το κεφάλαιο ξεκινά με μια ιστορική παρουσίαση των γεγονότων που δείχνουν την αλλαγή στη στάση των ανθρώπων απέναντι στους πολιτικούς, απέναντι στους ψηφοφόρους άλλων κομμάτων, γεγονότα που δείχνουν τον αυξημένο κατακερματισμό εντός των κοινοβουλίων, την ολοένα και πιο πολωμένη ρητορική από την πλευρά των πολιτικών που ενίοτε έχουν οδηγήσει ακόμα και σε βίαια ξεσπάσματα. Εν συντομία, παρατηρούμε μια εξέλιξη της πολιτικής πόλωσης που μπορεί να χωριστεί χονδρικά σε τρεις εποχές. Πρώτον, λίγο μετά το τέλος του Β΄ Παγκοσμίου Πολέμου (μετά το τέλος της περιόδου των αντιποίνων, που διήρκεσε περίπου δύο χρόνια) και σε όλη τη δεκαετία του 1950-60. Κατά την διάρκεια των ετών αυτών παρατηρούμε ότι υπάρχει μια ευρεία τάση για συναίνεση μεταξύ των κομμάτων. Ενώ υπάρχει αντιπολίτευση, η διάσπαση μεταξύ των κομμάτων αριστερά και δεξιά δεν είναι ακόμη τόσο βαθιά και τα κόμματα δεν είναι ακόμη τόσο ομοιογενή, οι εσωτερικές γραμμές και κατευθυντήριες οδηγίες δεν είναι ακόμα τόσο δεσμευτικές. Στην Ευρώπη, η εποχή αυτή χαρακτηρίζεται από την δημιουργία της Ευρωπαϊκής Ένωσης Άνθρακα με την υπογραφή της συνθήκης του Παρισίου το 1951, που αργότερα θα οδηγήσει στην Ευρωπαϊκή Οικονομική Κοινότητα, για να γίνει σήμερα η Ευρωπαϊκή Ένωση. Άλλο ένα χαρακτηριστικό παράδειγμα αυτού είναι η ψήφιση του Νόμου για τα Πολιτικά Δικαιώματα στις ΗΠΑ το 1964. Πρόκειται σαφώς για ένα θέμα έντονα διχαστικό, για μια πολύ μεγάλη διαφορά στις πεποιθήσεις των ανθρώπων σχετικά με το πώς πρέπει να κυβερνάται μια χώρα, τι είναι ηθικά σωστό και πως αυτό πρέπει να αποτυπώνεται στο Σύνταγμα και στους νόμους της χώρας. Στην ψήφιση του εν λόγω νόμου όμως παρατηρούμε πως το σχέδιο αυτό στηρίχθηκε τόσο από Ρεπουμπλικάνους και Δημοκράτες ενώ παράλληλα δέχτηκε έντονες κρητική τόσο εντός όσο και εκτός κόμματος. Χαρακτηριστικά αναφέρεται πως η ψήφος των Δημοκρατικών του Νότου ήταν ξεκάθαρα πιο επηρεασμένη από τον γεωγραφικό προσδιορισμό των γερουσιαστών παρά από την γραμμή του κόμματος. Η γραμμή του κόμματος τότε δεν είχε ακόμα τόσο ισχυρό αντίκτυπο, όσο μπορεί να είχαν άλλα χαρακτηριστικά της ταυτότητας όσων συμμετείχαν στην ιστορική αυτή ψηφοφορία. Εν συνεχεία, καθ΄ όλη τη διάρκεια της δεκαετίας 1970-80 και μέχρι το τέλος της δεκαετίας του 1990 παρατηρούμε μια πιο ξεκάθαρη ταξινόμηση μεταξύ δεξιάς και αριστεράς. Κατά έναν τρόπο, θα μπορούσαμε να πούμε ότι είναι σαν η δεξιά γίνεται πιο «δεξιά» και η αριστερά γίνεται «αριστερή». Προς το παρόν, αυτό δεν είναι τόσο με την έννοια ότι τα κόμματα απομακρύνονται και η απόσταση που τα χωρίζει γίνεται μεγαλύτερη, αλλά σχετίζεται περισσότερο με την ομοιογένεια εντός των κομμάτων. Οι διαχωριστικές γραμμές μεταξύ των κομμάτων αρχίζουν να γίνονται ολοένα και πιο ξεκάθαρες, χωρίς πλέον να υπάρχουν και πολλά κοινά σημεία τομής. Η εποχή των μεγάλων συνασπισμών έχει φύγει. Καθώς βαδίζουμε προς τον 21ο αιώνα, μπαίνουμε στην τρίτη εποχή της πόλωσης και τα πράγματα γίνονται πιο έντονα και παρατηρείται ακόμη και ένα αίσθημα εχθρότητας, μεταξύ των οπαδών των κομμάτων. Αυτό φαίνεται ακόμα και από την αλλαγή πάνω σε συμπεριφορές κοινωνικοποίησης, όπως αυτή εκφράζεται μέσα από φαινόμενα όπως η αυξημένη πολιτική συμφωνία μεταξύ συζύγων καθώς και μεταξύ γονέων και παιδιών «Iyengar *et al.*, 2015). Η πολιτική πόλωση αναφέρεται πλέον συχνά στις καθημερινές συζητήσεις, στους τίτλους των εφημερίδων, στις ειδήσεις κ.λπ. Με άλλα λόγια, δεν είναι ότι σήμερα οι διαφορές μεταξύ των ψηφοφόρων είναι πιο μεγάλες ή αφορούν σε πιο σημαντικά ζητήματα (άλλωστε πιο ζήτημα θα μπορούσε να είναι πιο σημαντικό, για παράδειγμα, από τον συστηματικό και συστημικό φυλετικό διαχωρισμό των ανθρώπων σε φυλετικές ή άλλες εθνοτικές ομάδες στην καθημερινή ζωή, όπως ίσχυε στις ΗΠΑ μέχρι το 1964). Το πρόβλημα είναι πως πλέον παρατηρείται μια σαφής και ξεκάθαρη ταξινόμηση των πεποιθήσεων και των απόψεων των ανθρώπων γύρω από τις γραμμές του εκάστοτε κόμματος. Στα πιο πρόσφατα γεγονότα, αυτό έγινε αρκετά ξεκάθαρο όσον αφορά το ζήτημα διαχείρισης της πανδημίας της COVID-19. Είδαμε εκεί πως οι απόψεις των ψηφοφόρων γύρω από ένα ξεκάθαρα υγειονομικό ζήτημα συντάσσονταν με την γραμμή του κόμματος που υποστήριζαν. Ενδεικτικά αναφέρεται ότι στις ΗΠΑ η πλειονότητα των ανεμβολίαστων ήταν ψηφοφόροι του Trump ενώ η πλειονότητα των εμβολιασμένων ήταν Δημοκρατικοί. Επίσης, το θέμα των εμβολιασμών έδωσε πάτημα για νέα όξυνση της πολιτικής ρητορικής, με τον Biden να ισχυρίζεται πως το μείζον πλέον πρόβλημα είναι η πανδημία των ανεμβολίαστων. Γύρω από το θέμα της πολιτικής πόλωσης έχει αναπτυχθεί μια εκτενής βιβλιογραφία. Έχουν αναπτυχθεί πολλές θεωρίες και έχουν πραγματοποιηθεί αρκετές μελέτες, κυρίως στις ΗΠΑ και λίγες σε ευρωπαϊκές χώρες. Η βιβλιογραφία ανασκοπείται εκτενώς, παράλληλα με τις εμπειρικές μελέτες και τις θεωρίες που χρησιμοποιούνται για τη δημιουργία διάφορων δεικτών πόλωσης. Ξεκινώντας από την κλασική έννοια της κοινωνικής απόστασης από τον Bogardus (1947) και της κοινωνικής ταυτότητας από τον Tajfel (1970) και από τους Tajfel and Turner (1979), οι Iyengar et al. (2012), επισημαίνουν τη σημασία, όχι μόνο του συναισθήματος του να ανήκεις σε μια ομάδα, της ταύτισης με τα μέλη της, αλλά και συναισθήματος αποξένωσης προς τα μέλη των αντίπαλων ομάδων, τα οποία καμία φορά μπορούν να φτάσουν να χαρακτηρίζονται έως και εχθρικά. Οι «Iyengar *et al.*, 2012), βασιζόμενοι σε αυτή την θεωρία, αναπτύσσουν ένα νέο όρο, κάτι που αποκαλούν «συναισθηματική πολιτική πόλωση» (Affective Polarization) και το οποίο λαμβάνει υπόψη και τα δυο αυτά συναισθήματα που νοιώθει κάποιος (τα θετικά προς τα μέλη της ομάδας του και τα αρνητικά προς τα μέλη των άλλων ομάδων). Ουσιαστικά ισχυρίζονται πως αυτή η δημιουργία εξαιρετικά ομοιογενών κομμάτων μας έχει προχωρήσει ακόμη και ένα βήμα παραπέρα και έτσι, σύμφωνα με τους S. Iyengar et al. (2012) προκαλείται «συναισθηματική πόλωση», έναν όρο που κατωχήρωσαν στη θεμελιώδη εργασία τους Affect, Not Ideology. A Social Identity Perspective on Polarization. Σε ένα παρόμοιο πλαίσιο είχαν κινηθεί νωρίτερα και οι Esteban and Ray (1994) στην προσπάθεια τους να κατασκευάσουν έναν δείκτη που να μετράει την οικονομική πόλωση. Πάλι εκεί εξετάζεται η σημασία όχι μόνο της απόστασης μεταξύ των οικονομικών ομάδων αλλά και η συνοχή μεταξύ των ατόμων εντός των ομάδων αυτών. Συμφωνά με την θεωρία τους, η συσπείρωση μεταξύ των ατόμων μιας ομάδας, το αίσθημα ταύτισης που αναπτύσσεται μεταξύ τους, οξύνει το αίσθημα αποξένωσης που να νοιώθουν τα μέλη των διαφόρων ομάδων μεταξύ τους. Πατώντας πάνω σε αυτό, κατασκευάζουν ένα πλαίσιο εργασίας που ονομάζουν πλαίσιο «Ταύτισης – Αποξένωσης» (Identification – Alienation framework). Αυτό είναι και το πλαίσιο στο οποίο έχει βασιστεί και ο δείκτης πόλωσης στην παρούσα διδακτορική διατριβή. Το σορτάρισμα που έχει συμβεί τα τελευταία 50 περίπου χρόνια έχει ταυτίσει σχεδόν κομματικές και ιδεολογικές ταυτότητες, δημιουργώντας έντονα συναισθήματα ταύτισης και συμμαχίας μεταξύ των μελών κάθε ομάδας, αλλά ταυτόχρονα αύξησε τα επίπεδα κομματικής προκατάληψης, οργής και αντιπαλότητας προς τα μέλη των άλλων ομάδων (Mason, 2015). Όπως προτείνει η Mason (2015), ένας ψηφοφόρος σε ένα πολωμένο περιβάλλον, λειτουργεί περισσότερο σαν οπαδός, με την έννοια ότι ενδιαφέρεται για την ευημερία του κόμματός του, προτιμά να συναναστρέφεται με μέλη του κόμματός του και νιώθει θυμωμένος (και μπορεί ακόμη και να εκδηλωθεί με βίαιο τρόπο) όταν αισθάνεται ότι η ομάδα του απειλείται, παρόλο που μπορεί να μην συμφωνεί με την ομάδα σε όλα τα θέματα. Η ένταση της συμπεριφοράς αυτής ενός ατόμου εξαρτάται από διάφορους παράγοντες, όπως για παράδειγμα την προσωπικότητα ενός ατόμου, τις προσωπικές του εμπειρίες, τα κοινωνικά δίκτυα κ.λπ. Σύμφωνα με τους Roccas and Brewer (2002) το επίπεδο πολυπλοκότητας της ταυτότητας του ατόμου, δηλαδή πόσες από τις κοινωνικές του ταυτότητες συναντώνται σε μια μοναδική ομάδα ή κατά πόσο το άτομο αυτό συνδέεται με διάφορες διαφορετικές ομάδες, ανάλογα με ένα διαφορετικό συνδετικό κριτήριο κάθε φορά, επηρεάζει την ένταση του αισθήματος ταύτισης. Πέραν της ανάλυσης σχετικά με τις θεωρίες ταυτότητας του ατόμου και την επίδρασή της στα συναισθήματα συνοχής μια ομάδας καθώς και αντιπαλότητας μεταξύ των ομάδων, υπάρχουν πολυάριθμες εμπειρικές μελέτες που αποδεικνύουν την αύξηση της πόλωσης. Ενδεικτικά αναφέρεται ότι οι Webster and Abramowitz (2017) βρίσκουν ότι αύξηση της συσχέτισης μεταξύ ιδεολογίας και πολιτικής ταυτότητας έχει σχεδόν διπλασιαστεί από το 1972 έως το 2008. Οι Boxell *et al.* (2020) εξετάζουν συνολικά 12 χώρες του ΟΟΣΑ για να βρουν πως η πόλωση παρουσιάζει την δραματικότερη αύξηση στις ΗΠΑ και επιβεβαιώνουν την συσχέτιση με την οικονομική ανισότητα. Το δεύτερο μισό του πρώτου κεφαλαίου αφορά την αυξανόμενη οικονομική ανισότητα καθώς και τις κοινωνικές επιπτώσεις της. Αναλύεται τόσο από βιβλιογραφική σκοπιά όσο και από γεγονότα τα οποία επιβεβαιώνουν το αντίκτυπό της στις κοινωνίες. Η οικονομική ανισότητα ακολούθησε, λίγο πολύ, την ίδια εξέλιξη με την πολιτική πόλωση. Ενώ λίγο μετά το τέλος του Β′ Παγκοσμίου Πολέμου η οικονομική μεγέθυνση και ανάπτυξη που σημειώθηκε είχε ως αποτέλεσμα, μεταξύ άλλων, τη μείωση της ανισότητας, μετά τη δεκαετία του 1970 η ανισότητα (εντός των χωρών, όχι μεταξύ των χωρών) παρουσίασε αύξηση. Στη βιβλιογραφία, η οικονομική ανισότητα έχει επιχειρηθεί να συνδεθεί εμπειρικά με την πόλωση, αλλά λόγω της δυσκολίας μέτρησης της τελευταίας, οι αναφορές είναι σπάνιες και τα συμπεράσματα δεν είναι πολύ σαφή.

ΚΕΦΑΛΑΙΟ 2

Το δεύτερο κεφάλαιο ξεκινά με μια παρουσίαση βασικών ορισμών για να προσδιοριστούν οι μετρήσιμοι παράγοντες από τους οποίους εξαρτάται η πόλωση. Σύμφωνα με τον DiMaggio (1996) η πόλωση επηρεάζεται από τέσσερα βασικά μεγέθη. Δύο στατιστικά μεγέθη, τα οποία είναι η διασπορά της κατανομής και η κύρτωση της κατανομής, καθώς και δύο πιο θεωρητικές (και πιο δύσκολα μετρήσιμες) έννοιες. Η πρώτη είναι μια θεωρία του Converse (1964) που αφορά την συσχέτιση των πεποιθήσεων ενός ατόμου μεταξύ τους και η δεύτερη είναι η ανάλυση της διαφορετικότητας μεταξύ των ομάδων. Στην συνέχεια παρουσιάζεται αναλυτικά το πλαίσιο «Ταύτισης – Αποξένωσης» των Esteban and Ray (2004), καθώς και η προέκτασή του για την εφαρμογή σε συνεχείς μεταβλητές, από τους Duclos, Esteban and Ray (2004). Εν κατακλείδι, συμπεραίνει κανείς πως η πόλωση αυξάνεται με την αύξηση της ανομοιογένειας μεταξύ των ομάδων και της ομοιογένειας εντός των ομάδων. Σύμφωνα με το μοντέλο των Duclos *et al.* (2004), η ομοιογένεια εντός των ομάδων εκφράζεται με την πυκνότητα της κατανομής γύρω από ένα σημείο. Με άλλα λόγια, εξαρτάται από τον αριθμό των ατόμων μέσα σε μια ομάδα. Η ανομοιογένεια μεταξύ των ομάδων εκφράζεται μαθηματικά από την απόσταση μεταξύ των ομάδων. Στην συνέχεια παρουσιάζεται η αξιωματική περιγραφή του θεωρήματος. Τα αξιώματα δίνουν, θεωρητικά, μια περιγραφή για την εξέλιξη της τιμής του επιπέδου πόλωσης ανάλογα με κάποιες βασικές μεταβολές στην κατανομή της κοινής πολιτικής γνώμης. Έτσι, παράδειγμα, σε μια κοινωνία της οποίας η κατανομή εκφράζεται από μια και μοναδική κανονική κατανομή, αν τα άτομα της συσπειρωθούν εντονότερα γύρω από το μέσο της κατανομής, τότε, λόγω του ότι δεν υπάρχει, θεωρητικά, δεύτερη ομάδα στην κοινωνία (και έτσι η αποξένωση δεν αυξάνεται), το επίπεδο της πόλωσης δεν γίνεται να αυξηθεί (πάρα την αύξηση του επιπέδου ταύτισης των μελών της ομάδας). Προσοχή θα πρέπει να δοθεί όμως στο γεγονός ότι το προαναφερθέν αξίωμα αφορά πληθυσμό που κατανέμεται σε μια και μοναδική ομάδα. Σύμφωνα με το δεύτερο αξίωμα, στην περίπτωση κατά την οποία ένας πληθυσμός είναι χωρισμένος σε 3 ομάδες, η αύξηση της συσπείρωσης στις δυο ακριανές ομάδες (οδηγώντας σε αύξηση της ταύτισης και μείωση της αποξένωσης μιας και η συνολική απόσταση θα μικρύνει) δεν μπορεί να προκαλέσει πτώση του επιπέδου της πόλωσης. Τέλος, στην περίπτωση που μια κοινωνία κατανέμεται σε 4 ομάδες και οι δυο (εσωτερικές) ομάδες απομακρυνθούν (συνεπώς παρατηρείται μόνο αύξηση της αποξένωσης ενώ τα επίπεδο ταύτισης εντός ομάδων παραμένει σταθερό) το επίπεδο της πόλωσης θα πρέπει να αυξηθεί. Τα παραπάνω αξιώματα μπορούν να αποδοθούν σε ένα θεωρητικό μοντέλο, στου οποίου την πρακτική εφαρμογή προχωρήσαμε σε αυτήν εδώ την διδακτορική διατριβή. Θεωρητικά, τα αξιώματα εκφράζονται ως εξής:

$$P_\alpha \equiv \int \int f(x)^{1+\alpha} f(y)|y-x|dydx, \qquad (1)$$

Όπου ο δείκτης α κυμαίνεται στο $[0.25, 1]$. Μέσω πειραματικής διαδικασίας που εφαρμόσαμε, καταλήξαμε πως το ιδανικό επίπεδο ευαισθησίας του εκθέτη α είναι 0.25. Το μοντέλο εφαρμόστηκε σε δεδομένα έρευνας που πραγματοποιούνται από το European Election Studies (EES) κάθε πενταετία, παράλληλα με την διεξαγωγή των Ευροεκλογών. Στην έρευνα συμμετέχουν κάθε φορά περίπου 1000 ερωτώμενοι από κάθε χώρα. Αυτό μας δίνει ένα σύνολο 70000 πρωτογενών δεδομένων σχετικά με την υποκειμενική αντίληψη του κάθε ψηφοφόρου για το που τοποθετεί, πολιτικά, το εαυτό του. Τα δεδομένα αυτά, όπως αναφέρθηκε και πιο πάνω, έχουν πρόβλημα μεροληψίας. Για να αντιμετωπιστεί το πρόβλημα αυτό, στα δεδομένα εφαρμόστηκε μια πιθανοθεωρητική μέθοδος αναπροσαρμογής μεγέθους, η οποία αναπτύχθηκε από τους Aldrich and McKelvey (1977). Στα μετασχηματισμένα, πλέον, δεδομένα εφαρμόστηκε ο δείκτης PaF, όπως παρουσιάζεται παραπάνω. Αυτό έχει ως αποτέλεσμα τις εκτιμήσεις πολιτικής πόλωσης των δέκα χωρών κατά την τριακονταετία. Η ανάλυση όμως δεν περιορίζεται σε αυτό. Ο δείκτης PaF υπολογίστηκε για όλες τις πιθανές τιμές του εκθέτη α, δηλαδή $[0.25, 1]$ αυξάνοντας την τιμή του κατά 0,01 κάθε φορά. Αυτό ισούται με 76 τιμές του α και αυτή η διαδικασία δημιουργεί ένα μοναδικό πάνελ 5320 μοναδικών εκτιμήσεων πόλωσης, οι οποίες επιτρέπουν μια ολοένα πιο εκτεταμένη ανάλυση της επίδρασης της ομοιογένειας εντός των ομάδων και της ανομοιογένειας μεταξύ των ομάδων στην εξέλιξη του δείκτη της πόλωσης. Σε γενικές γραμμές, τα ευρήματα είναι σύμφωνα με τη βιβλιογραφία. Όσον αφορά τη διαχρονική ανάλυση στην περίπτωση του $\alpha = 0.25$, όντως επιβεβαιώνεται η ανοδική τάση της πολιτικής πόλωσης. Όσον αφορά την επιλογή της τιμής για εκθέτη α, τα αποτελέσματα για $\alpha = 0,25$ συνάδουν με τη γενική παρατήρηση ότι στην ΕΕ η πόλωση είναι αποτέλεσμα της παρατηρηθείσας ανόδου των κομμάτων που βρίσκονται στα άκρα του πολιτικού φάσματος.

ΚΕΦΑΛΑΙΟ 3

Στο τρίτο κεφάλαιο παρουσιάζεται το μοντέλο για τον έλεγχο της συσχέτισης μεταξύ της οικονομικής ανάπτυξης και της οικονομικής ανισότητας με την πολιτική πόλωση. Για να γίνει αυτό, κατασκευάστηκε ένας νέος και μοναδικός πίνακας διαμηκών διαστρωματικών δεδομένων με τις εκτιμήσεις πόλωσης που υπολογίστηκαν στο προηγούμενο κεφάλαιο. Όσον αφορά τις οικονομικές παραμέτρους, τα δεδομένα ανακτώνται από την βάση δεδομένων World Inequality Database. Για την οικονομική ανισότητα χρησιμοποιήθηκε ο συντελεστής Gini. Για τη οικονομική μεγέθυνση χρησιμοποιήθηκαν τα στοιχεία του ετήσιου κατά κεφαλήν ΑΕΠ και υπολογίστηκε ο ρυθμός οικονομικής μεγέθυνσης για διάφορα χρονικά διαστήματα (1 έτος, 3 έτη, 5 έτη, μέσος όρος τριετίας). Το μοντέλο εκτιμήθηκε τόσο με την μέθοδο των σταθερών επιδράσεων (FE) όσο και με αυτή των τυχαίων επιδράσεων (RE). Και τα δύο μοντέλα εκτιμώνται και επιτρέποντας για ατομικές επιδράσεις (Individual effects). Τα αποτελέσματα είναι τα αναμενόμενα. Η πολιτική πόλωση βρέθηκε να σχετίζεται θετικά με την εισοδηματική ανισότητα και αρνητικά με την οικονομική μεγέθυνση. Και οι δύο ανεξάρτητες μεταβλητές είναι στατιστικά σημαντικές στην εξήγηση της διακύμανσης της πολιτικής πόλωσης. Το εύρημα αυτό είναι σύμφωνο με τη βιβλιογραφία σχετικά με τη σχέση μεταξύ εισοδηματικής ανισότητας και πολιτικής πόλωσης συγκεκριμένα, αλλά και με την ευρύτερη βιβλιογραφία γύρω από το γενικότερο θέμα της πολιτικής αστάθειας. Η κατανομή του εισοδήματος επηρεάζει την απόφαση των ψηφοφόρων, όσο πιο άνιση είναι η κατανομή τόσο πιο ακραία είναι η επιλογή του κόμματος στην κάλπη. Το αίσθημα αδικίας, καθώς περισσότερος πλούτος συγκεντρώνεται στην κορυφή και η μεσαία τάξη τείνει να εξαφανιστεί, αντανακλάται από το ισχυρότερο αίσθημα ταύτισης των ψηφοφόρων με τα ακραία κόμματα. Από την άλλη πλευρά, η ανάπτυξη φαίνεται να λειτουργεί προς την αντίθετη κατεύθυνση. Όσο υψηλότερο είναι το κατά κεφαλήν ΑΕΠ σε μια χώρα, τόσο περισσότερο οι ψηφοφόροι τείνουν να τοποθετούνται προς το κέντρο και τόσο χαμηλότερο είναι το αίσθημα εχθρότητας προς τους ψηφοφόρους των αντίπαλων κομμάτων.

Extended Summary

Introduction

This thesis analyzes the development of political polarization after the end of WWII, analyzes issues regarding its measurement and proposes an appropriate index for its monitoring and lastly examines its interrelation with economic growth and economic inequality.

In the first chapter an extensive literature review is offered, which provides an in-depth analysis of the phenomenon over the past seventy years, which helps one gain an understanding of the existing research and debate around the matter. Alongside the literature review, real-life examples starting from the past-WWII era until the latest elections in the EU as well as the USA as well as the response of governments and opposition towards the COVID-19 pandemic are presented. This offers a clear view of the gravity of the situation as well as the severity of the problem nowadays, caused by the increased fragmentation of parliaments and the constantly deepening cleavage between the left and right. The, at tandem, increasing economic inequality is analyzed as well and its hypothesized causal linking with the increasing polarization is theoretically presented.

In the second chapter a clear and concrete definition of political polarization is attempted. Despite the recent popularity of the subject, the literature lacks a clear definition of political polarization as well as a concrete framework to work around. After presenting the prevailing theories, the two main factors on which political polarization depends are presented and analyzed, namely homogeneity within clusters (the identification sentiment between members of the same groups) and heterogeneity between clusters (the alienation sentiment between members of different groups). The index applied in this study in order to retrieve the polarization estimations is presented as well as the individual level survey data on ten EU countries over the period of thirty years, from 1989 until 2019. The difficulties arising when dealing with perceptual data are presented as well as how to overcome them with the help of a probabilistic scaling method.

The third chapter presents a regression analysis to uncover the causal relationship between political polarization and economic factors like economic growth and economic inequality. A novel data set is constructed with the polarization estimations calculated in the previous chapter and economic data retrieved for the World Inequality Database. The expected results are confirmed, i.e. economic inequality increases political polarization and, on the contrary, economic growth hinders political polarization.

Structure

This thesis consists of three separate but interrelated chapters. The three chapters combined offer a thorough understanding of political polarization, how this qualitative, continuous and perceptual variable can be measured and compared (throughout time for a specific country or between countries for a given moment in time) as well as its relationship with economic measures like economic growth and economic inequality. More analytically, each chapter is structured as follows:

Chapter 1

In the first chapter an analytical literature review is offered regarding the subject of political polarization and the subject of the rising economic inequality as well as their interrelation. The chapter begins with a historical presentation of events which show the change in people's attitudes towards politics, towards partisans of the opposite party, the increased fragmentation in parliaments, the increasingly polarized rhetoric by politicians and on some occasions even violent breakouts. In brief, we observe an evolution of political polarization which can be roughly divided in three eras. Firstly, shortly after the end of WWII (after the end of the retaliation period, which roughly lasted a couple of years) and throughout the 1950-60s, we observe that there is a broad tendency for consensus between parties. While there is opposition, the cleavage between parties in the left and the right is not yet so deep, and parties are not yet so homogeneous. Throughout the 1970-80s and until the end of the 1990s we observe a sorting of a kind, the right is becoming "righter" and the left is becoming "lefter". This is not so much in the sense of the parties moving further apart as it is in the homogeneity within parties. The lines between the parties are beginning to become more and more clear, with no softening between them. The time of the great coalition-builders is gone. As we walk towards the change of the century, we enter the third era of polarization and things get more intense and even a feeling of animosity is observed, among party members. This is evident even from the change in socialization attitudes, as expressed by the increased inter-marital political agreement and inter-generational political agreement between parents and off-spring. The increase in political polarization is often referred to in everyday conversation, in newspapers' headlines, in the news, etc. As could be anticipated, it has sparked the interest of political scientists as well and a vast literature has developed surrounding the matter. Many theories have been developed and several studies have been performed, mostly in the USA and few in European countries. The literature is extensively reviewed, alongside the empirical studies and the measures used for the creation of polarization indices. In the second half of the first chapter an overview of the growing economic inequality is attempted, both regarding the literature as well as events that confirm what the numbers are saying. Economic inequality has followed, more or less, the same progress as political polarization. While shortly after the end of WWII the economic boom that took place resulted, among other things, in the decline of inequality, after the 1970s inequality (within nations, not between nations) has shown an increase. In the literature, economic inequality has been attempted to be linked with polarization but given the difficulty in measuring the latter the mentions are scarce, and the conclusions are not very clear.

Chapter 2

The second chapter deals extensively with the measurement of political polarization. First the ground is laid for a broad definition of political polarization and secondly, based on said definition, what polarization basically depends on is analyzed, i.e. the homogeneity within groups as well as the heterogeneity between groups. On this account, the "Identification-Alienation" framework is utilized, firstly developed by Esteban and Ray (1994) for discrete variables and further extended to be applied to continuous variables as well (like income and political opinion) by Duclos et al. (2004). The framework is analytically presented alongside its axiomatic approach. This is the index (PaF index) applied to the individual level preferential data on voters' self-placement for 10 countries, members of the European Union (namely Denmark, Germany, Greece, France, Ireland, Italy, Luxembourg, the Netherlands, Spain, and the UK) for a thirty-year period, spanning from 1989 until 2019 (the most recently held European Parliament elections). Additionally, the problems arising with the processing of individual level perceptual data are presented (regarding the voter's perception of their country's parties as well as their own self-placement). The data suffers from two types of biases: the interpretation by the voter of the scale he can allocate the parties as well as how he conveys this information to the interviewer. This problem is dealt with by applying a probabilistic scaling model to the data. After this procedure, a vector with the voter's ideal point estimate is retrieved, which is then used to non-parametrically (with the help of a Gaussian kernel procedure) estimate the probability distribution function of the public political opinion for each country for each time interval (the data is divided in five-years' intervals, the years during which European Parliament elections were held). Additionally, the PaF index used, assigns, by way of an α exponent a weight to the identification component. The experiments performed so as to determine the optimal level of the α parameter (which in this case was $\alpha = 0.25$) are described. After resolving the bias issue and determining the level of the α parameter, the PaF index is applied on the data on self-placement. This results in the political polarization estimates of the ten countries over the thirty-years period. The analysis though, is not limited to this. The PaF index is calculated for all possible values of the α parameter, i.e. [0.25, 1] by 0.01. This equals 76 values of α and this procedure creates a unique panel of 5320 unique polarization estimates, which allow for an ever more extensive analysis of the effect of identification versus the effect of alienation. In general, the findings are in accordance with the literature. Regarding the longitudinal analysis in the case of $\alpha = 0.25$, indeed the upward trend of political polarization is confirmed. Regarding the choice of the value for the α parameter, the choice of $\alpha = 0.25$ produces the results in line with the general observation that in the EU polarization is a result of the rise of parties lying in the extremes of the “left-right” political axis.

Chapter 3

In the third chapter the model for testing the causal link between economic growth and economic inequality with political polarization is presented. To do so a novel and unique panel data is constructed with the polarization estimates calculated in the previous chapter. Regarding the economic parameters, the data is retrieved from the World Inequality Database. For economic inequality the Gini coefficient was used and for the growth variable we used the annual GDP per capita data and calculated the growth rate for various time intervals. The model is run with both Fixed Effects (FE) as well as Random Effects (RE). In order to account and test for heterogeneous effects, both models were run allowing for individual slopes for one of the independent variables as well as for both. The results are as expected. Political polarization is found to be positively related with income inequality and negatively related with economic growth. Both independent variables are statistically significant in explaining the variation of political polarization. This finding is in accordance with the literature regarding the relationship between income inequality and political polarization specifically, but also with the vaster literature surrounding the more general subject of political instability. The distribution of income affects the voters’ decision, the more unequal the distribution the more extreme the choice at time of election. The injustice sentiment, as more wealth is concentrated in the top and the middle class diminishes, is reflected by the stronger identification sentiment of voters with extreme parties. On the other hand, growth seems to work towards the opposite direction. The higher the per person average GDP in a country, the more towards the centre voters tend to place themselves and the lower the animosity sentiment towards partisans of the opposing parties.

Chapter 1

Political Polarization

1.1 Political Polarization

Human beings are social individuals. Group affiliation is an essential characteristic, and even a necessity, to our sense of self, to our identity. Thousands of years ago, belonging to a group, was literally a matter of life or death. Nowadays this may no longer be the case but, with time, the need of belonging has been hardwired to our DNA. This sense of belonging still feels highly important, as if our life depends on it. In this context, individuals perceive of themselves as representing a broad socioeconomic and cultural bundle and not a clear-cut distinctive package of traits. Political parties form along these lines and try to capture those characteristics exactly because they understand the importance of this sense of identity (Brewer 1991, Tajfel 1978, Lipset and Rokkan 1967, Iyengar *et al.* 2019).

As we evolve over the course of time, so do political parties, partisanship and of course political polarization. In the past 80 years, after the end of World War II, things, in the political arena (and not only) have changed dramatically.

The end of World War II, left the world deeply scared. As a result leaders of that time searched for ways to unite nations and their people in order to avoid such atrocities reoccurring in the future. This signified the beginning of an era of political consensus and willingness for collaboration and coalitions.

In Europe this period was marked by the formation of the European Coal and Steel Community, established with the treaty of Paris in 1951, which later evolved into the European Economic Community, established by the treaty of Rome in 1957.

The political landscape in the USA is also moving along the same lines during those years. A clear example of that is the passing of the Civil Rights Act in 1964. This is clearly a matter of a very large difference in beliefs about how the country should be governed, what needs to be done and what is the right thing to do. What is remarkable about that moment in American history, and very different from what is happening today, is that these differences in opinions are not clearly sorted by party. Observing votes by party and region in the Senate as well as the House of Representatives, we observe that almost all Southern Democrats voted against the bill. We observe a truly bipartisan effort to pass the bill as well as a truly bipartisan opposition.

Healthcare also offers a striking example of bipartisan support. The Medicare program was presented in 1965 by a Democratic president and involved a single-player health-care system for the elderly, a quite liberal idea. Nonetheless, it received 70 Republican votes in the House and 13 in the Senate.

As we move further along, in the 1980's we observe that differences and ideologies start to sort by party. In the 1950s and 60s parties are quite heterogeneous. In the USA, in the Republican party there is the so-called ≪liberal Rockefeller≫ wing and the Southern Democrats are more on the conservative side. But in the 1980s and more so in the 1990s, we observe a slow but steady shift of the political elites from one party to the other, leading to extremely homogeneous parties. Diversity within parties starts to vanish.

In Europe, towards the end of the 1980s we witness one of the most major post-war events, the fall of the Berlin wall. Germany is finally reunited. So, we see the end of the geographical division (like with the Civil Rights Acts in the USA, where, in a way, we have a North vs South division instead of Democrats vs Republicans) but slowly but steadily we observe a sorting of a kind, along political party lines within the country.

Matthew Dowd, Bush's chief campaign strategist in 2000, came to the same conclusion with his own research on data regarding undecided voters and what they actually voted for in the end for the 2000 elections. He then compared it to what had happened in the past five to six elections. What Dowd found was that, by his own calculations, the percentage of actual independent voters had fallen sharply from 22% to only 7%. This was a finding of great importance as it changed the game; it was no longer important to convince the undecided, there were not that many of them left anyway. What would win the elections was to motivate voters that were going to vote for you, to actually go and vote (given the fact that in the USA voting is not compulsory). This of course, had a dramatic effect on how Dowd run the 2004 campaign, which won Bush the election: instead of trying to win over the undecided median voter they did their best to excite Republican voters and convince them that Bush was a Republican that Republican voters would appreciate (in contrast to the 2000 campaign which was focused on convincing undecided Democrats that Bush was a Republican president that Democrats would appreciate) (Klein, 2020). The findings of Glaeser *et al.* (2005) confirm that this tactic had a statistically significant effect on voter turnout in 2004. The Republican platform in 2004 regarding the divisive subject of abortion read:

> That is why we say the unborn child has a fundamental individual right to life which cannot be infringed. We support a human life amendment to the Constitution and we endorse legislation to make it clear that the Fourteenth Amendment's protections apply to unborn children. Our purpose is to have legislative and judicial protection of that right against those who perform abortions. We oppose using public revenues for abortion and will not fund organizations which advocate it [Republican Party Platform 2004].

This is a very clearly, very conservative opinion, leaving no doubt for any Republican voter to assume that Bush might even, in the farthest, deepest corner of his mind, be pro-choice.

Additionally, this sorting, meaning that most of liberals are Democrats and most of the conservatives are Republicans, also affects people's opinion on non-political matters, as can be seen from above excerpt of the Republican platform regarding abortion, or other issues, like for example gun control. The results of the 2008 ANES confirm this clearly. Regarding abortion, only 23% of those identifying as conservatives claimed to be pro-choice compared to 70% of those who identified as liberals. Regarding gun control, only 32% of those identified as conservatives were in favor of stricter control compared to 60% of those identifying as liberals.

The point being made here is not that the distribution of public opinion on all of these (social, political, economic, cultural) matters has changed but the consistency of opinions across issues and the relationship between these issue positions and party affiliation has changed dramatically. The groups are more and more homogeneous and members of each group have started sharing opinions on a large variety of matters, creating a stronger identification sentiment amongst group members.

This point can easily be ascertained by observing the evolution of the candidates' campaigns, like we saw in the case of Bush Jr. One can observe how some matters were approached in a more moderate manner in the past (or sometimes even closer to the opposite side's ideology). Another prominent example are the Clintons.

Bill Clinton's campaign in the 1990's was much more moderate, in terms of the extremity of his positions on for example fiscal policy and abortion, than was his wife's, Hillary Clinton, campaign somewhat 20 years later. Bill Clinton, in his campaign claimed that he would ≪reform welfare and balance the budget≫ and regarding abortion he stated that it should be ≪legal, safe and rare≫. When Hilary Clinton run for president, she ran more than Obama than like her husband, or actually even more to Obama's left (who was already on B. Clinton's left) (Klein, 2020). Regarding Obama, we have another striking case of homogeneity in votes in the case of the Obamacare program. Despite the fact that it relied heavily on Mitt Romney's reforms in Massachusetts and was constructed on conservative ideas (like relying on private insurance for the bulk of its coverage), the program did not receive a single Republic vote in either the Senate or the House (quite a difference with the, of much more liberal ideology, Medicare program in 1965) (Klein, 2020).

The increase of political polarization is reflected not only by the intense sorting of opinion on all matters around party lines but also by the increase in the popularity of populist voices.

In a polarized environment politicians tend to propose more hardcore solutions, radical programs and policies. It could be described as the right getting more right and the left becoming more left and so the cleavage between them is growing ever bigger. The public seems to move in the same direction; the division between party supporters seems to be growing deeper and deeper and this does not seem to be confined only to the most informed (although the phenomenon is even stronger there) but it extends to large segments of the public (Abramowitz and Saunders, 2008). The latest severe example of the manifestation of feelings of hatred and hostility is that of former US president, D. Trump, who followed an extremely polarized rhetoric, particularly exacerbated after the loss of the presidential elections of November 2020. In the end, his provocative claims of voter fraud and his questioning of the elections' results led to an armed riot and invasion of the US Capitol by Trump's supporters, which cost the lives of five people, on the day of the Electoral College vote count to formalize Biden's victory.

Characteristic of the deep divide in the USA today are the results of the University of Massachusetts Amherst and YouGov national poll, conducted in December 2021. Still, only 58% of Americans believe that Biden was (definitely or probably) legitimately elected with the remaining 42% still in doubt about the procedure and the legitimacy of the results.

President Biden, on the other hand, in an attempt to recapture political momentum and support from moderate voters horrified by the harsh partisanship and extremist rhetoric that crested on Jan. 6, directly blamed former president D. Trump, in his speech on January 7th, 2022, for the insurrection at the U.S. Capitol one year earlier. President Biden is watching his approval rates sag: according to ongoing Gallup Analytics, J. Biden's approval is steadily diminishing since he took office, reaching 43% in the first two weeks of December, from 57% when he took office. Engaging in this divisive rhetoric is the simplest way to increase those rates.

Last but not least on the subject of presidential campaigns and how things have shifted in the last 3 to 4 decades, current USA's president, Joe Biden, has stated, in respect with the Republican's party effort to overturn the decision in *Roe vs Wade* that ≪It is wrong. It is pernicious. We have to stop it≫. This is in total line with the Democratic Party's line on the subject. But, in 1982, as a US senator, he voted for a constitutional amendment that would do exactly that. He called it, at the time, ≪the single most difficult vote≫ he had cast as a U.S. Senator and blamed it on his Catholic upbringing.

This sorting of opinions on various topics across party lines is the major difference between today and the post-WWII era. Societies are getting more and more polarized and it seems as if every issue that comes up nowadays only contributes to this phenomenon. The most recent example of this is the understanding of and reaction towards the current COVID-19 pandemic. We see, again here, that people's beliefs around an extremely serious matter are neatly sorted along party lines. There is growing evidence that, despite calls for political consensus, the public response to the COVID-19 pandemic has been politicized in Europe as well as the USA.

The Pew Research center published two social attitudes reports, both in 2020 as well as in

2021. Both reports show that the people's perception of how their country handled the crisis as well as their opinion on how strict the restrictions should have been are quite politicized. Regarding Europe, in 2020 the findings show that, in most countries, people believe that their country has handled the COVID-19 pandemic well. But a closer look into those numbers shows a division of the people. In all countries researched, majorities of governing party supporters, believe their country handled the pandemic well. Regarding Europe, France, Spain and the UK have the leading difference among countries researched; In France 89% of governing party supporters find that the government did a good job handling the pandemic as opposed to only 55% of non-governing supporters, in Spain the numbers are 74% and 40% respectively and in the UK the percentages are 70% and 37% respectively.

In June 2021, two years well in the pandemic, the Pew Research Center report revealed that the divide got deeper. In most European countries researched, majorities still believed that the government did a good job in handling the pandemic but in all countries those majorities have shrunk (in some countries no longer being a majority). Additionally it revealed the trend that supporters of far-right parties tended to be in favor of fewer restrictions in all countries researched but one, namely Greece. For example in Germany 37% of those on the right of the political spectrum are in favor of less restrictions, whereas only 17% of voters on the left favored less restrictions, a difference of 20 points. Italy, Spain and the Netherlands follow along the same lines; with the differences being 17%, 15%and 11% points between proportions of voters on the right being in favor of fewer measures and proportions of those on the left. In Greece the situation is reversed, with the majority of voters on the left in favor of fewer restrictions, 55%, and only 34% of voters on the right in favor of fewer restrictions. These numbers are probably driven by the left-populist Syriza party supporters (55% of Syriza's supporters wish for fewer restrictions), whose leader, Alexis Tsipras, has accused the center-right government of taking advantage of the pandemic in order to suppress democratic rights. We observe that again opinions tend to sort around party lines and that party leaders seize this opportunity to increase their share of votes.

According the same reports by the Pew Research Center, the divide in the USA is even larger. In the 2020 report it was found that 76% of Republicans or independents leaning towards the Republican Party believe that Trump's administration had done a good job dealing with the corona-virus outbreak, whereas only 25% of Democrats and those leaning towards the Democratic Party share that opinion. This trend continues in almost the same levels in 2021; 52% of voters on the right part of the political spectrum believe there should have been fewer restrictions in the course of the pandemic contrary to only 7% of voters on the left.

Momentarily the largest dispute is about the COVID-19 vaccine. Even in this case, societies seem to be divided and the tension seems to keep rising, as the pandemic does not seem to end. Of course, even though the matter of the vaccination is not a purely political one, politicians (of the governing party) are the ones deciding on the measures that are implemented and the oppositions try to capitalize on the people's discontent. There seems to be a lack, nowadays, of coalitions in the name of the *common good*.

This is exacerbated by politicians' rhetoric. Biden's administration's phrase "pandemic of the unvaccinated" implies that now the new (political) enemy is no longer COVID-19 but the unvaccinated. It seems that the battle has shifted; instead of us (all of us) against the virus it has turned into us versus them, where us in this case in Biden's voters / Democrats / the vaccinated and them, the outer group, which is consisted of Trump voters / Republicans / the unvaccinated. A national tracking poll by Morning Consult and POLITICO, shows that as of the second week of September 2021, the majority of the vaccinated were Biden's voters (55% of those having received at least one dose of the vaccine had voted for Biden in the last elections contrary to only 28% of the vaccinated Trump voters) and the leading group of the unvaccinated were Trump's supporters (42% of the unvaccinated had voted for Trump and only 22% had voted for Biden). These results confirm that unfortunately what Americans vote is, at some level, correlated with their decision to be vaccinated or not. A matter of public health is affected by political affiliation. Again, difference of opinion has always been there, that's not something new. But what strikes us nowadays is that difference of opinion surrounding, what could be characterized as, a non-political issue tends to be sorted according to partisanship or political affiliation.

To sum up, we observe different cycles of partisanship and political polarization. In the 1st era, let's say roughly after the end of WWII and during the 1960's and 70's, we see a more pragmatic partisanship where boundaries would shift and as a result, we would see coalitions along party lines. Some major examples of this are the Medicare bill, which was passed by Republicans as well as Democrats and the Civil Rights Act, of which several Republicans were leaders in passing. The 2nd era, which is a bit more recent, i.e. the 1980s and'90s as well as perhaps the first decade of the 21st century, we see a sorting of liberals and conservatives into the Democratic and Republican parties, respectively. This has led to a conflation of parties and ideologies (Democrat and Republican have nearly become synonyms for the term liberal and conservative). We start to observe a profound divergence in ideology. Nowadays, we observe a 3rd evolution in partisan attitudes, where we see animosity (perhaps even hatred on some occasions) felt towards opponents, outsiders, or simply anybody who does not share our point of view. There is a sense of fear and loathing across the party divide (Iyengar and Westwood, 2015). Already from the early 1990's political speeches are characterized by a harsh and critical sentiment towards opponents. Today studies show that, by some measures, partyism (the devotion to a political party; party spirit) exceeds racism (Iyengar and Westwood, 2015). We observe that nowadays there is party prejudice, and it has penetrated many aspects of life, personal as well as professional. Spousal agreement on political affiliation has increased over the past 50 years, as has inter-generational agreement between parents and children (Iyengar *et al.*, 2015). So we see that this sorting we have been talking about has spread at all aspects of our live, even the most personal ones.

We do not have bigger ideological differences today than we did in the past. People have always disagreed on what we should do or what is the right thing for the government to do, but today those differences are neatly sorted along party lines. For example the difference over issues like: ≪Is segregation by law okay or even a commendable thing to have?≫ is a much more dramatic ideological difference than is most of what we are dealing with today. But we observe a truly bipartisan effort on the passing of the Civil Rights Act as well as a truly bipartisan opposition. The difference is that today a lot more of our distinctions drop across party lines. People have always been disagreeing and there have been much more dangerous moments of societal fracture than the one we are currently living in. The dynamic of political conflict has changed. It is not that what people fight over today did not exist 50 or 80 years ago but back then these conflicts existed as cleavages within the parties as well. And when a cleavage exists within a party, the party will try to compromise it or suppress it. But when it is between parties, then it is escalated and it can be used, it can be capitalized on, to increase share of votes. It is as if today all ideological diversity within parties is basically gone.

1.2 Literature Review

Based on the classic concept of social distance by Bogardus (1947) and of social identity by Tajfel (1970) and by Tajfel and Turner (1979), Iyengar *et al.* (2012), point out the importance, not only of the sentiment of belonging to a group, but also of the negative sentiment towards members of opposing groups and outsiders.

This creation of extremely homogeneous parties has moved us even a step further, in what S. Iyengar *et al.* (2012) call ≪affective polarization≫, a term coined in their seminal paper *Affect, Not Ideology. A Social Identity Perspective on Polarization* . We observe that today people do not only feel attracted to their own group and to people similar to them, they seem to feel an extreme hostility towards the members of the outer group, the opposition or however we choose to call it.

This definition, one could perhaps argue, better conceptualizes what McCarty *et al.*, (2016), imply when they say that there are no longer any skilled coalition builders in politics. The problem is not just that the right moves more to the right and the left more to the left, it is also that the members of one party have started to feel hostile towards members at the other end of the spectrum. This sentiment of hostility, alienation, rivalry, of not wanting to have anything to do with those on the *other side* (regardless which side you are on) makes it impossible nowadays for functional coalitions to emerge and for the sense of obligation towards the common good to prevail.

In this respect, Iyengar *et al.* (2012), show that mass polarization has increased steeply during the last decades in the USA. They question the assertion of some of the scholars of political polarization that this is a matter that only concerns the political elites. Iyengar *et al.* (2012), triggered by the notion of group psychology and the common finding that members of groups, even when groups are created by trivially assigned characteristics, create a sentiment of belonging amongst the members and of dislike towards members of opposite groups.

Iyengar *et al.* (2012), research concludes that affective polarization (in the sense of negative feeling towards other groups) caused by political affiliation can de as strong as, or even stronger than, the one created by other prominent social issues like gender, race and religion. The main source behind this strong negative sentiment towards *outsiders* seems to be primarily triggered by the recent trend of negative advertising as well as negative campaigning (i.e. campaigns focused on a great deal on defaming the opponent and his supporters rather than focusing on positive traits of the running candidate himself). One potential explanation for the increase of mass polarization is the increase of exposure to this negative campaigns through news (where journalists seem to perpetually repeat any provocative, stressing or negative messages), media and the internet. The results of Iyengar *et al.* (2012), indicate that while the within party affect remains somewhat unchanged, the affect towards the outsiders has increased a lot. Iyengar *et al.* (2012), also check for the assertion made by Fiorina *et al.* (2005) that it is only the polarization within partisans which increases, and while they find that indeed polarization among partisans increases, so does mass polarization as well (at a slightly lower rate) and at the same time they find that the number of partisans has increased sharply.

The sorting that has occurred in the last 50 or so years has brought partisan and ideological identities into alignment, creating strong feelings of identification and alliance among the members of each group but at the same time it has increased the levels of partisan bias, of anger towards members of the out-group (Mason, 2015). As Mason (2015) suggests, a partisan in a polarized environment, acts more like a sports fan, in the sense that he cares about the well-being of his party, he prefers to associate with members of his own party and feels angry (and can even manifest in a violent way) when he feels his group is being threatened, even though he may not agree with the group on all matters per ce. The intensity of a person's partisanship may increase due to various reasons, like for example a person's own personality, personal experiences, social networks etc. According to Roccas and Brewer (2002) another factor that intensifies bias towards in-group members and hostility towards out-group members is the level of identity complexity of the individual, meaning how many of his or hers social identities are met within one sole group or does the individual connect with various different groups, depending on a different connecting criterion each time. In the neatly sorted groups, one person's various social identities (for example race, religion, economic status etc) overlap. This creates a stronger identification sentiment among group members. Group members are characterized by low identity complexity which, according to Roccas and Brewer (2002) results in lower tolerance of outer groups. The research by Mason (2015) confirms that sorting in homogeneous groups has a substantial effect on anger and bias. Even more, Mason's research shows that the intensity of the identification sentiment with one's party affects how biased and angry that person is, even (and I believe that is the most important point) if that person's issue positions are moderate. This confirms what is mentioned above, meaning that it is not that people did not disagree in the past or that the matter on which the disagreement evolves around are more important today, it is the homogeneity of the groups that has intensified the us versus them feeling of hostility. As per Mason (2015) the alignment of ideology and partisanship intensifies the bias, activism and anger of the individual. So even without change in the issue opinion's distribution, the electorate can be more polarized and regard the out-group partisans with increasing prejudice.

This increasing prejudice and bias, described in one word by the term ≪partyism≫, is recent subject of ongoing research by many scholars of the political science.

Based on the findings of data of various surveys, Iyengar *et al.* (2012), find that Americans, increasingly, dislike, even loathe, their opponents. According to their research, the loathing between members of the various groups does not evolve (focus) only around the policies proposed by each party. Political campaigns play a very significant role in the shaping of today's polarized environment. The polarized political rhetoric helps exacerbate the problem. Hostile messages towards the outsiders reinforces partisan's biased opinions. They rely their measure on the level of dislike between opposing groups' members. As per the theory of Tajfel (1970) the positive sentiment for one's own group alone is not enough, but the negative sentiment for the out-group must be taken into account as well. According to Iyengar *et al.* (2012) simply identifying with a political party suffices for creating negative feelings for the opponent's followers. Political campaigns play a significant role in this. Older studies of social psychology have already shown that any form of group membership instantly creates positive feelings for one's own group and negative ones for the outsiders (Tajfel *et al.* (1971); Billig and Tajfel (1973)).

The study Iyengar *et al.* (2012) shows an increasing dislike between members of groups with different political affiliation in the USA, for both activists as well as not activists (with the former presenting a somewhat stronger trend). As mentioned above, this increase in polarization has penetrated many aspects of social life. In their study, Iyengar *et al.* (2012), find an increase in inter-generational political agreement over the past fifty years for both the USA and the UK. According to the 1960 Almond-Verba survey shows that in that period only 5% of Republicans and 4% fo Democrats would feel ≪displeased≫ if their son or daughter married someone of a different party affiliation. At the same time, the respective percentage for the UK are 12% for the Conservative party supporters and only 3% for followers of the Labor party. Fifty years later those number have increased for both countries, with the USA leading the difference. More specifically, according to the YouGov survey in the USA in 2008, 27% of Republicans and 20% of Democrats would be ≪very≫ or ≪somewhat upset≫ with the possibility of an out-party marriage for their child. In the UK the corresponding percentages were 10% and 19% for the supporters of the Conservative party and the Labor party respectively.

The increase of polarization has also affected family socialization. Iyengar *et al.* (2018) find an increased marital agreement as well as inter-generational agreement (between parents and offspring). In the past 50 years, from 1965 till 2015, Iyengar *et al.* (2018), using data from the YouGov survey find that spousal agreement has increased by 8%, from 73.2% to 81.5%, and inter-generational has increased by 6%, from 68.6% to 74.2%. While the increase in marital partisan agreement might seem relatively low, if we further examine the characteristics of the interviewees, we will realize that the difference is quite important. In the 1965 sample couples with lengthy marriages are over-represented (all couples had children in high school). This is not the case for the 2015 sample, where the higher percentage of agreement is due to assortative mating. According to a press release published in June 2020 by *Dating.com*, part of the Dating.com Group and the company behind numerous online dating sites, an analysis of their user data revealed that 84% of singles would not even consider dating some with opposite political views.

Polarization based on political affiliation is momentarily so strong in the USA that Iyengar and Westwood (2015) assert that partisan animus nowadays exceeds even racial hostility, which per general agreement represents the deepest divide in American society, at least up to now (Pager and Shepherd 2008, Schuman *et al.* 1997). They design and perform three experiments which show that partisan affect strongly affects judgment and behavior on non-political issues. They test the following hypotheses: (1) is partisan affect sufficiently embodied in citizen consciousness so that it is obvious in partisan attitudes, (2) does the size of the partisan affect exceed the size of affect due to other social characteristics where discrimination is discouraged by society (like, for example, race or gender), (3) does partisanship and partisan affect create bias in favor of co-partisans which leads to discrimination and affects judgments and (4) is favoritism towards co-partisans weaker than animosity towards outsiders. For the testing of these hypotheses they use an Implicit Association Test (IAT) as well as its Brief version (BIAT).

For the first question Iyengar and Westwood (2015) compare the time people require to implicitly respond to pairings of ≪ingroup + good≫, ≪outgroup + good≫, ≪ingroup + bad≫ and ≪outgroup + bad≫. For example it should take a Democrat less time than a Republican to dissociate the democratic mascot with the word ≪good≫. They, afterwards, use the widely accepted ≪D-score≫ (detailed description of its computation at Greenwald *et al.*, 2003) to interpret the results of the BIAT times. The results show a strong polarization, in the sense that Republicans are strongly positively biased towards other Republicans and Democrats are also positively biased towards fellow Democrats, yet to a lesser extent than Republicans. The D-scores, when the same experiment is conducted using pairings of racial in- and out-groups, are surprisingly lower than the partisan D-score. According to Iyengar and Westwood (2015) the magnitude of this difference reveals a lot. For the partisan bias to reach, and even surpass, the racial bias unveils a substantial underlying animosity.

For the second hypothesis Iyengar and Westwood (2015) designed an experiment where candidates would have to make a non-political choice (offer a scholarship to a student). The study was designed so that participants had to choose between two candidates of the either the same academic achievements or varying. This allowed for partisan and racial bias to be measured both when candidates were equally qualified as well as one candidate was better qualified than the other. The race was indicated by a stereotypical African/American or European/American name and a particular membership in an extracurricular group. Partisanship was indicated by membership in an extracurricular partisan group. According to the results, it was the indicator that revealed the partisanship that exerted the strongest impact on the selection of candidate. Almost 80% of the time a Democrat participant or a Repuiblican one, would choose to choose to grant the scholarship to a candidate of the same partisanship. Even when accounting for candidate qualification, still every participant would more often than not, choose a candidate of the same partisanship. Specifically, when the Republican candidate was more qualified there was only a 30% that the Democrat participant would grant him the scholarship. Similarly, when the Democrat candidate was more qualified, the probability of the Republican participant choosing him was only 15%. Regarding bias based on candidates' race, the results show a lower effect compared to partisanship! In general most European Americans as well as African Americans selected the African American candidate during the experiment. In-group preference did not have as large an effect in this case as it did in respect with the partisanship criterion. An African-American participant would choose an in-group candidate at 76% and 78% chance in case the two candidates were equally qualified or in case the in-group candidate was less qualified, respectively. European Americans would choose an in-group candidate a little less than half of the time (42%) in case both candidates were equally qualified. When the in-group candidate was less qualified then the probability fell to 29%.

Regarding the question if the bias created by co-partisanship affects partisans' judgment, Iyengar and Westwood (2015) perform a dictator's game and a trust game on a group of 814 individuals. The results show that in both games co-partisans favor each other. In the dictator's game players allocate, on average 24% more money in case they are dealing with a co-partisan and in the case of the trust game that percentage was a mere 10%.

In order to check if the animosity towards out-group members exceeds favoritism for the forth hypothesis, Iyengar and Westwood (2015) repeated the dictator's game and the trust game, only this time player 2 could be 1 of following four: Republican, Democrat, Independent or described without any partisanship. Results indicated that indeed out-group animosity is more consequential than in-group favoritism. In both games, if player 2 belonged to the opposing party they were assigned, on average, the lowest amount of all cases and when player 2 was a co-partisan then he was, on average assigned the largest amount of all cases. Additionally, if player 2 was an independent or if there was no mention of his partisanship (which served as a control ID), player 2 was given, on average, the same amount in both cases, an amount lower than that given to the co-partisan and higher than the amount given to the member of the opposing party.

According to McCarty(2019) it is around 1977 - 1982 when the intense sorting of the political elites is starting to become obvious, in the USA, according to their analysis of the roll call votes. While during the period starting in the 1930s and throughout the 1960s we observe a somewhat steady trend in polarization, after the 1970s polarization is increasing. McCarty's (2019) data on roll call votes show the Republicans having shifted more to the right than the Democrats having shifted to the left, but this might be caused by the nature of the agenda of congress, which mainly focuses on issues of economic or regulatory nature. Social and cultural issues tend to be handled by the courts. This is to say that, if we had data on that too, we might observe a movement towards the left of perhaps almost similar magnitude of Democrats. In other words, the cleavage has deepened by an outward shift of both parties.

The widening of the gap and the increase in mass polarization is also supported by the data provided by the American National Election Study (ANES). As Abramovitz (Abramovitz and Fiorina, (2013)) points out between 1972 (when a question about ideology was first included in the ANES) and 2008, the correlation between the party identification scale and the ideology scale increased from .36 to .66 among all voters. Additionally the cleavage between the average Democrat and the average Republican widened significantly; on a 7-point ideology scale their distance increased from 0.9 to 2.2 units. The distance more than doubled, with the score of the average Republican of 4.7 and the average Democrat's score of 3.7 in 1972 going to 5.4 and 3.2 respectively in 2008. We observe that the Republicans, who already were well right of the center moved even more into the right and the democrats, who were slightly left of the center moved well into the left, in this 36 year period. During this same period the voters' distribution has become more clearly bimodal, with the voters in the center diminishing. The percentage of voters placing themselves in the center fell from 35% to 27% and the percentage of voters placing themselves distinctively in the left or the right increased from 29% to 46%.

These findings are in accordance with Abramowitz and Saunders (2008), who put Fiorina and Abrams' (2008) works to the test. While Fiorina and Abrams (2008) claim that there is no strong evidence that proves that the mass public is polarized, Abramowitz and Saunders (2008) knock down their arguments one by one, asserting that the American people have become increasingly more polarized from the 1970s and onward. This assertion is true for both the politicians as well as the public, though the phenomenon is more intensely observed among the most informed, interested and active citizens. Additionally Abramowitz and Saunders (2008) find that ≪Conservative Democrats≫ as well as ≪Liberal Republicans≫ are becoming more and more difficult to find, deepening the division between the left and the right. Additionally, the change of various demographic characteristics, like for example the higher education level of voters, has perhaps aided to this end.

As the study of Panagopoulos (2016) points out, campaign strategies have shifted from trying to persuade undecided voters to vote for their party to strategies which aim at mobilizing the base and increasing turnout. The same point is also being made by Smidt (2017); as was exemplified by the 2004 and 2012 winning campaigns, it is a better strategy to focus on mobilization of people that have already made up their mind than to try and persuade the, steadily less and less, undecided ≪Floating American≫ voters. This is so, because, as the study showed, between 2000 and 2004 self-proclaimed independent voters were more consistent in which party they supported than were self-proclaimed partisans from 1972 to 1976!

Closely related to the claim of McCarty *et al.* (2016), that in the years following WW II, politicians and vote-maximizing strategies used to move around the center of the political spectrum, as years go by it seems that politicians are moving in the opposite direction. It seems that the public today is more intrigued by the ≪edges≫. As per Downs' (1957) central hypothesis, parties form their policies with sole purpose to gain votes (and consequently office) and not vice versa (i.e. try to win so as to implement the, what they they consider to be right and beneficial, policy). In other words, the policy is the *good* sold for votes (like a firm sells products for revenue) and it is the objective of every party to maximize votes. While the objective remains unchanged, the means have changed; no longer do campaign and policies of parties (even in a two party system) try to resemble each other. Quite the opposite actually; candidates spend more and more time and money in the effort to distinguish themselves from their opponent (Glaeser, 2005).

Glaeser (2005) attempts to develop a model to explain ≪the political economy of hatred≫. Glaeser (2005) constructs a supply and demand model for hatred animosity is cultivated by some groups towards out-groups. While hatred towards the out-group is often based on false news (like stories of false crimes, falsely attributing evil characteristics etc), in reality the relationship between hatred and the criminality of the targeted groups is often minimal. The model developed by Glaeser (2005) is a supply and demand model of hatred, where politicians supply hatred through the spread of hate-creating stories and the people are on the demand side, which increases when the people do not have any incentives in investigating the truth behind the stories supplied and just take them as granted. For example, depending on which side of the political spectrum the politician is, he will follow a certain policy to handle economic inequality. In order to back their case, they supply hatred-filled arguments which voters will investigate only in the case they have a personal benefit from finding out the truth (Glaeser, 2005). The general prediction of the model is that candidates who are opposed to redistribution will spread hatred towards poor minorities while pro-redistribution candidates will spread hate-creating stories towards the rich elites. As the minorities income changes and they become either more rich or more poor, the incentive of the candidates to spread hatred will increase, as the effect of redistribution on the resources of the minority gets larger. The demand of hatred depends on the private incentives of the minority to find out the truth about the out-group. For example, strong integration on a minority into a society, increases the benefits of knowing the truth and there for makes its more difficult to spread hate-creating stories. Glaeser (2005) also examines what happens in case the hated reply with hatred and vicious circle of ≪hating the haters≫ is created. According to Glaeser (2005) changes in technology can affect the level of hatred in both directions, as on the one hand it makes it quite easier to spread the hate (as we clearly witnessed happening during the Trump campaign) but at the same time it makes it easier and less costly to investigate the truth of the story.

Glaeser *et al.* (2005) construct a model to predict when it is in the politician's self-interest to resort to extreme rhetoric, what Glaeser *et al.* (2005) call ≪strategic extremism≫. The thinking behind this is close to Dowd's conclusion, in 2000. Just as Dowd figured out, there must be a ≪intensive≫ margin, i.e. a respectable number of partisans who know what they would vote for but are not motivated enough to actually go and cast a vote. Then the extreme rhetoric aims at motivating them and convincing them that the matters are serious enough so that it is worth the voter's trouble to vote. Additionally, there must be enough ≪extensive≫ margin, i.e. the extreme to which the rhetoric can extend to should be such so that it will cost the politician as few as possible votes of his median voters. Of course, for this strategy to be profitable, the cost in median voters cannot exceed the gain in mobilized voters. They key assumption of Glaeser *et al.* (2005) 's model is that a politician's campaign and rhetoric reach first of all his own supporters. The new, extreme, message may never even reach the out-group member's ears. This means that this strategy affects the politician's own supporters more than the out-group's members. The model aims to explain in a formal way the motives for the formation of a strategy like the one designed by Dowd, which won Bush the elections: is it plausible that politicians adjust their rhetoric to what voters need to hear in order to motivate them to vote and increase turnout. If the audience is more sensitive to immigration subjects then the rhetoric will evolve around that. If the audience, on the other hand is more sensitive regarding economic matters like rising inequality and low wages, then the rhetoric will evolve around redistribution policies and protection of minimum wages. Glaeser *et al.* (2005) focus their study on the interrelation between extremism of political campaign and political platforms regarding the subject of abortion and the change in voter turnout.

This point can easily be ascertained by observing the evolution of the candidates' campaigns. One can observe how some matters were approached in a more moderate manner (or sometimes even closer to the opposite side's ideology). Examples of this were presented in the previous section.

Language reflects many things. The way people speak, which words are chosen, active or passive voice for certain actions, they all reveal how a society thinks and feels about many things. Among other things, language reveals tension, hostility and, of course, polarization. Gentzkow *et al.* (2019) analyze speeches of congressmen and how easy it is to correctly predict their partisanship after hearing only one minute of their speech. To do so they use the text of the United States Congressional Record starting with the 43rd Congress (which met in Washington D.C. from 1873 till 1875) to the 114th Congress (which met in Washington D.C. from 2015 till 2017). Performing an analysis using key words representative of each era, they find that around the early 1990s speeches begin to get more polarized. A characteristic example of that time is the 1994 Republican campaign, when campaign consultant Frank Lutz helped develop the ≪Contract with America≫ slogan. It was the election than won the Republicans majority in the House for the first time after more than forty years.

Boxell *et al.* (2020) study the increase of political affective polarization in 12 OECD countries, namely the USA, Switzerland, France, Denmark, Canada, New Zealand, Japan, Australia, Britain, Norway, Sweden and Germany. They examine the evolution of polarization starting from the 1980's till now using data from 149 different studies. The fact that all the data does not come from the same source creates the need for data harmonization with the accompanying limitations (for example different wording of questions can extract different answers on the same subject). They use surveys which include questions like how much a person likes a given party, how they feel toward it or if they sympathize with it and scale the responses (so data analyzed is homogeneous) with 0 being the minimum response and 100 being the maximum one. They construct a model using information on the set of parties in given country. Respondents answers are utilized only when they provide a valid party identification as well as a valid affect towards at least one more party other than the one the claim to be affiliated to. Boxell *et al.* (2020) calculate the partisan affect π_i, which expresses respondents' more favorable attitude towards their own party than toward the other ones. Following this, Boxell *et al.* (2020), calculate affective polarization as the weighted average of respondents' partisan affect.

Boxell *et al.* (2020) also study the correlation between affective polarization and a number of explanatory variables like openness to trade, share of getting news online, fraction of foreign-born, fraction of non-white and a measure of ethnic fractionalization. For most of these they find a weak or a negative association. Regarding economic inequality, they use the Gini coefficient and find that only two countries (namely Switzerland and France) have a negative relationship. All remaining countries exhibit a positive relationship, as inequality rises so does polarization. Especially the USA and Australia exhibit a quite strong correlation (the USA with a steeper slope of 0.09 compared to Australia, where the slope is 0.15).

Among other things, they restrict their sensitivity analysis of their findings on the years when the top two parties of a country remain the same. Additionally, responses to questions about how much a person likes a given party, how they feel toward it or if they sympathize with it are standardized to a 0-100 scale with 0 being the minimum response and 100 being the maximum one. They find the largest positive trend in the USA. They also find a positive trend in 5 other countries, namely Switzerland, France, Denmark, Canada and New Zealand. Switzerland's trend is near the USA's level but the finding is statistically significant only for Denmark. The remaining six countries (i.e. Japan, Australia, Britain, Norway, Sweden and Germany) all exhibit a negative trend in affective polarization. The largest effect is found in Germany and this result statistically significant. It is the only significant negative result together with that of Sweden.

In their 1994 paper, Esteban and Ray, analyze how a society divided into groups based on income becomes economically polarized (they use this term to differentiate their index from the Gini coefficient of economic inequality). The main assumption is that the members of each group are a lot alike but the various groups are quite dissimilar amongst them. The stronger the inter-group *alikeness*(the resemblance of the group's members, if one prefers, based on a specific characteristic or trait, like for example, their political self-placement) as well as the intra-group *dissimilarity* (how far, for example, politically speaking one group places it self from the others), the higher the polarization. These feelings of belonging and hostily are translated into a ≪Identification - Alienation≫ (IA) framework. The density of a group accounts for the *Identification* part and the distance between groups represents the *Alienation* sentiment. The combination of the two is translated into ≪effective antagonism≫. The sum of all *effective antagonisms* (so *alienation* between all possible pairs of groups and their respective *identifications*) constitutes the country's polarization. Based on this narrative, Esteban and Ray (1994) construct a theoretical model, where polarization increases with the higher density within each group as well as the greater distance among groups. In their subsequent paper Duclos, Esteban and Ray adjust the model so it can be applied to continuous variables. This index is applied in this study on data of self-placement of voters in order to assert the level of political polarization.

In this study, we adopt said polarization index which takes into account both the distance between groups but also the density of each group, as both seem to be playing an important role. In other words, we calculate a weighted average. We take into account where each voter places himself and the distance between the average voter in the left and the right respectively, but this average is weighted by the number of members in each of these clusters.

The importance of homogeneity within groups of same ideological orientation and how it can affect sentiments towards the outsiders has been extensively discussed and proven in many studies (for example Iyengar *et al.*, 2012, Iyengar *et al.*, 2019). A more detailed description of the index, its derivation and the intuition behind our choice of this specific index is presented in section...

As per Benabou (2008) an ideology can be a mixture of beliefs, a bundle of issue-positions, which is usually placed somewhere along the left-right political axis. But, in reality, it seems that people might hold opinions on various matters from both sides of the political spectrum (Converse 2006, Kinder and Kalmoe 2017). Additionally, the recent surge of populist voices further complicates the issue, as it is not clear how to handle these in relation to the simple left-right scale. To this end Draca and Schwarz (2021) use the Latent Dirichlet Allocation (LDA) topic models (Blei *et al.*). They apply the model on data from individual-level surveys on a typical set of socio-economic issues. This allows for a mixture of ideologies to be explored as topic models, which are usually used in the social science on text data due to their ability to identify a latent topic structure, which may underpin the data. Draca and Schwarz (2021) apply the LDA technique to the individual-level responses (instead of using text analysis). The latent topics are then interpreted as the political ideology which creates individual political beliefs in the general public, allowing this way for a calculation of mass polarization. The framework has two main advantages: it provides information on the individual-level ideology mixture (the share of each type of ideology in one person) as well as the nature of the ideologies (the type vectors). With this information it is possible to predict the probability of a person taking a certain position on an issue, given his ideology mixture. With above methodology, Draca and Schwarz (2021), aim at answering two main questions. Firstly, can the public's political beliefs be summarized in a statistically coherent fashion and does this conform with the typical left-right axis, so widely used both in popular discussions as well as formal models. Secondly, how has the public's ideology evolved over time and among countries. The data used for above-described analysis is drawn from the World Values Surveys (WVS) and results reveal that indeed there exists a series of coherent citizen ideologies. The ideologies are consistent with the traditional left-right axis as well as a for- and anti- establishment classification. The latter aims at describing the populist voices that have emerged and gained popularity recently and have been the focus of new studies. The model reveals four types of ideologies: ≪Liberal Centrist≫, ≪ Conservative Centrist≫, ≪Left Anarchist≫, and ≪Right Anarchist≫. Regarding the questions of their research, Draca and Schwarz (2021), firstly find that over time the ideologies have varied little and that they are relatively stable. The only change worth mentioning is an increase in the intensity of socially liberal attitudes across most types. Draca and Schwarz (2021), find for example such a shift around matters of homosexuality and abortion regarding the Conservative Centrist group. Secondly, regarding the evolution of the distribution of political opinion within a country over time, Draca and Schwarz (2021), find that things remain relevantly stable over time, with exception of the US. They find that northern European countries are more liberal where as in countries where religious ties are strong, people tend to be more conservative. Specifically, Draca and Schwarz (2021), find that the aggregate type shares remain relatively stable within each country but in the USA the Anarchists type's shares increase from 30% in the 1989-1993 to 50% in 2005-2009 (when the fifth wave of the WVS is conducted). Additionally the model reveals a high correlation between the types share and the ≪left-right≫ self-positioning of interviewees. Furthermore, the model predicts that an individual with more than 50% type share in either one of the Anarchist type is 38% more likely to vote for a populist party. The above-mentioned results refer to the total share of types in a population. More analytically this means that, for example, in country A we may find 50% of the ≪Liberal Centrist≫ and 50% of the ≪ Conservative Centrist≫ types. This can mean that in country A either half of the population is 100% ≪Liberal Centrist≫ and the other half is 100% ≪ Conservative Centrist≫, (i.e. a society split into two perfectly homogeneous groups), or the entire population consists of people who's type is a vector of 50% of the ≪Liberal Centrist≫ and 50% of the ≪ Conservative Centrist≫ types (i.e. a perfect homogeneous society were all people are equally as much≪Liberal Centrist≫ as well as ≪ Conservative Centrist≫) or all possible in between combinations. In other words, we have no indication of the societal tensions and the vulnerability of country A for political conflict. In order to clarify these findings, Draca and Schwarz (2021), construct a Gini based index to measure the within-person concentration in each country. The results of this analysis indicate two main results. Firstly that within-person concentration is relatively high in all countries and secondly that between waves 2 and 5 (in years 1989-2003 and 2005-2009 respectively) the changes are not very big. The country which presents the largest change is Denmark, Finland followed by the USA and Canada at the same level. They also put to work an analysis based on the ≪identification-alienation≫ framework by Esteban and Ray (1994) and Duclos *et al.* (2004) which allows them to develop a multi-polar analysis of ideology, regarding in-group identification and out-group alienation. They incorporate information regarding the group size in the information extracted by the application of the LDA model and study cross-country trends in polarization, which complements the work already existing on affective polarization. They find that, on aggregate, polarization remains the same with exception of the US once more, which is in this case better described by the notion of the disappearing center.

Reiljan (2020) introduces the Affective Polarization Index (API) in measuring affective polarization in 22 European Democracies and the USA between 2005 and 2016. The results indicate that affective polarization is stronger in Eastern and Southern European countries than in the USA and least present in the Northwestern European countries. Reiljan (2020) distinguishes the notion of affective polarization with that of ideological polarization, in the sense that the latter does not incorporate the notion of hostility towards the members of the out-group. Reiljan (2020) studies the correlation between the two and finds that there is a significant degree of correlation but it is conditional. In some cases (like for example in the West European political systems) ideological polarization does not lead directly to hostility whereas in Central Eastern European countries a high degree of hostility is present despite having ideologically centrist party structures.

Reiljan (2020) constructs the API to measure first of all affective polarization and then the measure is extended for application to multi party systems. Reiljan (2020) starts by defining affective polarization as the combination of the favorable evaluation towards the in-party members and the negative evaluation towards the out-party members. According to Reiljan (2020) the difference between the two types of polarization is that in the case of affective polarization the attributes comes from the direct attitude of partisans towards own party members and outsiders, where as political polarization, as defined by Esteban and Ray (1994) is determined by the distance between the groups (alienation) and their within homogeneity, whose intensity increases with the size of the group (identification).

For measuring the API in the USA things are quite simple since there are only two competing parties. Data can be used from the American National Election Study (ANES), where a question of how the respondent feels towards the opposite party, based on a «thermometer» scale, ranging from 0 to 100 degrees, where 0 indicates most negative attitude and 100 indicates the most positive one. However, in the multi-party case of Europe, things are not that simple, as data needs to be collected regarding the sentiment towards one's own party as well as all other parties participating in the electoral process. The API measure, in order to be objective and usable for cross-country comparison, must take into account not only the respondent's attitude towards all parties involved but the respective size of the parties in question matters as well (as Esteban and Ray (1994) mention, all parties are assigned weights relevant to their size so as to make sure that insignificant outliers don't alternate the index).

The process of Reiljan's API is comprised of two steps.

First the relative affective polarization of one party is calculated as follows:

$$AP_n = \sum_{\substack{m=1 \\ m \neq n}}^{N} \left[(Like_n - Like_m) \times \left(\frac{Voteshare_m}{1 - Voteshare_m} \right) \right], \tag{1.1}$$

where «Like» represents the thermometer rating described above, n refers to the in-party and m refers to the out-party. The $1 - voteshare$ is necessary so as not to avoid calculating the in-party twice (so that all shares add up to 100 per cent).

The total affective polarization is the weighted sum of all relative affective polarizations described by above equation:

$$API = \sum_{n=1}^{N} (AP_n \times Voteshare_n). \tag{1.2}$$

Further, Reiljan (2020) wishes to compare the differences between ideological and affective polarization and their relationship, as it seems that there is a strong correlation. The data used by Reiljan (2020) in the study are taken from the Comparative Study of Electoral Systems (CSES). Due to lack of continuity in the questionnaire throughout the waves, Reiljan (2020), is able to utilize data only from the third and forth wave (covering the period from 2005 until 2016). The sample is consisted out of 22 European countries and the USA.

For the ideological polarization index, Reiljan (2020), uses the index proposed by Dalton (2008), which is a standard deviation based measure of party positions. The index is applied both for the supply side (the voters' perception of the party position) as well as the demand side (the voters' self positioning).

The Ideological Polarization Index (IPI) looks as follows:

$$IPI = \sqrt{\sum (Partyvoteshare_i) \times \left(\frac{PartyLRScore_i - PartySystemAverageLRScore}{5} \right)^2}, \tag{1.3}$$

where i, in the supply side equation, represents a party (on the demand side equation i represents an individual), the $PartyLRScore$ is calculated from the CSES data by using the responses of the interviewees and their perception of a party's placement along a left-right axis. These responses are not corrected for the respondent's bias or noise in communication between interviewer and interviewee. According to Dalton (2008) when all answers are aggregated these effects will be canceled out. The same formula is used to calculate the demand side of polarization, but instead of a $PartyLRScore$ we have an $VoterLRScore$ which is calculated based on the self-placement responses of the voters. The total ideological polarization is simply the average of the above two, given as follows:

$$IPI_{total} = (IPI_{supply} + IPI_{demand})/2$$

According to the findings of Reiljan (2020), affective polarization is quite high in many countries of Europe, exceeding even the USA. Amongst the European countries, the variation is very high. Countries of central East Europe and South Europe (namely Bulgaria, Portugal, Czech Republic, Slovakia, Montenegro, Spain ans Greece) all score above 5. The countries on Northwest Europe have an average score of 4, with not one country exceeding the milestone of 5 points. The USA scores around the average (below the median), at only 4.38. As explained by the formulas given above, the AP is a weighted average of the ≪like≫ and ≪dislike≫ feeling of voters. Taking a closer look and examining the ≪like≫ and ≪dislike≫ scores separately reveals interesting facts. We observe that in southern Europe the animosity feelings are the strongest of all European regions and the USA. Looking into the southern European countries, we see how this result comes up. In Greece, after the 2012 elections (a period, as described earlier with extremely high social unrest in the country, perhaps the highest in years) the ratings for the out-group were very low, hitting almost bottom (0 represents the highest rate of dislike and that year in Greece the governing party supporters rated the main competitor with 1.38 and the opposition's supporters rated the ruling party with 1.61). In Bulgaria the ratings are even lower, with the ruling right-wing party's supporters rating the second and third in row parties with 1.06 and 0.53 respectively. In the USA in 2012 things look somewhat better regarding the animosity levels, with Democrats rating Republicans with 3.09 and the Republicans giving the Democrats a rating of 2.91

Regarding the relationship between ideological and affective polarization, based on the indices used in his study, Reiljan (2020), does find a positive correlation but still the majority of the cross-national variation remains unexplained. Indeed, in many cases a high level of ideological polarization does not necessarily lead to a high level of affective polarization.

Harteveld (2021) tests the hypothesis that when a society is sorted along lines that exceed the strictly political ideology then affective polarization will increase as a result of the division of people in more homogeneous groups, as has been the case in recent years in the USA. This increase in homogeneity will increase the hostile feeling towards the out-group. To test this hypothesis, Harterveld (2021), constructs two models. In the first one he uses data on aggregate level from CSES on 40 countries during the period 1996 until 2018 and finds that indeed when social sorting increases, so does affective polarization. Secondly, Harteveld (2021), investigates if these results are confirmed on an individual level, and therefore uses individual level data from the Netherlands for the period from 2007 until 2018, which allow for deeper insight into the accordance of individuals with the socio-demographic profile of their party. Indeed, results are once more confirmed as Harteveld (2021) finds that when individuals are sorted along more dimensions affective polarization is more likely to increase.

For the first model, Harteveld (2021) uses two measures. To measure the levels of affective polarization he uses the API, as per Reiljan (2020) in above mentioned equation (1.2). To measure the social sorting he uses *Cramer*'s V in the same way as in the work of Selway (2011) and Knutsen (2010). He uses this *Social Sorting Score* to calculate the association between political affiliation and four other socially important variables, namely income, religion, education and region. These four variables broadly cover the main social issues which tend to be expressed politically as follows: class (income), affected or not by globalization (education), center or periphery (region) and religious opposition (religion) which we have seen often is correlated with conservatism as well. The score is calculated as follows:

$$C = \frac{CV_{ideology,income} + CV_{ideology,education} + CV_{ideology,region} + CV_{ideology,religion}}{4} \tag{1.4}$$

Additionally a couple more control variables are used, namely elite and mass political polarization measures.

The results regarding affective polarization are in accordance with Reiljan (2020) regarding its level in European countries and the USA. Harteveld (2021) also reaches the same conclusion regarding the relationship between affective polarization and ideological polarization, i.e. the latter does not always explain the movement of the first. The same conclusion is reached regarding the relationship between social sorting and affective polarization; while there seems to be a positive correlation it does not always explain all the variation in affective polarization. More specifically, a more detailed look into the regression results reveals that while the social sorting measure over time correctly predicts the level of affective polarization, the same is not true for the between-country coefficient. The data on individual level makes it clear that when citizens start to sort around socio-demographic characteristics as well, then affective polarization increases. Both studies confirm that hostility between partisans depends on the interplay of non-political matters.

In another study, Wagner (2021), tries to tackle the issue of measuring affective polarization in multi-party systems, since in this case things are a little bit more complicated as an individual might have positive feelings towards more than one parties and similarly have negative feelings towards more than one parties again. Among other things, Wagner (2021) notes that the size of the party is an important factor. As in Esteban and Ray (1994) negative or positive feelings towards a single, small outlier should not significantly affect the levels of polarization (despite the large distance between the outlier and everybody else). Wagner (2021) proposes a measure on the individual level, which also includes non-partisans. He also uses the like-dislike questions of the CSES but suggests to measure the spread of their scores for each respondent, which is simply the standard deviation of the like-dislike score:

$$Spread_i = \sqrt{\frac{\sum_{1=p}^{P}(like_{ip} - \overline{like_i})^2}{n_p}},$$

where p is each party, i each individual and $like_{ip}$ is the like-dislike score assigned to each party by each respondent.

Additionally, following the logic of many other measures, Wagner (2021) also uses the weighted standard deviation, based on each party's vote share:

$$WSpread_i = \sqrt{\sum_{1=p}^{P} v_p(like_{ip} - \overline{like_i})^2},$$

where v_p is the vote share of each party and $\overline{like_i} = \sum_{1=p}^{P}(v_p \times like_{ip})$.

Wagner (2021) proposes another measure as well, where the average distance between one's favorite party and all other parties is calculated:

$$Distance_i = \sqrt{\frac{\sum_{1=p}^{P}(like_{ip} - \overline{like_{max,i}})^2}{n_{p-1}}},$$

where this time the denominator is the number of parties without the individual's most favorable party.

The measure can again be weighted:

$$WDistance_i = \sqrt{\sum_{1=p}^{P} v_p(like_{ip} - \overline{like_{max,i}})^2}.$$

Lastly, for calculating ideological polarization Wagner (2021) uses the standard deviation of the placement of parties on a 0 to 10 scale:

$$WIdeology_i = \sqrt{\sum_{1=p}^{P} v_p(position_{ip} - \overline{position_i})^2},$$

where $\overline{position_i} = \sum_{1=p}^{P}(v_p \times position_{ip})$.

Results of the regressions show that affective polarization positively affects partisanship. Specifically an increase of one standard deviation in affective polarization increases probability for partisanship by 15%. Negative partisanship (this is captured by the answer to the question for which party the voter would never vote for) is also more likely with higher levels of affective polarization, although the effect is not as strong as with positive partisanship. A clear positive relationship is also found between ideological and affective polarization. Last but not least, Wagner (2021) also examines the relationship between affective polarization and democratic values and participation. He finds that the higher the polarization in a society the lower the satisfaction with democracy (perhaps because the disappointment in losing is stronger) and at the same time people, in highly polarized societies, tend to believe it is important who they vote for and who is in power. Participation in political activities (like putting up posters etc) as well as voter turnout (self-reported) are positively related with affective polarization. Regarding the four measures developed and described, not surprisingly, it does not seem to make much of a difference which measure of affective polarization is used, the relationship is always in the same direction. Wagner (2021) concludes that research of affective polarization in multi party countries (like Europe, contrary to the USA) opens up a large variety or research topics and questions. Foremost perhaps, what are the drivers behind polarization and what is the explanation behind the different levels in, otherwise similar, countries. A cross country longitudinal analysis, like the one performed in this thesis, will shed light in questions like that.

Using evidence for the American National Election Studies (ANES), Webster and Abramowitz (2017), show the increasingly dislike of partisans towards voters of the opposite party in the USA as well as towards the respective presidential candidates. Additionally, they show that this dislike is caused by widening difference around policy matters, especially regarding matters of social welfare. Matters regarding the size and the role of the federal government are of highest importance. This increasing distance in ideological positions has resulted in increasing feeling of ≪fear and loathing≫ amongst Americans, as described by Iyengar *et al.*, (2012). According to the findings of Webster and Abramowitz (2017), these feelings are the strongest between liberal Democrats and conservative Republicans. According to Levendusky (2009) the ideological divergence between the two sides started first among the elites, but as is evident from the data analyzed, the voters followed. According to Webster and Abramowitz (2017), the data from the

ANES show that the distance between Democrats and Republicans more than doubled from 1972 till 2012, according to where respondents place themselves. Furthermore, data shows that while respondents seem to feel more or less the same distance with their own party (despite the more polarized rhetoric of the elites, another sign that the electorate has shifted as well), their feeling of remoteness to the opposing party has increased by more than one entire point on a scale ranging from 1 to 6 (the distance has moved from 2.0 to 3.2 points). Voters' self placement and the distance they perceive to have form the opposing party are notions which are highly correlated. In the USA the distance perceived by liberals and the Republican party increased in the above mentioned period from 0.85 points to 0.66; on the other side of the aisle the distance between conservatism and the Democratic party has increased from 0.71 to 0.77. Additionally the relationship between ideological distance and negative affect towards the opposing party has grown even stronger; the correlation between the perceived distance and the rating of the feeling thermometer has risen from -0.34 to -0.48. the shred variance of this relationship has more than doubled in the aforementioned 34 years period. There seems to be a deeply rooted belief by voters, that in case the opposing party wins and enacts on its promised policies, then these will harm the nation and everyone's overall well-being (Webster and Abramowitz, 2017). The authors also analyze the specific stance of Republicans towards president Obama, a president who was challenged on all levels, even on his religious beliefs and whether or not he was legally entitled to the presidency (Klein, 2005). They find that former president Obama indeed evoked intense feelings of fear and especially anger at Republicans. When asked if the feel angry towards Obama, 49% of Republicans (including those leaning towards the Republican party) reported feeling angry at least half of the time while 34% reported feeling angry most of the time or always. Data show that Republicans were not as fearful, as angry, since only 31% reported feeling fear at least half of the time and 20% reported feeling fear most of the time or always.

The implications of the sorting, of the ideological polarization are severe. As Levendusky (2009) shows for USA, not only were partisans more sorted in the opinions they held, or perhaps even better more polarized, but the people who *become* sorted also follow suit in becoming more polarized and more adamant in the views they hold. Momentarily office holders in America express, more often than not, views that can be considered quite more extreme than 30 or 40 years ago and they will continue to do so, as there is not only a lack of fear for electoral defeat, to the contrary we observe that sometimes this behavior is rewarded, even with the highest of positions, like in the case of Trump. Klein, in his book ≪Why We're Polarized≫ says that when he asked political analyst Bartels what went wrong in 2016 and Hilary Clinton lost the election, his reply was that actually nothing unusual had happened, and indeed a deeper analysis of the voting data shows exactly that. Trump's percentages on the groups of white males, Hispanics, even with white Christians were pretty close with the percentages of the Bushes, father and son.

The overall conclusion drawn from all above is that polarization is on the rise and has been for a while now. For the moment, we have no indications of things getting any better. The increase of polarization is of extreme importance as it seems to be penetrating all aspects of life, political or not, and has serious consequences, perhaps most importantly the intense societal fraction and the political gridlock. These are things that shape the quality of our every day life and it would be in the best interest of everyone to combat this phenomenon.

1.3 Economic Inequality - Events

While income has risen in the past decades almost all over the world since 1980, it has not risen equally for everybody. According to the World Inequality Report (Alvaredo *at al.*, 2018) in 2016 the top 10% of the world owned 47% of national income, compared to 34% in 1980. This uneven distribution of income is not equally severe all over the world: in Europe the top 10% accounts for 37% of total income, in the US and Canada it accounts for 47% of total income whereas in the Sub-Saharan Africa this percentage is equal to 54% and the Middle East with the higher score at 64% (World Inequality Report, Alvaredo *at al.*, 2018). One observes, thus, that inequality is a world wide problem. This pattern is also noticeable at the top 1% share; in 1980 their share in Europe corresponded to 10% and in the USA 11% whereas in 2016 those percentages were 12% and 20% respectively. Over this period, it is estimated that the top 1% captured 27% of total growth (Alvaredo *at al.*, 2018).

In Europe, as is clear by above numbers, things are a little bit better than in the USA, but the general pattern is unfortunately the same. Taking a closer look into Europe's data we see that in France, from 1983 until 2014, the top 10% captured 42% of total growth whereas the bottom 50% captured only 21% (Alvaredo *at al.*, 2018). In Germany, in 1980 the top 10% captured 32% of income which, by 2014, had amounted up to 41%. On the opposite side of the distribution, the bottom 50% captured 24% of total income in 1980 but only 16% in 2014, being at an even worse point than France (Alvaredo *at al.*, 2018).

The same pattern, but more intensely, is followed by wealth inequality (Alvaredo *at al.*, 2018). While data on wealth inequality are more difficult to find, the conclusion remains the same: wealth is concentrated among the few and that gets more and more so nowadays. In the USA wealth inequality has almost doubled from 1984 till 2014, from 22% it has risen to 39%. In France and the UK the top 10% owned approximately 50% of total wealth (almost a, historically, lowest point for the 20^{th} century) but by 2014 this had risen to 55% for France and 52% for the UK (small increase after many fluctuations but in line with the ascending trend of wealth accumulation of the upper classes).

History has shown that economic inequality offers ground for the rise of such populist voices (Acemoglu, 2013). Financial crises are every bit as much about politics as they are about economics (Green, 2018). People search for someone to blame and the government is an easy target. Additionally financial crises are often perceived to be the fault of the few (i.e. the 1%, the ≪golden boys≫ of the financial institutions) but are paid by the many (i.e. the remaining 99%). This anger is fueled by an increase of economic inequality which exacerbates the feeling of injustice. Populist leaders, regardless if they are on the left or the right side, claim to be fighting against the current injustices and unequal distribution issues their country faces at that point in time. Such leaders promise to implement macroeconomic policies of deficit financing, generalized controls with a disregard for basic economic equilibrium having, more often than not, resulted in major macroeconomic crises that have ended up hurting exactly those they promise to help (numerous examples are analyzed in ≪Macroeconomics of Populism in Latin America≫ by Dornbusch and Edwards, 1991).

The financial collapse of the Lehman Brothers in 2008 created a sequence of events which offered opportunities for left as well as right populists in the USA. Regulators allowed Lehman Brothers to bankrupt in September of 2008, a decision which, even a few days earlier, was not anticipated even by the bankruptcy lawyers of the bank itself, as they claim that up to a couple of days before the failure of the agreement with Barclays, no-one from the Fed was telling them to file (Sorkin, 2012). Just one day after the bankruptcy of Lehman, the Fed bailed out the AIG and a few weeks later the USA Congress passed the Troubled Asset Relief Program ("TARP"). This was a bipartisan bill, signed by G. Bush only a month before the end of his presidency. This bill (the TARP bill) was presented by the Republican Kentucky senator McConnell as a highlight in the history of the Senate. But, when the Obama administration took over, senator McConnell fought hard to make the distribution of the fund as difficult as possible. As he told journalist Green in 2010, his aim of this was to make it crystal clear to the public that the proposals were not bipartisan. The goal of this strategy was, which ultimately failed though (for many and complicated reasons, this effort was not enough), to keep Obama of being re-elected for a second term (Green, 2012).

The financial turmoil ignited by the collapse of the Lehmann Brothers set the path for Obama's administration, together with Timothy Geithner as head of Treasury, to believe that it was in the American's people best interest to not allow such a disastrous event to take place again. In order to do so, they did their best to make the most out of TARP as well as arrange mergers, Fed lifelines and bailouts for big corporations like CityGroup, General Motors and Goldman Sacks, to name a few (Green, 2012). While their moves succeeded in setting the American economy back on track and send the American stock market on a rising trend, at the same time those moves created a sentiment of injustice, betrayal and disappointment in the American people (Green, 2018, Rodrik, 2021). Many Americans had lost their jobs, their homes, or their pension funds while large banks were being bailed out.

Events like these can work as a driving force behind social fractionalization based on income classification. Egalitarian politicians, favoring redistribution, will try to built a dislike sentiment towards the rich elites, whereas opponents of redistribution will turn their rhetoric against poor minorities (Glaeser, 2005), creating this way a polarized environment.

This gave birth to movements like ≪Occupy Wall Street≫ and created a feeling of resentment towards the financial sector and a sentiment of distrust towards the government (according to the polls of the Gallup Institutions "Government" had replaced "economic issues" as a top concern of Americans in 2013). Events like these gave rise to left wing populist voices in the USA, like the Democratic candidate Bernie Sanders. This polarization and accumulated political anger laid the ideal floor for voices like Trump's to be heard. While his campaign is nowadays mainly remembered to have been driven by anti-immigrant hostility, Trump also spent an enormous time attacking the Wall Street Golden Boys in the name of the honest hard-working American earner.

The "Occupy Wall Street" movement was characterized by a strong fractionalization sentiment. A group of people, describing themselves as the 99% clearly (and perhaps even in a hostile way one may add) distinguished themselves from the top 1%. The criterion in this case was not a religious matter (as Glaeser,2005, suggests is often the case), but it was the distribution of income and wealth. Indeed the rising inequality of the USA has been widely documented by many. As a matter of fact, Stiglitz (2012) points out that 95% of the gains of income in the USA after 2009 have gone to the top 1%. Wealth inequality has also risen in the USA over the past decades: the top 1% owned 7% of total wealth in the late 1970's but its share increased to 22% by 2012 (Saez and Zucman, 2016). In the following elections of 2016, D. Trump, with his populist "Make America Great Again" campaign, was elected.

In the EU, in 2009, ten years after the launch of the Euro, economists and politicians were celebrating the success story of the common monetary union (Gibson *et al.*, 2014). But, by the end of that year, the first financial crisis of the EU broke out. By 2012, 8 out of the 28 EU member states had received some form of financial aid accompanied by strict reform programs. These austerity programs, which were implemented as a remedy to the financial crisis created an environment favorable for extremism. The austerity programs applied to the countries in distress affected both the financial aspect of the EU and its member states (for both those receiving as well as those providing economic aid) but also the political one, as it created an environment favorable for extremism and populism.

Greece and Spain were two countries struck especially hard by the economic crisis and they witnessed harsh consequences. Unemployment returned to post-war levels (Hobolt and de Vries, 2016) and the anti-EU sentiment grew vastly in that period, as was reflected in the 2014 European parliament elections but also in their respective national elections, with populist parties gaining groung. A closer look into the events that unfolded in in these two countries during that period gives a better understanding of the severity of the situation.

In Greece, social unrest unfolded quickly and many events which were meant to form the country's recent history took place. It started with 2010's extremely severe riots, which targeted anything systemic (the EU, the banking system etc). Athens was on fire. Among the buildings that were burnt was also a bank, whose employees were still working inside. Four people died there. Witnesses say that protesters were shouting: ≪Burn the rich!≫, regardless from the fact that the employees who died, probably weren't that rich. In 2012 early national elections were held twice (since the first time no parties were able to form a coalition), due to raptures and resignations caused by the austerity programs implemented. In said elections, for the first time since its founding in 1980, the extremist, right nationalist, neonazi party "Golden Dawn" enters the Greek parliament, winning 7% of total votes (today the Golden Dawn party does no longer exist, as in October 2020 its leadership was convicted as running a criminal organization under the pretenses of a political party. Its founders and numerous members were imprisoned). Three years later, in the again early, elections of 2015, SYRIZA (the coalition of the left-wing and radical left parties) took power with the aid of the right-wing "Independent Greeks" (ANEL) party, with whom they formed a coalition government.

Respectively, in Spain, the traditional two-party system was put to an end, after three decades, by the 2015 general elections (Orriols and Cordero, 2016). Two new parties, "Podemos", a left-wing party founded by political scientist P.I. Turrión in 2014, and "Ciudadanos" a center or center-right party, together obtained 34.6% of total votes. It was the first time, after the re-establishment of democracy in Spain in 1977, that the party which received most votes was not able to form a government.

Besides left-wing populism, which was more popular in the south of Europe, as mentioned above, the countries of central of northern Europe, witnessed mainly the surge of right-wing populism. France witnessed the surge of the right-wing party of Marine Le-Pen, National Rally, who in the presidential elections of 2017 came in second in the first round with 21.30% and thus competed with Emmanuel Macron in the second round for the presidency (which she lost with 33.9%). Likewise, in the Netherlands the Eurosceptic party of Geert Wilders, PVV, gained support during the years of the crisis, mostly as a result of the economic insecurity felt by Dutch voters (De Vries, 2016). In the last elections of 2017, PVV, gained 20 (out of a total of 150) seats in the Dutch House of Representatives. In the UK, UKIP rose in popularity, under the leadership of Farage.

According to Rodrik (2021), while globalization may have reduced global inequality, it gave rise at the same time to domestic inequality and it deepened cleavages amongst certain groups, like for example skilled and unskilled workers, globally mobile professionals and local producers, industries and regions with comparative advantages and those without. Such domestic inequalities have led to one of the, arguably, most serious example of polarization and Euroskepticism in the EU during the years of the Great Recession: the BREXIT. In the UK, the UK Independence Party (UKIP), a party that always supported the UK's exit from the EU, was founded in the late 1990's but saw its support growing only after 2010. This coincides with the time when austerity-led welfare reforms (with cuts ranging from46.3% to 6.2% from 2010 till 2015) took place throughout the UK, striking harder on poorest districts (Innes and Tetlow, 2015, and Fetzer, 2019). This situation, resulted in the end, in a close-call referendum which revealed a highly polarized society. A society split in two. "Leave" prevailed with only 51.9%. This result follows a period of rising income inequality and worsening living conditions for the poorer areas (due to austerity-led welfare cuts), where the majority of voters of "Leave" come from (Fetzer, 2019). As the study of Colantone and Stanig (2016) points out, globalization seems to have a direct effect on the results of the ≪Leave≫ vote. The rise in unemployment it created, in the areas affected the hardest, as well as the low income wages there, seem to be 2 of the strongest explanatory variables of the ≪Leave≫ vote. Guiso *et al.* (2017) find economic insecurity driving the popularity of populist parties, like UKIP, up. All said economic parameters, like unemployment, low wages for certain groups and economic insecurity, are factors increasing economic inequality. It is this economic inequality which drives the sentiment of unfairness and pushes people towards punishment of the established elite through a vote towards the populist parties. In the case of the BREXIT, unfortunately is quite likely though, that in the end the results of the referendum will only worsen said conditions for the "Leave" supporters. This could lead to further increase of polarization and so on.

The pandemic was a global shock, to say the least, for everyone on every aspect of our lives. Inevitably it affected the global economy enormously. As we all know, around February or March 2020, all countries started imposing strict lock-downs and most people and businesses were forced to restrict their activities. This sent the global economy into a brief but damaging free fall. Offices and stores closed, many factories suspended production and companies laid off workers in masses. With less money earned, demand was expected to drop. But the opposite happened, starting first of all with a sudden and steep increase in demand for masks and other supplies (like disinfectants, protective gear, etc). In other words, demand was shifted from eating out to medical supplies. This was quickly followed by another trend which had already been emerging but was accelerated by these unique conditions: online shopping. According to Amazon.com, their sales increased by 57% from April to June 2020, compared to the same period the previous year.

This trend was facilitated by government stimulus which were handed out to households to help them cope with the difficulties created by the pandemic, which took the form of stimulus checks and from other programs like the expanded child tax credit, extra unemployment benefits, the Paycheck Protection Program etc. In the graph of the disposable income growth this effect is represented by the spikes in the disposable income graph, which are especially size-able for the bottom 50%. But, while we watch spikes in disposable income, the American society still remains a highly unequal one.

During the pandemic, Blanchet, Saez and Zucman created an online tool, called ≪Realtime Inequality≫, (available at *https://realtimeinequality.org/*) which provides timely statistics on how economic growth is distributed across various economic groups in the USA. According to the data they provide, we observe that income from labor and wealth fell significantly during the first months of the pandemic. Specifically, we see that in April 2020 income fell by an average of 14%. The bottom 50% was hit the hardest, as they saw their income plummet by 30.20%, explained perhaps by the job loss of many blue-collar workers, while the upper 10% saw it decrease by only 10.40%, affected mainly by the fall in profits by many businesses which is the main source of their income. By August 2021 income in all economic classes had fully recovered, by an average of 3.4%, but again we witness an unequal distribution of this growth: the bottom 50%'s income increased by 1% while the top 10% enjoyed an increase of 5.1%. So, on the one hand we have a quite faster recovery of income in this crisis (compared to the Great recession of 2008, when the income of the bottom 50% recovered after 11 or so years) but still, it was unequal.

The same is true for wealth growth. In the first quarter of 2020, wealth plummeted for everyone. The effect was less strong for the poorer than it was for the rich. Growth in wealth in the first quarter of 2020 fell by -1.6% for the middle 40%, by -3.5% for the top 10% and by -8.6% for the top 0.01%. But by the end of the same year, in the fourth quarter of 2020, things had started recovering and they had returned back to the usual unequal growth. Wealth grew by 16% for the middle 40%, by 17.4% for the top 10% and by 21.1% for the top .0.01%. By the end of the next year this trend is exacerbated even more. Wealth for the top 0.01% grows almost twice as fast than for the middle 40% (i.e. by 39.6% and 25.8% respectively).

The explanation behind this unequal rise in income lies at the fact that the wealthy disproportionately own stocks and thus have gained more by the stock market increase, as can been seen in figure, whereas wealth of the middle class lies mainly in housing assets (Krugman, 2022).

Capital One recently launched the Capital One Insights Center, and its first release, ≪Capital One Marketplace Index: The Road to Recovery≫ sheds some more light into the factors thta might have affected the slower income recovery of the bottom 50% and the low-income class in general. The survey provides data that explain how the impact of the pandemic has not been the same for all working individuals. While income and job losses have been widespread across all income groups, it has not affected all of them equally and the recovery has been significantly slower for lower earners. According to the "Capital One Marketplace: Road to recovery" survey, among all Americans, 32% to 36% have seen their incomes disappear decrease at some point since the pandemic started. The pandemic has been tougher on the middle and lower class and it seems that the middle class is getting thinner and thinner: while 13% of the higher income households still has a lower income, for the middle class this is 19% still. In July 2021, 8% of the middle class still considered themselves as underemployed versus4% of the higher income class. Additionally, more than half of the middle and higher class earners managed to save part or all of the stimulus payments handed out but only 43% of the lower class were able to do so. Regarding difficulties to cope with change due to at-home work and closed schools, we see that lower earners had greater difficulties with coping with the new circumstances: 50% of lower earners and30% of middle earners had to cut on job hours or quit altogether due to lack of outside child care. For higher earners the percentage is only 18%. Data from the Federal Reserve also indicate the widening of the gap. Net worth of college graduates has increased by almost $25trillion were as for those with only a high school degree or not even that their net worth has increased by only $3.49 trillion, resulting in the latter representing an even smaller percentage of the US economy today than when the pandemic started.

1.4 Economic Inequality

In their very popular political analysis book, Levitsky and Ziblatt (2018) wonder ≪*How Democracies Die*≫ and conclude that one of the factors that has a significant impact is economic inequality. The book evolves around the election of Trump and how this could have happened and what should be done, and by whom, in order to protect our democracies (not only in the USA, but in all democratic nations). The authors explain how economic inequality can act as a divisive factor and how policies aiming to reduce it can be polarizing (or they could be de-polarizing if presented in an other way). They explain how in the USA *welfare* has become a pejorative term, often implying that those who receive it are *free-riders*, taking advantage of the rich. Politicians, like Reagan, have helped create this picture, using terms like ≪Welfare queens≫ and ≪young bucks≫ buying steaks with food stamps. Unfortunately today in the USA partisan animosities are a mix of racial divide (which is still very much present) together with slowed economic growth, wages at the bottom which have been stable and the resulting rising economic inequality. According to Levitsky and Ziblatt (2018) today's increased polarization in the USA, which is affected by ethnic diversity, increased during a period of rising inequality which, negatively, affected those at the bottom of the distribution. In the USA social welfare policies have heavily relied on means tests, meaning that only those who fall below a certain threshold are entitled to the benefits. This has created a stigma around the benefits. Policies like Medicare, which benefit everyone, would be a solution towards depolarization. According to Levitsky and Ziblatt (2018), regardless of how difficult such a task may be, it is very important for policies aiming at combating racial as well as economic inequality to be put forward.

In the same spirit, Voorheis *et al.*, (2015), show that in the USA during the last years polarization and inequality have both been rising and that there is a causal relationship between the two. With the aid of a new data set on polarization and inequality in the USA, they show that indeed the latter has a large, positive and statistically significant effect on polarization. Inequality makes Democrats more liberal and Republicans more conservative. Along the same line with McCarty *et al.* (2016), they find that it is actually the fact that moderate politicians, or what one may characterize as skilled coalition builders, have gone scarce. More specifically, Voorheis *et al.*, (2015) find that within-district income inequality has a stronger effect on Republicans, moving them more to the right whereas between-district inequality affects Democrats more, shifting them more to the left. In any case, the cleavage between Republicans and Democrats is getting bigger.

McCarty *et al.* (2016) observe the similar pattern followed by political polarization, income inequality and immigration in the USA in the 20th century. They observe that while these three factors declined sharply in the beginning and the middle of the 20th century, after the

1970s they started to increase and have not stopped since. This pattern is also followed by the pattern of the state's economic policies regarding progressive taxation of high incomes as well as minimum wage policy.

While McCarty *et al.* (2016) begun their research thinking that polarization is elite-driven, their findings point out otherwise. The preferences of voters might have influenced the surge of polarization. It seems that the increase of association between demographic characteristics and the voting behavior of member of the House districts has had a positive effect on polarization. As the one increases so does the other. Additional findings indicate that there is also a positive relationship between the stronger association of economic well-being of a district and the representative's voting behavior on roll call votes. District income has become a direct predictor of the conservatism of the House member, while in 1973 this was a non-factor. McCarty *et al.* (2016) also find an increase in importance of identity politics, close to the idea of Iyengar *et al.* (2012). Regarding income levels, McCarty *et al.* (2016), discover a sorting here as well. High-income Americans have consistently voted for the Republican party whereas low-income Americans consistently vote for the Democratic party. The statistical importance of the effect of income on the voting decision remains, even after controlling for other demographic factors. Results of the regressions show that the increase of average income, in the second half of the 20th century, has contributed to an increase in the competitiveness between the two parties. Income has become a lot more important than it was in the 1950s and, even though voting cannot be characterized as exclusively class-based, there has been an intense political sorting based on income levels. Regarding income inequality and the voter's incentive for redistribution, McCarty *et al.* (2016) show that up to the Great Recession of 2008-09, that had not changed much for the median income voter. One explanation for this might be that the increase in immigration shifted the location of voters on the income distribution. Non-voters are significantly poorer than those eligible to vote. Mobilization levels of poorer citizens, eligible to vote, regarding voting has not changed dramatically. The disenfranchisement of poorer citizens has increased mobilization of most voters. But, at the same time, a voter is less pro-redistribution if the results of this are to be shared with poorer non-citizens. According to McCarty *et al.* (2016) though immigration is not an explanatory factor for the commencement of inequality and polarization in the 1970s. In that time the fraction of non-citizens was very small and their income profile close to that of citizens'. In the 1990s, with the increase in immigration, the median income voter's preferences on redistribution changed. There is evidence of change in voter's preference regarding redistribution, again, after the Great Recession in 2008-09.

Gidron *et al.* (2018) test four hypotheses regarding ideological, economic, and institutional factors that may affect polarization. Regarding economic factors they test the hypothesis if economic inequality, when controlling for elite-level polarization, intensifies affective polarization. This study also uses the CSES data, from waves one to four, for all Western countries for which at least two election surveys are available. This creates an unbalanced panel of 20 countries over the period from 1996 until 2015. Again, the question used is that of the ≪feeling thermometer≫ to measure affective polarization. Due to the assertion that polarization is mainly driven by the hostile feeling of partisans towards the out-group, in their calculation of the affective polarization index, Gidron *et al.* (2018) only include the feelings' score of the voter towards all parties except of the positive feelings towards their own. This may create some issues though, as in a multiparty system a voter might hold positive feelings towards more than one parties (Wagner, 2018, Weisberg, 1980; Garry, 2007). So, the mean evaluation assigned by voters of party A to all other parties, in other words a weighted out-party dislike measure, is expressed as follows:

$$APP_a = \sum_{i=B}^{K} \left((thermometerScoreParty_i) \times (VoteShareParty_i) \times (1 - VoteSharePartyA) \right),$$

where the $thermometerScoreParty_i$ denotes the thermometer's rating that voter of party

A has assigned to parties $i = B, ..., K$ and $VoteShareParty_i$ is party i's vote share in current elections.

Total out-party dislike feeling of all partisans (of all parties) towards all out-parties for a given country, is calculated as follows:

$$CountryAP = \sum_{i-A}^{K} (AffectivePolarizationParty_i \times VoteShareParty_i)$$

According to above measures, as in Reiljan (2020), the USA is close to the median value. Greece and Spain, two countries struck especially hard by the financial crisis, feature on the top of the list as the most polarized. On the contrary northern European and Scandinavian countries seem far less polarized, with the Netherlands being at the bottom of the list.

Gidron *et al.* (2018) perform various regressions to test their hypotheses. Regarding ideological polarization they use the Dalton (2008) standard deviation based measure, as is described above by equation (1.3). For the economic inequality they use the Gini coefficient (as is the case in this dissertation), with values from the Standardized World Income Inequality Database. Unemployment data are retrieved from the World Bank. The highest unemployment rate was for Greece in 2012 (24.4%). Among other things, the data's descriptive statistics reveal a high level of between and within variation for ideological polarization but regarding income inequality, while there is a high variation between countries, the within country variation is very little, showing the ≪stickiness≫ of the variable.

The results of the regressions indicate that there is, as expected, a positive relationship between economic inequality and affective polarization, when controlling for political polarization, unemployment and majoritarian institutions. The effect is substantial: one standard deviation change in the Gini inequality (the equivalent of a change from 25.4% to 33.8%) would intensify partisan's out-party dislike by 0.5 to 0.6 units, on a $0 - 10$ scale, in other words a 5% to 6% change. When controlling for country Fixed Effects, the income inequality variable loses statistical significance (and also changes from positive to negative, but since it is not statistically significant this does not pose a matter of concern).

Duca and Saving (2016) test the hypothesis of the existence of a causal link between rising income inequality and rising political polarization in the USA. They aim to tackle various issues that have arisen over time in the study of the matter, like correlations between the multiple factors driving the two phenomena, measurement issues, potential feedback between inequality and polarization etc. In doing so, they use a different measure for inequality, namely the inverted Pareto-Lorenz coefficient (which measures income inequality in the top 1% of families), as they find that it is statistically more consistently correlated to polarization (as measured by the DW Nominate scores of Poole and Rosenhtal, 1997 and 2007) in the short and the long run, compared to another popular measure for inequality, that of the top 1% share of income. Duca and Saving (2012a) found evidence regarding bi-directional feedback when using the Gini coefficient with above mentioned polarization index.

In their study, Duca and Saving (2016) run regressions using both measures of inequality (i.e. the Pareto-Lorenz coefficient and the the top 1% share of income) and the Congressional polarization indexes of Poole and Rosenthal. Indeed, they find that long-run relationship between income inequality and political polarization does not offer statistically significant predictions on polarization in the House or the Senate in the short run, when income inequality is measured by the top 1% share of income. When two political realignment variables are included, one for the New Deal era and one for the, more recent, Tea Party, the results become highly statistically significant and point to a consistent relationship between inequality and polarization, the latter as given by the top 1% share of income.

Nevertheless, when the inverted Pareto-Lorenz curve is used to measure inequality then, regardless if the two political realignment variables are included or not, changes in polarization in the short-run are better explained.

Last but not least, the study confirms that in the long-run, the is a bi-directional causal effect between inequality and polarization, meaning that in the long run the two variables, inequality and polarization, are endogenous and the one is affected by the other.

The findings of the study imply, that unless we see some structural changes in congress or in policies combating inequality, a high level of political polarization in congressional voting is likely to remain.

In a subsequent study, Duca and Saving (2017), investigate the relationship between polarization (as measured by the DW-Nominate index of Poole and Rosenthal for the House and for the Congress) and inequality (this time represented by the Gini coefficient) as well as increased fragmentation of American media (the share of households with cable or pay TV is used as a proxy for this). Additionally a dummy variable for the Tea Party phenomenon is added (taking the values of 1 for the 2011 and 2012 Congresses and zero otherwise).

They find that the share of people owning cable or pay TV is negatively correlated with political polarization in both the House and Congress in the long run. They also find statistically significant evidence for the long-run effect of income inequality on polarization. But when both explanatory variables are included in the same model, then income inequality, as expressed by the Gini coefficient, loses its statistical significance and switches signs, compared to other models.

Pontusson and Reuda (2008) analyze the relationship between income inequality and political polarization of twelve OECD countries. Seeking to explain why some countries have more redistributive welfare states than others, they construct a theoretical model driven by the logic that left and right political parties most probably address constituencies in different segments of the income distribution, since their distribution policies affect voters differently. The intuition behind this idea is that voters of left parties usually are in favor of more redistributive politics and voters of right parties are usually against those policies (like incremental taxation for example) as inequality rises. To measure political polarization, Pontusson and Reuda (2008), use election manifestos as measured by the Comparative Manifesto Project (CMP) and study the movement of those parties on the left-right axis. They utilize data starting from the mid-1940s through 2003. The countries included in their set are Australia, Belgium, Britain, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, USA. An unbalanced data set is created as data are not available for the same years for all countries. Regarding wage inequality, they use data from the OECD set on relative wages and from the Luxembourg Income Study. The index constructed is the ratio of the wage of someone in the bottom of the top 10% percentile to the earnings of someone on the top of the bottom 10%. They also derive the Gini coefficient from data from the LIS datacenter on disposable income among working-age households. The countries studied are Australia

Pontusson and Reuda (2008) proceed under the logical assumption that left-side parties' voters would most probably earn the largest share of their income from dependent employment and that the core constituency consists of voters whose income does not come from mainly from wages. The results of Pontusson and Reuda (2008)'s research shows that wage inequality mainly creates left-skewed polarization (i.e. all parties tend to move to the left but the leftist parties will move relatively more to the left) and specifically when low-income workers are characterized by medium or high levels of mobility. On the other hand, household income inequality creates right-skewed polarization (i.e. all parties move to the right but the rightist parties move relatively more to the right) when low-income workers are characterized by low mobilization. The reasoning behind the results is that the main constituency of left-wing parties are mainly blue-collar workers who are not that greatly interested in income inequality (which includes income from assets and dividends as well) as they probably don't feel that this affects them as much as wage inequality. The results indicate that an increase in income inequality does not increase the interest of the left-wing voters for more redistribution. This of course holds only in the case of high mobility of low income workers. At the same time though right-wing parties, when faced with high low-income workers mobility, are less likely to adjust their policies to their constituency's preferences.

Grechyna (2016) uses the Bayesian Model Averaging (BMA) on a sample of 66 countries to determine the underlying factors that affect political polarization. To measure political polarization she uses data on voter's self-reported political preferences, based on their answers on 3 questions regarding income inequality, government spending and state-ownership from the World Value Survey (WVS). the period under consideration are five waves of polls in the years 1990 until 2013. Ten variables are used from 3 different categories (economic, socio-historical and geographic): the real gross domestic product (GDP) per capita, income inequality as expressed by the Gini coefficient, globalization as expressed by the foreign direct investment share, government expenditure as percentage of the GDP, media status in the sense of how free the media is in the specific country, ethnolinguistic fractionalization, trust (which is calculated from the answers to the relevant questions in the WVS), the level of democracy, population density and lastly absolute latitude as a geographical factor. The cross section panel estimates show that the most robust measures are trust and income inequality. High trust in a society and low inequality reduce polarization. Government expenditure and GDP are also significant factors. The higher their levels the lower the polarization.

Garand (2010) performs various regressions to prove the relationship between income inequality, mass polarization and roll-call voting. Using individual data from the Cumulative American National Election Study (CANES), for the period 1974-2004, on self-placement and the feeling thermometer, he constructs the dependent variables of individual ideological identification and affect towards conservatives and republicans. Regarding the independent variables, the Gini coefficient on family income inequality is used, as calculated by the data compiled by Guetzkow *et al.* (2007) and regarding partisan identification the standard 7-point scale on the partisanship question (Democrat or Republican) is used. Garand (2010) shows that higher income inequality increases ideological polarization on the individual level. But, as he proves, things are a little bit more complicated than that. Inequality seems to work as an amplifier of the feeling of dislike between partisans. His cross-state's analysis shows that in states with higher inequality Republicans are significantly more conservatives than Democrats. The other finding, of even bigger interest, is the finding that Republicans like conservatives more than liberals significantly more than Democrats do. We observe here that this is close to the notion of affective polarization, and we have a linking between the feeling towards people of different ideology being affected by income inequality. Furthermore, Garand (2010), shows that income inequality increases the ideological distance between Republican voters and Democrat voters. Additionally, the research shows that mass polarization affects the position of senators. This means that in states with high income inequality, where mass polarization has increased, Republican senators are much more conservative than the non-southern Democrats (a group omitted for standing out). Additionally, when checking from lowest to highest inequality states, Garand (2010), observes that as inequality increases, so does ideological distance between Republican and Democratic senators and so does the effect of Republican partisan ship on roll-call voting. Finally, regressions indicate that inequality and constituency's polarization both have an independent effect on the senator's party and their roll-call behavior.

While there are many studies examining the increase of polarization in the USA, they are scarce in the case of the EU. But the matter is of high significance nowadays as Euro-skepticism is in the rise and we already observed the first exit of a member state. Given the assumption that after the end of the pandemic we will most probably be faced with a new recession which will require new measures, the chances of Euroskeptisism rising again a very high.

After the end of the pandemic inequality indices will most probably have risen. The pandemic stroke unevenly.

Chapter 2

Political Polarization: Definition and Measurement

2.1 Defining Polarization and its Measures

The division of a country's society in groups that are characterized by animosity and its causal relationship with given country's economic prosperity was brought to the forefront of attention by the seminal paper of Easterly and Levine (1997) who examined the underlying reasons of Africa's growth tragedy. While Africa is a continent with high potentials, it never reached the predicted growth rates. According to Easterly and Levine (1997) the main cause behind this is the high degree of ethnolinguistic fractionalization.

This result is confirmed again in a later study by Alesina *et al.* (2003), where the number of countries examined is widely extended and so proves that a divided society hinders growth.

Above mentioned studies show the important consequences of a divided society based on ethnolinguistic characteristics. As was extensively analyzed in the previous section, nowadays in the USA as well as in Europe we observe a different type of societal fragmentation, namely the sorting of people along party lines. In the past forty something years researchers, journalists as well as people in their everyday lives discuss how political opinion polarization has increased and the consequences and challenges this brings along, like political conflict and social volatility (DiMaggio, 1996).

In order to proceed with the analysis of political polarization and to test its correlation with economic factors such as growth and inequality we first need to define it. A concrete framework defining polarization is hard to find, despite its recent popularity. We will define below the main axes around which, based on the previously presented literature, we built our definition and which we find are important to take under consideration when measuring political polarization.

Perhaps, in order to provide an accurate measure for polarization, we should broadly outline what it depends on. In a few words, one could argue, polarization (as a special type of societal fragmentation) depends on the number of clusters in a society (how fragmented a society is), their density (how many people are sharing the same political opinion) and the distance between said groups. In other words, polarization depends on the extremity of the views expressed, the extent of the disagreement as well as the size of each of the groups holding those opinions (this is to say that a unique outlier does not affect tensions in society as a whole).

According to DiMaggio (1996) polarization is both a state, referring to the extent to which opinions are opposed to each other, compared to a theoretical maximum, as well as a process, which refers to the evolution of this phenomenon over time. DiMaggio (1996) describes four main principles which should be taken under consideration, when measuring polarization. These are:

1. *The Dispersion Principle.* This refers to the distance between opinions, in other words it refers to the variance of the distribution. The further away opinions move (the more the

spread of the distribution increases and the more dispersed it becomes), ceteris paribus, the more away we move from the position of the median voter and thus the more difficult it is to reach any kind of consensus or to built a coalition. The increase of variance in the distribution of political opinions, either in the elites or in the mass public, is what is shown by the empirical studies of McCarty *et al.* (2016) and McCarty (2019). The conclusion of both studies was that the left is getting *left-er* and the right is getting *right-er*. The chasm in the roll-call votes between Republicans and Democrats is getting bigger and according to Abramowitz and Webster (2016) this is reflected in the the mass public as well. It is a straightforward assumption that the further people's opinions move from one another, the more alienated people will feel with people from outside groups and at the same time more closely related they will feel with people from their own group. This is confirmed by the study of Iyengar *et al.* (2018) which shows an increase in spousal agreement in political opinions as well as an increase in inter-generational agreement, between parents and their offspring. Socialization is restricted within the limits of one's own group.

A natural measure for the dispersion of the distribution is the distribution's variance.

2. *The Bimodality Principle.* In addition to the range of the distribution, what also affects the level of polarization is how the data is distributed within that range. A society where all people are scattered evenly around the center will be less polarized than a society where people are distributed around two modes. Two modes create, in a sense, two opposite poles and reinforce the ≪us versus them≫ feeling. In the most severe form of polarization a society would be divided between the two ends of the political spectrum. Any additional in-between modes would *soften* this effect, as they would work as a mediator of tensions.

3. *The Constraint Principle.* The term ≪*constraint*≫ is a notion developed by Converse in his seminal 1964 paper. By this term the interrelation between the bundle of beliefs and opinions a persons holds on various topics, seemingly irrelevant, is meant. The more sorted these bundles become, along party lines, the more possible it is that conflict between groups may arise. Polarization between groups increases as the bundles of opinions of members of a group become more homogeneous. To illustrate the ≪*constraint principle*≫, I will quote the line of the campaign run by the ≪Club for Growth≫, a conservative club in the USA, in 2004 against the then candidate for the Democratic nomination, Howard Dean. The ad shows an elderly white couple who are asked what their opinion is about Dean. The man starts by saying: ≪*I think Howard Dean should take his tax-hiking, government-expanding, latte-drinking, sushi-eating, Volvo-driving, New York Times–reading -* ≫, and his wife continues: ≪*Body-piercing, Hollywood-loving, left-wing freak show back to Vermont, where it belongs.*≫ This example very clearly shows this sorting and how it is perceived by the public. For example, the older couple essentially asserts that all Democrats, who are in favor of tax-cutting, drink lattes, drive Volvos, read the New York Times etc. The more intense this kind of sorting becomes, the more intense the degree of polarization. There is no common ground to be found between members of the two groups on any matter (not even what type of coffee one drinks!). As Mason explains in her book ≪Uncivil Agreement: How Politics became our Identity≫ (2018), momentarily what a person votes for (in the USA) may reveal his race, religion, ethnicity, gender, neighborhood and even his favorite grocery store. According to the principle, the higher the association between the social characteristics, *ceteris paribus*, the higher the probability of conflict. In other words, the increase of homogeneity within each of the groups will inevitably result in the increase of the alienation between groups and this will increase polarization.

4. *The Consolidation Principle.* The ≪*Consolidation Principle*≫ examines the extent to which, all other things being equal, when the differences between various social characteristics increases, so does polarization. The ≪*Consolidation Principle*≫ essentially refers

to the heterogeneity between groups. Heterogeneity between groups means that there are more points to disagree on and the cleavage between the groups becomes deeper.

Essentially it boils down to two things: the degree of homogeneity within groups and the degree of heterogeneity between groups. What DiMaggio essentially asserts is that polarization basically depends on these two things and polarization increases when: (*a*) homogeneity within groups increases (the dispersion of opinion within a cluster gets smaller) and (*b*) heterogeneity between groups increases (the distance between the means of each cluster becomes larger).

2.2 The Identification - Alienation Framework

Esteban and Ray (1994) develop an index regarding economic polarization. According to Esteban and Ray (1994), just like politics, income can also work as a characteristic of one's sense of identity. As was very evident with movements like the ≪Occupy Wall Street≫ movement, income can create clear boundaries around which the ≪us vs them≫ rhetoric can be built. Esteban and Ray (1994) focus on the importance of this identity sentiment by taking under consideration the number of people in each cluster. In contrast with simple income inequality measures, which usually only consider distance and perhaps number of clusters, Esteban and Ray (1994) argue that the frequency of observations in each cluster is very important. To describe this idea, they construct the ≪*Alienation-Identification*≫ framework (henceforth AI framework). *Alienation* represents distance between clusters and *Identification* represents the frequency based identity sentiment developed in each cluster. Their combination results in what Esteban and Ray (1994) call ≪effective antagonism≫. It is the combination of both the heterogeneity between groups (*Alienation*) as well as the homogeneity within groups (*Identification*) that have to co-exist in order for polarization to be manifested. The adjective *effective* distinguishes this index from the more *passive*, more *static* notion of inequality. This index of polarization embodies this feeling of animosity between clusters.

More formally expressed, Esteban and Ray (1994) present the following three features, basic to a polarization measure:

1. *High Degree of Homogeneity Within Groups.* This refers to the the sentiment of *Identification* an individual feels with the other members of the same cluster. The higher the homogeneity, the higher the sense of identification. In the model this is expressed by the *Identification function.*

2. *High Degree of Heterogeneity Between Groups.* The larger the distance between the clusters, the higher the *Alienation* sentiment between groups. In the model this is expressed by the *Alienation function.*

3. *Few Insignificant Small Groups* (i.e. outliers of the distribution). This ensures that their weight in the model will be insignificant.

In a subsequent paper, Duclos *et al.* (2004) adapt the index of Esteban and Ray (1994), so that it is applicable on continuous data. This is the index we have chosen to use for measuring political polarization in this study. The concept of the index remains the same, as the AI framework is once again the basis: polarization is related to the degree of heterogeneity between groups and the alienation felt between their members but this sentiment of alienation is accentuated by the degree of homogeneity within groups. Differences between groups are not enough on their own, the sentiment of group identity between members of the same group triggers the sentiment of animosity, i.e. the ≪us versus them≫ feeling of hostility. A tighter scope of opinions or, otherwise put, a smaller range of opinions within a group, creates a clearer and stronger sense of identity between members of the same group. This is in accordance with much of the literature in social psychology and the literature reviewed in the previous section. This is the concept of ≪collective identity≫, which was extensively studied by Tajfel (1970). When groups are formed, even on the most trivial and meaningless characteristic then this creates a sense of ≪us≫, a group identity, which automatically puts anybody else on the opposite side, classified as ≪they≫. Tajfel's experiments showed that it takes almost nothing to trigger group identity. Tajfel conducted a series of famous experiments to test two of his most prominent hypotheses. The first hypothesis was that people are so prone, or even so eager, to classify themselves into a group, that they will do so even on the most trivial or meaningless characteristic. The second hypothesis tests that once people have sorted themselves into some group, then they act in favor of the in-group and they will discriminate towards the out-group, even when there is no actual reason to do so. To test these hypotheses, they gathered 64 boys from the same school (so they all knew each other), aged between fourteen and fifteen. First of all the boys were told they would undergo a visual judgment test: they had to guess how many dots were in each cluster they saw. Secondly, the boys were told that, since they were there, the researchers would like to perform a second test (irrelevant of the first one). They would split the boys into groups, allegedly based on the number of dots they guessed (in reality the boys were allocated randomly) and they would be given real money to hand out (they could not keep any of the money themselves). This second test was just a procedure to set the baseline and Tajfel did not expect it to generate any results (as the creation of groups was totally random in reality) (Robinson, 1996). In the previous section, we have already described the experiments of Iyengar and Westwood (2015), which were already by design more politically oriented. These first experiments by Tajfel were very simple and, as expected by Tajfel himself, should essentially create no bias between in- and out-group members. But this was not the case. Despite it not being the optimum strategy, the boys chose most of the time to allocate more money to their group-mate. Their decision was biased. When they had to choose between two members of their own group, then they did allocate the money in such way as to maximize the benefit (indicating that they did understand the concept, they simply chose to ignore it when having to chose between in- and out-group members) (Robinson, 1996). And it is this feeling of group identity which is correlated with social unrest phenomena like strikes, demonstrations, widespread violence etc (Duclos *et al.*, 2004).

The index constructed by Duclos *et al.* (2004) for this purpose is one to be applied on distributions described by densities. In order to do so, they first present an axiomatic definition of polarization, whose central idea is close to the one of DiMaggio (1996) described above.

2.2.1 General Assumptions for the Index

We assume all continuous density functions in $\mathbb{R}_+$, whose integrals equal the corresponding populations under study. If we assume a density f, then $P(f)$ is defined as the polarization of the density. This polarization depends on the *identification* and *alienation* of each individual in the density f.

Each individual located at point x feels *alienated* towards an individual located at y. This alienation increases monotonically in distance $|x - y|$.

According to the AI framework presented above, for the said *alienation* to be translated into effective voice, the element of *identification* is required, which refers to the group identity feeling created amongst the members of the same group. It is assumed that an individual located at x feels a sense of *identification* which depends on the density at that point, $f(x)$.

The combination of the *identification* and *alienation* felt by an individual at x towards the one situated at y produces the effective antagonism, defined as

$$T(i, a),$$

where $i = f(x)$ and $a = |x - y|$. We assume that $T(i, a)$ is increasing in a and that $T(i, 0) = T(0, a) = 0$. The latter condition implies that people at the same point (zero distance, as implied by $a = 0$) feel no alienation with each other and single, unique outliers ($f(x) = 0$) have no effect on the polarization within a society.

Total polarization in a society is the average weighted sum of all effective antagonisms:

$$P(F) = \int \int T\left(f(x), |x-y|\right) f(x)f(y)\ dxdy \tag{2.1}$$

Above general equation broadly expresses the main assumptions of the AI framework. Hereunder follow the axioms, which help specify this formula and make it applicable to our data set.

2.2.2 Basic Definitions

For simplicity reasons, the axioms are largely based on the normal or the uniform distribution due to the fact that the properties we are interested in do not depend on the type of distribution but on the effects a *slide* or a *squeeze* has on the polarization of the distribution.

By the term *slide*, we mean the *movement* of the density function to either the left or the right with no further change in any of its properties (variance, skewness, kurtosis). What changes is the mean of the distribution as well as its support, as a result of the change of mean while keeping the same variance. Let's say, for example, that we have a density function $f(x)$ with $x \sim N(2, 1)$, a slide to the right by z, would produce a new density function $f_1(x)$, with $x \sim N(2+z, 1)$. Correspondingly, a slide to left by z would create a density function $f_2(x)$ with $x \sim N(2-z, 1)$.

A *squeeze* of the density function implies that what changes is only its variance (and range consequently). So if, for example, we have a density function $g(x)$ with $x \sim N(0, 2)$, a *squeeze* would produce a new function, $g_1(x)$ with $x \sim N(0, z^2 \times 2)$, where $z \in (0, 1)$. In other words the *squeeze* is a mean-preserving reduction in the range of $g(x)$.

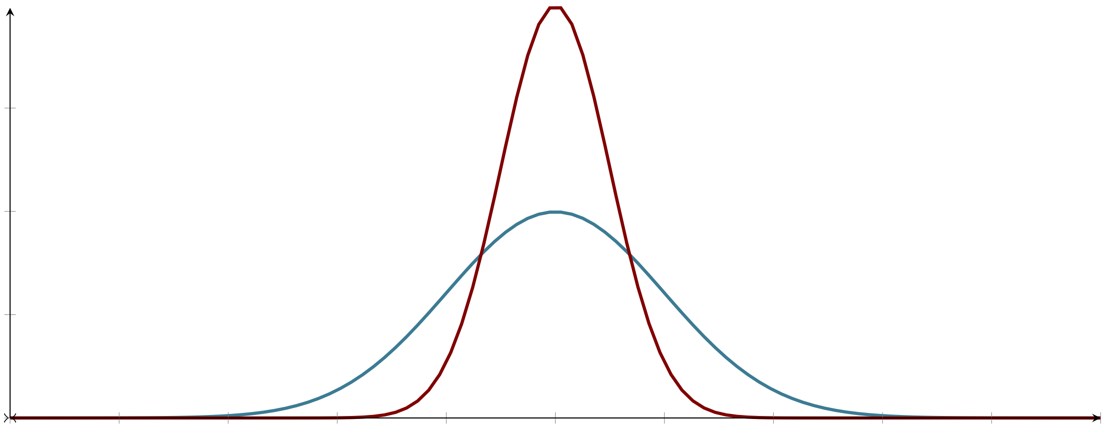
2.2.3 Statement of Axioms

Axiom 2.2.1. *When a distribution is described by a unimodal density function, then a squeeze of that density function cannot increase polarization.*

In a single-party society, a stronger ideological consistency cannot increase polarization (the key word here being *single-party*). The intuition behind this is straightforward. In the case of the squeeze of the unimodal distribution we have a reduction of the alienation sentiment since the range of the distribution, and thus the distance between individuals, is smaller. At the same time, a more homogeneous cluster implies a stronger identification sentiment, which would increase polarization. According to axiom 2.2.1, the effect of the increase in identification, only in this specific set, cannot exceed the effect of decrease in alienation, so that polarization cannot increase. This axiom will define the upper bound of the α exponent, which defines the weight of identification. It is very important to note that we are talking about a population whose distribution would be described by a single unimodal density function, because this is the main reason the effect of identification does not exceed that of alienation. Essentially in this setting, due to the fact that we have only one mode, we are talking about in-group alienation reduction. As we have explained previously, identification is the key element which essentially converts alienation into animosity. But in the single mode distribution a *global squeeze* can be translated as an increase of people feeling politically closer. The effect of the increase of the *identification* sentiment is faded out by the fact that there is no opposite political group. There are only people not so close to the median voter.

Axiom 2.2.2. *We assume a symmetrical distribution, comprised out of two identical unimodal density functions at the opposite ends of the spectrum and one unimodal distribution in the center. In case the two outer densities undergo a squeeze, polarization cannot decrease.*

Figure 2.1: Axiom 2.2.1: Squeeze of single unimodal density



This axiom, in a sense, holds the essence of the measure as it involves the comparison between the effect of *identification* and *alienation*. The squeeze of the two outer densities will increase the *identification* sentiment in the outer distributions while at the same time it will reduce *alienation* sentiment between the two densities. Axiom 2.2.2 ensures that the effect of the heterogeneity reduction between the two groups cannot outweigh the effect of the increase of homogeneity within each group. It is axiom 2.2.2 which defines the lower bound of the α parameter, so that identification has a greater impact than alienation.

It is this axiom, that separates this measure from measures of simple fractionalization of a society. It also distinguishes our index from other measures of polarization or inequality, because it takes under consideration the stronger effect of homogeneity (compared to heterogeneity) within a society. This measure ensures that identification carries a bigger weight in the equation than does alienation.

This is total line with the theories of *group identity* and affective polarization analyzed above. It is not just the fact that the parties are moving further away, which might sometimes not even be the case, it is the fact that partisans have become neatly sorted along party lines. This is the trigger of social tensions. As per Converse's *constraint* notion, what one votes determines his or hers point of view on various subjects, not just political matters or issues regarding economic policy. While voters of different parties used to disagree on most points but still agree on some, nowadays we observe that agreement is more and more rare. It is this strong feeling of identification which triggers social tensions, hostility and even animosity and ignites unrest.

Figure 2.2: Axiom 2.2.2: Symmetric squeeze of outer densities

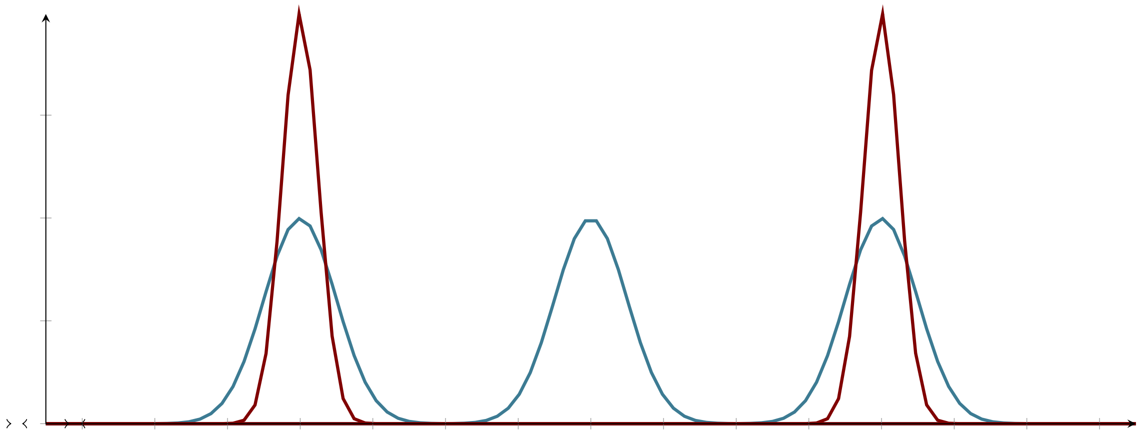
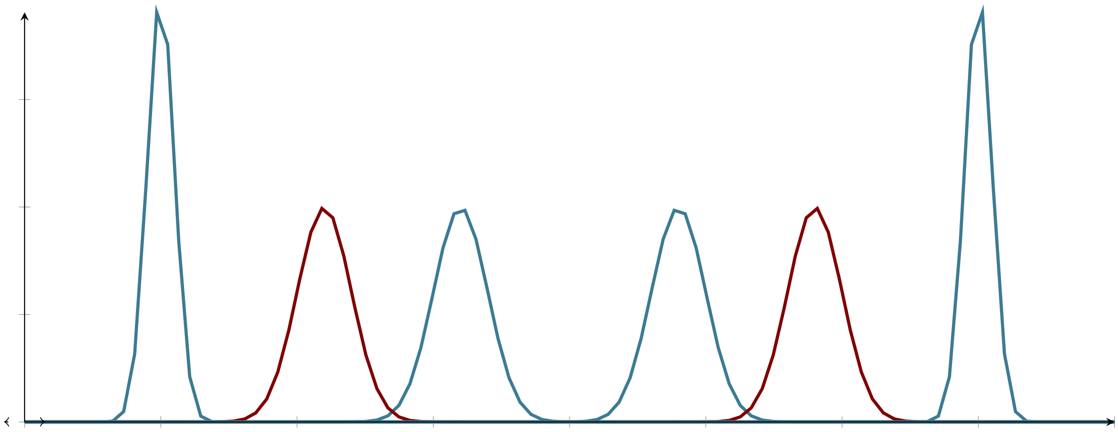


Figure 2.3: Axiom 2.2.3: Outward swift of inner densities



Axiom 2.2.3. *We assume a symmetric distribution comprised out of four unimodal density functions, as shown in figure 2.3. A slide of the two middle densities outwards must increase polarization.*

Axiom 2.2.3 is straightforward. While we keep identification stable, alienation between some of the members of society increases with the increase of distance between the two middle distances. This of course results in an increase of polarization. We must point out of course that in this set we have a reduction of alienation as well (between the outer density and the middle density that moves outwards, towards the former) but it is counterbalanced by the respective increase in alienation between the outer density and the outward move of the middle density which moves further away. Simply put, the cleavage between all four densities becomes larger, the chasm dividing the left and the right is bigger. While there were two middle parties (a center-right party and a center-left party), after the outwards slide of the two middle densities we are left with parties situated clearly on the left or on the right, resulting in a more polarized society.

Axiom 2.2.4. *If $P(F) \geq P(G)$ and $p > 0$, then $P(pF) \geq P(pG)$, where pF and pG represent identical population scalings of F and G, respectively.*

Axiom 2.2.4 implies that the polarization index is population-invariant. Assume a society, whose distribution is described by density F, is more polarized than a society whose distribution is described by the density function G. This will remain unchanged if the populations of both societies are scaled up (or down) by the same factor p and the distributions remain unchanged.

Theorem 2.2.1. *A measure of polarization that satisfies above axioms and is in accordance with the IA framework must to be proportional to:*

$$P_\alpha \equiv \int \int f(x)^{1+\alpha} f(y)|y-x|dydx, \tag{2.2}$$

where $\alpha \in [0.25, 1]$.

The analytical proof of the index, theorem 2.2.1 as well as the bounds of α are presented in the accompanying appendix (section A).

2.2.4 On The Bounds of α

The exponent α expresses the importance of the identification sentiment, which accentuates the effect of alienation. Essentially α increases the weight of identification compared to alienation.

The bounds set make sure that all the requirements posed by the axioms are met. More analytically, according to axiom 2.2.2, when we have a movement like the one of the double squeeze, which creates two more strongly identified groups and at the same time we have a reduction in the total range of the distribution (a decrease in alienation), the polarization cannot decrease. This implies that the effect of the increase in identification must outweigh the effect of the decrease in distance. This is ensured by the lower bound of α. By restricting α not to take any values below .25 we make sure that identification will have a stronger effect. At the same time, axiom 2.2.1 states that, in the case of the unimodal distribution, if people's opinions become more closely related then polarization cannot go up. This means that the positive effect of the increase in identification within the cluster must not exceed the reduction of overall alienation. This is ensured by setting the upper bound of α equal to 1.

2.3 Empirical Application of the PaF index and Statistical Inference of Political Polarization in the EU

We proceed with the empirical application of the PaF index and dealing with the associated issues regarding its statistical inference.

Starting, let's note that for every cumulative distribution function F which corresponds to the density f of a continuous random variable, like the distribution of political opinion, and μ the mean, then we have that:

$$P_\alpha F = \int_y f(y)^\alpha a(y) dF(y), \tag{2.3}$$

where $\alpha(y) \equiv \mu + y(2F(y) - 1) - 2\int_{-\infty}^{y} x dF(x)$.

Suppose now that we wish to estimate $P_\alpha F$ using a random sample of n i.i.d. observations on political opinion y_i, where i is the random individual interviewed and $i = 1, ..., n$.

A natural estimator of $P_\alpha F$ is:

$$P_\alpha(\hat{F}) = n^{-1} \sum_{i=1}^{n} \hat{f}(y_i)^\alpha \hat{a}(y_i), \tag{2.4}$$

where μ is the sample mean, $\hat{a}(y_i)$ is given as:

$$\hat{a}(y_i) = \hat{\mu} + y_i(n^{-1}(2i-1) - 1) - n^{-1}\left(2\sum_{j=1}^{i-1} y_j + y_i\right), \tag{2.5}$$

and $\hat{f}(y_i)^\alpha$ is estimated non-parametrically using a Gaussian kernel function. This procedure is based on a symmetric kernel function $K(u)$ such that $\int_{-\infty}^{\infty} K(u)du = 1$ and $K(u) \geq 0$. This will produce the estimated probability density function of the random variable in question, i.e. voter's self placement on the political left-right axis. The estimator $\hat{f}(y)$ is then defined as:

$$\hat{f}(y) \equiv n^{-1} \sum_{i=1}^{n} K_h(y - y_i), \tag{2.6}$$

with $K_h(z) \equiv h^{-1}K(\frac{z}{h})$ and h representing the bandwidth. The proposed bandwidth which minimizes the mean square error is given by the formula

$$h^* \cong 4.7 n^{0.5} \sigma \alpha^{0.1} \tag{2.7}$$

2.3.1 Data

Since 1979 the members of the European Parliament have been elected directly by the citizens of the EU through universal suffrage. Results of said elections are often believed to closely reflect citizens' current political opinion and sentiment regarding their own country's parties. Furthermore, these elections represent a unique opportunity for cross country studies as all European citizens are called to the ballot simultaneously every five years and so results offer an important insight on the people's opinion and mindset throughout the EU at that given point in time.

A main part of the European Election Study (EES) is the Voter Study. Some years include both pre-election and post-election surveys or are a part of the Euromanifesto Studies. Despite the fact that the surveys evolved over the years and focused on contemporary matters, they were always designed along similar principles and included some identical questions, like the questions of self-placement. This results in data that are completely harmonized/aligned and thus allows for a in depth and reliable analysis. The wording around the subject of self-identification of participants as well as their perception of the placement of the political parties of their countries has remained the same throughout the years. Additionally all countries are surveyed the same year, i.e. the year of the European Parliament elections. This alignment of the data allows for a reliable comparison between countries as well as an in depth analysis of the longitudinal examination of the evolution of polarization within a country.

For our study, we have used the set of questions pertaining to matters of political opinion, self-placement of voters and their perception of their country's parties placement. The data for creating a discrete distribution of political opinion of a country's people were produced by the answer to the following question: ≪In political matters people talk of "the left" and "the right". How would you place your views on this scale ?≫. The scale ranges from one to ten, with one representing the left and ten the right. This question was included in all studies. Along this question, people were also asked to place, on the same scale, the parties of their country, according to their perception. We collected data on 10 countries, namely Denmark, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Spain, and the UK for a thirty years period, from 1989 until 2019, in five-year intervals (the years of European Parliament Elections).

The answers to these questions create a set of discrete data on political opinion, which must be handled with caution and care as there are some methodological difficulties inherent in the collection of individual level perceptual data (in contrast with preferential data). Perceptual data present two types different challenges: first we have the true error in perception of the interviewee and secondly there is the distortion in the actual survey situation (Aldrich and McKelvey, 1977).

Before proceeding with the presentation of the formal formula of the probabilistic model developed by Aldrich and McKelvey (1977) to tackle this issue, I would like to explain in more detail the issues that might occur during such surveys. First of all, let's consider, for example, a voter on the left of the scale (not the far-left per ce, but not the center left as well). Such a voter will most probably allocate a center right party more to the right than would a voter who places himself on the right (let alone a voter in the far-right). Additionally, there is a tendency of voters to place candidates as well as themselves on the ≪prominent≫, or ≪clear≫ if you prefer, points of the scale, i.e. the middle point and the two ends. Often the in-between points are avoided because either voters cannot make such fine distinctions between various candidates or because respondents do not know how exactly to interpret the in-between points. So for example, from a scale of one to ten, one is clearly right, ten is clearly left and five represents the center, but what is the difference between seven and eight or eight and nine?... This might be a too fine distinction for a random voter to make. In other words, the problem arises due to the different interpretation each individual might give to the terms ≪left≫ and ≪right≫ and even more to the relative classification of who is more left/right than whom. Another reason might be cognitive bias. People tend to listen to news and stories that confirm what they already believe. So if a voter, for some reason has a wrong impression of a candidate's true position, this will most probably not only hardly change, but often be reinforced (this is what makes fake news so popular nowadays). This also affects the quality of information one is receptive to. In any case, whatever the reason, each voter has his own perceptual space, which is a result of the true space subject to an observation error (by the voter).

So, the data are contaminated by two types of variation: (a) the variation in the placement of one's self as well as the candidates and (b) the variation in interpretation of the scale and how the interviewee communicates his perception to the interviewer. The two types of variation essentially stem from the same problem, the subjective interpretation of information by the people who receive it. But the two types of variation which contaminate the data happen at two different time intervals. The first happens on a daily basis throughout the voter's life and the second one is at the time of the interview.

2.3.2 Aldmck Scaling Procedure for Perceptual Data

Below follows the presentation of the formal model, constructed by Aldrich and McKelvey, (1977), which we applied to the voter study data we retrieved from the EES.

Each candidate, j, is assumed to occupy a true position $Y_1, Y_2, ..., Y_j$, for $1 \geq j \geq J$ on a one-dimensional issue continuum, meaning that $Y_j \in \mathbb{R}$. Additionally, there are n individuals. Respectively, each individual, i, has his own perception of the candidate's true position. The i^{th} individual's perception of the j^{th} candidate's true position is denoted as Y_{ij}, which is randomly distributed around the candidate's true position, Y_j. This perception is modeled as the candidate's true position distorted by some randomly distributed variation. This is the first stage of the analysis and can be expressed as follows:

$$Y_{ij} = Y_j + u_{ij} \tag{2.8}$$

for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant J$ and u_{ij} is a randomly distributed variable, satisfying below Gauss-Markov assumptions, i.e.

$E(u_{ij}) = 0$ and $V(u_{ij}) = s^2$ for all i, j

$E(u_{ij}u_{kl}) = 0$ for all i, j, k, l with either $i \neq k$ or $j \neq l$

Afterwards, we deal with the second variation, the one between what the individual truly believes and what he communicates to the interviewer (for whatever reasons that may be, as mentioned earlier). It is assumed that what the interviewee tells the interviewer is a linear transformation of what he actually has in mind and can be expressed as follows:

$$c_i + w_i X_{ij} = Y_{ij}, \tag{2.9}$$

where c_i and w_i are scalars.

From equations 2.8 and 2.9 it is obvious that:

$$c_i + w_i X_{ij} = Y_{ij} = Y_j + u_{ij} \tag{2.10}$$

Simply put, we have the following: each voter has a true perception of each candidates true position. Both the candidates true position, Y_j distorted by some random noise, u_{ij}, as well as the, again by some random noise distorted but also linearly transformed, reply of the voter to the questionnaire, must equal the voter's true perception of the candidate's position, Y_{ij}.

The only data that is observed is X_{ij}, i.e. the individual's answers to the survey, and from that the true parameters Y_j, c_i and w_i must be derived.

All the above regards the candidates' positions.

Regarding the citizen's true positions, the same is assumed as with the candidate's true position. Of course in this case, only the second "noise" has to be dealt with (i.e. what the individual says to be his true position does not accurately reflect what he has in mind). Again, some linear transformation is assumed to explain the distance between the i^{th} individual's true position and what he tells the interviewer.

Thus is X_{i0} denotes the i^{th} individual's true position, then

$$\hat{Y}_{i0} = \hat{c}_i + \hat{w}_i X_{i0}, \tag{2.11}$$

where $\hat{Y}_{i0}$ is the estimate of the i^{th} citizen's ideal point and $\hat{c}_i, \hat{w}_i \in \mathbb{R}$ are estimates of the true parameters.

Aldrich and McKelvey (1977) create a two-fold procedure, which deals first with the scaling and calculation of the true parameters regarding the true candidates' positions and secondly with obtaining the ideal point estimate of the individuals' true position, which is what is of concern in this study (although we calculated both stages, as we had available data for both the individual's perception of the party's true position as well as the true self-placement of the individual).

The ideal point estimate of the individual's true position is then used in the Gaussian kernel procedure for the non-parametrical estimation of the probability density function of the distribution of public political opinion.

2.3.3 Data Processing

With the aid of the free statistical software environment and programming language R we perform a spatial analysis of the self placement and perceptual data we obtained from the EES. Specifically we apply the Aldrich-McKelvey scaling procedure, analytically described above, which allows for the interviewees' true location to be inferred from their biased answers to the questionnaire (this procedure is also known as the ideal point estimation). In order to do so the procedure also makes use of the respondents' replies about the parties' placements and with the help of this additional information, the package estimates the perceptual bias of each respondent. With the application of the Aldrich-McKelvey scaling procedure we obtain the estimated positions of both the respondents and the parties on one single issue scale dimension (e.g. *left-right*).

From the results created by this procedure we extract the ideal point estimators of each respondent. Herewith we create a new data set of the continuous variable of public political opinion, which essentially is the y_i variable of equation 2.4.

Non-parametrical Estimation of $\hat{f}(y_i)^\alpha$

The ideal point estimate of the individual's true position is then used in the Gaussian kernel procedure for the non-parametrical estimation of the probability density function of the distribution of public political opinion.

Specifically we use below specification:

$$\hat{g}_h(y) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{y - y_i}{h}\right), \tag{2.12}$$

where K is the normal Gaussian distribution kernel, y_i is the ideal point estimation retrieved with the Aldrich-McKelvey scaling procedure and h is the bandwidth for which we have chosen the one recommended by Duclos et al. (2004):

$$h^* \cong 4.7 n^{-0.5} \sigma \alpha^{0.1}.$$

2.3.4 Experiment 1: Selection of appropriate α level

Duclos *et al.* (2004) propose an acceptable range of values, from which one might chose an appropriate level for the α exponent. The α parameter, as in detail analyzed above, determines the significance of the identification sentiment.

In order to chose the level of α which we would deem appropriate for our study, we conducted below experiment on the predictive power of the index, based on the chosen level of α.

Two sets of randomly normally distributed data were generated. In the beginning, the means of the two distributions coincided (figure 2.4, the population is almost not polarized). Afterwards, the means of the populations moved away by 0.5 points every time, until a maximum distance fo 5 points was created (as shown in figures 2.5 till 2.14).

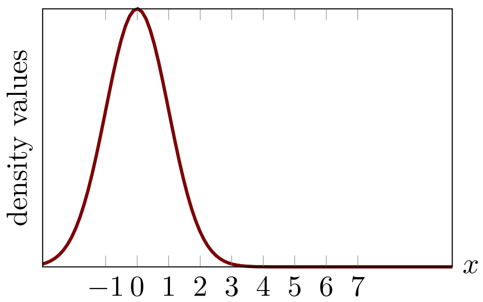


Figure 2.4: Difference = 0

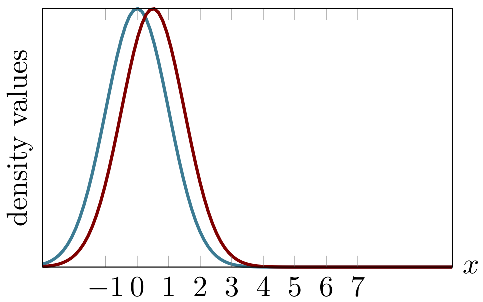


Figure 2.5: Difference = 0.5

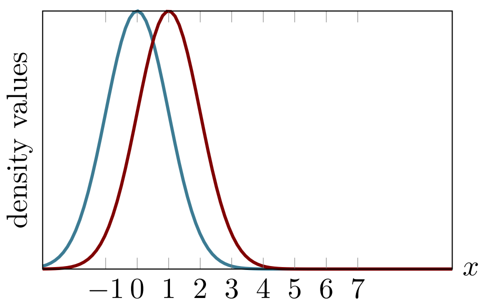


Figure 2.6: Difference = 1

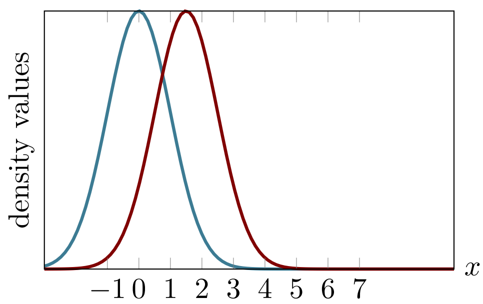


Figure 2.7: Difference = 1.5

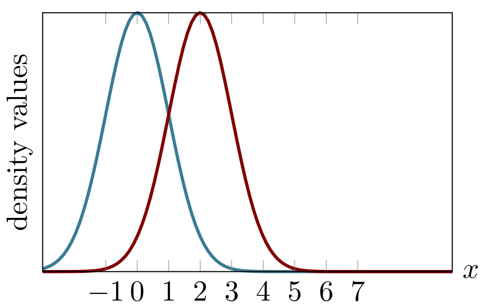


Figure 2.8: Difference = 2

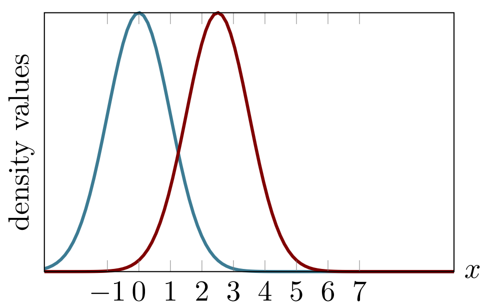


Figure 2.9: Difference = 2.5

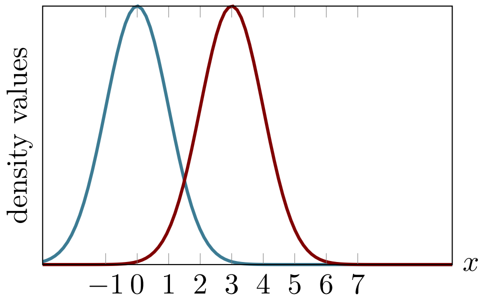


Figure 2.10: Difference = 3

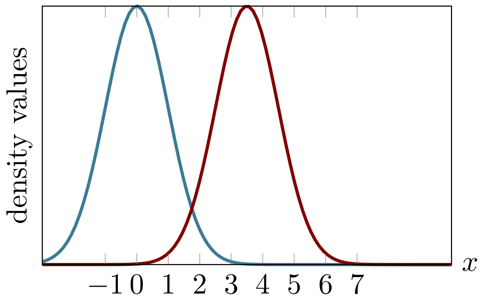


Figure 2.11: Difference = 3.5

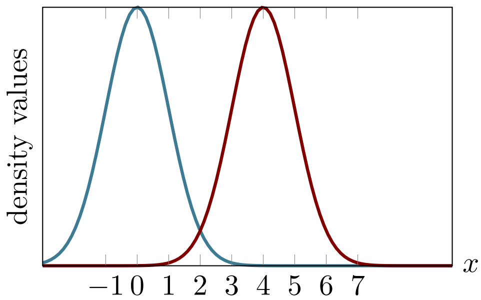


Figure 2.12: Difference = 4

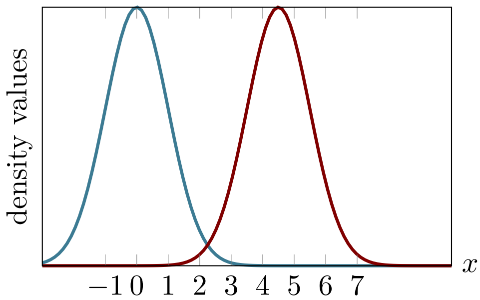


Figure 2.13: Difference = 4.5

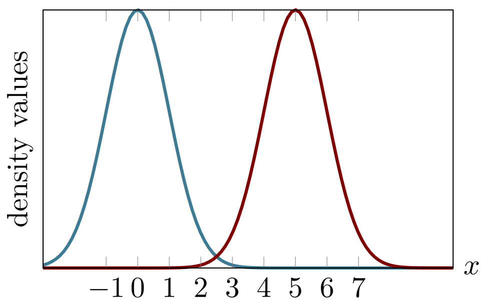


Figure 2.14: Difference = 5

For the distributions shown in figures 2.4 till 2.14, we calculated the probability that the PaF estimator, for various values of α, would take a higher value as the means of the distributions moved away (so theoretically, the distributions could be characterized as more polarized).

This was performed for both proposed bandwidths (i.e. $h^* \cong 4.7n^{-0.5}\sigma\alpha^{0.1}$ as well as $h^*_{alt} \cong n^{-0.5}IQ\frac{(3.76+14.7\sigma_{ln})}{(1+1.09\times10^{-4}\sigma_{ln})^{(7268+15323\alpha)}}$).

Results are presented in tables 2.1 and 2.2 respectively. We observe that for both bandwidths the estimator has a higher probability in correctly predicting the evolution of polarization correctly when $\alpha = 0.25$.

Table 2.1: Probability of PaF estimator taking higher value as distance grows for h^*

	difference between means									
alpha	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.25	0.75	0.95	0.99	1	1	1	1	1	1	1
0.3	0.73	0.95	0.98	1	1	1	1	1	1	1
0.35	0.72	0.95	0.98	1	1	1	1	1	1	1
0.4	0.7	0.95	0.98	1	1	1	1	1	1	1
0.45	0.7	0.95	0.98	1	1	1	1	1	1	1
0.5	0.71	0.96	0.98	1	1	1	1	1	1	1
0.55	0.7	0.96	0.98	1	1	1	1	1	1	1
0.6	0.69	0.96	0.97	1	1	1	1	1	1	1
0.65	0.7	0.96	0.97	1	1	1	1	1	1	1
0.7	0.69	0.96	0.97	1	1	1	1	1	1	1
0.75	0.68	0.95	0.95	0.99	1	1	1	1	1	1
0.8	0.65	0.92	0.94	0.99	1	0.99	1	1	1	0.99
0.85	0.64	0.91	0.91	0.97	1	0.98	1	1	1	0.99
0.9	0.6	0.81	0.88	0.97	0.98	0.98	1	0.99	0.99	0.99
0.95	0.54	0.75	0.81	0.91	0.96	0.98	0.99	0.99	0.99	0.98
1	0.45	0.63	0.66	0.82	0.93	0.95	0.99	0.99	0.98	0.94

Table 2.2: Probability of DER estimator taking higher value as distance grows for h^*_{alt}

	difference between means									
alpha	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.25	0.68	0.97	1	1	1	1	1	1	1	1
0.3	0.68	0.97	1	1	1	1	1	1	1	1
0.35	0.68	0.97	1	1	1	1	1	1	1	1
0.4	0.67	0.97	1	1	1	1	1	1	1	1
0.45	0.68	0.98	1	1	1	1	1	1	1	1
0.5	0.68	0.98	1	1	1	1	1	1	1	1
0.55	0.69	0.97	1	1	1	1	1	1	1	1
0.6	0.7	0.97	1	1	1	0.99	1	1	1	1
0.65	0.73	0.95	1	1	1	0.99	1	1	1	1
0.7	0.72	0.94	1	0.99	1	0.99	1	1	1	1
0.75	0.69	0.93	0.99	0.99	1	0.99	1	1	1	1
0.8	0.65	0.91	0.99	0.99	1	0.99	1	1	1	1
0.85	0.64	0.9	0.98	0.99	1	0.99	1	1	1	1
0.9	0.63	0.82	0.94	0.99	1	0.98	1	1	1	1
0.95	0.62	0.8	0.87	0.96	0.99	0.96	1	1	1	1
1	0.58	0.7	0.81	0.9	0.99	0.96	1	1	1	1

2.3.5 Experiment 2: Testing for goodness of fit between one bimodal distribution and a Gaussian Mixture Model

We performed a Bayesian Information Criterion (BIC) analysis to see if a Gaussian Mixture Model of two normal distributions would give better results than a single normal distribution.

To do that we generated a set of randomly normally distributed data. We followed again the procedure described above by starting with a uni-modal distribution, as shown in figure 2.4

and afterwards we created a population who's distribution can either be expressed by a single bi-modal distribution or two unimodal distributions, with the two means/modes moving away (again until a maximum distance of 5 points was created (as shown in figures 2.5 till 2.14).

On the above described set of randomly normally distributed data, we run a BIC analysis to see if the single normal distribution fits the data better than the mixture of two Gaussians. In the appendix the analytical results of the BIC analysis are presented (tables A.1 and A.2). Table 2.3 contains the main results of the BIC analysis, i.e. the percentage of times the BIC of one single normal distribution (BIC_{sgl}) is larger than that of the GMM (BIC_{gmm}) (i.e. $BIC_{gmm} - BIC_{sgl} \leq 0$) indicating that the BIC_{gmm} fits the data better . We see that indeed the predictions are correct, as for the first sets of data, when the two peaks are quite close to each other , the BIC_{sgl} takes mostly lower values (the proportion of times the BIC_{gmm} was smaller than BIC_{sgl} in all, of a total of one hundred iterations, for the first four columns). As the peaks move further away from each other, we observe the the BIC_{gmm} takes more often lower values than the BIC_{sgl}.

Table 2.3: Main Results of BIC Analysis

	Proportion of times when $BIC_{gmm} - BIC_{sgl} \leq 0$										
difference	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
percentage	0%	0%	0%	0%	20%	94%	100%	100%	100%	100%	100%

2.3.6 Application of PaF Estimator

After having processed the data with the appropriate scaling procedure (the Aldrich-McKelvey technique described above) and after having determined what the most appropriate level is for the α exponent, we proceed with the calculation of the polarization index for the said 10 European countries over the period 1989 until 2019. This results in a unique panel data set with 70 polarization estimates which allows for both longitudinal examination of the evolution of polarization within a unique country as well as a cross-country comparison on a given point in time.

In table 2.4 the results of the PaF estimator for $\alpha = 0.25$ are presented. The same results are also graphically presented in figure 2.15. A closer look into the results reveals some interesting things. First of all, we observe that the highest polarization estimator for each country does not lie in 1989 or 1994, for all countries it is in the period between 1999 till 2019 (in table 2.4 the asterisk next to the estimation indicates which is the highest estimation for the specific country). This is in accordance with the reviewed literature. It seems that indeed after the 2000,s approximately, we enter the third, and most severe, era of polarization. Even more in line with this is the fact that for all countries except the UK, polarization went up from 2014 to 2019. If we examine the average fluctuation of the PaF estimator for each country under study, we observe that for the eight out of ten countries (the exceptions are the United Kingdom and Spain) polarization has an upwards trend (the average of the increases or decreases observed for each of the five-years period is positive for said countries), as can be seen in table 2.8. All above confirms what is already mentioned in the literature on the subject, i.e. polarization is in an upward trend and that the current situation is the most dire to date.

In addition to the above analysis for $\alpha = 0.25$, we also run the code for calculating the PaF estimator on our data for all other values of α in the range $[0.25, 1]$ with a 0.01 interval. In total, we repeated the procedure for 76 values of the α exponent on all 10 countries for all 5-year interval in the 30-years period under examination. This created a total of 5320 polarization estimations. For the sake of brevity we present here the results for the additional three prominent values of α, namely $0.50, 0.75, 1.00$. All analytical results for all 76 values are extensively presented in the appendix (section C). What is clear from these additional results is the effect the α exponent has on the estimator and its evolution over time in the countries under study. In table 2.8 we present the average evolution over the thirty year period of the PaF estimator for the four main values of α $(0.25, 0.50, 0.75, 1.00)$. What can be seen is that (with the sole exception of the UK and Spain) on average polarization exhibits the largest increase when $\alpha = 0.25$. This result validates what our experiment indicated about the correct choice of α. According to the literature reviewed in the previous section, the rise of polarization in the EU is mostly due to the rise of populism, either left or right, like for example Marine Le Pen for France, Wilders and his Party for Freedom for the Netherlands, Farage's UKIP for the UK, Tsipras's SYRIZA on the left and the Golden Dawn on the right for Greece, Podemos for Spain, etc. The rise of populism is translated in an increase in distance between the two ends of the political axis. It can be said that the total range of political opinion (the variance of its distribution) increased. In other words, we observe an increase in alienation rather than an increase in the identification within the existing parties. Therefor, it is in accordance with the literature and our experiments that for the case of the EU, the value of 0.25 regarding the exponent α works best. It would be an interesting addition to juxtapose this finding with an application of the PaF index on data regarding the USA, where we have only two parties. It would be of interest, as in the USA we have both an increase in distance, as the parties and their candidates keep moving further and further from the political centre but at the same time there is stronger sorting between partisans, translated in this case as an increase in identification, which could lead to the need for an increased α.

Figure 2.15: PaF Estimators for $\alpha = 0.25$ from 1989 till 2019

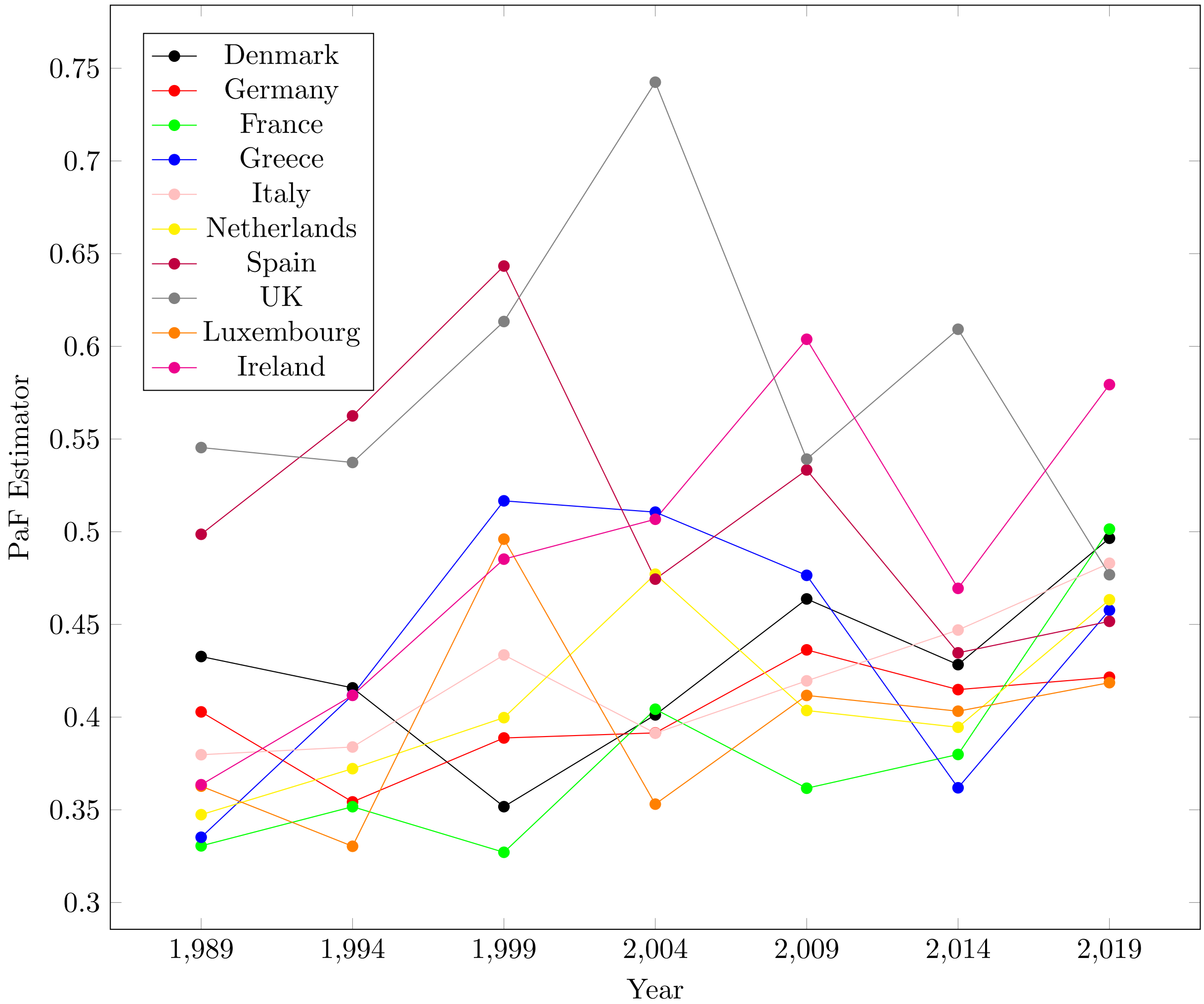


Table 2.4: PaF Values for a=0.25 and Respective Countries Ratings

	1989		1994		1999		2004		2009		2014		2019
France (1)	0.330547	4	0.330325	1	0.327083	4	0.353056	1	0.361637	2	0.361877	4	0.418581
Greece (2)	0.335196	1	0.35167	8	0.351693	6	0.391339	3	0.403569	1	0.379853	7	0.421526
Netherlands (3)	0.347373	7	0.354261	7	0.38875	7	0.39149	4	0.411693	3	0.394525	3	0.451671
Luxembourg (4)	0.362748	3	0.372192	3	0.399708	8	0.401194	6	0.419561	4	0.403193	2	0.457684
Ireland (5)	0.363533	6	0.383872	6	0.433526	1	0.404259	7	0.436274*	7	0.414835	3	0.463288
Italy (6)	0.379718	5	0.411739	5	0.485216	9	0.474434	8	0.463745	8	0.428308	10	0.476817
Germany (7)	0.402817	2	0.411829	4	0.496002*	3	0.477092*	2	0.476486	9	0.434669	6	0.483005*
Denmark (8)	0.432683	8	0.415776	2	0.516661*	5	0.506725	9	0.533329	6	0.446981	8	0.496484*
Spain (9)	0.498621	10	0.537357	10	0.61339	2	0.510566	10	0.539173	5	0.469451	1	0.501419*
UK (10)	0.545398	9	0.562493	9	0.643317*	10	0.742483*	5	0.603819*	10	0.609224	5	0.579356

Table 2.5: PaF Values for a=0.50 and Respective Countries Ratings

	1989		1994		1999		2004		2009		2014		2019
France (1)	0.311148	3	0.302447	1	0.308113	3	0.321604	1	0.325324	4	0.324413	3	0.356458
Netherlands (2)	0.318431	7	0.320538	8	0.321441	7	0.342842	2	0.351355	1	0.339094	7	0.360387
Luxembourg (3)	0.322942	1	0.321585	7	0.339378	8	0.352586	3	0.354233	2	0.344981	4	0.380583
Greece (4)	0.328104	2	0.339122	2	0.348778	1	0.353474	7	0.368627	3	0.346952	2	0.385205
Ireland (5)	0.331476	6	0.343793	6	0.373643	6	0.360399	6	0.374501	7	0.353538	9	0.386007
Italy (6)	0.339675	5	0.3554	5	0.393936	9	0.398799	4	0.388599	8	0.366267	10	0.39081
Germany (7)	0.353361	4	0.360812	3	0.398762	2	0.39985	8	0.391834	9	0.372315	6	0.400428
Denmark (8)	0.380014	8	0.36268	4	0.414129	5	0.407158	9	0.417386	6	0.379982	1	0.408169
Spain (9)	0.404837	10	0.421368	10	0.472679	4	0.409769	10	0.421376	5	0.38771	8	0.411539
UK (10)	0.426897	9	0.440994	9	0.482821	10	0.526364	5	0.457793	10	0.45917	5	0.445274

Table 2.6: PaF Values for a=0.75 and Respective Countries Ratings

	1989		1994		1999		2004		2009		2014		2019
Luxembourg (1)	0.296872	1	0.283921	3	0.294745	1	0.300503	3	0.298304	7	0.297869	1	0.313648
Netherlands (2)	0.297175	6	0.297237	9	0.299668	6	0.310403	1	0.311383	3	0.307082	6	0.318942
France (3)	0.298615	3	0.299538	6	0.307656	9	0.314948	2	0.312738	1	0.307611	7	0.325799
Ireland (4)	0.308948	5	0.311992	2	0.310292	3	0.315411	6	0.320477	2	0.308565	2	0.326062
Italy (5)	0.312206	2	0.316364	5	0.327683	5	0.335337	7	0.326408	6	0.310388	10	0.330757
Germany (6)	0.316878	4	0.316921	1	0.328544	4	0.337895	8	0.334545	9	0.319111	8	0.336005
Greece (7)	0.325274	9	0.321633	4	0.332039	7	0.337905	9	0.33564	8	0.324547	5	0.337111
Spain (8)	0.337585	7	0.325404	7	0.33928	2	0.340442	5	0.338367	4	0.327981	3	0.33931
Denmark (9)	0.337841	10	0.341218	8	0.370184	8	0.343681	10	0.344386	5	0.328083	9	0.346573
UK (10)	0.342818	8	0.351197	10	0.380472	10	0.389941	4	0.362105	10	0.356595	4	0.357534

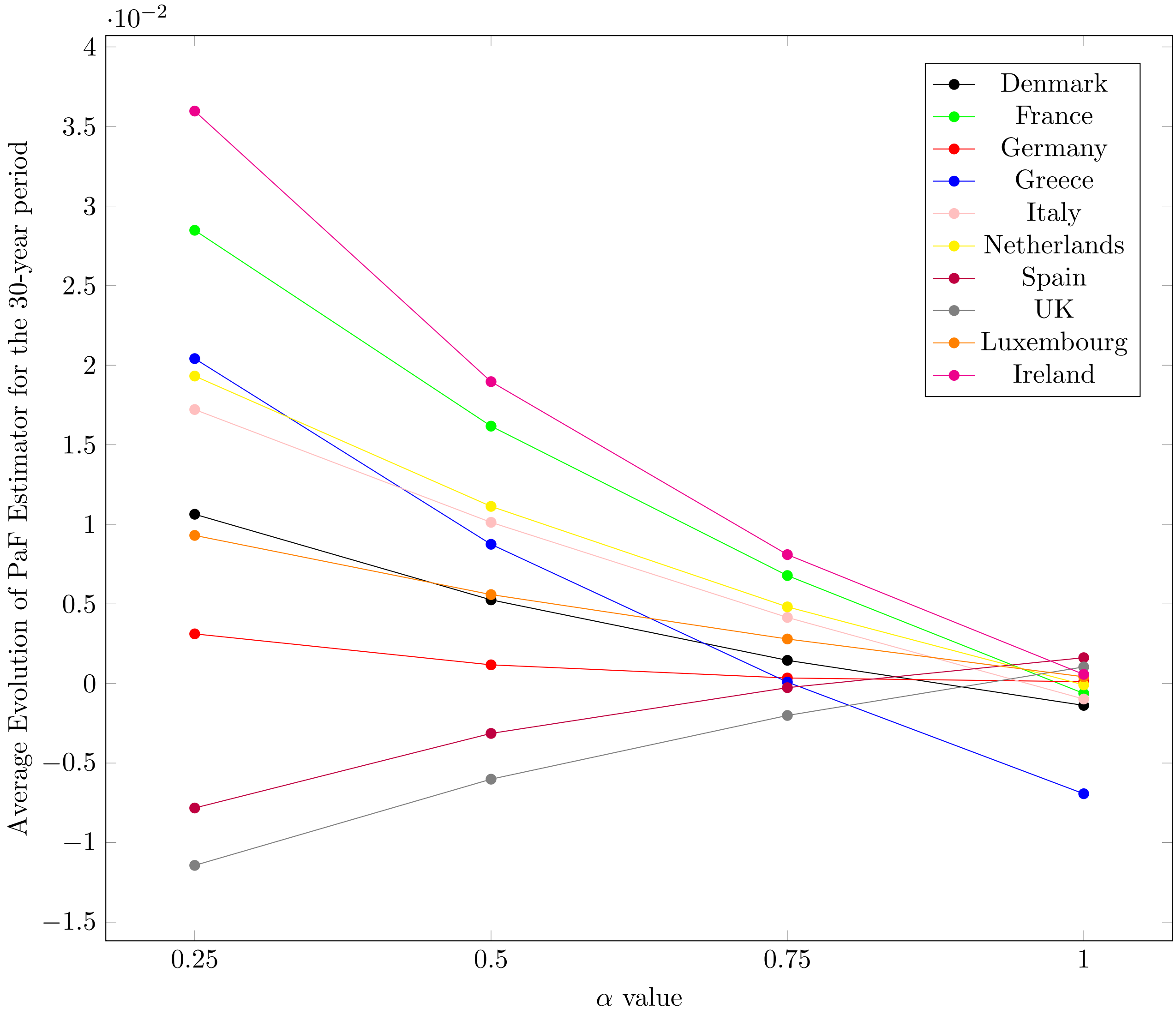
Table 2.7: PaF Values for a=1.00 and Respective Countries Ratings

	1989		1994		1999		2004		2009		2014		2019
Luxembourg (1)	0.279659	1	0.270931	1	0.275486	10	0.283766	4	0.273116	1	0.27857	3	0.28016
UK (2)	0.280428	5	0.28068	3	0.279889	9	0.284542	6	0.277626	5	0.278614	1	0.282184
Netherlands (3)	0.280687	6	0.282694	10	0.281992	6	0.285532	1	0.277942	10	0.278686	10	0.284099
Spain (4)	0.286305	2	0.282913	9	0.283388	1	0.286092	10	0.2801	3	0.28045	2	0.286665
Germany (5)	0.288206	4	0.283247	6	0.284991	7	0.286399	3	0.282728	6	0.281133	8	0.286831
France (6)	0.2906	8	0.28592	5	0.286239	5	0.287708	5	0.284048	9	0.281771	6	0.286855
Ireland (7)	0.292372	7	0.288259	7	0.287176	3	0.293025	9	0.290055	7	0.281974	5	0.288892
Italy (8)	0.292689	9	0.288403	4	0.287808	2	0.297288	2	0.290256	2	0.282982	9	0.294498
Denmark (9)	0.302761	3	0.299826	8	0.290666	4	0.302029	7	0.294924	8	0.286138	7	0.295809
Greece (10)	0.32567	10	0.30009	2	0.314598	8	0.314327	8	0.308067	4	0.286312	4	0.29603

Table 2.8: Average Evolution of PaF Estimator for the Period 1989 - 2019, depending on the value of α

	0.25	0.5	0.75	1
Denmark	0.010633	0.005254	0.001455	-0.00138
France	0.028479	0.01617	0.006782	-0.00062
Germany	0.003118	0.001171	0.000344	0.000114
Greece	0.020415	0.008747	8.75E-05	-0.00693
Ireland	0.03597	0.018966	0.008098	0.000573
Italy	0.017214	0.010126	0.004151	-0.00098
Luxembourg	0.009306	0.005586	0.002796	0.000421
Netherlands	0.019319	0.011129	0.004815	-8.8E-05
Spain	-0.00783	-0.00314	-0.00026	0.001621
UK	-0.01143	-0.00601	-0.00201	0.00104

Figure 2.16: Average Evolution of PaF Estimator for the Period 1989 - 2019, depending on the value of α



Chapter 3

Political Polarization and Income Inequality

3.1 Income Inequality

As described in the first chapter, it has been observed that income inequality and political polarization follow a similar trend in the USA and most Western European countries after the end of WWII. Both these phenomena have been a concern of people and politicians for a while now and have entered everyday discussions. As such they have, of course, attracted academic interest as well and both subjects have, separately, been studied at length. At a lesser frequency (mainly due to the difficulties that arise in the measurement of political polarization) their causal relationship has been studied as well.

Economic growth and inequality are two measures highly connected. Growth on its own is only one side of the story. The other part is how this growth is distributed. The distribution of income and the groups this creates often leads to societal tension between said groups (like the Occupy Wall Street movement, mentioned in the first chapter). As is extensively discussed in the existing literature, inequality is growing, globally but also within countries. Accessing and analyzing such data provides useful information and can work as a tool to reinforce democracy. Such kind of socio-economic disparities, like economic inequality and unequal distribution of wealth, create social unrest and reinforce the social divide.

Just recently, the World Inequality Report of 2022 (Cancel *et al.*, 2022) was released. It analyzes the growing world inequality, presenting some very interesting statistics on a global level. For example, worldwide in the past twenty-five years, wealth has grown. But for the ultra-rich (the 0.001%) it has grown at a quite faster pace. While in 1995, the 0.001% owned about 3.4% of global wealth, by 2021 they had almost doubled their wealth to 6.5%. The same is not true for the bottom 50%. While it experienced a small increase from 1.5% to slightly above 2% (around 2007) it has now stagnated at just 2%. Of course, these statistics reflect the entire world, and so it includes the vast differences amongst regions like Europe (the most equal region in the world) and North Africa and the Middle East (the two most unequal regions of the world).

When examining Europe alone, things are of course better, but they are not good. The WIR 2022 (Cancel *et al.*, 2022) offers a closer insight to some of the European countries included in our analysis as well. Looking at their inequality statistics, we see that the path followed is once again along the same lines; it reduces shortly after WWII and around the 1980s it starts rising again. In France inequality dropped after the WWII with the expansion of the social state and various pro-labor policies implemented after the 1968 social protests. But inequality started rising again after 1983, when a wave of deregulatory policies was put forward. In Germany, according to the WIR (Cancel *et al.*, 2022), income inequality was reduced between the 1960s and 1980s. After that, and until today, the share of the top 10% income started rising disproportionately, affected in part by liberalization policies implemented. The same evolution, but even more intensely, is found in Italy. The top 10% - bottom 50% ratio fell sharply until the beginning of the 1980s, when it started rising again. The austerity policies that followed the European financial crisis exacerbated this phenomenon even further. In Spain again we observe the reduction of the ratio until the 1980s, after which it follows an unstable trend. The economic growth of the 1995-2005 period negatively influenced inequality but the financial crisis of the Eurozone depressed average income. Lastly, according to the WIR 2022 (Cancel *et al.*, 2022), income inequality in the UK also witnessed a sharp decrease until the 1970s. The change towards more liberal policies in the 1980s resulted in an increase of the total income share help by the top 10% by approximately 10%.

The above analysis goes to show that the course of inequality follows a similar path with that of political polarization. We could again say that, just as with the evolution of political polarization, the evolution of income inequality is divided in three eras as well. In the first era, following the end of WWII and roughly until the end of the 1970s, we observe a decline in economic inequality. Upon entering the 1980s, this changes and inequality starts to rise. Alongside the beginning of the political divide and the sorting in societies analyzed in the previous chapter, we observe that inequality increases. One might argue that inequality increased as a result of policies implemented. It is not too far a stretch to say that it is the resulted, by said policies, inequality that led to (or at least aided) the sorting procedure. In other words, people blamed the governments for their impoverishment compared to the increased concentration of money in the top percentages. In the third, so to speak, era, following the Great Recession, which was as much of a financial crisis as well as a political one (at least for the most part in Europe, this was a period when Euroscepticism bloomed), we see a deepening of the cleavage between the rich and the poor, and what is perhaps more distinct is the narrowing of the middle class. In this era, politically speaking, we observe an exacerbation of the political polarization phenomenon.

This has led to the hypothesis we test in this chapter, i.e. if inequality positively affects political polarization.

3.2 Hypothesis testing

In this chapter we test the hypothesis that economic inequality has a positive effect on the increase of political polarization and economic growth has a negative impact.

It is often the case, in everyday talk, that people blame the government for the worsening of their living conditions, for unemployment, for increased inflation levels etc. As such, we want to test if the increase in the economic divide within a society affects people's vote. During the Great Recession, we observed many phenomena that lead to this hypothesis. We have already referred to the Occupy Wall Street movement, whose slogan was ≪We are the 99%≫, clearly making a reference to the concentrated wealth in the top 1%. This movement, as indicated by its name, started in the financial district of Wall str. in New York, but soon expanded over the rest of the USA. The movement affected, among other things, Obama's campaign for re-election. In a speech he gave in December 2013 he characterized rising inequality as the ≪defining challenge of our time≫, capitalizing on the popularity of the matter. So, we already witness a direct link between inequality and politics.

Besides the example of the ≪Occupy Wall Street≫ that we have referred to earlier, we can name many others as well in the EU. In the years of the financial crisis in Greece, when the IMF, the EU and the Central European Bank imposed austerity measures to solve the sovereign debt crisis, people blamed the Greek government for the wage cuts, the invoked privileges etc. This was a very turbulent period for Greek politics as well, as a direct consequence of said measures. In 2012, two rounds of elections were held, in May and then in June, as no party was able to form government. It is also the first time for the far-right ultra-nationalist party of the Golden

Dawn to enter the Greek parliament. In 2015, before the end of term, elections are again held in Greece, again two times (first in January and then again in September). In both elections no party managed to obtain the majority of votes, leading to a coalition government (both times the same) between Tsipras' Coalition of the Radical Left (SYRIZA) and the right-wing Independent Greeks (ANEL), not the most expected combination of parties, to say the least. In 2015 also a referendum was held in Greece, the first one since 1974. As Greece started to gain again economic stability, so does the political life of the country, as in the next elections of 2019 we have again a one-party government, with the party of New Democracy obtaining the majority of votes.

Spain, another country hit especially hard by the financial crisis, also faced a politically unstable period when unemployment sky-rocketed, together with inflation and income inequality. Spain followed, more or less, a path similar to Greece. It officially entered recession by the first quarter of 2012 (preceded by contractions in growth in the third quarter of 2008, in 2009 and in 2010). The main cause behind Spain's financial crisis was the housing bubble and by 2012 the country had officially entered the bailout program. By 2015, three years later, came the end of the two party system that prevailed in the country since the transition to democracy in the 1970s.

In the next section, the regressions performed to test our hypothesis are presented.

3.2.1 Regression Specifications

We test four different independent variables to describe the economic growth:

- Rate of economic growth compared to the previous year,
- Rate of economic growth compared to the three years back (for example 1986 to 1989),
- Rate of economic growth compared to five years back (in this case we use the rate of economic growth during the period between two European Parliament Elections), and lastly
- Average growth rate for the past three years (for example average of the growth rate of the years 1987, 1988 and 1989)

We estimate the model using the Fixed and Random Effects methods firstly and then allowing for individual effects, including either one or two independent variables in each separate regression.

Analytically, we estimate the following specifications:

- Fixed Effects (FE) model:
$$y_{it} = X_{1it}b_1 + a_{1i} + e_{it},$$
where y_{it} is the political polarization index ($P_\alpha(\hat{F})$, for $a = 0.25$), X_{1it} is the $1x2$ time-variant independent variable vector (containing the Gini index and the economic growth rate variable for 1 year, 3 years, 5 years and the average of 3 years respectively in each regression performed), b_1 is the $2x1$ respective parameter matrix, a_{1i} is the unobserved time-invariant individual effect and e_{it} is the error term
- Random Effects (RE) model:
$$y_{it} = b_0 + X_{1it}b_1 + a_{1i} + e_{it},$$
where b_0 is the random intercept,assuming in this case that the time-invariant unobserved heterogeneity term a_{1i} is uncorrelated with the independent variables, thus allowing for random intercepts.

Additionally, we estimate the FE and RE models where we allow for individual slopes. We do so by allowing for individual slopes first only for the economic growth variable and secondly for both independent variables (Gini coefficient and growth rate).

So, the models, as mentioned in the result tables, correspond to the following:

- FEIS I:
$$y_i = X_{1it}b_1 + W_{Ii}a_{1i} + e_i,$$
where W_{Ii} is a matrix of slope variables, containing for $ModelI$ only the growth variable

- FEIS II:
$$y_i = X_{1it}b_1 + W_{Ii}a_{1i} + e_i,$$
where W_{IIi} is a matrix of slope variables, containing for $ModelII$ both independent variables, i.e. the growth rate variable as well as the Gini coefficient

- Model Random Slopes I:
$$y_{it} = b_0 + X_{1it}(b_1 + u_{Iij}) + a_{1i} + e_{it},$$
where u_{Iij} represents the random individual effect for the growth rate independent variable, and

- Model Random Slopes II:
$$y_{it} = b_0 + X_{1it}(b_1 + u_{IIij}) + a_{1i} + e_{it},$$
where u_{IIij} represents the random individual effect for the growth rate and the gini coefficient independent variables

Table 3.1 summarizes the countries and the years under study.

Table 3.1: Group of Countries and Year Intervals

Group of Countries		Years	
1	Denmark	**5-years' intervals over a thirty-years period**	
2	France		
3	Germany		
4	Greece	**Years of European Parliament Elections**	1989
5	Ireland		1994
6	Italy		1999
7	Luxembourg		2004
8	Netherlands		2009
9	Spain		2014
10	UK		2019

The results of above-described regressions are analytically presented in sets of three tables per regression. In the first table the main results (coefficients as well as the detailed results regarding the random effects of each regression) of each regression are presented. In the second table the individual slopes are presented. In the third and last table the results of the various test are included. In all cases, the results of the Hausman tests indicate that the appropriate method of choice are the Random Effects.

In our analysis we have also included a plot of Actual versus Fitted values of the dependent variable of political polarization. We observe that the regression line neatly follows the real data indicating the high explanatory power of the models.

Table 3.2: Estimation of 1 Year Growth Rate and Gini Index on Political Polarization

	Dependent Variable: $P_{\alpha}(\hat{F})$					
	Panel Linear Models		*Linear Mixed-Effects Models*			
	FE	RE	Random Slopes I	Random Slopes II	FEIS I	FEIS II
1 year Growth	-0.0136^{*}	-0.0126^{*}	-0.0139^{*}	-0.0117	-0.0244	-0.0222
	(0.0074)	(0.0073)	(0.0073)	(0.0073)	(0.0242)	(0.0259)
Gini	1.2027^{***}	1.1526^{***}	1.1528^{***}	1.0318^{***}	1.3749^{***}	0.7584
	(0.3492)	(0.2972)	(0.2946)	(0.3366)	(0.3918)	(1.2965)
Constant		-0.1399	-0.1393	-0.0170		
		(0.2991)	(0.2968)	(0.3113)		
Random Effects						
Number of Groups (Country):	10	10	10	10	10	10
sd(Country)		0.1125	0.1166	0.5869		
sd(1 year Growth)			0.0031	0.0065		
sd(Gini)				0.6920		
Observations	70	70	70	70	70	70
R^2	0.198				0.9887	0.9898
Adjusted R^2	0.046				0.9839	0.9821
Log Likelihood		37.303	37.377	39.996		
Akaike Inf. Crit.		-64.605	-60.754	-59.993		
Bayesian Inf. Crit.		-53.363	-45.015	-37.508		
F Statistic (df = 2; 58)	7.171^{***}					

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 3.3: Individual Slopes

Fixed Effects' Individual Slopes

	Model I		*Model II*			
	Estimate	t value	Estimate of growth	t value	Estimate of gini	t value
Denmark	-0.02	-1.01	-0.02	-0.86	0.76	0.58
France	0.00	0.11	0.00	0.09	0.02	0.00
Germany	0.03	0.76	0.01	0.28	-0.21	-0.14
Greece	0.00	0.12	0.00	0.06	0.60	0.33
Ireland	-0.01	-0.45	-0.02	-0.57	1.62	0.97
Italy	0.02	0.64	0.03	0.62	1.04	0.54
Luxembourg	0.02	0.52	0.02	0.71	-0.77	-0.37
Netherlands	0.03	0.68	0.03	0.70	1.37	0.64
Spain	0.02	0.59	0.02	0.43	1.13	0.53
United Kingdom	0.09	1.66	0.10	1.70	1.67	0.92

Random Effects' Individual Slopes

	Model I		*Model II*			
	intercepts	NA	deviation from slopes of growth	NA	deviation from slopes of gini	NA
Denmark	-0.00		-0.00		0.25	
France	0.00		0.01		-0.83	
Germany	0.00		0.01		-0.64	
Greece	0.00		0.00		-0.43	
Ireland	-0.00		-0.01		0.83	
Italy	0.00		0.00		-0.05	
Luxembourg	0.00		0.01		-0.74	
Netherlands	-0.00		-0.00		0.18	
Spain	-0.00		-0.00		0.49	
UK	-0.01		-0.01		0.93	

Table 3.4: ANOVA and RANOVA test results for choice of model

Statistic	FE vs FEIS I	FEIS I vs FEIS II	FE vs FEIS II	RE	REIS I	REIS II	Hausman
P-value	0.708	0.904	0.924				
P-value(country)				2.035e-06***			
P-value(1 year growth)					0.928	0.886	
P-value(gini)						0.155	
P-value							0.7304

Figure 3.1: Actual vs Fitted Values for Growth Rate for 1 year

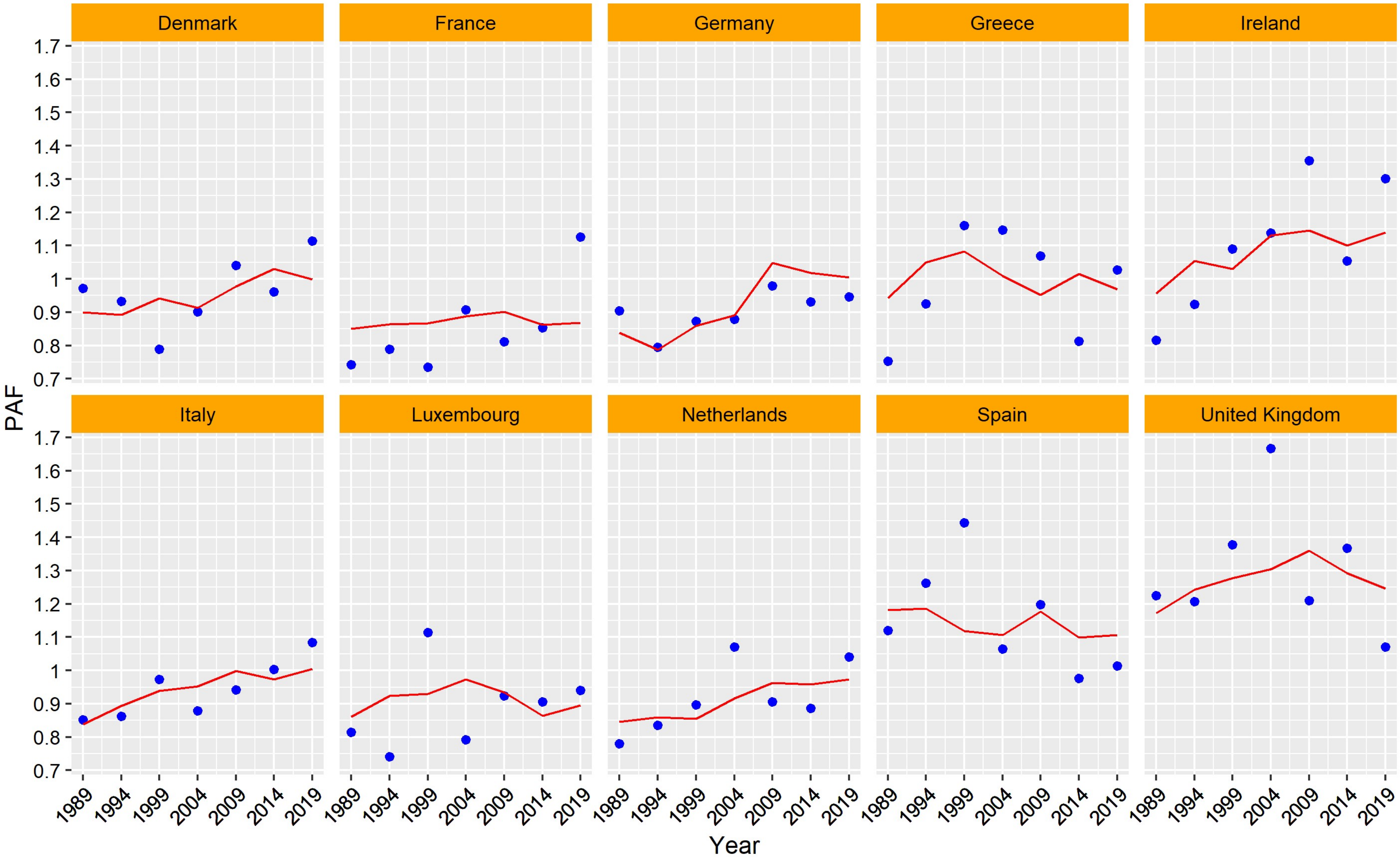


Table 3.5: Estimation of 3 Years' Growth Rate and Gini Index on Political Polarization

	Dependent Variable: $P_\alpha(\hat{F})$					
	Panel Linear Models		*Linear Mixed-Effects Models*			
	FE	RE	Random Slopes I	Random Slopes II	FEIS I	FEIS II
3 years' Growth	−5.6633*	−5.6160*	−5.7620*	−6.5096*	−4.6975	−19.1282
	(3.2510)	(3.1314)	(3.4146)	(3.7800)	(9.9121)	(16.2500)
Gini	0.9132**	0.9279***	0.8824***	0.6656*	0.8964*	−1.3356
	(0.3760)	(0.3108)	(0.3211)	(0.3955)	(0.5286)	(2.0147)
Constant		5.6881*	5.8797*	6.8313*		
		(3.2439)	(3.5104)	(4.0213)		
Random Effects						
Number of Groups (Country):	10	10	10	10	10	10
sd(Country)		0.1092	4.5394	7.4880		
sd(3 years Growth)			4.6489	6.6738		
sd(Gini)				0.9072		
Observations	70	70	70	70	70	70
R^2	0.194				0.9898	0.9909
Adjusted R^2	0.042				0.9854	0.984
Log Likelihood		37.425	38.553	42.398		
Akaike Inf. Crit.		−64.850	−63.106	−64.795		
Bayesian Inf. Crit.		−53.608	−47.367	−42.310		
F Statistic (df = 2; 58)	6.997***					
Note:				*p<0.1; **p<0.05; ***p<0.01		

Table 3.6: Individual Slopes

Fixed Effects' Individual Slopes

	Model I		*Model II*			
	Estimate	t value	Estimate of growth	t value	Estimate of gini	t value
Denmark	-4.70	-0.47	-19.13	-1.18	-1.34	-0.66
France	-15.07	-1.10	-1.29	-0.07	3.57	0.67
Germany	9.26	0.68	28.68	0.98	2.61	0.99
Greece	-4.78	-0.34	8.89	0.47	2.56	1.14
Ireland	-16.35	-1.24	-17.77	-0.76	0.28	0.11
Italy	-2.51	-0.19	10.53	0.34	2.03	0.51
Luxembourg	-6.51	-0.45	8.32	0.42	2.14	0.86
Netherlands	-2.23	-0.17	19.88	0.47	3.42	0.55
Spain	15.96	1.09	43.16	1.72	-0.09	-0.02
United Kingdom	12.03	0.91	31.82	1.64	3.79	1.60

Random Effects' Individual Slopes

	Model I		*Model II*			
	intercepts	NA	deviation from slopes of growth	NA	deviation from slopes of gini	NA
Denmark	1.31		-1.67		-0.23	
France	-5.10		-8.49		-1.15	
Germany	-3.75		-4.99		-0.68	
Greece	-3.58		-1.97		-0.27	
Ireland	2.86		5.57		0.76	
Italy	-0.29		-0.49		-0.07	
Luxembourg	-5.17		-5.01		-0.68	
Netherlands	0.48		0.06		0.01	
Spain	4.55		6.52		0.89	
UK	8.70		10.46		1.42	

Table 3.7: ANOVA and RANOVA test results for choice of model

Statistic	FE vs FEIS I	FEIS I vs FEIS II	FE vs FEIS II	RE	REIS I	REIS II	Hausman
P-value	0.2404	0.8297	0.5928				
P-value(country)				3.471e-06			
P-value(3 years' growth)					0.3237	0.20358	
P-value(gini)						0.0528	
P-value							0.9978

Figure 3.2: Actual vs Fitted Values for Growth Rate of 3 years

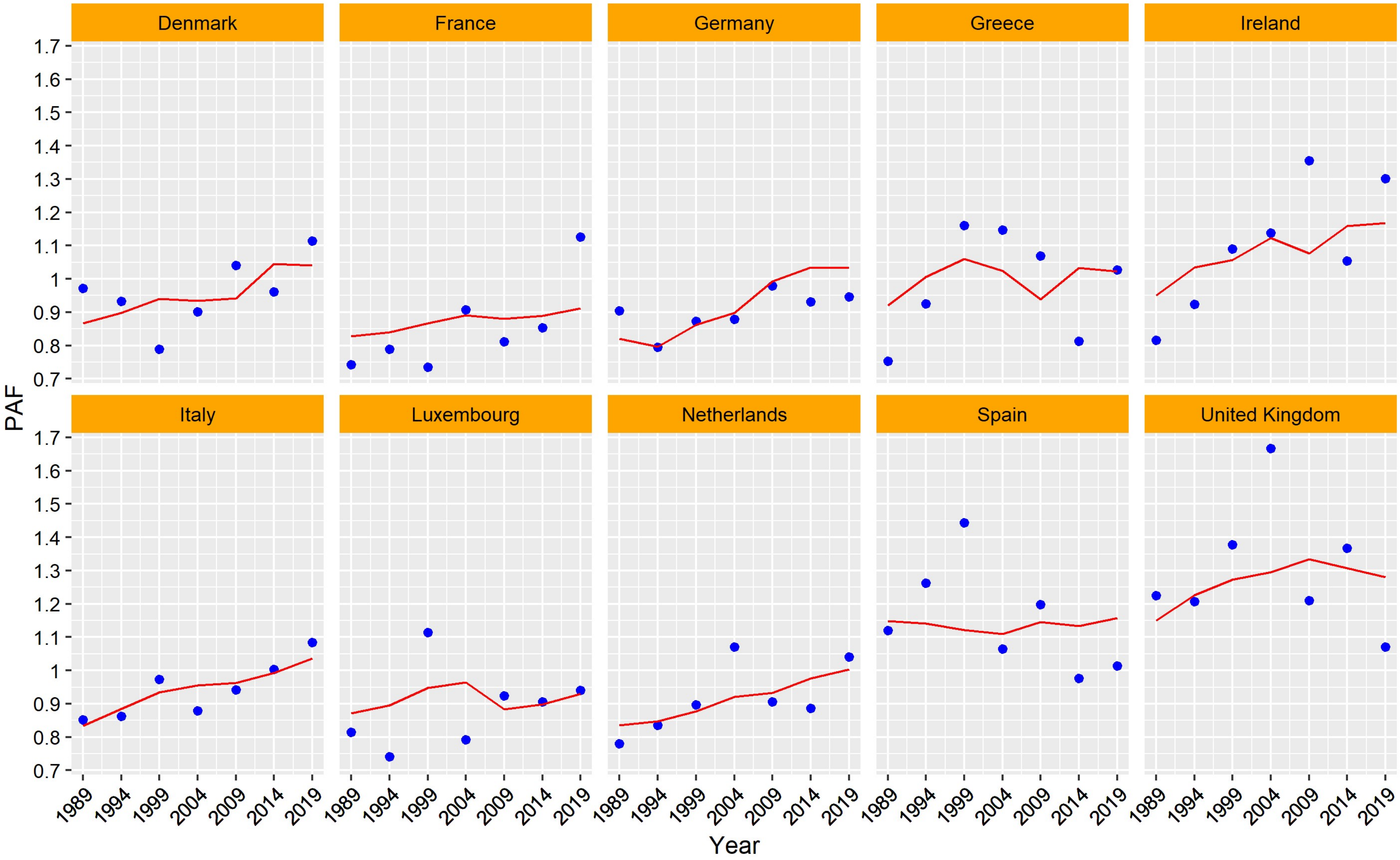


Table 3.8: Estimation of 5-Years' Growth Rate and Gini Index on Political Polarization

	Dependent Variable: $P_\alpha(\hat{F})$					
	Panel Linear Models		*Linear Mixed-Effects Models*			
	FE	RE	Random Slopes I	Random Slopes II	FEIS I	FEIS II
5-years' Growth Rate	-5.6575^{*}	-5.6102^{*}	-5.7605^{*}	-6.5026^{*}	-4.6927	-19.1110
	(3.2479)	(3.1283)	(3.4301)	(3.7760)	(9.9025)	(16.2348)
Gini	0.9132^{**}	0.9279^{***}	0.8810^{***}	0.6657^{*}	0.8964^{*}	0.9132
	(0.3760)	(0.3108)	(0.3214)	(0.3955)	(0.5286)	(0.3763)
Constant		5.6823^{*}	5.8795^{*}	6.8243^{*}		
		(3.2408)	(3.5251)	(4.0173)		
Random Effects						
Number of Groups (Country):	10	10	10	10	10	10
sd(Country)		0.1092	4.6792	7.4795		
sd(5 years Growth)			4.7890	6.6654		
sd(Gini)				0.9069		
Observations	70	70	70	70	70	70
R^2	0.194				0.9898	0.9909
Adjusted R^2	0.042				0.9854	0.984
Log Likelihood		37.425	38.554	42.397		
Akaike Inf. Crit.		-64.850	-63.108	-64.795		
Bayesian Inf. Crit.		-53.607	-47.368	-42.310		
F Statistic (df = 2; 58)	6.997^{***}					
Note:				$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$		

Table 3.9: Individual Slopes

Fixed Effects' Individual Slopes

	Model I		*Model II*			
	Estimate of growth	t value	Estimate of growth	t value	Estimate of gini	t value
Denmark	-4.69	-0.47	-19.11	-1.18	-1.34	-0.66
France	-15.05	-1.10	-1.29	-0.07	3.57	0.67
Germany	9.26	0.68	28.65	0.98	2.61	0.99
Greece	-4.77	-0.34	8.88	0.47	2.56	1.14
Ireland	-16.34	-1.24	-17.75	-0.76	0.28	0.11
Italy	-2.51	-0.19	10.52	0.34	2.04	0.51
Luxembourg	-6.50	-0.45	8.31	0.42	2.14	0.86
Netherlands	-2.23	-0.17	19.87	0.47	3.42	0.55
Spain	15.95	1.09	43.12	1.72	-0.09	-0.02
United Kingdom	12.02	0.91	31.79	1.64	3.79	1.60

Random Effects' Individual Slopes

	Model I		*Model II*			
	intercepts	NA	deviation from slopes of growth	NA	deviation from slopes of gini	NA
Denmark	1.34		-1.67		-0.23	
France	-5.24		-8.48		-1.15	
Germany	-3.84		-4.98		-0.68	
Greece	-3.68		-1.97		-0.27	
Ireland	2.92		5.57		0.76	
Italy	-0.30		-0.49		-0.07	
Luxembourg	-5.31		-5.00		-0.68	
Netherlands	0.48		0.06		0.01	
Spain	4.68		6.51		0.89	
UK	8.94		10.44		1.42	

Note: FE: estimate of coefficient refers to deviation from base country (Denmark)
RE: deviation refers to deviation from mean slope, as mentioned in regression results

Table 3.10: ANOVA and RANOVA test results for choice of model

Statistic	FE vs FEIS I	FEIS I vs FEIS II	FE vs FEIS II	RE	REIS I	REIS II	Hausman
P-value	0.24042	0.8297	0.5928				
P-value(country)				3.472e-06 ***			
P-value(5 years growth)					0.3233	0.20358	
P-value(gini)						0.05294	
P-value							0.9978

Figure 3.3: Actual vs Fitted Values for Growth Rate of 5 years

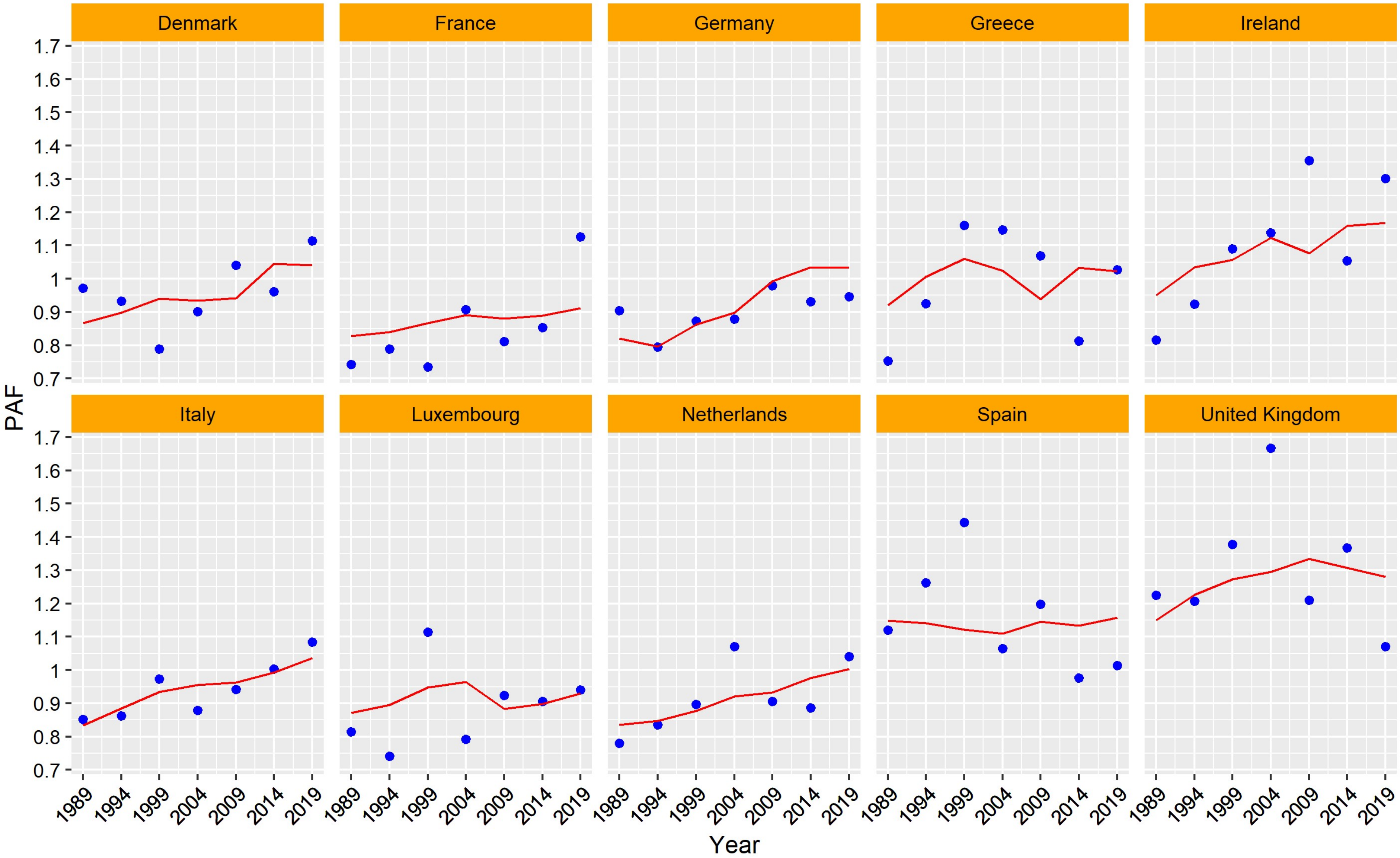


Table 3.11: Estimation of Average 3 years' Growth Rate and Gini Index on Political Polarization

	Dependent Variable: $P_\alpha(\hat{F})$					
	Panel Linear Models		*Linear Mixed-Effects Models*			
	FE	RE	Random Slopes I	Random Slopes II	FEIS I	FEIS II
Average 3 years' Growth	−0.0027	−0.0008	−0.0015	0.0014	−0.0525	−0.0496
	(0.0102)	(0.0099)	(0.0099)	(0.0099)	(0.0476)	(0.0523)
Gini	1.1670***	1.1066***	1.1384***	1.0145***	1.1852***	0.8984***
	(0.3603)	(0.3029)	(0.2996)	(0.3424)	(0.3814)	(1.3505)
Constant		−0.1058	−0.1356	−0.0101		
		(0.3042)	(0.3012)	(0.3161)		
Random Effects						
Number of Groups (Country):	10	10	10	10	10	10
sd(Country)		0.1093	0.1175	0.6103		
sd(Average 3 years Growth)			0.0062	0.0101		
sd(Gini)				0.7162		
Observations	70	70	70	70	70	70
R^2	0.1533				0.9885	0.9893
Adjusted R^2	−0.0073				0.9835	0.9813
Log Likelihood		35.8600	36.0092	38.8592		
Akaike Inf. Crit.		−61.7199	−58.0185	−57.7184		
Bayesian Inf. Crit.		−50.4775	−42.2790	−35.2335		
F Statistic (df = 2; 58)	5.2499***				5.2499***	5.2499***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.12: Individual Slopes

Fixed Effects' Individual Slopes

	Model I		*Model II*			
	Estimate	t value	Estimate of growth	t value	Estimate of gini	t value
Denmark	-0.05	-1.10	-0.05	-0.95	0.90	0.67
France	0.03	0.42	0.03	0.37	-0.42	-0.07
Germany	0.07	0.74	0.05	0.50	-0.23	-0.15
Greece	0.09	1.70	0.09	1.57	-0.42	-0.24
Ireland	0.02	0.45	0.02	0.35	1.15	0.67
Italy	0.06	0.89	0.06	0.90	1.05	0.51
Luxembourg	0.04	0.78	0.06	0.91	-1.09	-0.48
Netherlands	0.06	0.90	0.06	0.89	1.21	0.57
Spain	0.06	1.03	0.05	0.80	0.91	0.40
United Kingdom	0.13	1.69	0.13	1.55	1.00	0.54

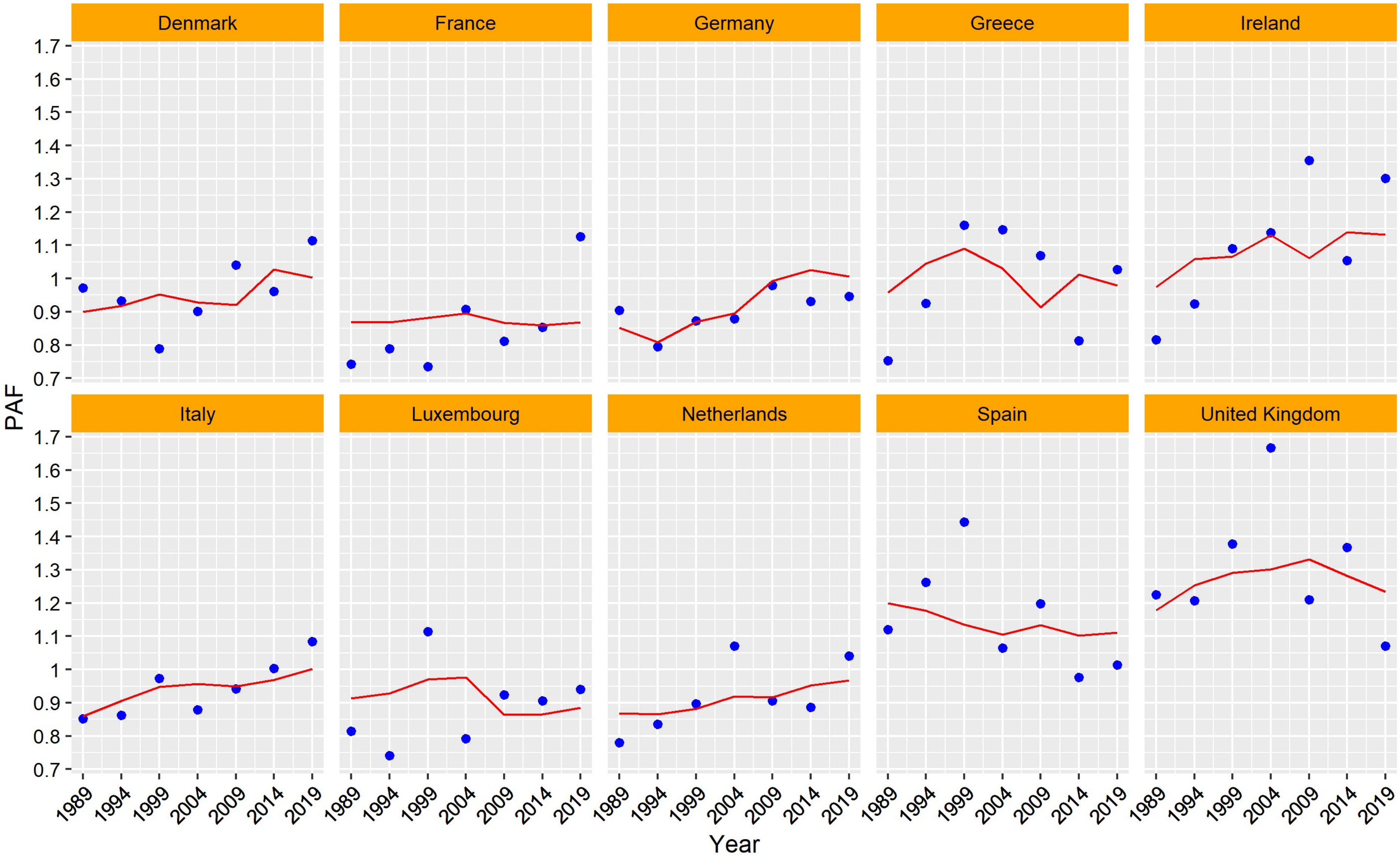
Random Effects' Individual Slopes

	Model I		*Model II*			
	intercepts	NA	deviation from slopes of growth	NA	deviation from slopes of gini	NA
	intercepts	NA	slopes of growth	slopes of gini	NA	NA
Denmark	-0.00		-0.00		0.27	
France	0.01		0.01		-0.85	
Germany	0.01		0.01		-0.62	
Greece	0.01		0.01		-0.49	
Ireland	-0.01		-0.01		0.76	
Italy	-0.00		-0.00		0.05	
Luxembourg	0.01		0.01		-0.83	
Netherlands	-0.00		-0.00		0.25	
Spain	-0.00		-0.01		0.50	
UK	-0.01		-0.01		0.96	

Table 3.13: ANOVA and RANOVA test results for choice of model

Statistic	FE vs FEIS I	FEIS I vs FEIS II	FE vs FEIS II	RE	REIS I	REIS II	Hausman
P-value	0.5413	0.9503	0.9012				
P-value(country)				8.373e-06			
P-value(average 3-years' growth)					0.8613	0.8437	
P-value(gini)						0.1272	
P-value							0.6564

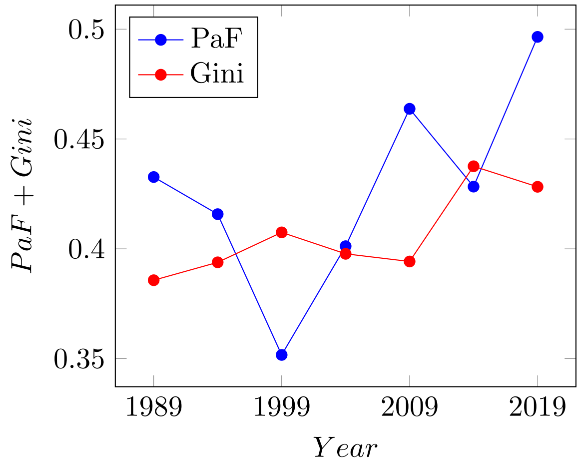
Figure 3.4: Actual vs Fitted Values for Growth Rate of the Average of the Past 3 Years



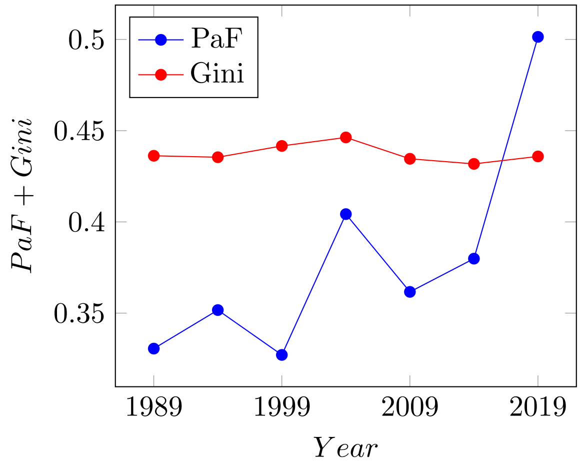
As can be seen from the results presented above, the best models are the ones where the growth variables are calculated as the growth of the past three or five years. This was expected and is in accordance with the observations of the historic events. Indeed, it seems to usually take a while for the various financial events to impact voting behavior. For example, in the USA the financial crisis started with the collapse of the Lehman Brothers and the Occupy Wall Street phenomenon started in 2011, 3 years later. Greece signed the first bailout memorandum in 2010, but it is not until 2012 that we start observing multiple rounds of elections and the entrance of the Golden dawn party in the Greek parliament. Similarly in Spain: the program started in 2012 but the two-party system broke down in 2015, the same year the Podemos entered the Spanish parliament. In the UK, the political tension culminated in 2016 with the announcement of the referendum regarding the EU membership. The poubt being made here is obvious. People do not change their mind about which party to vote overnight. It takes sometime for the various political developments to sink in and affect voter's decisions. The same is true, on the positive side, for economic growth. It takes some time for the effects on economic growth to soften to political stand of the mass public. Economic growth has a positive effect on people's lives (for some more than others, thus the increase in inequality but still the overall effect is positive) and this is affects voting patterns. As pointed out in a vast number of papers, economic growth is positively affected by political stability (the most known paper probably being that of Barro, 1996) but as our results indicate, growth negatively impacts political polarization and deepening of the cleavage between parties and partisans.

Political polarization is entering dire times. Actually, to be more accurate, it has entered dire times a while now and the situation does not seem to get any better. On the contrary, given the current situation as it has been shaped by the recent events (the COVID pandemic, which does not seem to be over quite yet and even more the ongoing war between Russia and Ukraine) we have dark times ahead of us. Momentarily, regarding the war, major political decisions are being taken on a daily basis and the effects of these decisions have a harsh and immediate impact on everyday life of everyone (in Europe but also elsewhere). Growth was dramatically slowed down on a world wide level due to the pandemic and the war will have its negative impact as well. Additionally, due to the sharp increase in daily life goods caused by the war, inequality will be most evident (it might also increase but even if it does not, it will be felt more harshly due to the upcoming shortages in gas and wheat). In the upcoming winter the phenomenon will most probably be exacerbated by the mentioned circumstances. These factors indicate that political polarization will most probably rise a little bit more in the future. It is therefor very important to thoroughly understand the forces that drive it and what we can do to put a break on its upward trance. More growth stimulating policies need to be adopted and governments need to work towards a more equal distribution of said growth.

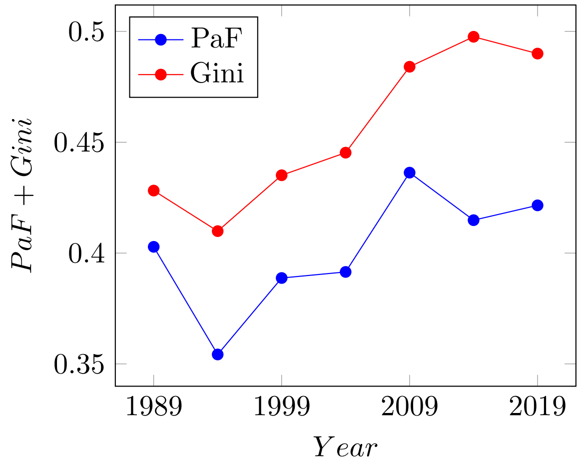
Figure 3.5: PaF and Gini



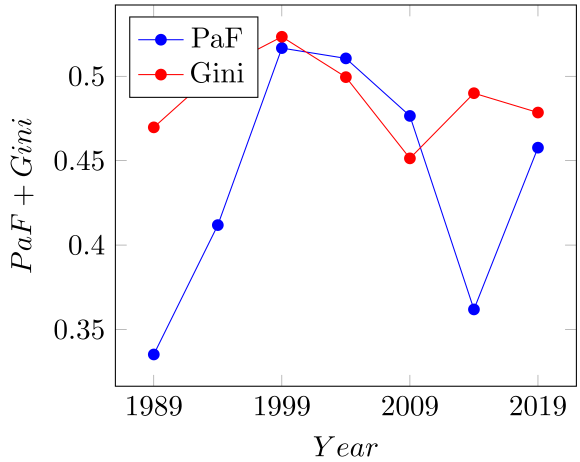
(a) PaF Estimators and Gini - Denmark



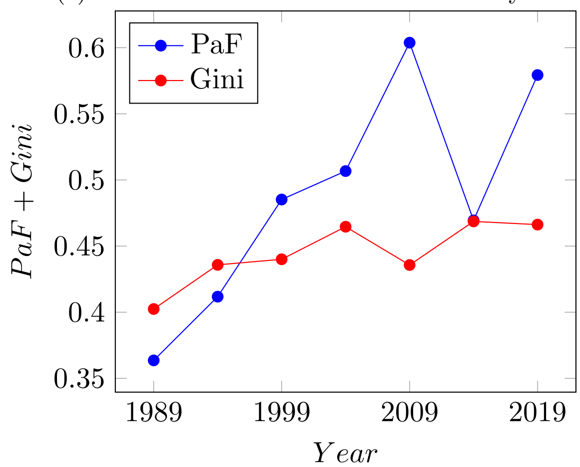
(b) PaF Estimators and Gini - France



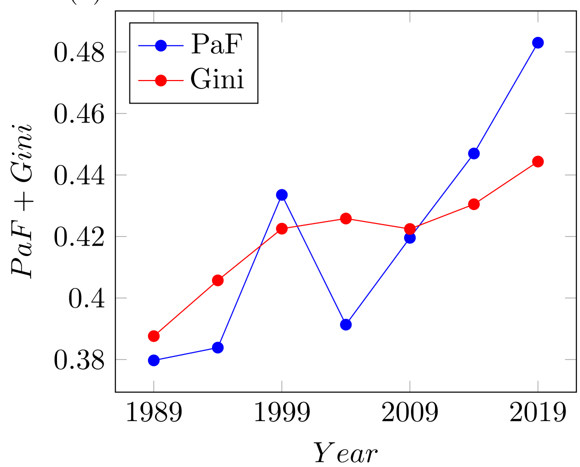
(c) PaF Estimators and Gini - Germany



(d) PaF Estimators and Gini - Greece

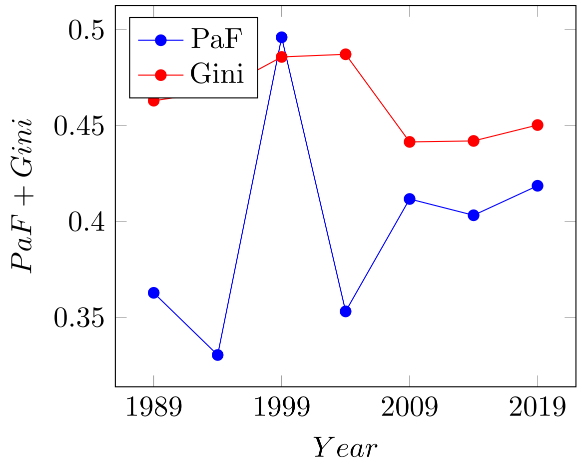


(e) PaF Estimators and Gini - Ireland

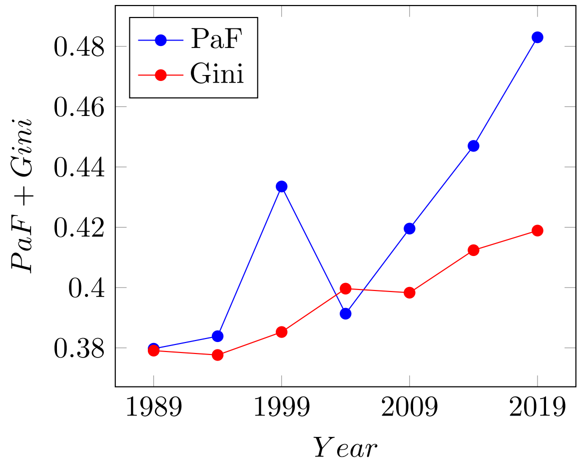


(f) PaF Estimators and Gini - Italy

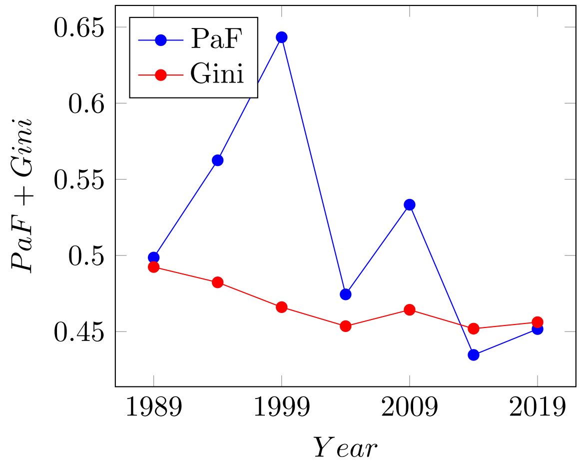
Figure 3.5: PaF and Gini - Cont'd



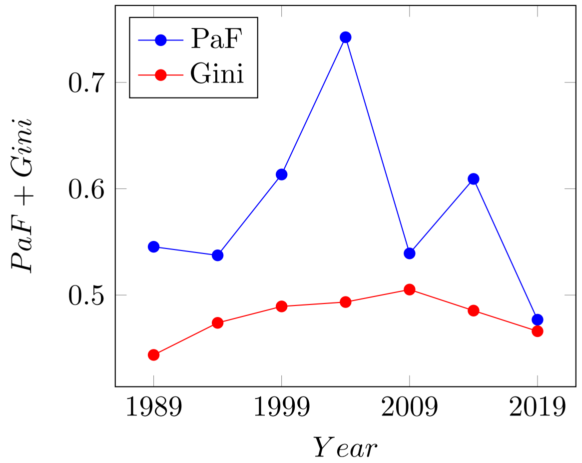
(g) PaF Estimators and Gini - Luxembourg



(h) PaF Estimators and Gini - Netherlands



(i) PaF Estimators and Gini - Spain



(j) PaF Estimators and Gini - UK

Chapter 4

Conclusion

Political polarization has been in the epicenter of political discussions for some time now. Recently it has made the headlines of newspapers and entered everyday discussions. To summarize the evolution of political polarization after the end of WWII, we could simply put it as follows. Firstly, in the 1950s and 1960s, the years after the end of the war, there is a wide feeling of consensus. Perhaps due to the recent memories of the atrocities of war and the immense pain it had caused worldwide, the numerous loses of human lives and the endless suffering, people were determined to protect peace (within countries as well as between countries). That's the period when we witness events of major political collaborations; in the USA the Medicaid was passed as well as the Civil Rights Acts. In Europe the European Economic Community was formed. As we move through time, in the 1970s and 1980s we start to see a decline in both political concession as well as a slowing down of the exploding growth that followed the end of World War II. Slowly, but steadily, clearer groups start to form around political lines, and this starts to spill over into the social aspect of people's lives. The clusters become more homogeneous and the distance between them grows bigger.

As we approach the change of the millennium and especially as we move towards the bankruptcy of Lehman Brothers things take an obvious turn for the worse. Not only are people neatly sorted along party lines but there seems to be a sentiment of animosity between groups. The rhetoric of hatred becomes the way to motivate voters. The Great Recession makes sure that growth is stagnant. At the same time, it becomes very clear that some people still profit on the expense of others, that the little wealth that is being produced is more and more concentrated among a the very few.

So, we observe, over this period of 70something years that we have a decline in political polarization in the beginning, alongside steep economic growth. But, unfortunately, as with all good things, this does not last forever. Political polarization start to rise, first simply as a more clear divide between groups and later on as animosity between groups. At the same time growth slows down until we enter the Great Recession, when we observe that in many countries the GDP shrinks.

The aim of this thesis is twofold. First, we wish to tackle the difficulties in the measurement of polarization. We do so by applying an index for economic polarization, which is based on the ≪Identification-Alienation≫ framework. This index captures the essence of polarization, as it is mainly based on the assumption that as clusters become more homogeneous within and more heterogeneous between them, polarization increases. We apply this index on individual-level preferential data regarding voters' self-placement. The data has to be processed first so as to extract the ideal point estimator, which is later used to generate non-parametrically, with the help of a Gaussian kernel, the probability distribution function of public political opinion. With this procedure we estimate the polarization level of a set of ten European countries over a thirty-years period. We observe, that, on average, political polarization in said ten European countries is indeed moving upwards. Secondly, we wish to test the dependence of political polarization on income inequality and economic growth. We do so by regressing various models. We try four different growth variables, the growth level compared to the previous year, the growth level compared to three years back (the average business cycle), the average growth rate of the past three years as well as the growth rate of the past five years (the time-interval between European Parliament elections). We regress the model using both Fixed as well as Random Effects. As expected, due to the nature of the data, the Hausman test indicates that the Random effects is the most consistent estimate. Additionally, as intuitively expected and in accordance with the analyzed literature, we find that political polarization is positively affected by the increase in income inequality and negatively affected by the increase of GDP levels.

Chapter 5

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Appendix A

Appendix

A.1 Section A - Proof of Theorem 2.2.1

Introduction

$$P(F) = \int \int T\left(f(x), |x-y|\right) f(x) f(y) dy dx \tag{A.1}$$

Characterization Theorem

Theorem A.1.1 (Polarization Index)**.** *A measure P, as described in A.1, satisfies relevant Axioms, if and only if it is proportional to*

$$P_a(F) = \int \int f(x)^{1+a} f(y) |y-x| dy dx, \tag{A.2}$$

where $a \in [0.25, 1]$.

Proof of Theorem A.1.1

Lemma A.1.2 (Jensen's Inequality)**.** *Let f be an integrable function defined on $[a, b]$ and let ϕ be a continuous convex function defined at least on the set $[m, M]$ where m is the inf of f and M is the sup of f. Then*

$$\phi\left(\frac{1}{b-a}\int_a^b f(x)dx\right) \leq \frac{1}{b-a}\int_a^b \phi(f(x))dx$$

Based on this, below lemma holds:

Lemma A.1.3. *Let g be a continuous real-valued function defined on $\mathbb{R}$ such that for all $x_i > 0$ and all δ with $0 < \delta < x_i$,*

$$g(x_o) \geq \frac{1}{2\delta}\int_{x_o-\delta}^{x_o+\delta} g(y)dy. \tag{A.3}$$

Then $g(x)$ must be a concave function.

Lemma A.1.4. *Function T must be concave in a for every $i > 0$.*

Proof. We assume three basic densities, as in Axiom 2, distributed as shown in figure A.1. The width of the densities is 2δ and 2ϵ, which of course implies that $\delta > 0$ and $\epsilon > 0$. Further more, $\delta \in (0, x_1)$ and the inequality $\delta + \epsilon < x_1$ must hold so that densities have disjoint supports. The height of each density is given by the following constant functions:

$$f_1(x) = h$$

$$f_2(x) = \frac{h}{\lambda}, \quad where \quad 0 < \lambda < 1$$

$$f_3(x) = i$$

According to Axiom 2, a λ-squeeze of the side densities, which causes them to be less wide and taller, cannot lower polarization.

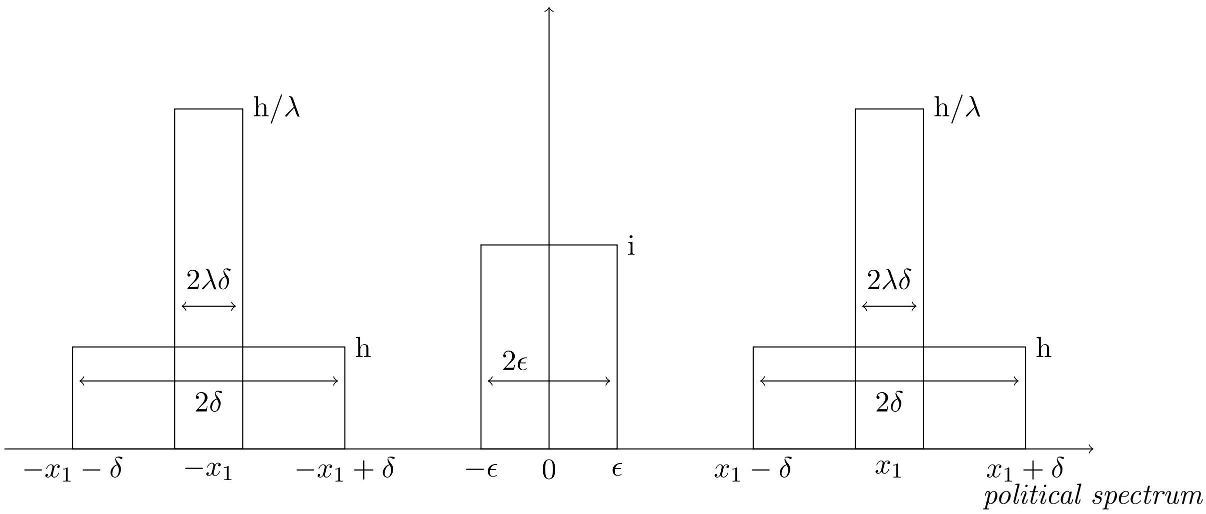


Figure A.1: Double Squeeze

We can estimate the polarization of the two distributions before and after the squeeze. Using measure A.1, total polarization of each distribution can be broken down into five components:

1. The internal polarization in the middle density,P_m, which is not affected by the λ-squeeze,
2. The internal polarization in each of the side densities, P_s,
3. The effective antagonism felt from each of the two side densities towards the middle density, P_{sm},
4. The effective antagonism felt from the middle density to each side density, P_{ms},
5. The effective antagonism felt from one side density to the other, P_{ss}.

Each of above mentioned components, except the first one, is observed twice and affected by the λ-squeeze.

For *Axiom 2* to hold, polarization after the squeeze cannot be lower, meaning that

$P(\lambda) \geq P(1)$, where $P(\lambda)$ is the value of the polarization index after the squeeze and $P(1)$ is the polarization for the original distribution of the population. For this, we will calculate each term, beginning with the first one:

$$P(\lambda) = P_m + 2P_s(\lambda) + 2P_{sm}(\lambda) + 2P_{ms}(\lambda) + 2P_{ss}(\lambda) \tag{A.4}$$

We compute every component of the right hand side of equation A.4, except for the first term, which remains unchanged after the squeeze and thus adds nothing to the comparison of the two indices.

The internal polarization of each side density is given by:

$$P_s(\lambda) = \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(f_2(x), |y-x|) f_2(x) f_2(y) dy dx \Rightarrow$$

$$P_s(\lambda) = \frac{1}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(\frac{h}{\lambda}, |y-x|)h^2 dydx,$$

where (here and in all subsequent cases) x will stand for the ≪origin≫ income (to which identification is applied) and y for the ≪destination≫ income (towards which antagonism is felt).

Note: Theoretically, each variable, x and y, should be on a different axis. We then, hypothetically, have some density function $f_1(x)$ whose integral gives us the population up to some value x_1 on the x-axis ($F(x_1) = \int_{-\infty}^{x_1} f_1(x)dx$) and some other density function,say $f_2(x)$ whose integral gives us the population up to some point y_1 of the y-axis. So, in the case of two distinct groups we would have three axes: one for each variable and one for polarization (which only depends on these two variables x and y). In figure A.4, and as in most figures presented here, except figure A.2, both densities seem to be on the same axis, that of political spectrum, and graphically this may be accepted because of the hypothesis of disjoint supports.

Similarly,

- For the antagonism from side densities to middle:

$$P_{sm}(\lambda) = \int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon} T(f_2(x), |y-x|)f_2(x)f_3(y)dydx \Rightarrow$$

$$P_{sm}(\lambda) = \frac{1}{\lambda}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon} T(\frac{h}{\lambda}, |y-x|)hidydx$$

- For the antagonism felt from the middle to the side densities:

$$P_{ms}(\lambda) = \int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(f_3(x), |y-x|)f_3(x)f_2(y)dydx \Rightarrow$$

$$P_{ms}(\lambda) = \frac{1}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i, |y-x|)ihdydx$$

- For the antagonism felt from one side density to the other:

$$P_{ss}(\lambda) = \int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta} T(f_2(x), |y-x|)f_2(x)f_2(y)dydx \Rightarrow$$

$$P_{ss}(\lambda) = \frac{1}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta} T(\frac{h}{\lambda}, |y-x|)h^2 dydx$$

Similarly, the polarization before the squeeze is given by:

$$P(1) = P_m + 2P_s + 2P_{sm} + 2P_{ms} + 2P_{ss} \tag{A.5}$$

We calculate again each element on the right hand side of the equation:

- For the internal polarization felt inside each side density:

$$P_s = \int_{x_1-\delta}^{x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(h, |y-x|)h^2 dydx$$

- For the antagonism from side densities to middle:

$$P_{sm} = \int_{x_1-\delta}^{x_1+\delta}\int_{-\epsilon}^{\epsilon} T(h, |y-x|)hidydx$$

- For the antagonism felt from the middle to the side densities:

$$P_{ms} = \int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i, |y-x|)ihdydx$$

- For the antagonism felt from one side density to the other:

$$P_{ss} = \int_{x_1-\delta}^{x+\delta}\int_{-x_1-\delta}^{-x_1+\delta} T(h, |y-x|)h^2dydx$$

As mentioned above, following *Axiom 2*, the following must hold:

$$P(\lambda) \geq P(1) \Rightarrow$$

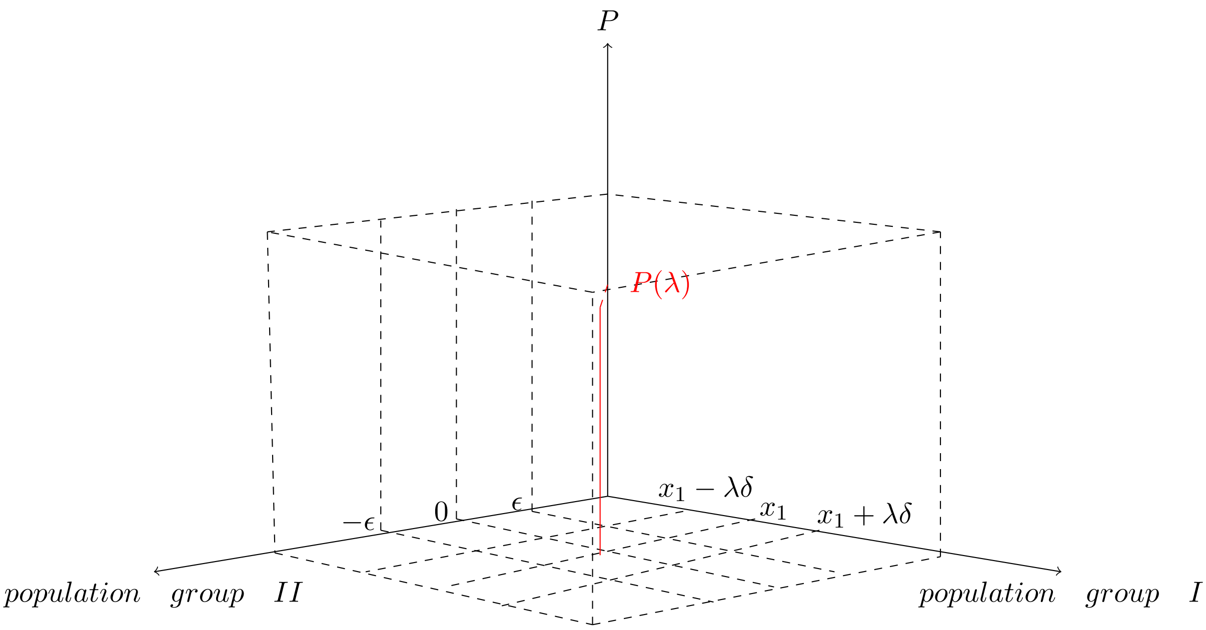


Figure A.2: Proof Lemma 3.3

$$\frac{1}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(\frac{h}{\lambda}, |y-x|)h^2dydx + \frac{1}{\lambda}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon} T(\frac{h}{\lambda}, |y-x|)hidydx+$$
$$\frac{1}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i, |y-x|)ihdydx + \frac{1}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta} T(\frac{h}{\lambda}, |y-x|)h^2dydx \geq$$
$$\int_{x_1-\delta}^{x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(h, |y-x|)h^2dydx + \int_{x_1-\delta}^{x_1+\delta}\int_{-\epsilon}^{\epsilon} T(h, |y-x|)hidydx$$
$$\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i, |y-x|)ihdydx + \int_{-x_1-\delta}^{x_1+\delta}\int_{-x_1-\delta}^{x_1+\delta} T(h, |y-x|)h^2dydx \Rightarrow$$

$$\frac{h^2}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx+\frac{hi}{\lambda}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon}T(\frac{h}{\lambda},|y-x|)dydx+$$
$$\frac{ih}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(i,|y-x|)dydx+\frac{h^2}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx\geq$$
$$h^2\int_{x_1-\delta}^{x_1+\delta}\int_{x_1-\delta}^{x_1+\delta}T(h,|y-x|)dydx+hi\int_{x_1-\delta}^{x_1+\delta}\int_{-\epsilon}^{\epsilon}T(h,|y-x|)dydx+$$
$$ih\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta}T(i,|y-x|)dydx+h^2\int_{x_1-\delta}^{x_1+\delta}\int_{-x_1-\delta}^{-x_1+\delta}T(h,|y-x|)dydx\Rightarrow$$

dividing with h on both sides:

$$\frac{h}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx+\frac{i}{\lambda}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon}T(\frac{h}{\lambda},|y-x|)dydx+$$
$$\frac{i}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(i,|y-x|)dydx+\frac{h}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx\geq$$
$$h\int_{x_1-\delta}^{x+\delta}\int_{x_1-\delta}^{x+\delta}T(h,|y-x|)dydx+i\int_{x_1-\delta}^{x_1+\delta}\int_{-\epsilon}^{\epsilon}T(h,|y-x|)dydx+$$
$$i\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta}T(i,|y-x)dydx+h\int_{x_1-\delta}^{x_1+\delta}\int_{-x_1-\delta}^{-x_1+\delta}T(h,|y-x|)dydx\Rightarrow$$

Taking limit of $h\to 0$ and invoking $T(i,0)=T(0,a)=0$ we are left with:

$$\lim_{h\to0}\frac{h}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx+\lim_{h\to0}\frac{i}{\lambda}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon}T(\frac{h}{\lambda},|y-x|)dydx+$$
$$\lim_{h\to0}\frac{i}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(i,|y-x|)dydx+\lim_{h\to0}\frac{h}{\lambda^2}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx\geq$$
$$\lim_{h\to0}h\int_{x_1-\delta}^{x_1+\delta}\int_{x_1-\delta}^{x_1+\delta}T(h,|y-x|)dydx+\lim_{h\to0}i\int_{x_1-\delta}^{x_1+\delta}\int_{-\epsilon}^{\epsilon}T(h,|y-x|)dydx+$$
$$\lim_{h\to0}i\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta}T(i,|y-x|)dydx+\lim_{h\to0}h\int_{x_1-\delta}^{x_1+\delta}\int_{-x_1-\delta}^{-x_1+\delta}T(h,|y-x|)dydx\Rightarrow$$
$$0\times\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(0,|y-x|)dydx+\frac{i}{\lambda}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-\epsilon}^{\epsilon}T(0,|y-x|)dydx+$$
$$\frac{i}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(i,|y-x|)dydx+0\times\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}\int_{-x_1-\lambda\delta}^{-x_1+\lambda\delta}T(\frac{h}{\lambda},|y-x|)dydx\geq$$
$$0\times\int_{x_1-\delta}^{x_1+\delta}\int_{x_1-\delta}^{x_1+\delta}T(0,|y-x|)dydx+i\int_{x_1-\delta}^{x_1+\delta}\int_{-\epsilon}^{\epsilon}T(0,|y-x|)dydx+$$
$$i\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta}T(i,|y-x|)dydx+0\times\int_{x_1-\delta}^{x_1+\delta}\int_{-x_1-\delta}^{-x_1+\delta}T(h,|y-x|)dydx\Rightarrow$$

$$\frac{i}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(i,|y-x|)dydx\geq i\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta}T(i,|y-x|)dydx\Rightarrow$$

$$\frac{1}{\lambda}\int_{-\epsilon}^{\epsilon}\int_{x_1-\lambda\delta}^{x_1+\lambda\delta}T(i,|y-x|)dydx\geq\int_{-\epsilon}^{\epsilon}\int_{x_1-\delta}^{x_1+\delta}T(i,|y-x|)dydx\Rightarrow$$

$$\int_{-\epsilon}^{\epsilon} \frac{1}{\lambda} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i,|y-x|)dydx \geq \int_{-\epsilon}^{\epsilon} \int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)dydx, \tag{A.6}$$

which essentially means that we require that $P_{ms}(\lambda) \geq P_{ms}$ and this must be true for all $\lambda \in (0,1)$ and all $\epsilon \in (0, x-\delta)$.

Due to the fact that the functions f_1, f_2 and f_3 are constants, thus having the same slope everywhere, if above inequality holds throughout the entire range $(x_1 - \lambda\delta, x_1 + \lambda\delta)$ for all $\lambda \in (0,1)$ then it must also hold at the center of the density, x_1, i.e. when λ tends to 0.

Following the *mean value theorem for definite integrals* over the interval $(x_1 - \lambda\delta, x_1 + \lambda\delta)$ we have:

$$T(i, x_1 - x) = \frac{1}{x_1 + \lambda\delta - (x_1 - \lambda\delta)} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i,|y-x|)dy \Rightarrow$$

$$T(i, x_1 - x) = \frac{1}{2\delta\lambda} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i,|y-x|)dy$$

For all $\delta \leq \frac{1}{2}$ it holds that:

$$\frac{1}{2\delta\lambda} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i,|y-x|)dy \geq \frac{1}{\lambda} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i,|y-x|)dy \Rightarrow$$

$$T(i, x_1 - x) \geq \frac{1}{\lambda} \int_{x_1-\lambda\delta}^{x_1+\lambda\delta} T(i,|y-x|)dy \Rightarrow$$

By making the necessary substitutions in A.6 and dividing both sides with 2ϵ, we get:

$$\frac{1}{2\epsilon} \int_{-\epsilon}^{\epsilon} T(i, x_1 - x)dx \geq \frac{1}{2\epsilon} \int_{-\epsilon}^{\epsilon} \int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)dydx \tag{A.7}$$

for every $\epsilon \in (0, x-\delta)$.

Again, since above holds for the entire interval $(-e, e)$ then it must also hold at the center of the density, i.e. at point 0. So, invoking again the *mean value theorem for definite integrals*, we get that:

$$T(i, x_1 - 0) = \frac{1}{\epsilon - (-\epsilon)} \int_{-\epsilon}^{\epsilon} T(i, x_1 - x)dx \Rightarrow$$

$$T(i, x_1) = \frac{1}{2\epsilon} \int_{-\epsilon}^{\epsilon} T(i, x_1 - x)dx$$

Making the right substitutions in A.7, we have:

$$T(i, x_1) \geq \int_{x_1-\delta}^{x_1+\delta} T(i, y)dy \tag{A.8}$$

As A.8 must hold for every $x > 0$ and every $\delta \in (0, x)$, we may invoke Lemma A.1.3 to conclude that T is concave in every x for every $i > 0$.

□

Lemma A.1.5. *Let g be a concave, continuous function on $\mathbb{R}_+$, with $g(0) = 0$. Suppose that for each a and a' with $a > a' > 0$, there exists $\bar{\Delta} > 0$ such that*

$$g(a + \Delta) - g(a) \geq g(a') - g(a' - \Delta) \tag{A.9}$$

for all $\Delta \in (0, \bar{\Delta})$. Then g must be linear.

Proof. In the case of a concave function, equation A.9 would hold with reversed inequality:

$$g(a+\Delta) - g(a) \leq g(a') - g(a'-\Delta),$$

as can be seen in figure A.3.

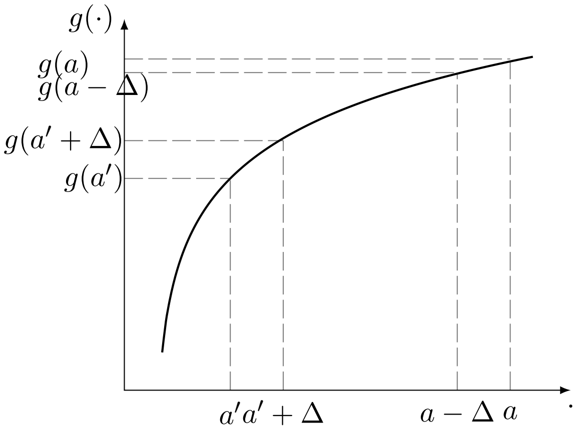


Figure A.3: Concave Function

For inequality A.9 to hold, the first derivative of function g needs to be monotonically increasing (i.e. $g'' > 0$), implying that function g is convex. We have already proven that function g is concave. If it is also convex, then g is a linear function.

□

Lemma A.1.6. *There is a continuous function $\phi(i)$ such that $T(i,a) = \phi(i)a$ for all i and $|a|$ in $\mathbb{R}_+$.*

Proof. Fix a and a' with $a > a' > 0$, $i > 0$. We consider that population is distributed along four basic densities, as shown in figure A.4 (based on Axiom 3), centered at $-y_1$, $-x_1$, x_1 and y_1 respectively, where $x_1 \equiv \frac{a-a'}{2}$ and $y_1 \equiv \frac{a+a'}{2}$.

The ≪outer≫ densities are of width 2ϵ and their height is given by a constant function $f_1(x) = i$.

The ≪inner≫ densities are of width 2δ and their height is given by a constant function: $f_2(x) = h$.

To ensure disjoint support we assume that $\epsilon < x_1$ and $\delta + \epsilon < y_1 - x_1 - \bar{\Delta}$ for some $\bar{\Delta} > 0$.

We assume a symmetric slide outwards of the two center densities by an amount $\bar{\Delta}$.

As in the previous case, the polarization measure can be decomposed in several distinct components, of which some are affected by the symmetric slide and some are not.

$$P(x) = \sum_{i=1}^{4} P_i(x) + \sum_{i=1}^{4} \sum_{j \neq i} A_{ij}(x)$$

Since the width and height of each density remains the same, internal polarization in all densities remains unchanged. Also, since the symmetric slide only concerns the two middle densities, 2 and 3, the effective antagonism felt from one side density to the other will remain unchanged as well. The components of total polarization that are affected by the symmetric slide are the following:

- The effective antagonism felt from density 1 to 2, $A_{12}(x)$, and from 4 to 3, $A_{43}(x)$, which should decrease as the distance between them decreases
- The effective antagonism felt from density 1 to 3, $A_{13}(x)$, and from 4 to 2, $A_{42}(x)$, which should increase since the distance between them gets wider

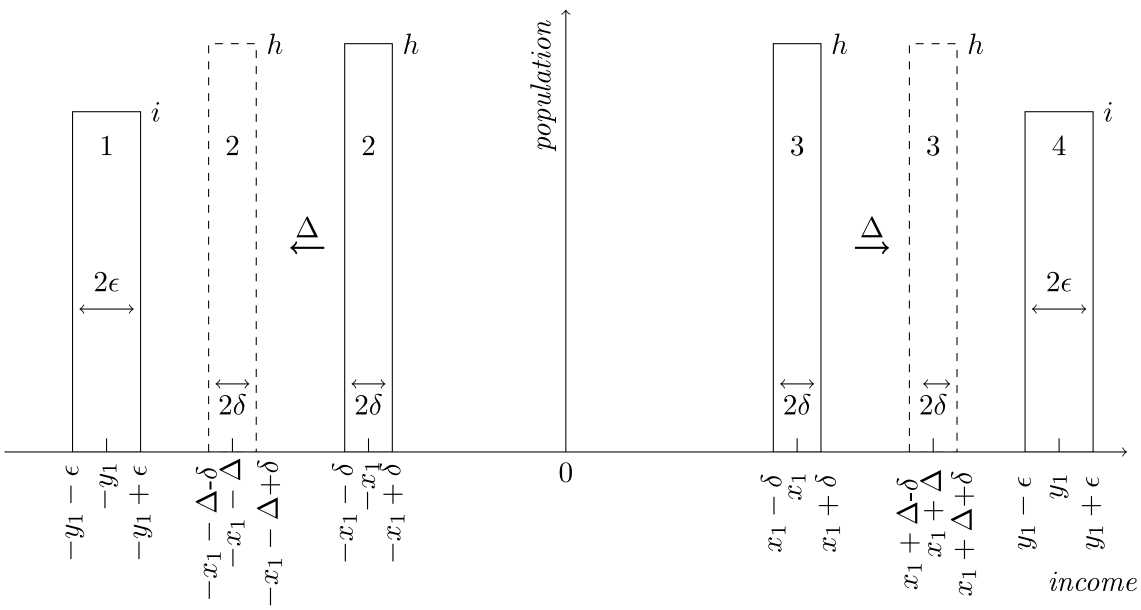


Figure A.4: Symmetric Outward Shift

- The effective antagonism felt from density 2 to 1, $A_{21}(x)$, and from 3 to 4, $A_{34}(x)$, which should decrease as the distance between them decreases
- The effective antagonism felt from density 2 to 3, $A_{23}(x)$ and vise versa, $A_{32}(x)$, which should rise due to the increase in distance
- The effective antagonism felt from density 2 to 4, $A_{24}(x)$, and from 3 to 1, $A_{31}(x)$, which should increase as well since the distance between them gets longer

For *Axiom 3* to hold, the positive impact of the two former effects needs to outweigh the negative effect of the latter three, so as polarization after the slide to be higher than before the slide.

So, we have:

$$P(x + \Delta) \geq P(x), \tag{A.10}$$

where $P(x)$ is the polarization before the outward shift and $P(x + \Delta)$ the polarization afterwards.

The components mentioned in each bullet above are equal with each other (before and after the shift respectively). Analytically:

- $A_{12}(x) = A_{43}(x)$ and $A_{12}(x + \Delta) = A_{43}(x + \Delta)$,
- $A_{13}(x) = A_{42}(x)$ and $A_{13}(x + \Delta) = A_{42}(x + \Delta)$,
- $A_{21}(x) = A_{34}(x)$ and $A_{21}(x + \Delta) = A_{34}(x + \Delta)$,
- $A_{23}(x) = A_{32}(x)$ and $A_{23}(x + \Delta) = A_{32}(x + \Delta)$,
- $A_{24}(x) = A_{31}(x)$ and $A_{24}(x + \Delta) = A_{31}(x + \Delta)$,

where $A_{12}(x)$ is the effective antagonism felt from a person in density 1 towards a person in density 2 before the shift and $A_{12}(x + \Delta)$ is the effective antagonism felt from a person in density 1 towards a person in density 2 after the symmetric shift and so forth for the rest. So total polarization, before the outward slide, is:

$$A_{12}(x) + A_{13}(x) + A_{21}(x) + A_{23}(x) + A_{24}(x) + A_{31}(x) + A_{32}(x) + A_{34}(x) + A_{43}(x) =$$

$$2A_{12}(x) + 2A_{13}(x) + 2A_{21}(x) + 2A_{23}(x) + 2A_{24}(x)$$

And this should be smaller or equal to the polarization after the outwards shift. Utilizing above equalities for the components of polarization after the outward shift, we have the following:

$$2A_{12}(x+\Delta) + 2A_{13}(x+\Delta) + 2A_{23}(x+\Delta) + 2A_{21}(x+\Delta) + 2A_{24}(x+\Delta) \geq$$
$$2A_{12}(x) + 2A_{13}(x) + 2A_{23}(x) + 2A_{21}(x) + 2A_{24}(x)$$

Dividing throughout with 2 gives us:

$$A_{12}(x+\Delta) + A_{13}(x+\Delta) + A_{23}(x+\Delta) + A_{21}(x+\Delta) + A_{24}(x+\Delta) \geq$$
$$A_{12}(x) + A_{13}(x) + A_{23}(x) + A_{21}(x) + A_{24}(x) \Rightarrow \quad (A.11)$$

$$\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta} T(f_1(x),|y-x|)f_1(x)f_2(y)dydx + \int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(f_1(x),|y-x|)f_1(x)f_2(y)dydx +$$
$$\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(f_2(x),|y-x|)f_2(x)f_2(y)dydx + \int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(f_2(x),|y-x|)f_2(x)f_1(y)dydx +$$
$$\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(f_2(x),|y-x|)f_2(x)f_1(y)dydx \geq$$
$$\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\delta}^{-x_1+\delta} T(f_1(x),|y-x|)f_1(x)f_2(y)dydx + \int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(f_1(x),|y-x|)f_1(x)f_2(y)dydx +$$
$$\int_{-x_1-\delta}^{-x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(f_2(x),|y-x|)f_2(x)f_2(y)dydx + \int_{-x_1-\delta}^{-x_1+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(f_2(x),|y-x|)f_2(x)f_1(y)dydx +$$
$$\int_{-x_1-\delta}^{-x_1+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(f_2(x),|y-x|)f_2(x)f_1(y)dydx \Rightarrow$$
$$\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta} T(i,|y-x|)ihdydx + \int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(i,|y-x|)ihdydx +$$
$$\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(h,|y-x|)h^2dydx + \int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(h,|y-x|)ihdydx +$$
$$\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(h,|y-x|)hidydx \geq$$
$$\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\delta}^{-x_1+\delta} T(i,|y-x|)ihdydx + \int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)ihdydx +$$
$$\int_{-x_1-\delta}^{-x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(h,|y-x|)h^2dydx + \int_{-x_1-\delta}^{-x_1+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(h,|y-x|)ihdydx +$$
$$\int_{-x_1-\delta}^{-x_1+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(h,|y-x|)hidydx$$

Pulling out the constants from the integrals and dividing everything by h we have:

$$
\begin{aligned}
& i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta} T(i,|y-x|)dydx + i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(i,|y-x|)dydx+ \\
& h\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(h,|y-x|)dydx + i\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(h,|y-x|)dydx+ \\
& i\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(h,|y-x|)dydx \geq \\
& i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\delta}^{-x_1+\delta} T(i,|y-x|)dydx + i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)dydx+ \\
& h\int_{-x_1-\delta}^{-x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(h,|y-x|)dydx + i\int_{-x_1-\delta}^{-x_1+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(h,|y-x|)dydx+ \\
& i\int_{-x_1-\delta}^{-x_1+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(h,|y-x|)dydx
\end{aligned}
$$

Taking $h \to 0$, invoking $T(i,0) = T(0,a) = 0$ and dividing by i we are left with:

$$
\begin{aligned}
& \lim_{h\to 0} i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta} T(i,|y-x|)dydx + \lim_{h\to 0} i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(i,|y-x|)dydx+ \\
& \lim_{h\to 0} h\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(h,|y-x|)dydx + \lim_{h\to 0} i\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(h,|y-x|)dydx+ \\
& \lim_{h\to 0} i\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(h,|y-x|)dydx \geq \\
& \lim_{h\to 0} i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\delta}^{-x+\delta} T(i,|y-x|)dydx + \lim_{h\to 0} i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)dydx+ \\
& \lim_{h\to 0} h\int_{-x_1-\delta}^{-x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(h,|y-x|)dydx + \lim_{h\to 0} i\int_{-x_1-\delta}^{-x_1+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(h,|y-x|)dydx+ \\
& \lim_{h\to 0} i\int_{-x_1-\delta}^{-x_1+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(h,|y-x|)dydx \Rightarrow \\
& i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta} T(i,|y-x|)dydx + i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(i,|y-x|)dydx+ \\
& 0\times\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(0,|y-x|)dydx + i\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(0,|y-x|)dydx+ \\
& i\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(0,|y-x|)dydx \geq \\
& i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\delta}^{-x_1+\delta} T(i,|y-x|)dydx + i\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)dydx+ \\
& 0\times\int_{-x_1-\delta}^{-x_1+\delta}\int_{x_1-\delta}^{x_1+\delta} T(0,|y-x|)dydx + i\int_{-x_1-\delta}^{-x_1+\delta}\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(0,|y-x|)dydx+ \\
& i\int_{-x_1-\delta}^{-x_1+\delta}\int_{y_1-\epsilon}^{y_1+\epsilon} T(0,|y-x|)dydx \Rightarrow
\end{aligned}
$$

Dividing with i:

$$\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\Delta-\delta}^{-x_1-\Delta+\delta} T(i,|y-x|)dydx + \int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1+\Delta-\delta}^{x_1+\Delta+\delta} T(i,|(y-x|)dydx \geq$$
$$\int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{-x_1-\delta}^{-x_1+\delta} T(i,|y-x|)dydx + \int_{-y_1-\epsilon}^{-y_1+\epsilon}\int_{x_1-\delta}^{x_1+\delta} T(i,|y-x|)dydx \quad \text{(A.12)}$$

We may assume that $y > x$ and afterwards apply the mean value theorem for definite integrals to the inner integrals of equation A.12. This gives us:

$$2\delta\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,-x_1-\Delta-x)dx + 2\delta\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,x_1+\Delta-x)dx \geq$$
$$2\delta\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,-x_1-x)dx + 2\delta\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,x_1-x)dx \Rightarrow$$

Dividing with 2δ:

$$\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,-x_1-\Delta-x)dx + \int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,x_1+\Delta-x)dx \geq$$
$$\int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,-x_1-x)dx + \int_{-y_1-\epsilon}^{-y_1+\epsilon} T(i,x_1-x)dx \quad \text{(A.13)}$$

Applying again the mean value theorem for definite integrals on the remaining integral in equation A.13:

$$2\epsilon T(i,-x_1-\Delta+y_1) + 2\epsilon T(i,x_1+\Delta+y_1) \geq$$
$$2\epsilon T(i,-x_1+y_1) + 2\epsilon T(i,x_1+y_1) \Rightarrow \quad \text{(A.14)}$$

Dividing with 2ϵ:

$$T(i,-x_1-\Delta+y_1) + T(i,x_1+\Delta+y_1) \geq$$
$$T(i,-x_1+y_1) + T(i,x_1+y_1) \Rightarrow \quad \text{(A.15)}$$

Using $x_1 \equiv \frac{a-a'}{2}$ and $y_1 \equiv \frac{a+a'}{2}$, above can be rewritten as:

$$T(i,a'-\Delta) + T(i,a+\Delta) \geq T(i,a') + T(i,a) \Rightarrow$$
$$T(i,a+\Delta) - T(i,a) \geq T(i,a') - T(i,a'-\Delta) \quad \text{(A.16)}$$

According to lemma A.1.5 and equation A.16 we conclude $T(i,.)$ is linear for every $i > 0$, meaning that there is a function $\phi(i)$ such that $T(i,a) = \phi(i)a$ for every a and i. Given that T is a continuous function, the same must hold for ϕ.

□

Lemma A.1.7. *$\phi(i)$ must be of the form Ki^a, for constants $K > 0$ and $a > 0$ where $K, a \in \mathbb{R}$*

Proof. First we observe that

$$\phi(i) > 0 \quad \forall \quad i > 0 \quad \text{(A.17)}$$

We start with proving that ϕ satisfies the fundamental Cauchy equation

$$\phi(p)\phi(p') = \phi(pp')\phi(1), \tag{A.18}$$

for every $(p, p') \gg 0$.

We assume a configuration consisted out of two basic densities, both of width 2ϵ. The first centered at 0 and the second at 1 and the height is given by a constant function $f_1(x) = p$ and $f_2(x) = h$ respectively, as shown in figure A.5.

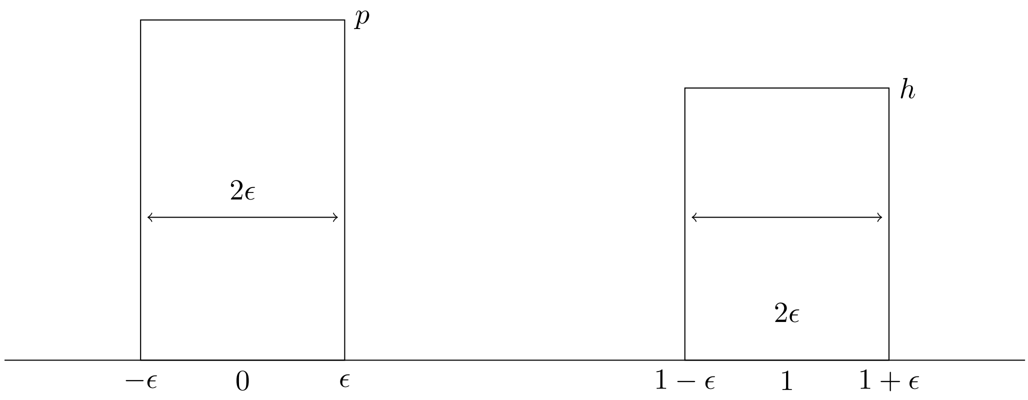


Figure A.5

Polarization in this case would be given by:

$$P(x) = \sum_{j=1}^{2} P_j(x) + \sum_j \sum_{k\neq j} A_{jk}(x) =$$
$$\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} T(f_1(x), |x-y|)f_1(x)f_1(y)dydx + \int_{1-\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} T(f_2(x), |x-y|)f_2(x)f_2(y)dydx+$$
$$\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon} T(f_1(x), (y-x))f_1(x)f_2(y)dydx + \int_{1-\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon} T(f_2(x), (y-x))f_1(x)f_2(y)dydx =$$
$$\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} \phi(p)|x-y|ppdydx + \int_{1-\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} \phi(h)|x-y|hhdydx+$$
$$\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon} \phi(p)(y-x)phdydx + \int_{1-\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon} \phi(h)(y-x)phdydx =$$

We pull out the constants:

$$\begin{aligned}&\phi(p)p^2\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon}|x-y|dydx + \phi(h)h^2\int_{1-\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon}|x-y|dydx+\\&\phi(p)ph\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx + \phi(h)ph\int_{1-\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx\end{aligned} \tag{A.19}$$

We want to manipulate two of the four terms of the right part of above equation:

$$\phi(h)h^2 \int_{1-\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx + \phi(h)ph\int_{1-\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx =$$
$$\phi(h)h^2 \left[\int_{1-\epsilon}^{-\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx + \int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx + \int_{\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx\right] +$$
$$\phi(h)ph \left[\int_{1-\epsilon}^{-\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx + \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx + \int_{\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx\right] =$$
$$\phi(h)h^2 \left[\int_{1-\epsilon}^{-\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx + \int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{-\epsilon} |x-y|dydx + \right.$$
$$\left.\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} |x-y|dydx + \int_{-\epsilon}^{\epsilon}\int_{\epsilon}^{1+\epsilon} |x-y|dydx + \int_{\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx\right] +$$
$$\phi(h)ph \left[\int_{1-\epsilon}^{-\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx + \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{1-\epsilon}(y-x)dydx + \right.$$
$$\left.\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx + \int_{-\epsilon}^{\epsilon}\int_{1+\epsilon}^{\epsilon}(y-x)dydx + \int_{\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx\right]$$

We pull apart the terms we wish to keep unaltered and manipulate the rest:

$$\phi(h)h^2 \left[\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} |x-y|dydx\right] + \phi(h)ph\left[\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx\right] +$$
$$\phi(h)h^2 \left[\int_{1-\epsilon}^{-\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx + \int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{-\epsilon} |x-y|dydx + \right.$$
$$\left.\int_{-\epsilon}^{\epsilon}\int_{\epsilon}^{1+\epsilon} |x-y|dydx + \int_{\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx\right] +$$
$$\phi(h)ph \left[\int_{1-\epsilon}^{-\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx + \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{1-\epsilon}(y-x)dydx + \right.$$
$$\left.\int_{-\epsilon}^{\epsilon}\int_{1+\epsilon}^{\epsilon}(y-x)dydx + \int_{\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx\right] =$$
$$\phi(h)h^2 \left[\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} |x-y|dydx\right] + \phi(h)ph\left[\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx\right] +$$
$$\phi(h)h^2 \left[-\int_{-\epsilon}^{1-\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx - \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{1-\epsilon} |x-y|dydx + \right.$$
$$\left.\int_{-\epsilon}^{\epsilon}\int_{\epsilon}^{1+\epsilon} |x-y|dydx + \int_{\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx\right] +$$
$$\phi(h)ph \left[-\int_{-\epsilon}^{1-\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx + \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{1-\epsilon}(y-x)dydx - \right.$$
$$\left.\int_{-\epsilon}^{\epsilon}\int_{\epsilon}^{1+\epsilon}(y-x)dydx + \int_{\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx\right] =$$
$$\phi(h)h^2 \left[\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} |x-y|dydx\right] + \phi(h)ph\left[\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx\right]$$

Thus we see that:

$$\phi(h)h^2 \int_{1-\epsilon}^{1+\epsilon}\int_{1-\epsilon}^{1+\epsilon} |x-y|dydx + \phi(h)ph\int_{1-\epsilon}^{1+\epsilon}\int_{-\epsilon}^{\epsilon}(y-x)dydx =$$
$$\phi(h)h^2 \int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon} |x-y|dydx + \phi(h)ph\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx$$

We make the necessary replacement in equation A.19:

$$
\begin{aligned}
&\left[\phi(p)p^2+\phi(h)h^2\right]\int_{-\epsilon}^{\epsilon}\int_{-\epsilon}^{\epsilon}|x-y|dydx+\\
&[\phi(p)ph+\phi(h)ph]\int_{-\epsilon}^{\epsilon}\int_{1-\epsilon}^{1+\epsilon}(y-x)dydx=\\
&\left[\phi(p)p^2+\phi(h)h^2\right]\int_{-\epsilon}^{\epsilon}\left(\int_{-\epsilon}^{x}(x-y)dy+\int_{x}^{\epsilon}(y-x)dy\right)dx+\\
&[\phi(p)ph+\phi(h)ph]\int_{-\epsilon}^{\epsilon}\left[\frac{1}{2}y^2-xy\right]_{1-\epsilon}^{1+\epsilon}dx=\\
&\left[\phi(p)p^2+\phi(h)h^2\right]\int_{-\epsilon}^{\epsilon}\left(\left[xy-\frac{1}{2}y^2\right]_{-\epsilon}^{x}+\left[\frac{1}{2}y^2-xy\right]_{x}^{\epsilon}\right)dx+\\
&[\phi(p)ph+\phi(h)ph]\int_{-\epsilon}^{\epsilon}\left(\frac{1}{2}(1+\epsilon)^2-x(1+\epsilon)-\frac{1}{2}(1-\epsilon)^2+x(1-\epsilon)\right)dx=\\
&\left[\phi(p)p^2+\phi(h)h^2\right]\int_{-\epsilon}^{\epsilon}(\epsilon^2+x^2)dx+[\phi(p)ph+\phi(h)ph]\int_{-\epsilon}^{\epsilon}(2\epsilon-2x\epsilon)dx=\\
&\left[\phi(p)p^2+\phi(h)h^2\right]\left[\epsilon^2x+\frac{1}{3}x^3\right]_{-\epsilon}^{\epsilon}+[\phi(p)ph+\phi(h)ph]\left[2\epsilon x-x^2\epsilon\right]_{-\epsilon}^{\epsilon}=\\
&\left[\phi(p)p^2+\phi(h)h^2\right]\frac{8}{3}\epsilon^3+[\phi(p)ph+\phi(h)ph]\,4\epsilon^2
\end{aligned}
$$

Dividing by $4\epsilon^2$ we get:

$$\left[\phi(p)p^2+\phi(h)h^2\right]\frac{2}{3}\epsilon+[\phi(p)ph+\phi(h)ph]$$

If the first rectangle were to be shorter, for example of height r, where as mentioned above $r\leq p$, then there exists a (unique) height $h(e)$ for the second rectangle so that total polarization of the two configurations are equated. In other words, there exists height $h(\epsilon)$ such that:

$$
\begin{aligned}
&ph[\phi(p)+\phi(h)]+\frac{2\epsilon}{3}[p^2\phi(p)+h^2\phi(h)]=\\
&rh(\epsilon)[\phi(r)+\phi(h(\epsilon))]+\frac{2\epsilon}{3}[r^2\phi(r)+h(e)^2\phi(h(\epsilon))]
\end{aligned}
\tag{A.20}
$$

By *Axiom 4*, it follows that for all $\lambda>0$,

$$
\begin{aligned}
&\lambda^2ph[\phi(\lambda p)+\phi(\lambda h)]+\frac{2\epsilon}{3}[(\lambda p)^2\phi(\lambda p)+(\lambda h)^2\phi(\lambda h)]\\
&=\lambda^2rh(e)[\phi(\lambda r)+\phi(\lambda h(e))]+\frac{2\epsilon}{3}[(\lambda r)^2\phi(\lambda r)+[\lambda h(e)]^2\phi(\lambda h(e))]
\end{aligned}
\tag{A.21}
$$

As can be noticed, the limit of $h(\epsilon)$ as $\epsilon\downarrow 0$ cannot take infinite values, thus it lies in some bounded set. Therefore we may extract a convergent subsequence with limit h' as $\epsilon\downarrow 0$.

Taking thus the limit of ϵ to zero from the right side (since ϵ cannot take negative values this is the only possible limit in this case), we have:

$$
\begin{aligned}
&\lim_{\epsilon\to 0^+}\lambda^2ph[\phi(\lambda p)+\phi(\lambda h)]+\lim_{\epsilon\to 0^+}\frac{2\epsilon}{3}[(\lambda p)^2\phi(\lambda p)+(\lambda h)^2\phi(\lambda h)]=\\
&\lim_{\epsilon\to 0^+}\lambda^2rh(e)[\phi(\lambda r)+\phi(\lambda h(e))]+\lim_{\epsilon\to 0^+}\frac{2\epsilon}{3}[(\lambda r)^2\phi(\lambda r)+[\lambda h(e)]^2\phi(\lambda h(e))]\Rightarrow\\
&\lambda^2ph[\phi(\lambda p)+\phi(\lambda h)]+0=\lambda^2rh'[\phi(\lambda r)+\phi(\lambda h')]+0\Rightarrow
\end{aligned}
\tag{A.22}
$$

$$\lambda^2 ph[\phi(\lambda p)+\phi(\lambda h)] = \lambda^2 rh'[\phi(\lambda r)+\phi(\lambda h')] \tag{A.23}$$

and

$$ph[\phi(p)+\phi(h)] = rh'[\phi(r)+\phi(h')], \tag{A.24}$$

where $h' = \lim_{\epsilon \to 0^+} h(e)$.

Dividing A.24 with A.23, we get:

$$\frac{\phi(p)+\phi(h)}{\phi(\lambda p)+\phi(\lambda h)} = \frac{\phi(r)+\phi(h')}{\phi(\lambda r)+\phi(\lambda h')} \tag{A.25}$$

We have assumed that $T(0,a) = T(i,0) = 0$ and that $T(i,a)$ is of the for $\phi(i)a$. Above implies that for $i=0$, $\phi(i)a = 0$ and there for we may conclude that $\phi(0) = 0$. So, when we take the limit $h \to 0$ in equation A.25 we get:

$$\begin{aligned}
&\lim_{h\to 0}\left(\frac{\phi(p)+\phi(h)}{\phi(\lambda p)+\phi(\lambda h)}\right) = \lim_{h\to 0}\left(\frac{\phi(p)+\phi(h)}{\phi(\lambda p)+\phi(\lambda h)}\right) \Rightarrow \\
&\frac{\phi(p)+\phi(0)}{\phi(\lambda p)+\phi(\lambda 0)} = \frac{\phi(p)+\phi(0)}{\phi(\lambda p)+\phi(\lambda 0)} \\
&\frac{\phi(p)}{\phi(\lambda p)} = \frac{\phi(r)}{\phi(\lambda r)}
\end{aligned} \tag{A.26}$$

Setting $\lambda = \frac{1}{p}$ and using the fact that $r = pp'$, then equation A.26 yields the fundamental Cauchy equation, as presented in equation A.18.

$$\begin{aligned}
&\frac{\phi(p)}{\phi(\frac{1}{p}p)} = \frac{\phi(pp')}{\phi(\frac{1}{p}pp')} \Rightarrow \\
&\frac{\phi(p)}{\phi(1)} = \frac{\phi(pp')}{\phi(p')} \Rightarrow \\
&\phi(p)\phi(p') = \phi(pp')\phi(1)
\end{aligned} \tag{A.27}$$

Additionally ϕ is a continuous function and A.17 holds. Given all above, the class of solutions to A.18 is completely described by

$$\phi(p) = Kp^a,$$

for constants $(K,a) \gg 0$, according to *Aczél, 1966, p41, Theorem 3.* □

Lemmas A.1.6 and A.1.7 together establish necessity but not sufficiency of the index. The bounds on "a" also remain to be set. These are shown with below lemmas.

Lemma A.1.8. *Let f be a basic density with mass p and mean μ on support $[a,b]$. Let $m \equiv \mu - a$ and let f^* denote the root of f. Then if f^λ denotes some λ-squeeze of f,*

$$P(F^\lambda) = 4kp^{2+a}(m\lambda)^{1-a}\int_0^1 f^*(x)^{1+a}\left(\int_0^1 f^*(y)(1-y)dy + \int_x^1 f^*(y)(y-x)dy\right)dx \tag{A.28}$$

for some constant $k > 0$.

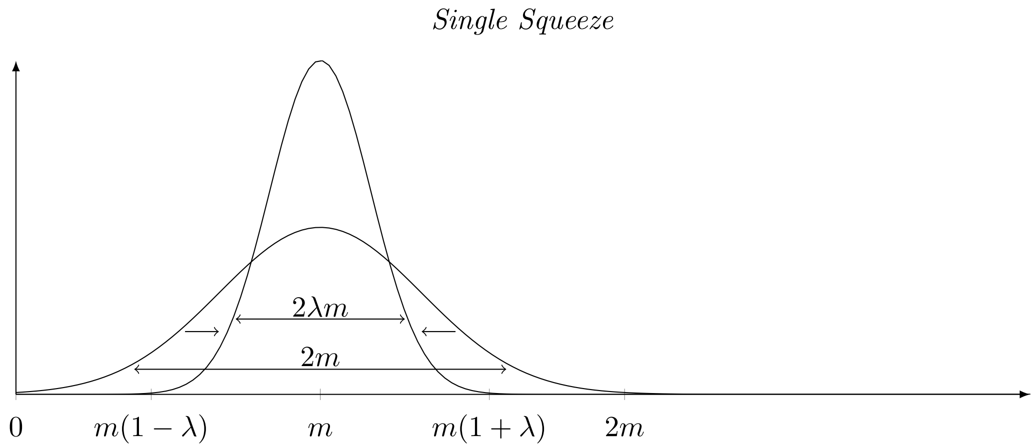


Figure A.6: Lemma 6

Proof. Given the fact that a distribution can be slided, we may set $a = 0$ and $b = 2m$, where m is now the new mean, as $m \equiv \mu - a$, as is shown in figure A.6.

Given

$$P(F) = k \int \int f(x)^{1+a} f(y)|y - x| dy dx, \tag{A.29}$$

for some $k > 0$, we have:

$$\begin{aligned} P(F) =& k \int_0^m \int_0^m f(x')^{1+a} f(y')|y' - x'| dy' dx' + k \int_0^m \int_m^{2m} f(x')^{1+a} f(y')|y' - x'| dy' dx' + \\ & k \int_m^{2m} \int_m^{2m} f(x')^{1+a} f(y')|y' - x'| dy' dx' + k \int_m^{2m} \int_0^m f(x')^{1+a} f(y')|y' - x'| dy' dx' \end{aligned} \tag{A.30}$$

Due to symmetry:

$$k \int_m^{2m} \int_m^{2m} f(x')^{1+a} f(y')|y' - x'| dy' dx' = k \int_0^m \int_m^{2m} f(x')^{1+a} f(y')|y' - x'| dy' dx'$$

and

$$k \int_m^{2m} \int_0^m f(x')^{1+a} f(y')|y' - x'| dy' dx' = k \int_0^m \int_0^m f(x')^{1+a} f(y')|y' - x'| dy' dx'$$

So A.30 can be rewritten as:

$$\begin{aligned}P(F) =&k\int_0^m\int_0^m f(x')^{1+a}f(y')|y'-x'|dy'dx' + k\int_0^m\int_m^{2m} f(x')^{1+a}f(y')|y'-x'|dy'dx' +\\ &k\int_0^m\int_m^{2m} f(x')^{1+a}f(y')|y'-x'|dy'dx' + k\int_0^m\int_0^m f(x')^{1+a}f(y')|y'-x'|dy'dx' =\\ &2k\int_0^m\int_0^m f(x')^{1+a}f(y')|y'-x'|dy'dx' + 2k\int_0^m\int_m^{2m} f(x')^{1+a}f(y')|y'-x'|dy'dx' =\\ &2k\int_0^m\int_0^{2m} f(x')^{1+a}f(y')|y'-x'|dy'dx' =\\ &2k\int_0^m f(x')^{1+a}\left(\int_0^{x'} f(y')(x'-y')dy' + \int_{x'}^m f(y')(y'-x') + \int_m^{2m} f(y')(y'-x')dy'\right)dx'\end{aligned} \tag{A.31}$$

We can set $z \equiv 2m - y'$ and replace the last term of above equation. before doing so we have to adjust the limits of the last integral and find the equivalent of dy' in terms of dz:

- For $y' = m$ the lower limit changes to: $z = 2m - y = 2m - m = m$ and for $y' = 2m$ the upper limit changes to: $z = 2m - 2m = 0$.
- $$z = 2m - y'$$ $$dz = -dy'$$ $$-dz = dy'$$

So, the last integral in equation A.31 becomes:

$$\int_m^{2m} f(y')(y'-x')dy' = \int_m^0 -f(z)(2m-x'-z)dz = \int_0^m f(z)(2m-x'-z)dz \tag{A.32}$$

Substituting A.32 in A.31 and below steps (and taking advantage of the fact that $f(y)$ and $f(z)$ are symmetric as can be seen in figure A.7), we get:

Symmetry of $f(y)$ and $f(z)$

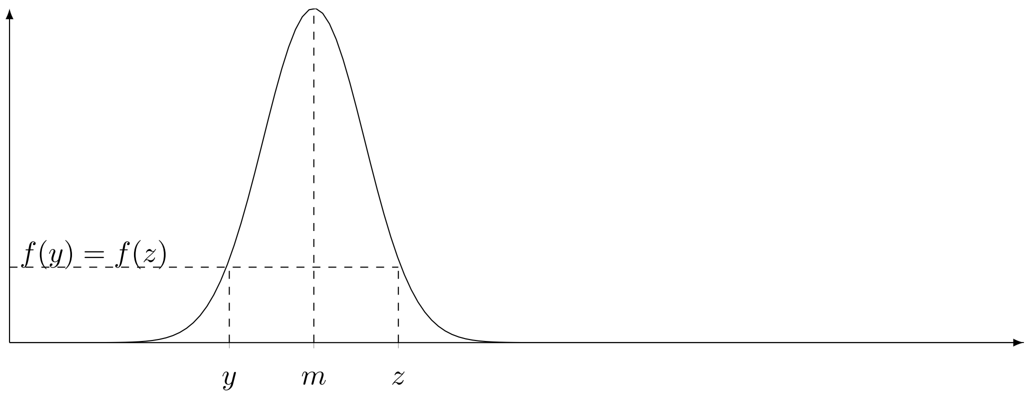


Figure A.7: Symmetry of $f(y)$ and $f(z)$

$$\begin{aligned}
P(F) = 2k \int_0^m & f(x')^{1+a} \times \\
& \left(\int_0^{x'} f(y')(x'-y')dy' + \int_{x'}^m f(y')(y'-x') + \int_0^m f(z)(2m-x'-z)dz \right) dx' = \\
& = 2k \int_0^m f(x')^{1+a} \times \\
& \left(\int_0^m f(y')(x'-y')dy' - \int_{x'}^m f(y')(y'-x')dy' + \int_{x'}^m f(y')(y'-x')dy' + \int_0^m f(y')(2m-x'-y')dy' \right) dx' \\
& = 2k \int_0^m f(x')^{1+a} \times \\
& \left(\int_0^m f(y')(x'-y'+2m-x'-y')dy' + \int_{x'}^m f(y')(y'-x'+y'-x')dy' \right) dx' \\
& = 2k \int_0^m f(x')^{1+a} \left(\int_0^m f(y')(2m'-2y')dy' + \int_{x'}^m f(y')(2y'-2x')dy' \right) dx' \\
& = 2k \int_0^m f(x')^{1+a} \left(2\int_0^m f(y')(m'-y')dy' + 2\int_{x'}^m f(y')(y'-x')dy' \right) dx' \\
& = 4k \int_0^m f(x')^{1+a} \left(\int_0^m f(y')(m'-y')dy' + \int_{x'}^m f(y')(y'-x')dy' \right) dx'
\end{aligned} \tag{A.33}$$

$$P(F) = 4k \int_0^m f(x')^{1+a} \left(\int_0^m f(y')(m-y')dy' + \int_{x'}^m f(y')(y'-x')dy' \right) dx' \tag{A.34}$$

We can transform f by a λ-squeeze, which as is previously mentioned is a special sort of mean-preserving reduction in the spread of f, as can be seen in figure A.6.

So if we assume that above function, $f(x')$, represents a *squeezed* distribution, $f_\lambda(x')$, then the equivalent *un-squeezed* form would be given by the following expression:

$$f_\lambda(x') \equiv \frac{1}{\lambda} f\left(\frac{x' - [1-\lambda]\mu}{\lambda}\right). \tag{A.35}$$

We make the necessary replacements and adjust the limits of the integrals as is shown in figure A.6 and we get:

$$\begin{aligned}
P(F^\lambda) = & 4k \int_{(1-\lambda)m}^m \left(\frac{1}{\lambda}\right)^{1+a} f\left(\frac{x'-(1-\lambda)m}{\lambda}\right)^{1+a} \times \\
& \left(\int_{(1-\lambda)m}^m \frac{1}{\lambda} f\left(\frac{y'-(1-\lambda)m}{\lambda}\right)(m-y')dy' + \int_{x'}^m \frac{1}{\lambda} f\left(\frac{y'-(1-\lambda)m}{\lambda}\right)(y'-x')dy' \right) dx' = \\
& 4k\lambda^{-(2+a)} \int_{(1-\lambda)m}^m f\left(\frac{x'-(1-\lambda)m}{\lambda}\right)^{1+a} \times \\
& \left(\int_{(1-\lambda)m}^m f\left(\frac{y'-(1-\lambda)m}{\lambda}\right)(m-y')dy' + \int_{x'}^m f\left(\frac{y'-(1-\lambda)m}{\lambda}\right)(y'-x')dy' \right) dx'
\end{aligned} \tag{A.36}$$

We substitute x' by $x'' = \frac{x'-(1-\lambda)m}{\lambda}$ and y' by $y'' = \frac{y'-(1-\lambda)m}{\lambda}$. We must adjust limits and differentials as follows:

- For the upper limit: when $x' = m$:

$$x'' = \frac{m - (1-\lambda)m}{\lambda} = m$$

- For the lower limit, when $x' = (1-\lambda)m$:

$$x'' = \frac{(1-\lambda)m - (1-\lambda)m}{\lambda} = 0$$

- The parenthesis at the second integral:

$$y'' = \frac{y' - (1-\lambda)m}{\lambda} \Rightarrow y' = y''\lambda + (1-\lambda)m$$

- Similarly to above we see that

$$x' = x''\lambda + (1-\lambda)m$$

- The $(m - y')$ term will become:

$$m - y' = m - y''\lambda + (1-\lambda)m = \lambda(m - y'')$$

- The $(y' - x')$ will become:

$$y' - x' = \lambda y'' + (1-\lambda)m - \lambda x'' - (1-\lambda)m = \lambda(y'' - x'')$$

- The differential:

$$x'' = \frac{x' - (1-\lambda)m}{\lambda} \Rightarrow dx'' = \frac{1}{\lambda}dx' \Rightarrow dx' = \lambda dx''$$

- Similarly:

$$dy' = \lambda dy''$$

Replacing all above in A.36:

$$\begin{aligned}
P(F^{\lambda}) = 4k\lambda^{-(2+a)} &\int_0^m f(x'')^{1+a} \times \\
&\left(\int_0^m f(y'')\lambda(m - y'')\lambda dy'' + \int_{x''}^m f(y'')\lambda(y'' - x'')\lambda dy''\right) \lambda dx'' = \\
&4k\frac{\lambda^3}{\lambda^{-(2+a)}} \int_0^m f(x'')^{1+a} \times \\
&\left(\int_0^m f(y'')(m - y'')dy'' + \int_{x''}^m f(y'')(y'' - x'')dy''\right) dx'' = \\
&4k\lambda^{1-a} \int_0^m f(x'')^{1+a} \left(\int_0^m f(y'')(m - y'')dy'' + \int_{x''}^m f(y'')(y'' - x'')dy''\right) dx''
\end{aligned} \tag{A.37}$$

As a final step, we must substitute $f(x'')$ and $f(y'')$ with the root function f^*, which is a basic density with mean 1 and support $[0, 2]$. First we population-scale f to h, where h has mass 1, in other words:

$$f(z) = ph(z) \quad \forall z$$

So A.37 becomes:

$$P(F^{\lambda}) = 4k\lambda^{1-a}\int_0^m p^{1+a}h(x'')^{1+a}\left(\int_0^m ph(y'')(m-y'')dy'' + \int_{x''}^m ph(y'')(y''-x'')dy''\right)dx''$$
$$= 4kp^{2+a}\lambda^{1-a}\int_0^m h(x'')^{1+a}\left(\int_0^m h(y'')(m-y'')dy'' + \int_{x''}^m h(y'')(y''-x'')dy''\right)dx'' \quad \text{(A.38)}$$

To derive A.28 we will use the following relations: $x = \frac{x''}{m}$, $y = \frac{y''}{m}$ and $f^*(x) = mh(mx)$. Limits etc for the substitution will become:

- For the differentials:
$$x = \frac{x''}{m} \Rightarrow xm = x'' \Rightarrow mdx = dx''$$
- Similarly for y''
$$y = \frac{y''}{m} \Rightarrow ym = y'' \Rightarrow mdy = dy''$$
- $$f^*(x'') = mh(mx'') = mh(m\frac{x''}{m}) = mh(x'') \Rightarrow$$
$$\frac{1}{m}f^*(x'') = h(x'')$$

Substituting all above in A.38 we have:

$$P(F^{\lambda}) = 4kp^{2+a}\lambda^{1-a}\int_0^1 \left(\frac{1}{m}f^*(x)\right)^{1+a)}$$
$$\left(\int_0^1 \frac{1}{m}f^*(y)m(1-y)mdy + \int_x^1 \frac{1}{m}f^*(y)m(y-x)mdy\right)mdx =$$
$$= 4kp^{2+a}\lambda^{1-a}\frac{m^2}{m^{1-a}}\int_0^1 f^*(x)^{1+a} \quad \text{(A.39)}$$
$$\left(\int_0^1 f^*(y)(1-y)dy + \int_x^1 f^*(y)(y-x)dy\right)dx =$$
$$= 4kp^{2+a}(m\lambda)^{1-a}\int_0^1 f^*(x)^{1+a}\left(\int_0^1 f^*(y)(1-y)dy + \int_x^1 f^*(y)(y-x)dy\right)dx$$

which is equation A.28 of lemma A.1.8. □

Lemma A.1.9. *Let f and g be two basic densities with disjoint support, with their means separated by distance d and with population masses p and q respectively. Let f have mean μ on support $[a, b]$. Let $m \equiv \mu - a$ and let f^* denote the root of f. Then for any λ-squeeze f^{λ} of f,*

$$A(f^{\lambda}, g) = 2kdp^{1+a}q(m\lambda)^{-a}\int_0^1 f^*(x)^{1+a}dx, \quad \text{(A.40)}$$

where $A(f^{\lambda}, g)$ denotes the total effective antagonism felt by members of f^{λ} towards members of g.

Proof. We assume a function f with support $[0, 2m]$ (with mean m) and a function g with support $[d, d + 2m]$ (where $d \geq 2m$ for disjoint supports). Using A.29 and without loss of generality, we have:

$$\begin{aligned} A(f,g) =& k\int_0^{2m} f(x)^{(1+a)}\left(\int_d^{d+2m} g(y)(y-x)dy\right)dx = \\ & k\int_0^{2m} f(x)^{(1+a)}\left(\int_d^{d+m} g(y)(y-x)dy + \int_{d+m}^{d+2m} g(y)(y-x)dy\right)dx \end{aligned}$$

Due to symmetry we know that:

$$\int_d^{d+m} g(y)(y-x)dy = \int_{d+m}^{d+2m} g(y)(y-x)dy$$

Substituting in above equation, we get:

$$\begin{aligned} A(f,g) =& k\int_0^{2m} f(x)^{(1+a)}\left(\int_d^{d+m} g(y)2(m+d-x)dy\right)dx = \\ & 2k\int_0^{2m} f(x)^{(1+a)}\left(\int_d^{d+m} g(y)(m+d-x)dy\right)dx \end{aligned}$$

We know that:

$$\int_d^{d+m} g(y)(m+d-x)dy = \frac{q}{2}$$

Substituting again in above equation we get:

$$\begin{aligned} A(f,g) =& 2k\frac{q}{2}\int_0^{2m} f(x)^{(1+a)}(m+d-x)dx = \\ & kq2\int_0^{m} f(x)^{(1+a)}(m+d-x)dx = \\ & 2kq\int_0^{m} f(x)^{(1+a)}(m+d-m)dx = \\ & 2kq\int_0^{m} f(x)^{(1+a)}(d)dx = \\ & 2kqd\int_0^{m} f(x)^{(1+a)}dx \end{aligned}$$

As mentioned earlier, a distribution may undergo a squeeze with below properties:

$$f_\lambda(x) \equiv \frac{1}{\lambda} f\left(\frac{x-[1-\lambda]\mu}{\lambda}\right). \tag{A.41}$$

Above equation must hold for the case of the *squeezed* distribution as well:

$$A(f_\lambda, g) = 2kqd\int_0^{m} f_\lambda(x')^{(1+a)}dx'$$

Or in terms of the f function (after necessary adjustment of limits and differential):

- Upper limit: For $x = 0$

$$0 = \frac{x' - (1-\lambda)m}{\lambda} \Rightarrow x' = (1-\lambda)m$$

- Lower limit: For $x = m$:

$$m = \frac{x' - (1-\lambda)m}{\lambda} \Rightarrow x' = m$$

- Differential:

$$x = \frac{x' - (1-\lambda)m}{\lambda} \Rightarrow dx' = \lambda dx$$

We have:

$$\begin{aligned} A(f,g) =& 2kqd \int_{(1-\lambda)m}^{m} \left[\frac{1}{\lambda} f\left(\frac{x' - (1-\lambda)m}{m}\right)\right]^{(1+a)} dx' = \\ & 2kqd \int_{(1-\lambda)m}^{m} \frac{1}{\lambda^{1+a}} f\left(\frac{x' - (1-\lambda)m}{m}\right)^{(1+a)} dx' = \\ & 2kqd\lambda^{-(1+a)} \int_{(1-\lambda)m}^{m} f\left(\frac{x' - (1-\lambda)m}{m}\right)^{(1+a)} dx' \end{aligned}$$

To simplify, we substitute as follows:

$$x'' = \frac{x' - (1-\lambda)m}{\lambda}$$

and we adjust respective limits and differential:

- Upper limit: For $x' = m$

$$x'' = \frac{m - (1-\lambda)m}{\lambda} = m$$

- Lower limit: For $x' = (1-\lambda)m$:

$$x'' = \frac{(1-\lambda)m - (1-\lambda)m}{\lambda} = 0$$

- Differential:

$$x'' = \frac{x' - (1-\lambda)m}{\lambda} \Rightarrow dx' = \lambda dx''$$

Substituting in above equation we have:

$$\begin{aligned} A(f,g) =& 2kdq\lambda^{(1+a)} \int_{0}^{m} f(x'')^{1+a} \lambda dx'' = \\ & 2kdq\lambda^{-a} \int_{0}^{m} f(x'')^{(1+a)} dx'' \end{aligned} \tag{A.42}$$

To transform this into the basic root function $f^*(x)$, we use the following analogies:

- $x = \frac{x''}{m} \Rightarrow x'' = xm \Rightarrow dx'' = mdx$
- $f^*(x) = mh(mx) = mh(m\frac{x''}{m}) = mh(x'')$, where $f(x'') = ph(x'')$
- Lower limit: $x'' = 0 \Rightarrow x = 0$
- Upper limit: $x'' = m \Rightarrow x = \frac{m}{m} = 1$

$$A(h,g) = 2kdq\lambda^{-(1+a)} \int_{0}^{m} (ph(x''))^{(1+a)} dx'' \Rightarrow$$

$$\begin{aligned} A(f^*, g) =& 2kdqp^{1+a}\lambda^{-a} \int_0^1 \left(\frac{1}{m} f^*(x) \right)^{1+a} m dx = \\ & 2kdq\lambda^{-a} \int_0^1 m^{-a} f^*(x)^{1+a} dx = \\ & 2kdqp^{1+a}(m\lambda)^{-a} \int_0^1 f^*(x)^{1+a} dx \end{aligned}$$

□

Lemma A.1.10. *Define, for any root f and $a > 0$,*

$$\psi(f, a) \equiv \frac{\int_0^1 f(x)^{1+a} dx}{\int_0^1 f(x)^{1+a} \left(\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy \right) dx}. \tag{A.43}$$

Then - for any $a > 0$ - $\psi(f, a)$ attains its minimum value when f is the uniform root, and this minimum value equals 3.

Proof. We work with the inverse function:

$$\zeta(f, a) \equiv \psi^{-1}(f, a) = \frac{\int_0^1 f(x)^{1+a} \left(\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy \right) dx}{\int_0^1 f(x)^{1+a} dx} \tag{A.44}$$

Since f is a uniform root, then it is of the form $f(x) = c$. then A.44 can be rewritten as follows:

$$\begin{aligned} \zeta(f, a) =& \frac{\int_0^1 c^{1+a} \left(\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy \right) dx}{\int_0^1 f(x)^{1+a} dx} = \\ & \frac{c^{1+a} \int_0^1 \left(\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy \right) dx}{\int_0^1 f(x)^{1+a} dx} \end{aligned}$$

We set $L = \int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy$, which is decreasing in x (due to term $(y-x)$).

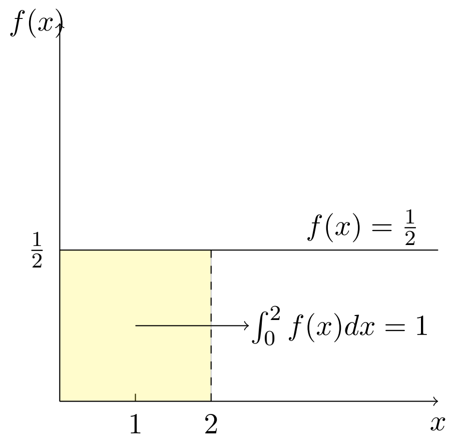
Furthermore, we assume that:

$$\zeta(f, a) \leq \int_0^1 L(x) dx,$$

which is indeed the case, as can be shown as follows:

$$\begin{aligned} & \zeta(f, a) \leq \int_0^1 L(x) dx \Rightarrow \\ & \frac{c^{1+a} \int_0^1 \left(\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy \right) dx}{\int_0^1 f(x)^{1+a} dx} \leq \int_0^1 \left(\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy \right) dx \end{aligned} \tag{A.45}$$

For above to hold, the following must be true:



$$\frac{c^{1+a}}{\int_0^1 f(x)^{1+a}dx} \leq 1$$

Utilizing again that $f(x)$ is a constant function, above can be rewritten as follows:

$$\frac{c^{1+a}}{c^{1+a}\int_0^1 1dx} \leq 1$$
$$\frac{c^{1+a}}{c^{1+a}}\frac{1}{[x]_0^1} = \frac{c^{1+a}}{c^{1+a}}\frac{1}{1} \leq 1,$$

which is true.
Further:

$$\begin{aligned}
L(x) = &\int_0^1 f(y)(1-y)dy + \int_x^1 f(y)(y-x)dy = \\
&= \int_0^1 f(y)(1-y)dy + \int_x^0 f(y)(y-x)dy + \int_0^1 f(y)(y-x)dy = \\
&= \int_0^1 f(y)(1-y+y-x)dy + \int_0^x f(y)(x-y)dy = \\
&= \int_0^1 f(y)(1-x)dy + \int_0^x f(y)(x-y)dy = \\
&= \int_0^1 f(y)dy - \int_0^1 f(y)xdy + \int_0^x f(y)(x-y)dy = \\
&[f(y)y]_0^1 - [f(y)yx]_0^1 + \int_0^x f(y)(x-y)dy = \\
&= f(y) - f(y)x + \int_0^x f(y)(x-y)dy = \\
&\frac{1}{2} - \frac{1}{2}x + \int_0^x f(y)(x-y)dy = \\
&\frac{1-x}{2} + \int_0^x f(y)(x-y)dy =
\end{aligned}$$

Since $f(x)$ is non-decreasing and integrates to $\frac{1}{2}$ on [0,1], then:

$$\int_0^x f(y)(x-y)dy \leq \int_0^x \frac{1}{2}(x-y)dx \quad \forall x \leq 1$$

So, we have:

$$\begin{aligned}
\zeta(f,a) \leq & \int_0^1 \left(\frac{1-x}{2} + \int_0^x \frac{x-y}{2} dy \right) dx = \\
& = \int_0^1 \left(\frac{1-x}{2} [y]_0^1 + \int_0^1 \frac{x-y}{2} dy + \int_1^x \frac{x-y}{2} dy \right) dx = \\
& = \int_0^1 \left(\frac{1-x}{2} \int_0^1 1 dy + \int_0^1 \frac{x-y}{2} dy + \int_x^1 \frac{x-y}{2} dy \right) dx = \\
& = \int_0^1 \left(\int_0^1 \frac{1-x}{2} dy + \int_0^1 \frac{x-y}{2} dy + \int_x^1 \frac{x-y}{2} dy \right) dx = \\
& = \int_0^1 \left(\int_0^1 \frac{1-x+x-y}{2} dy + \int_x^1 \frac{x-y}{2} dy \right) dx = \\
& = \int_0^1 \left(\int_0^1 \frac{1-y}{2} dy + \int_x^1 \frac{x-y}{2} dy \right) dx = \\
& = \zeta(u,a)
\end{aligned}$$

Simple integration reveals that $\zeta(u,a) = \frac{1}{3}$

$$\begin{aligned}
& \int_0^1 \left(\int_0^1 \frac{1-y}{2} dy + \int_x^1 \frac{x-y}{2} dy \right) dx = \\
& = \frac{1}{2} \int_0^1 \left(\int_0^1 (1-y) dy + \int_x^1 (x-y) dy \right) dx = \\
& = \frac{1}{2} \int_0^1 \left(\left[y - \frac{1}{2} y^2 \right]_0^1 + \left[\frac{1}{2} y^2 - xy \right]_x^1 \right) dx = \\
& = \frac{1}{2} \int_0^1 \left(\frac{1}{2} + \frac{1}{2} - x + \frac{1}{2} x^2 \right) dx = \\
& = \frac{1}{2} \int_0^1 \left(1 - x + \frac{1}{2} x^2 \right) dx = \\
& = \frac{1}{2} \left[x - \frac{1}{2} x^2 + \frac{1}{6} x^3 \right]_0^1 = \\
& = \frac{1}{3} \times \frac{2}{3} = \frac{1}{3}
\end{aligned}$$

□

Lemma A.1.11. *Given that $P(f)$ is of the form A.29, Axiom 1 is satisfied if and only if $a \leq 1$.*

Proof. As can be seen from equation A.28:

$$P(F^\lambda) = 4kp^{2+a}(m\lambda)^{1-a} \int_0^1 f^*(x)^{1+a} \left(\int_0^1 f^*(y)(1-y) dy + \int_x^1 f^*(y)(y-x) dy \right) dx$$

$\forall a > 1$ the power of term $(m\lambda)^{1-a}$ becomes negative and thus it would be written as a fraction whose denominator would be $(m\lambda)^{a-1}$.

This would imply that a squeeze would decrease polarization, whereas according to *Axiom 1* polarization should be equal or larger after a single squeeze.

□

Lemma A.1.12. *Given that $P(f)$ is of the form A.29, Axiom 2 is satisfied if and only if $a \geq 0.25$.*

Proof. We assume again three densities, like described in *Axiom 2*, of which the two side ones undergo a squeeze.

Total polarization after the squeeze is given by:

$$P(\lambda) = P_m + 2P_s(\lambda) + 2P_{sm}(\lambda) + 2P_{ms}(\lambda) + 2P_{ss}(\lambda) \tag{A.46}$$

We set:

$$\psi_1(h,a) \equiv \int_0^1 h(x)^{1+a} \left(\int_0^1 h(y)(1-y)dy + \int_x^1 h(y)(y-x)dy \right) dx$$

and

$$\psi_2(h,a) \equiv \int_0^1 h(x)^{1+a} dx$$

Each component, except for P_m which remains unaffected by the squeeze, are computed, utilizing lemmas A.1.8 and A.1.9, as follows:

$$P_s(\lambda) = 4kp^{2+a}(m\lambda)^{1-a}\psi_1(f^*,a)$$
$$A_{ms}(\lambda) = 2kdq^{1+a}pn^{-a}\psi_2(f^*,a)$$
$$A_{sm}(\lambda) = 2kdp^{1+a}q(m\lambda)^{-a}\psi_2(f^*,a)$$
$$A_{ss}(\lambda) = 4kdp^{2+a}(m\lambda)^{-a}\psi_2(f^*,a)$$

Above can be summed up to given us polarization after the squeeze:

$$\begin{aligned}
P(\lambda) &= 4kp^{2+a}(m\lambda)^{1-a}\psi_1(f^*,a) + 2kdq^{1+a}pn^{-a}\psi_2(f^*,a) + \\
&+ 2kdp^{1+a}q(m\lambda)^{-a}\psi_2(f^*,a) + 4kdp^{2+a}(m\lambda)^{-a}\psi_2(f^*,a) = \\
&= 2k\psi_1[2p^{2+a}(m\lambda)^{1-a} + dq^{1+a}pn^{-a}\psi + dp^{1+a}q(m\lambda)^{-a}\psi + 2dp^{2+a}(m\lambda)^{-a}\psi] = \\
&= 2k\psi_1[2p^{2+a}(m\lambda)^{1-a} + d\psi(q^{1+a}pn^{-a} + p^{1+a}q(m\lambda)^{-a} + 2p^{2+a}(m\lambda)^{-a})] = \\
&= 2k\psi_1\left[2p^{2+a}(m\lambda)^{1-a} + \frac{d}{m}\psi\lambda^{-a}\left(q^{1+a}pn^{-a}m + p^{1+a}qm^{1-a}\lambda^{-a} + 2p^{2+a}m^{1-a}\lambda^{-a}\right)\right] \\
&= 2k\psi_1\left[2p^{2+a}(m\lambda)^{1-a} + \frac{d}{m}\psi\left(q^{1+a}pn^{-a}m\lambda^a + p^{1+a}qm^{1-a} + 2p^{2+a}m^{1-a}\right)\right] = \\
&= 2k\psi_1p^{2+a}\left[2(m\lambda)^{1-a} + \frac{d}{m}\psi\lambda^{-a}\left(q^{1+a}p^{-1-a}n^{-a}m\lambda^a + p^{-1}qm^{(1-a)} + 2m^{1-a}\right)\right] = \\
&= 2k\psi_1p^{2+a}m^{1-a}\left[2(\lambda)^{1-a} + \frac{d}{m}\psi\lambda^{-a}(q^{1+a}p^{-1-a}n^{-a}m^a\lambda^a + p^{-1}q + 2)\right] = \\
&= 2k\psi_1p^{2+a}m^{1-a}\left[2(\lambda)^{1-a} + \frac{d\psi}{m}\lambda^{-a}(q^{1+a}p^{-1-a}n^{-a}m^a\lambda^a) + \frac{d\psi}{m}\lambda^{-a}(\frac{q}{p}+2)\right] \\
&= 2k\psi_1p^{2+a}m^{1-a}2(\lambda)^{1-a} + 2k\psi_1p^{2+a}m^{1-a}\frac{d\psi}{m}\lambda^{-a}(q^{1+a}p^{-1-a}n^{-a}m^a\lambda^a) + 2k\psi_1p^{2+a}m^{1-a}\frac{d\psi}{m}\lambda^{-a}(\frac{q}{p}+2) = \\
&= 2k\psi_2pdq^{1+a}n^{-a} + 2k\psi_1p^{2+a}m^{1-a}\left[2\lambda^{1-a} + \frac{d\psi}{m}\lambda^{-a}\left(\frac{q}{p}+2\right)\right] = \\
&= D + c\left[2\lambda^{1-a} + \frac{d\psi}{m}\lambda^{-a}\left(\frac{q}{p}+2\right)\right]
\end{aligned}$$

Where D and c and positive constants independent of λ and $\psi(f^*,a) = \frac{\psi_2(f^*,a)}{\psi_1(f^*,a)}$.

For *Axiom 2* to hold, the term in the square brackets, $2\lambda^{1-a} + \frac{d\psi}{m}\lambda^{-a}\left(\frac{q}{p}+2\right)$ must be non increasing in λ in $(0,1]$.

It suffices to show that for every root f^*:

$$\lambda^{1-a} + \psi(f^*, a)\ lambda^{-(1+a)}$$

is non-increasing in every λ over $(0,1]$.

By simple differentiation with respect to λ we get:

$$(1-a)\lambda^{-a} - a\psi(f^*, a)\lambda^{-(1+a)}$$

must be non-negative for every $\lambda \in (0,1]$.

The necessary and sufficient condition for this is:

$$a \geq \frac{1}{1+\psi(f^*, a)} = \frac{1}{1+3} = \frac{1}{4} = 0.25$$

.

□

Lemma A.1.13. *Given that $P(f)$ is of the form A.29, Axiom 3 is satisfied.*

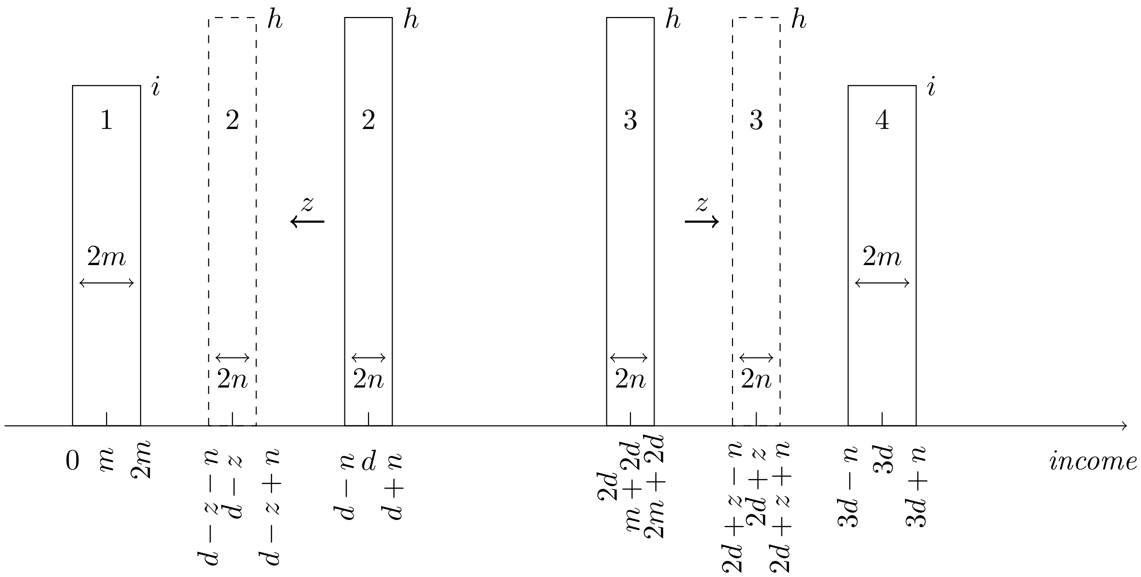


Figure A.8: Symmetric Outward Shift

Proof. As previously mentioned total polarization for a population divided into 4 densities is given by:

$$P(x) = \sum_{i=1}^{4} P_i(x) + \sum_{i=1}^{4}\sum_{j\neq i} A_{ij}(x)$$

where $f^*(x)$ denotes the root of f and $d_{ij}(0)$ denotes the distance between the means of each density.

As can be seen in figure an outwards slide of the inner densities by an amount z, will decrease distance between denisty 1 and 2 by z and increase distance between densities 1 and 3 by the same amount.

For *Axiom 3* to hold, we must have:

$$P(z) \geq P(0),$$

where $P(z)$ is the polarization after an outward slide by z of the inner densities and $P(0)$ is the original polarization of the population.

The internal polarization $P_i(x)$ remains unchanged since there is no squeeze of the densities. So, for *Axiom 3* to hold it suffices to show that:

$$\sum_{i=1}^{4}\sum_{j\neq i} A_{ij}(z) \geq \sum_{i=1}^{4}\sum_{j\neq i} A_{ij}(0)$$

Due to the symmetries mentioned in lemma A.1.6, it is sufficient to show that:

$$\begin{aligned} &A_{12}(z) + A_{13}(z) + A_{23}(z) + A_{21}(z) + A_{24}(z) \geq \\ &A_{12}(0) + A_{13}(0) + A_{23}(0) + A_{21}(0) + A_{24}(0), \end{aligned} \tag{A.47}$$

From equation A.40 and following lemma A.1.9 for two basic densities, f, with support $[0, 2m]$ and g with support $[m+d-n, m+d+n]$, with population masses p and q respectively we have that the total effective antagonism felt by members of f towards members of g before the slide equals:

$$A(f, g) = 2kd_{ij}(0)p^{1+a}q(m)^{-a}\int_0^1 f^*(x)^{1+a}dx, \tag{A.48}$$

Using this to calculate each element of equation A.47 we have

$$A_{12}(0) = 2kd_{12}(0)p^{1+a}qm^{-a}\int_0^1 f^*(x)^{1+a}dx$$

Due to the fact that the part $2kp^{1+a}qm^{-a}\int_0^1 f^*(x)^{1+a}dx$ is constant, above can be rewritten as follows:

$$A_{12}(0) = k_{12}d_{12}(0),$$

where $k_{12} = 2kp^{1+a}qm^{-a}\int_0^1 f^*(x)^{1+a}dx$.

In the same fashion, we calculate all remaining elements:

$$\begin{aligned} &A_{13}(0) = 2kd_{13}(0)p^{1+a}qm^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow \\ &A_{13}(0) = k_{13}d_{13}(0) \end{aligned}$$

$$\begin{aligned} &A_{21}(0) = 2kd_{21}(0)q^{1+a}pn^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow \\ &A_{21}(0) = k_{21}d_{21}(0) \end{aligned}$$

$$\begin{aligned} &A_{23}(0) = 2kd_{23}(0)q^{2+a}n^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow \\ &A_{23}(0) = k_{23}d_{23}(0) \end{aligned}$$

$$\begin{aligned} &A_{24}(0) = 2kd_{24}(0)q^{1+a}pn^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow \\ &A_{24}(0) = k_{24}d_{24}(0) \end{aligned}$$

$$A_{12}(z) = 2kd_{12}(z)p^{1+a}qm^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow$$
$$A_{12}(z) = k_{12}d_{12}(z) = k_{12}(d_{12}(0) - z)$$

$$A_{13}(z) = 2kd_{13}(z)p^{1+a}qm^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow$$
$$A_{13}(z) = k_{13}d_{13}(z) = k_{13}(d_{13}(0) + z)$$

$$A_{21}(z) = 2kd_{21}(z)q^{1+a}pn^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow$$
$$A_{21}(z) = k_{21}d_{21}(z) = k_{21}(d_{21}(0) - z)$$

$$A_{23}(z) = 2kd_{23}(z)q^{2+a}n^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow$$
$$A_{23}(z) = k_{23}d_{23}(z) = k_{23}(d_{23}(0) + 2z)$$

$$A_{24}(z) = 2kd_{24}(z)q^{1+a}pn^{-a}\int_0^1 f^*(x)^{1+a}dx \Rightarrow$$
$$A_{24}(z) = k_{24}d_{21}(z) = k_{24}(d_{24}(0) + z)$$

Further more we observe that:

$$k_{12} = k_{13}$$

and

$$k_{21} = k_{24}$$

Substituting all above in equation A.47 we have:

$$k_{12}d_{12}(0) - k_{12}z + k_{13}d_{13}(0) + k_{13}z + k_{21}d_{21}(0) - k_{21}z + k_{23}d_{23}(0) + 2k_{23}z + k_{24}d_{24}(0) + k_{24}z \geq$$
$$k_{12}d_{12}(0) + k_{13}d_{13}(0) + k_{21}d_{21}(0) + k_{23}d_{23}(0) + k_{24}d_{24}(0) \Rightarrow$$
$$[A_{12}(0) + A_{13}(0) + A_{23}(0) + A_{21}(0) + A_{24}(0)] + k_{21}z + 2k_{23}z \geq$$
$$A_{12}(0) + A_{13}(0) + A_{23}(0) + A_{21}(0) + A_{24}(0),$$

which is true, since K_{ij} and z are positive numbers.

□

A.2 Section B

Table A.1: $BIC_{gmm} - BIC_{sgl}$

	diff=0	diff=0.5	diff=1.0	diff=1.5	diff=2.0	diff=2.5	diff=3.0	diff=3.5	diff=4.0	diff=4.5	diff=5.0
1	17.0	14.7	17.5	15.3	9.5	-29.8	-17.5	-94.8	-159.4	-209.0	-333.6
2	18.1	18.4	16.0	13.2	13.5	-4.3	-61.7	-91.3	-131.0	-242.4	-296.8
3	16.6	15.4	17.4	17.9	10.2	-8.3	-60.8	-109.5	-146.1	-235.5	-274.5
4	14.8	13.4	16.8	14.6	13.3	-26.4	-60.9	-97.1	-138.6	-219.4	-305.3
5	18.5	18.4	17.5	9.4	-0.7	2.0	-58.1	-79.4	-137.1	-216.6	-300.7
6	18.7	17.4	18.0	16.5	5.2	-18.0	-42.0	-95.1	-122.3	-223.9	-271.4
7	18.7	17.9	18.4	11.7	5.2	4.5	-38.7	-119.6	-195.2	-319.7	-342.8
8	13.5	18.3	17.1	16.7	6.9	-11.0	-38.8	-78.0	-139.3	-210.0	-310.6
9	18.7	17.5	12.8	15.5	0.0	-5.7	-34.7	-83.4	-128.3	-258.6	-331.7
10	16.9	14.3	12.8	14.0	2.8	-8.6	-59.4	-98.1	-165.4	-212.1	-313.8
11	9.9	18.3	18.7	12.1	17.8	-0.4	-59.5	-94.3	-179.7	-252.2	-264.8
12	15.6	17.0	13.4	14.5	0.2	-7.7	-54.5	-122.1	-157.2	-221.2	-243.3
13	18.6	16.3	14.4	14.0	-0.1	-20.7	-32.0	-94.8	-164.0	-255.5	-360.6
14	9.6	16.5	18.2	12.0	3.1	-8.8	-51.7	-66.9	-165.4	-202.0	-293.3
15	17.4	18.1	17.6	14.8	13.3	-28.9	-39.0	-68.7	-194.2	-278.2	-356.4
16	18.5	18.6	18.0	13.4	4.7	-7.1	-39.3	-117.5	-171.6	-219.0	-279.5
17	17.4	12.1	15.6	11.3	5.8	-7.5	-48.7	-87.1	-154.7	-254.8	-224.4
18	18.2	17.2	18.7	11.7	-2.0	-7.8	-47.9	-82.5	-163.0	-242.0	-328.7
19	11.7	17.3	16.1	7.8	15.5	-17.3	-53.3	-82.5	-157.8	-238.7	-283.3
20	16.8	16.1	18.4	17.1	12.5	-18.6	-26.0	-92.9	-164.4	-251.3	-333.0
21	18.0	16.4	14.6	18.7	4.7	-18.1	-66.5	-80.7	-153.5	-208.9	-257.0
22	14.1	13.2	15.3	4.0	-1.7	-29.3	-7.8	-96.4	-172.0	-223.7	-324.4
23	18.7	16.6	12.8	17.9	0.6	-10.5	-34.5	-120.2	-113.8	-225.2	-356.0
24	17.8	17.6	9.3	15.3	13.3	-21.6	-44.3	-110.4	-143.3	-244.9	-278.5
25	18.6	18.7	18.2	5.1	7.5	-28.1	-49.5	-98.6	-143.9	-218.3	-270.9
26	17.6	17.7	18.4	11.3	9.7	-11.8	-41.4	-78.6	-122.8	-240.5	-312.5
27	16.8	15.9	17.5	14.1	2.6	-25.4	-57.3	-101.9	-185.4	-253.3	-291.2
28	18.8	16.8	16.7	10.7	8.9	-19.0	-53.4	-86.3	-126.9	-191.1	-308.8
29	17.7	17.2	16.4	15.1	14.4	-8.8	-29.9	-112.6	-147.4	-211.0	-294.5
30	11.5	12.2	18.6	11.8	-9.8	-13.5	-51.5	-86.2	-167.0	-182.7	-319.1
31	18.6	16.6	18.3	16.4	-3.5	-10.0	-47.6	-80.2	-176.3	-192.0	-269.3
32	14.7	18.7	16.2	18.6	-13.5	-7.2	-27.2	-91.3	-178.7	-265.2	-272.1
33	18.6	18.6	16.7	13.3	-2.0	0.6	-36.4	-107.0	-191.6	-189.6	-265.7
34	14.5	18.6	18.7	9.1	6.7	-8.8	-45.3	-121.5	-138.8	-223.0	-298.5
35	16.4	15.8	11.1	10.9	16.4	2.0	-50.6	-89.0	-178.3	-203.0	-300.1
36	17.9	15.9	16.6	15.1	11.5	-20.8	-65.5	-67.1	-148.8	-246.9	-294.9
37	17.3	16.2	17.9	1.4	10.2	-26.9	-29.6	-95.9	-147.2	-277.3	-271.6
38	17.2	16.2	17.3	11.5	-4.7	-9.4	-47.8	-123.0	-178.3	-234.1	-294.7
39	13.9	13.8	18.6	16.9	3.6	-4.2	-43.4	-114.0	-141.0	-258.8	-276.4
40	14.7	16.5	16.6	18.3	5.4	-35.7	-28.5	-83.6	-138.3	-227.1	-246.7
41	16.2	15.4	18.1	18.5	2.6	-0.2	-41.6	-113.5	-118.8	-228.1	-288.3
42	18.4	18.8	16.2	11.4	-2.7	-3.8	-46.2	-93.0	-198.6	-236.8	-326.9
43	16.4	17.4	18.6	12.2	-2.8	-21.3	-29.0	-72.9	-138.1	-214.4	-303.9
44	18.7	18.4	17.2	11.2	4.3	-24.7	-46.5	-119.3	-168.0	-237.4	-254.2
45	16.5	15.2	16.6	16.1	3.3	-7.9	-47.6	-101.5	-134.6	-248.8	-313.1

Table A.2: $BIC_{gmm} - BIC_{sgl}$ - Cont'ed

	diff=0	diff=0.5	diff=1.0	diff=1.5	diff=2.0	diff=2.5	diff=3.0	diff=3.5	diff=4.0	diff=4.5	diff=5.0
46	18.2	18.5	17.9	17.9	-8.2	-21.6	-40.4	-48.9	-85.1	-255.4	-315.2
47	17.0	16.8	18.4	13.8	-5.1	-1.2	-25.3	-123.8	-148.8	-216.4	-275.2
48	16.8	16.8	13.0	18.4	11.7	-7.8	-70.2	-117.5	-152.5	-263.4	-259.6
49	18.7	17.6	15.6	2.7	0.0	-4.3	-41.9	-77.2	-129.7	-185.6	-272.8
50	18.5	15.8	16.3	6.5	10.5	-42.0	-35.7	-88.0	-119.0	-229.9	-279.7
51	15.5	18.4	16.8	8.6	7.4	-17.7	-54.2	-83.0	-140.7	-229.7	-308.9
52	18.4	17.1	17.5	16.6	14.3	-0.5	-38.9	-88.0	-154.7	-218.8	-278.2
53	18.2	17.5	17.8	4.6	18.5	-28.8	-75.4	-96.0	-135.0	-235.1	-282.9
54	18.6	17.3	18.4	17.9	15.7	-46.1	-47.0	-108.0	-143.5	-225.2	-317.0
55	18.2	17.8	17.3	16.5	13.4	-3.0	-46.7	-89.4	-166.1	-272.7	-271.3
56	18.5	11.2	13.6	18.4	-1.0	-10.4	-35.8	-108.6	-130.6	-209.9	-302.7
57	17.7	17.7	16.8	13.1	-2.1	-16.1	-59.8	-88.0	-165.0	-249.3	-306.9
58	17.8	17.2	10.9	12.6	8.4	-10.3	-34.8	-90.7	-170.0	-246.6	-328.4
59	17.4	10.2	18.1	12.5	-3.3	-18.3	-21.0	-107.0	-145.3	-233.3	-294.0
60	15.6	18.7	18.0	12.4	3.9	-11.0	-60.9	-100.7	-169.6	-242.3	-295.7
61	18.5	16.9	17.6	16.1	6.4	-9.6	-40.7	-116.7	-150.5	-201.4	-307.0
62	18.0	18.1	9.8	15.3	3.9	-7.6	-86.1	-103.5	-153.2	-237.1	-279.9
63	16.1	17.6	18.0	12.9	16.0	-2.4	-31.0	-113.7	-136.6	-206.8	-339.0
64	15.2	18.0	15.9	9.3	7.1	-23.1	-38.9	-89.1	-153.4	-230.9	-336.4
65	17.5	16.4	16.3	12.4	7.7	-7.0	-50.8	-87.5	-156.3	-244.2	-373.2
66	17.7	17.1	3.8	18.5	14.0	-20.4	-35.4	-71.8	-142.5	-270.2	-325.5
67	16.9	18.7	18.5	14.0	4.1	-16.5	-30.0	-96.4	-173.1	-188.5	-275.7
68	18.1	16.6	17.8	15.9	14.3	-16.8	-69.1	-90.6	-142.0	-276.2	-304.8
69	17.2	16.9	9.0	17.3	8.6	-23.6	-29.9	-69.6	-169.9	-230.8	-300.7
70	15.0	18.7	16.2	17.2	9.0	-25.7	-52.8	-75.6	-186.4	-200.9	-244.1
71	15.1	18.6	18.2	13.2	0.9	-22.4	-51.2	-66.0	-175.4	-211.8	-264.3
72	15.0	18.2	18.4	16.7	9.9	-23.4	-63.8	-71.8	-154.8	-251.6	-311.0
73	18.7	16.9	13.6	18.1	0.3	1.5	-42.1	-94.5	-181.1	-248.5	-322.7
74	17.9	17.0	17.2	17.1	9.4	-5.7	-48.3	-78.0	-135.3	-203.9	-317.1
75	15.5	16.9	18.1	13.8	2.3	-24.4	-37.6	-133.1	-193.1	-186.6	-293.8
76	13.6	17.4	12.0	17.6	16.4	-16.8	-58.9	-71.0	-147.7	-242.2	-263.5
77	17.9	18.8	16.5	11.8	12.2	-0.2	-28.0	-95.3	-144.3	-202.8	-305.3
78	18.4	18.3	13.9	10.7	2.0	2.1	-43.8	-101.2	-130.4	-248.5	-336.0
79	18.2	18.0	9.3	17.8	11.4	-18.1	-55.8	-95.0	-165.1	-200.1	-249.7
80	11.5	17.2	16.7	6.5	3.9	-20.1	-44.9	-91.9	-193.1	-221.0	-249.2
81	15.5	18.1	15.1	13.1	0.2	-41.8	-94.3	-102.8	-154.7	-232.9	-261.1
82	15.9	18.7	14.9	18.6	5.5	-1.4	-55.9	-84.8	-154.2	-228.3	-310.8
83	18.1	18.8	18.1	7.2	1.3	-21.5	-31.1	-122.6	-183.2	-248.4	-251.7
84	16.1	18.6	17.3	17.0	-6.3	-6.4	-41.1	-61.9	-176.4	-249.1	-299.6
85	13.8	15.2	16.6	14.2	8.1	-19.4	-12.9	-126.3	-196.6	-253.6	-334.3
86	18.1	10.1	17.3	18.5	5.3	-15.4	-40.3	-98.0	-149.8	-256.3	-309.7
87	17.7	16.5	16.4	18.2	4.2	-12.0	-40.2	-139.2	-187.4	-236.9	-287.9
88	18.0	13.4	16.6	15.2	-9.1	-22.5	-69.7	-117.3	-158.1	-224.6	-300.9
89	16.6	15.9	18.7	16.3	8.2	-32.4	-62.5	-95.0	-159.0	-227.2	-376.0
90	17.2	17.5	18.6	0.3	-0.7	-16.7	-49.1	-92.3	-171.0	-204.3	-358.9
91	16.6	16.5	17.4	17.2	5.4	-20.1	-34.8	-98.1	-206.9	-235.6	-286.6
92	18.2	14.4	17.9	11.6	1.1	-10.9	-53.5	-123.7	-153.6	-218.9	-319.3
93	14.5	15.3	17.7	14.5	11.9	-20.3	-59.8	-82.9	-111.8	-231.4	-304.8
94	17.8	16.4	18.7	15.6	11.0	-9.2	-9.2	-52.7	-167.6	-249.4	-271.6
95	16.3	18.4	7.7	16.9	14.6	-7.1	-21.6	-72.8	-125.7	-237.2	-309.9
96	17.5	17.1	17.9	15.8	1.6	-23.9	-79.2	-95.5	-146.4	-246.3	-253.0
97	14.4	17.3	10.2	16.7	13.2	-16.2	-42.8	-104.7	-136.7	-193.5	-327.6
98	17.4	17.8	17.5	7.2	3.9	-21.8	-31.9	-78.2	-168.1	-181.2	-362.6
99	12.9	16.4	12.5	14.8	1.4	-24.6	-51.9	-109.8	-159.0	-222.8	-299.8
100	16.5	15.5	18.7	11.8	-5.0	-7.8	-53.3	-97.7	-159.6	-222.0	-323.3
times	0%	0%	0%	0%	20%	94%	100%	100%	100%	100%	100%

A.3 Section C

Table A.3: Results of PaF estimator for all values of α

	— α values —									
	0.25	0.26	0.27	0.28	0.29	0.3	0.31	0.32	0.33	0.34
	Year: 1989									
Denmark	0.4327	0.4303	0.4280	0.4256	0.4233	0.4211	0.4188	0.4166	0.4144	0.4122
France	0.3305	0.3296	0.3287	0.3277	0.3268	0.3260	0.3251	0.3242	0.3234	0.3226
Germany	0.4028	0.4005	0.3982	0.3959	0.3937	0.3915	0.3893	0.3872	0.3851	0.3830
Greece	0.3352	0.3348	0.3344	0.3341	0.3337	0.3334	0.3330	0.3327	0.3324	0.3321
Ireland	0.3635	0.3620	0.3605	0.3590	0.3575	0.3561	0.3547	0.3533	0.3519	0.3506
Italy	0.3797	0.3778	0.3759	0.3740	0.3722	0.3704	0.3686	0.3668	0.3651	0.3634
Luxembourg	0.3627	0.3608	0.3589	0.3570	0.3552	0.3534	0.3516	0.3498	0.3481	0.3464
Netherlands	0.3474	0.3460	0.3447	0.3433	0.3420	0.3408	0.3395	0.3382	0.3370	0.3358
Spain	0.4986	0.4941	0.4897	0.4854	0.4811	0.4769	0.4728	0.4687	0.4647	0.4608
UK	0.5454	0.5397	0.5342	0.5287	0.5233	0.5180	0.5128	0.5076	0.5026	0.4976
	Year: 1994									
Denmark	0.4158	0.4133	0.4109	0.4085	0.4062	0.4039	0.4016	0.3993	0.3971	0.3949
France	0.3517	0.3503	0.3489	0.3475	0.3461	0.3448	0.3435	0.3422	0.3409	0.3397
Germany	0.3543	0.3526	0.3510	0.3495	0.3479	0.3464	0.3449	0.3435	0.3420	0.3406
Greece	0.4118	0.4094	0.4070	0.4046	0.4023	0.4000	0.3977	0.3955	0.3933	0.3911
Ireland	0.4117	0.4090	0.4062	0.4036	0.4009	0.3984	0.3959	0.3934	0.3909	0.3886
Italy	0.3839	0.3821	0.3803	0.3785	0.3767	0.3750	0.3733	0.3716	0.3699	0.3683
Luxembourg	0.3303	0.3290	0.3276	0.3263	0.3250	0.3237	0.3225	0.3212	0.3200	0.3188
Netherlands	0.3722	0.3706	0.3690	0.3675	0.3659	0.3644	0.3630	0.3615	0.3601	0.3587
Spain	0.5625	0.5568	0.5513	0.5458	0.5404	0.5350	0.5297	0.5245	0.5193	0.5143
UK	0.5374	0.5318	0.5262	0.5208	0.5155	0.5103	0.5051	0.5000	0.4951	0.4902
	Year: 1999									
Denmark	0.3517	0.3503	0.3489	0.3475	0.3461	0.3447	0.3434	0.3421	0.3408	0.3395
France	0.3271	0.3262	0.3253	0.3244	0.3235	0.3227	0.3219	0.3210	0.3202	0.3194
Germany	0.3887	0.3863	0.3839	0.3815	0.3792	0.3769	0.3747	0.3725	0.3703	0.3682
Greece	0.5167	0.5118	0.5071	0.5024	0.4977	0.4932	0.4887	0.4843	0.4799	0.4756
Ireland	0.4852	0.4807	0.4764	0.4720	0.4678	0.4637	0.4596	0.4556	0.4516	0.4478
Italy	0.4335	0.4308	0.4280	0.4253	0.4227	0.4200	0.4175	0.4149	0.4124	0.4099
Luxembourg	0.4960	0.4914	0.4869	0.4824	0.4780	0.4737	0.4694	0.4652	0.4611	0.4570
Netherlands	0.3997	0.3973	0.3950	0.3927	0.3905	0.3882	0.3860	0.3838	0.3817	0.3795
Spain	0.6433	0.6356	0.6281	0.6206	0.6133	0.6061	0.5990	0.5920	0.5851	0.5784
UK	0.6134	0.6063	0.5994	0.5926	0.5860	0.5795	0.5731	0.5669	0.5608	0.5548

Table A.4: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.25	0.26	0.27	0.28	0.29	0.3	0.31	0.32	0.33	0.34
	Year: 2004									
Denmark	0.4012	0.3990	0.3968	0.3946	0.3924	0.3903	0.3882	0.3862	0.3841	0.3821
France	0.4043	0.4019	0.3996	0.3973	0.3950	0.3927	0.3905	0.3884	0.3862	0.3841
Germany	0.3915	0.3891	0.3868	0.3845	0.3822	0.3800	0.3778	0.3757	0.3736	0.3715
Greece	0.5106	0.5057	0.5010	0.4963	0.4918	0.4872	0.4828	0.4784	0.4741	0.4699
Ireland	0.5067	0.5019	0.4971	0.4925	0.4879	0.4834	0.4790	0.4746	0.4703	0.4661
Italy	0.3913	0.3900	0.3886	0.3872	0.3859	0.3845	0.3832	0.3819	0.3806	0.3794
Luxembourg	0.3531	0.3515	0.3500	0.3485	0.3471	0.3457	0.3443	0.3429	0.3415	0.3402
Netherlands	0.4771	0.4735	0.4700	0.4666	0.4632	0.4598	0.4565	0.4532	0.4499	0.4467
Spain	0.4744	0.4709	0.4674	0.4639	0.4605	0.4572	0.4539	0.4506	0.4474	0.4442
UK	0.7425	0.7315	0.7208	0.7103	0.7000	0.6899	0.6801	0.6705	0.6610	0.6518
	Year: 2009									
Denmark	0.4637	0.4605	0.4572	0.4540	0.4509	0.4477	0.4446	0.4416	0.4386	0.4356
France	0.3616	0.3600	0.3583	0.3566	0.3550	0.3534	0.3518	0.3503	0.3488	0.3472
Germany	0.4363	0.4330	0.4298	0.4267	0.4236	0.4206	0.4176	0.4147	0.4118	0.4089
Greece	0.4765	0.4723	0.4681	0.4641	0.4601	0.4561	0.4522	0.4484	0.4447	0.4410
Ireland	0.6038	0.5965	0.5894	0.5824	0.5756	0.5689	0.5623	0.5558	0.5495	0.5433
Italy	0.4196	0.4175	0.4155	0.4135	0.4115	0.4096	0.4077	0.4058	0.4039	0.4020
Luxembourg	0.4117	0.4090	0.4064	0.4038	0.4012	0.3987	0.3961	0.3937	0.3912	0.3888
Netherlands	0.4036	0.4011	0.3987	0.3963	0.3940	0.3917	0.3894	0.3871	0.3849	0.3827
Spain	0.5333	0.5278	0.5224	0.5171	0.5119	0.5067	0.5016	0.4966	0.4916	0.4867
UK	0.5392	0.5333	0.5275	0.5219	0.5164	0.5109	0.5056	0.5004	0.4953	0.4903
	Year: 2014									
Denmark	0.4283	0.4254	0.4226	0.4198	0.4171	0.4143	0.4117	0.4090	0.4064	0.4038
France	0.3799	0.3780	0.3762	0.3744	0.3726	0.3708	0.3691	0.3673	0.3656	0.3639
Germany	0.4148	0.4119	0.4090	0.4062	0.4034	0.4006	0.3979	0.3953	0.3926	0.3900
Greece	0.3619	0.3601	0.3584	0.3566	0.3549	0.3533	0.3516	0.3500	0.3484	0.3468
Ireland	0.4695	0.4656	0.4618	0.4580	0.4543	0.4507	0.4471	0.4436	0.4401	0.4367
Italy	0.4470	0.4439	0.4409	0.4379	0.4349	0.4320	0.4291	0.4262	0.4234	0.4206
Luxembourg	0.4032	0.4005	0.3978	0.3952	0.3926	0.3901	0.3876	0.3852	0.3828	0.3804
Netherlands	0.3945	0.3922	0.3899	0.3876	0.3854	0.3832	0.3810	0.3789	0.3768	0.3747
Spain	0.4347	0.4318	0.4290	0.4262	0.4234	0.4207	0.4180	0.4153	0.4127	0.4101
UK	0.6092	0.6019	0.5948	0.5877	0.5808	0.5740	0.5674	0.5608	0.5543	0.5480
	Year: 2019									
Denmark	0.4965	0.4925	0.4887	0.4848	0.4811	0.4773	0.4737	0.4700	0.4664	0.4629
France	0.5014	0.4971	0.4928	0.4885	0.4844	0.4803	0.4762	0.4722	0.4682	0.4643
Germany	0.4215	0.4185	0.4156	0.4127	0.4098	0.4070	0.4043	0.4016	0.3990	0.3964
Greece	0.4577	0.4540	0.4503	0.4467	0.4432	0.4397	0.4363	0.4330	0.4297	0.4264
Ireland	0.5794	0.5727	0.5661	0.5597	0.5534	0.5472	0.5411	0.5352	0.5294	0.5237
Italy	0.4830	0.4792	0.4755	0.4718	0.4681	0.4645	0.4609	0.4574	0.4539	0.4505
Luxembourg	0.4186	0.4156	0.4126	0.4097	0.4068	0.4040	0.4012	0.3985	0.3959	0.3932
Netherlands	0.4633	0.4597	0.4562	0.4527	0.4492	0.4458	0.4424	0.4391	0.4358	0.4325
Spain	0.4517	0.4486	0.4456	0.4427	0.4397	0.4369	0.4340	0.4312	0.4284	0.4257
UK	0.4768	0.4727	0.4686	0.4646	0.4607	0.4568	0.4530	0.4492	0.4455	0.4419

Table A.5: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.35	0.36	0.37	0.38	0.39	0.4	0.41	0.42	0.43	0.44
	Year: 1989									
	0.35	0.36	0.37	0.38	0.39	0.4	0.41	0.42	0.43	0.44
Denmark	0.4100	0.4079	0.4058	0.4037	0.4016	0.3995	0.3975	0.3955	0.3935	0.3915
France	0.3217	0.3210	0.3202	0.3194	0.3186	0.3179	0.3172	0.3165	0.3157	0.3151
Germany	0.3810	0.3790	0.3770	0.3750	0.3731	0.3712	0.3693	0.3674	0.3656	0.3638
Greece	0.3318	0.3315	0.3312	0.3309	0.3306	0.3304	0.3301	0.3298	0.3296	0.3294
Ireland	0.3492	0.3479	0.3466	0.3454	0.3441	0.3429	0.3417	0.3405	0.3393	0.3381
Italy	0.3618	0.3601	0.3585	0.3569	0.3554	0.3538	0.3523	0.3508	0.3493	0.3479
Luxembourg	0.3447	0.3431	0.3415	0.3399	0.3384	0.3369	0.3354	0.3339	0.3324	0.3310
Netherlands	0.3346	0.3334	0.3322	0.3311	0.3300	0.3288	0.3277	0.3267	0.3256	0.3245
Spain	0.4569	0.4530	0.4493	0.4456	0.4419	0.4383	0.4347	0.4312	0.4278	0.4243
UK	0.4926	0.4878	0.4830	0.4783	0.4737	0.4691	0.4646	0.4602	0.4558	0.4515
	Year: 1994									
Denmark	0.3927	0.3905	0.3884	0.3863	0.3842	0.3821	0.3801	0.3781	0.3761	0.3741
France	0.3384	0.3372	0.3360	0.3348	0.3336	0.3324	0.3313	0.3301	0.3290	0.3279
Germany	0.3392	0.3378	0.3364	0.3351	0.3338	0.3325	0.3312	0.3300	0.3287	0.3275
Greece	0.3890	0.3869	0.3849	0.3829	0.3809	0.3789	0.3770	0.3751	0.3732	0.3713
Ireland	0.3862	0.3839	0.3816	0.3794	0.3772	0.3751	0.3730	0.3709	0.3688	0.3668
Italy	0.3666	0.3650	0.3634	0.3618	0.3602	0.3586	0.3571	0.3555	0.3540	0.3525
Luxembourg	0.3177	0.3165	0.3154	0.3143	0.3132	0.3122	0.3111	0.3101	0.3091	0.3081
Netherlands	0.3573	0.3560	0.3546	0.3533	0.3520	0.3508	0.3495	0.3483	0.3471	0.3459
Spain	0.5092	0.5043	0.4994	0.4946	0.4898	0.4851	0.4804	0.4758	0.4713	0.4668
UK	0.4853	0.4806	0.4759	0.4713	0.4668	0.4623	0.4580	0.4536	0.4494	0.4452
	Year: 1999									
Denmark	0.3383	0.3371	0.3358	0.3346	0.3335	0.3323	0.3311	0.3300	0.3289	0.3278
France	0.3186	0.3179	0.3171	0.3163	0.3156	0.3149	0.3142	0.3134	0.3127	0.3121
Germany	0.3662	0.3641	0.3621	0.3602	0.3583	0.3564	0.3546	0.3527	0.3510	0.3492
Greece	0.4714	0.4672	0.4630	0.4590	0.4550	0.4510	0.4471	0.4432	0.4394	0.4357
Ireland	0.4440	0.4402	0.4365	0.4329	0.4294	0.4259	0.4225	0.4191	0.4158	0.4125
Italy	0.4074	0.4050	0.4026	0.4002	0.3978	0.3955	0.3932	0.3909	0.3887	0.3865
Luxembourg	0.4529	0.4490	0.4451	0.4412	0.4374	0.4336	0.4299	0.4263	0.4227	0.4191
Netherlands	0.3774	0.3754	0.3733	0.3713	0.3693	0.3673	0.3654	0.3634	0.3615	0.3596
Spain	0.5717	0.5651	0.5587	0.5523	0.5460	0.5399	0.5338	0.5278	0.5219	0.5160
UK	0.5489	0.5432	0.5375	0.5320	0.5265	0.5212	0.5159	0.5108	0.5057	0.5007
	Year: 2004									
Denmark	0.3801	0.3781	0.3762	0.3742	0.3723	0.3704	0.3685	0.3667	0.3649	0.3631
France	0.3820	0.3799	0.3779	0.3759	0.3739	0.3719	0.3700	0.3680	0.3661	0.3643
Germany	0.3695	0.3675	0.3655	0.3636	0.3617	0.3598	0.3580	0.3562	0.3544	0.3527
Greece	0.4657	0.4616	0.4575	0.4535	0.4496	0.4457	0.4419	0.4381	0.4344	0.4307
Ireland	0.4620	0.4579	0.4539	0.4500	0.4461	0.4423	0.4385	0.4348	0.4312	0.4276
Italy	0.3781	0.3768	0.3756	0.3744	0.3731	0.3719	0.3707	0.3695	0.3684	0.3672
Luxembourg	0.3389	0.3376	0.3363	0.3351	0.3339	0.3327	0.3315	0.3303	0.3291	0.3280
Netherlands	0.4435	0.4404	0.4373	0.4342	0.4312	0.4282	0.4252	0.4222	0.4193	0.4165
Spain	0.4411	0.4380	0.4350	0.4320	0.4290	0.4261	0.4232	0.4203	0.4175	0.4147
UK	0.6427	0.6339	0.6252	0.6167	0.6083	0.6001	0.5921	0.5842	0.5765	0.5689

Table A.6: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.35	0.36	0.37	0.38	0.39	0.4	0.41	0.42	0.43	0.44
	Year: 2009									
Denmark	0.4326	0.4297	0.4268	0.4239	0.4211	0.4183	0.4155	0.4128	0.4101	0.4074
France	0.3457	0.3443	0.3428	0.3413	0.3399	0.3385	0.3371	0.3357	0.3344	0.3330
Germany	0.4061	0.4033	0.4006	0.3979	0.3953	0.3927	0.3901	0.3876	0.3851	0.3827
Greece	0.4373	0.4337	0.4302	0.4267	0.4233	0.4199	0.4165	0.4133	0.4100	0.4068
Ireland	0.5372	0.5312	0.5253	0.5195	0.5139	0.5083	0.5029	0.4975	0.4922	0.4871
Italy	0.4002	0.3983	0.3965	0.3947	0.3930	0.3912	0.3895	0.3877	0.3860	0.3843
Luxembourg	0.3865	0.3841	0.3818	0.3795	0.3773	0.3750	0.3729	0.3707	0.3685	0.3664
Netherlands	0.3806	0.3784	0.3763	0.3743	0.3722	0.3702	0.3682	0.3662	0.3643	0.3624
Spain	0.4819	0.4772	0.4725	0.4679	0.4634	0.4589	0.4545	0.4501	0.4458	0.4416
UK	0.4854	0.4805	0.4758	0.4711	0.4666	0.4621	0.4577	0.4534	0.4491	0.4449
	Year: 2014									
Denmark	0.4012	0.3987	0.3962	0.3938	0.3913	0.3889	0.3865	0.3842	0.3818	0.3795
France	0.3623	0.3606	0.3590	0.3573	0.3557	0.3541	0.3526	0.3510	0.3495	0.3479
Germany	0.3875	0.3850	0.3825	0.3801	0.3777	0.3753	0.3730	0.3707	0.3685	0.3662
Greece	0.3453	0.3437	0.3422	0.3407	0.3393	0.3378	0.3364	0.3350	0.3336	0.3322
Ireland	0.4333	0.4300	0.4267	0.4234	0.4202	0.4171	0.4140	0.4109	0.4079	0.4049
Italy	0.4179	0.4151	0.4125	0.4098	0.4072	0.4045	0.4020	0.3994	0.3969	0.3944
Luxembourg	0.3781	0.3758	0.3735	0.3713	0.3691	0.3669	0.3648	0.3627	0.3606	0.3586
Netherlands	0.3727	0.3706	0.3687	0.3667	0.3647	0.3628	0.3609	0.3591	0.3572	0.3554
Spain	0.4075	0.4050	0.4025	0.4000	0.3975	0.3951	0.3927	0.3903	0.3880	0.3857
UK	0.5417	0.5356	0.5295	0.5236	0.5178	0.5120	0.5063	0.5008	0.4953	0.4899
	Year: 2019									
Denmark	0.4594	0.4559	0.4525	0.4491	0.4458	0.4425	0.4393	0.4360	0.4329	0.4297
France	0.4605	0.4567	0.4529	0.4492	0.4455	0.4419	0.4384	0.4348	0.4314	0.4279
Germany	0.3938	0.3913	0.3889	0.3865	0.3841	0.3818	0.3795	0.3772	0.3750	0.3728
Greece	0.4232	0.4201	0.4170	0.4139	0.4109	0.4080	0.4050	0.4022	0.3993	0.3965
Ireland	0.5181	0.5126	0.5072	0.5019	0.4967	0.4916	0.4866	0.4817	0.4768	0.4721
Italy	0.4471	0.4437	0.4404	0.4371	0.4339	0.4307	0.4275	0.4243	0.4212	0.4182
Luxembourg	0.3906	0.3881	0.3856	0.3831	0.3807	0.3783	0.3760	0.3737	0.3714	0.3692
Netherlands	0.4293	0.4261	0.4230	0.4199	0.4168	0.4137	0.4107	0.4078	0.4048	0.4019
Spain	0.4230	0.4203	0.4177	0.4151	0.4125	0.4099	0.4074	0.4049	0.4025	0.4000
UK	0.4383	0.4348	0.4314	0.4280	0.4246	0.4213	0.4180	0.4148	0.4117	0.4086

Table A.7: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.45	0.46	0.47	0.48	0.49	0.5	0.51	0.52	0.53	0.54
	Year: 1989									
Denmark	0.3896	0.3876	0.3857	0.3838	0.3819	0.3800	0.3782	0.3763	0.3745	0.3727
France	0.3144	0.3137	0.3131	0.3124	0.3118	0.3111	0.3105	0.3099	0.3093	0.3088
Germany	0.3620	0.3602	0.3585	0.3567	0.3550	0.3534	0.3517	0.3501	0.3484	0.3468
Greece	0.3291	0.3289	0.3287	0.3285	0.3283	0.3281	0.3279	0.3277	0.3276	0.3274
Ireland	0.3370	0.3358	0.3347	0.3336	0.3325	0.3315	0.3304	0.3294	0.3284	0.3274
Italy	0.3465	0.3451	0.3437	0.3423	0.3410	0.3397	0.3384	0.3371	0.3358	0.3346
Luxembourg	0.3296	0.3282	0.3269	0.3255	0.3242	0.3229	0.3217	0.3204	0.3192	0.3180
Netherlands	0.3235	0.3224	0.3214	0.3204	0.3194	0.3184	0.3175	0.3165	0.3156	0.3146
Spain	0.4210	0.4177	0.4144	0.4112	0.4080	0.4048	0.4017	0.3987	0.3956	0.3927
UK	0.4473	0.4431	0.4390	0.4349	0.4309	0.4269	0.4230	0.4191	0.4153	0.4116
	Year: 1994									
Denmark	0.3721	0.3702	0.3683	0.3664	0.3645	0.3627	0.3608	0.3590	0.3572	0.3554
France	0.3268	0.3258	0.3247	0.3236	0.3226	0.3216	0.3206	0.3196	0.3186	0.3176
Germany	0.3263	0.3251	0.3239	0.3228	0.3217	0.3205	0.3194	0.3184	0.3173	0.3162
Greece	0.3695	0.3677	0.3660	0.3642	0.3625	0.3608	0.3591	0.3575	0.3559	0.3543
Ireland	0.3649	0.3629	0.3610	0.3591	0.3572	0.3554	0.3536	0.3518	0.3501	0.3483
Italy	0.3510	0.3496	0.3481	0.3467	0.3452	0.3438	0.3424	0.3410	0.3396	0.3382
Luxembourg	0.3071	0.3061	0.3052	0.3043	0.3033	0.3024	0.3016	0.3007	0.2998	0.2990
Netherlands	0.3447	0.3436	0.3424	0.3413	0.3402	0.3391	0.3381	0.3370	0.3360	0.3349
Spain	0.4624	0.4580	0.4537	0.4494	0.4452	0.4410	0.4369	0.4328	0.4288	0.4248
UK	0.4411	0.4370	0.4330	0.4291	0.4252	0.4214	0.4176	0.4139	0.4102	0.4066
	Year: 1999									
Denmark	0.3267	0.3256	0.3245	0.3235	0.3225	0.3214	0.3204	0.3194	0.3185	0.3175
France	0.3114	0.3107	0.3100	0.3094	0.3087	0.3081	0.3075	0.3069	0.3063	0.3057
Germany	0.3475	0.3458	0.3442	0.3425	0.3409	0.3394	0.3378	0.3363	0.3348	0.3334
Greece	0.4320	0.4283	0.4247	0.4211	0.4176	0.4141	0.4107	0.4073	0.4039	0.4006
Ireland	0.4093	0.4061	0.4030	0.3999	0.3969	0.3939	0.3910	0.3881	0.3853	0.3825
Italy	0.3843	0.3821	0.3800	0.3778	0.3757	0.3736	0.3716	0.3695	0.3675	0.3655
Luxembourg	0.4156	0.4122	0.4087	0.4054	0.4020	0.3988	0.3955	0.3923	0.3892	0.3860
Netherlands	0.3578	0.3559	0.3541	0.3523	0.3505	0.3488	0.3470	0.3453	0.3436	0.3419
Spain	0.5103	0.5047	0.4991	0.4936	0.4882	0.4828	0.4776	0.4724	0.4672	0.4622
UK	0.4959	0.4911	0.4864	0.4817	0.4772	0.4727	0.4683	0.4639	0.4597	0.4555
	Year: 2004									
Denmark	0.3613	0.3595	0.3577	0.3560	0.3543	0.3526	0.3509	0.3492	0.3476	0.3460
France	0.3624	0.3606	0.3588	0.3570	0.3552	0.3535	0.3517	0.3500	0.3484	0.3467
Germany	0.3510	0.3493	0.3476	0.3460	0.3444	0.3428	0.3413	0.3398	0.3383	0.3368
Greece	0.4271	0.4235	0.4200	0.4166	0.4131	0.4098	0.4064	0.4032	0.3999	0.3967
Ireland	0.4241	0.4206	0.4172	0.4138	0.4104	0.4072	0.4039	0.4007	0.3976	0.3945
Italy	0.3660	0.3649	0.3638	0.3626	0.3615	0.3604	0.3593	0.3582	0.3571	0.3561
Luxembourg	0.3269	0.3258	0.3247	0.3237	0.3226	0.3216	0.3206	0.3196	0.3186	0.3177
Netherlands	0.4136	0.4108	0.4080	0.4053	0.4025	0.3998	0.3972	0.3945	0.3919	0.3893
Spain	0.4120	0.4093	0.4066	0.4040	0.4014	0.3988	0.3963	0.3938	0.3913	0.3888
UK	0.5615	0.5542	0.5471	0.5400	0.5331	0.5264	0.5197	0.5132	0.5068	0.5005

Table A.8: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.45	0.46	0.47	0.48	0.49	0.5	0.51	0.52	0.53	0.54
	Year: 2009									
Denmark	0.4047	0.4021	0.3995	0.3969	0.3944	0.3918	0.3893	0.3868	0.3844	0.3820
France	0.3317	0.3304	0.3291	0.3278	0.3266	0.3253	0.3241	0.3229	0.3217	0.3205
Germany	0.3802	0.3779	0.3755	0.3732	0.3709	0.3686	0.3664	0.3642	0.3620	0.3599
Greece	0.4037	0.4006	0.3975	0.3945	0.3915	0.3886	0.3857	0.3828	0.3800	0.3773
Ireland	0.4820	0.4770	0.4720	0.4672	0.4625	0.4578	0.4532	0.4487	0.4442	0.4399
Italy	0.3827	0.3810	0.3794	0.3777	0.3761	0.3745	0.3729	0.3713	0.3698	0.3682
Luxembourg	0.3643	0.3623	0.3602	0.3582	0.3562	0.3542	0.3523	0.3504	0.3485	0.3466
Netherlands	0.3605	0.3586	0.3568	0.3549	0.3531	0.3514	0.3496	0.3479	0.3461	0.3444
Spain	0.4374	0.4333	0.4292	0.4252	0.4213	0.4174	0.4135	0.4097	0.4060	0.4023
UK	0.4408	0.4368	0.4329	0.4290	0.4251	0.4214	0.4177	0.4140	0.4105	0.4069
	Year: 2014									
Denmark	0.3773	0.3750	0.3728	0.3706	0.3684	0.3663	0.3641	0.3620	0.3599	0.3579
France	0.3464	0.3449	0.3435	0.3420	0.3405	0.3391	0.3377	0.3363	0.3349	0.3335
Germany	0.3640	0.3619	0.3598	0.3577	0.3556	0.3535	0.3515	0.3495	0.3476	0.3456
Greece	0.3309	0.3296	0.3282	0.3269	0.3257	0.3244	0.3232	0.3219	0.3207	0.3195
Ireland	0.4019	0.3990	0.3961	0.3933	0.3905	0.3877	0.3850	0.3823	0.3796	0.3770
Italy	0.3919	0.3895	0.3871	0.3847	0.3823	0.3800	0.3777	0.3754	0.3731	0.3708
Luxembourg	0.3566	0.3546	0.3526	0.3507	0.3488	0.3470	0.3451	0.3433	0.3415	0.3397
Netherlands	0.3536	0.3519	0.3501	0.3484	0.3467	0.3450	0.3433	0.3417	0.3400	0.3384
Spain	0.3834	0.3811	0.3789	0.3767	0.3745	0.3723	0.3702	0.3680	0.3659	0.3639
UK	0.4846	0.4793	0.4742	0.4691	0.4641	0.4592	0.4543	0.4495	0.4448	0.4402
	Year: 2019									
Denmark	0.4266	0.4235	0.4205	0.4175	0.4145	0.4115	0.4086	0.4057	0.4029	0.4001
France	0.4245	0.4212	0.4179	0.4146	0.4114	0.4082	0.4050	0.4019	0.3988	0.3958
Germany	0.3706	0.3685	0.3664	0.3644	0.3624	0.3604	0.3584	0.3565	0.3546	0.3527
Greece	0.3938	0.3911	0.3884	0.3858	0.3832	0.3806	0.3780	0.3755	0.3731	0.3706
Ireland	0.4674	0.4628	0.4583	0.4539	0.4496	0.4453	0.4411	0.4369	0.4329	0.4288
Italy	0.4151	0.4121	0.4092	0.4062	0.4033	0.4004	0.3976	0.3948	0.3920	0.3892
Luxembourg	0.3670	0.3648	0.3627	0.3606	0.3585	0.3565	0.3544	0.3525	0.3505	0.3486
Netherlands	0.3991	0.3962	0.3934	0.3907	0.3879	0.3852	0.3825	0.3799	0.3772	0.3747
Spain	0.3976	0.3953	0.3929	0.3906	0.3883	0.3860	0.3838	0.3815	0.3793	0.3771
UK	0.4055	0.4025	0.3995	0.3966	0.3937	0.3908	0.3880	0.3852	0.3825	0.3798

Table A.9: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.55	0.56	0.57	0.58	0.59	0.6	0.61	0.62	0.63	0.64
	Year: 1989									
Denmark	0.3709	0.3691	0.3673	0.3656	0.3638	0.3621	0.3604	0.3587	0.3570	0.3554
France	0.3082	0.3076	0.3071	0.3065	0.3060	0.3055	0.3050	0.3045	0.3040	0.3035
Germany	0.3452	0.3437	0.3421	0.3406	0.3391	0.3376	0.3361	0.3346	0.3332	0.3317
Greece	0.3272	0.3271	0.3269	0.3268	0.3267	0.3265	0.3264	0.3263	0.3262	0.3261
Ireland	0.3264	0.3254	0.3244	0.3234	0.3225	0.3216	0.3207	0.3197	0.3189	0.3180
Italy	0.3334	0.3322	0.3310	0.3298	0.3286	0.3275	0.3264	0.3253	0.3242	0.3231
Luxembourg	0.3168	0.3157	0.3145	0.3134	0.3123	0.3112	0.3101	0.3091	0.3080	0.3070
Netherlands	0.3137	0.3128	0.3119	0.3110	0.3101	0.3092	0.3083	0.3075	0.3066	0.3058
Spain	0.3897	0.3868	0.3839	0.3811	0.3783	0.3755	0.3728	0.3701	0.3674	0.3648
UK	0.4079	0.4042	0.4006	0.3970	0.3935	0.3900	0.3866	0.3832	0.3799	0.3766
	Year: 1994									
Denmark	0.3537	0.3519	0.3502	0.3485	0.3468	0.3451	0.3434	0.3418	0.3402	0.3385
France	0.3166	0.3157	0.3148	0.3138	0.3129	0.3120	0.3111	0.3102	0.3093	0.3085
Germany	0.3152	0.3142	0.3132	0.3122	0.3112	0.3102	0.3093	0.3083	0.3074	0.3065
Greece	0.3527	0.3512	0.3496	0.3481	0.3466	0.3452	0.3437	0.3423	0.3409	0.3395
Ireland	0.3466	0.3449	0.3433	0.3417	0.3400	0.3385	0.3369	0.3353	0.3338	0.3323
Italy	0.3369	0.3355	0.3342	0.3329	0.3316	0.3303	0.3290	0.3277	0.3264	0.3252
Luxembourg	0.2982	0.2973	0.2965	0.2957	0.2950	0.2942	0.2934	0.2927	0.2919	0.2912
Netherlands	0.3339	0.3329	0.3319	0.3310	0.3300	0.3291	0.3281	0.3272	0.3263	0.3254
Spain	0.4209	0.4170	0.4132	0.4094	0.4056	0.4019	0.3983	0.3947	0.3911	0.3875
UK	0.4030	0.3995	0.3961	0.3927	0.3893	0.3860	0.3827	0.3795	0.3763	0.3732
	Year: 1999									
Denmark	0.3165	0.3156	0.3147	0.3137	0.3128	0.3119	0.3110	0.3102	0.3093	0.3084
France	0.3051	0.3045	0.3039	0.3033	0.3028	0.3022	0.3017	0.3011	0.3006	0.3001
Germany	0.3319	0.3305	0.3291	0.3278	0.3264	0.3251	0.3238	0.3225	0.3213	0.3200
Greece	0.3974	0.3941	0.3909	0.3878	0.3847	0.3816	0.3785	0.3755	0.3725	0.3696
Ireland	0.3797	0.3770	0.3743	0.3717	0.3691	0.3665	0.3640	0.3615	0.3591	0.3566
Italy	0.3635	0.3616	0.3597	0.3577	0.3558	0.3539	0.3521	0.3502	0.3484	0.3466
Luxembourg	0.3830	0.3799	0.3769	0.3739	0.3710	0.3681	0.3653	0.3624	0.3596	0.3569
Netherlands	0.3403	0.3386	0.3370	0.3354	0.3338	0.3322	0.3306	0.3291	0.3275	0.3260
Spain	0.4572	0.4523	0.4474	0.4426	0.4379	0.4333	0.4287	0.4241	0.4196	0.4152
UK	0.4513	0.4473	0.4433	0.4393	0.4354	0.4316	0.4279	0.4242	0.4205	0.4169
	Year: 2004									
Denmark	0.3443	0.3427	0.3411	0.3396	0.3380	0.3365	0.3349	0.3334	0.3319	0.3304
France	0.3450	0.3434	0.3418	0.3402	0.3386	0.3370	0.3355	0.3340	0.3324	0.3309
Germany	0.3353	0.3339	0.3325	0.3311	0.3297	0.3284	0.3271	0.3257	0.3245	0.3232
Greece	0.3936	0.3904	0.3874	0.3843	0.3813	0.3783	0.3754	0.3725	0.3697	0.3668
Ireland	0.3914	0.3884	0.3854	0.3825	0.3796	0.3767	0.3739	0.3711	0.3684	0.3656
Italy	0.3550	0.3539	0.3529	0.3518	0.3508	0.3498	0.3488	0.3478	0.3468	0.3458
Luxembourg	0.3167	0.3158	0.3149	0.3139	0.3131	0.3122	0.3113	0.3105	0.3096	0.3088
Netherlands	0.3868	0.3842	0.3817	0.3792	0.3768	0.3743	0.3719	0.3695	0.3672	0.3648
Spain	0.3864	0.3840	0.3817	0.3793	0.3770	0.3748	0.3725	0.3703	0.3681	0.3659
UK	0.4943	0.4882	0.4822	0.4763	0.4706	0.4649	0.4593	0.4538	0.4484	0.4431

Table A.10: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.55	0.56	0.57	0.58	0.59	0.6	0.61	0.62	0.63	0.64
	Year: 2009									
Denmark	0.3795	0.3772	0.3748	0.3724	0.3701	0.3678	0.3655	0.3633	0.3610	0.3588
France	0.3193	0.3181	0.3170	0.3158	0.3147	0.3136	0.3125	0.3114	0.3103	0.3093
Germany	0.3578	0.3557	0.3536	0.3516	0.3496	0.3476	0.3456	0.3437	0.3418	0.3399
Greece	0.3745	0.3718	0.3691	0.3665	0.3639	0.3613	0.3588	0.3563	0.3538	0.3514
Ireland	0.4356	0.4313	0.4272	0.4231	0.4190	0.4151	0.4111	0.4073	0.4035	0.3998
Italy	0.3667	0.3652	0.3637	0.3622	0.3607	0.3592	0.3577	0.3563	0.3548	0.3534
Luxembourg	0.3447	0.3429	0.3410	0.3392	0.3375	0.3357	0.3340	0.3322	0.3305	0.3288
Netherlands	0.3428	0.3411	0.3394	0.3378	0.3362	0.3346	0.3331	0.3315	0.3300	0.3284
Spain	0.3986	0.3950	0.3915	0.3880	0.3845	0.3811	0.3777	0.3744	0.3711	0.3678
UK	0.4035	0.4001	0.3967	0.3934	0.3902	0.3870	0.3838	0.3807	0.3777	0.3747
	Year: 2014									
Denmark	0.3558	0.3538	0.3518	0.3499	0.3479	0.3460	0.3441	0.3422	0.3403	0.3384
France	0.3321	0.3307	0.3294	0.3281	0.3267	0.3254	0.3241	0.3228	0.3216	0.3203
Germany	0.3437	0.3419	0.3400	0.3382	0.3364	0.3346	0.3328	0.3311	0.3294	0.3277
Greece	0.3184	0.3172	0.3161	0.3149	0.3138	0.3127	0.3116	0.3106	0.3095	0.3085
Ireland	0.3744	0.3718	0.3692	0.3667	0.3642	0.3618	0.3593	0.3569	0.3546	0.3522
Italy	0.3686	0.3664	0.3642	0.3620	0.3599	0.3578	0.3557	0.3536	0.3515	0.3495
Luxembourg	0.3380	0.3363	0.3346	0.3329	0.3313	0.3296	0.3280	0.3265	0.3249	0.3234
Netherlands	0.3368	0.3353	0.3337	0.3322	0.3307	0.3292	0.3277	0.3262	0.3248	0.3234
Spain	0.3618	0.3598	0.3577	0.3557	0.3538	0.3518	0.3499	0.3479	0.3460	0.3442
UK	0.4356	0.4311	0.4267	0.4223	0.4180	0.4137	0.4096	0.4054	0.4013	0.3973
	Year: 2019									
Denmark	0.3973	0.3945	0.3917	0.3890	0.3863	0.3837	0.3810	0.3784	0.3758	0.3733
France	0.3928	0.3898	0.3869	0.3840	0.3811	0.3782	0.3754	0.3727	0.3699	0.3672
Germany	0.3509	0.3491	0.3473	0.3455	0.3438	0.3421	0.3404	0.3387	0.3370	0.3354
Greece	0.3682	0.3659	0.3635	0.3612	0.3589	0.3567	0.3544	0.3522	0.3501	0.3479
Ireland	0.4249	0.4210	0.4172	0.4134	0.4097	0.4061	0.4025	0.3990	0.3955	0.3921
Italy	0.3865	0.3838	0.3811	0.3784	0.3758	0.3732	0.3707	0.3681	0.3656	0.3631
Luxembourg	0.3467	0.3448	0.3430	0.3411	0.3394	0.3376	0.3358	0.3341	0.3324	0.3307
Netherlands	0.3721	0.3695	0.3670	0.3645	0.3621	0.3597	0.3572	0.3549	0.3525	0.3502
Spain	0.3750	0.3729	0.3707	0.3686	0.3666	0.3645	0.3625	0.3605	0.3585	0.3565
UK	0.3771	0.3745	0.3719	0.3694	0.3669	0.3644	0.3619	0.3595	0.3571	0.3548

Table A.11: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.65	0.66	0.67	0.68	0.69	0.7	0.71	0.72	0.73	0.74
	Year: 1989									
Denmark	0.3537	0.3521	0.3504	0.3488	0.3472	0.3456	0.3440	0.3425	0.3409	0.3394
France	0.3030	0.3025	0.3020	0.3016	0.3011	0.3007	0.3003	0.2998	0.2994	0.2990
Germany	0.3303	0.3289	0.3275	0.3261	0.3248	0.3234	0.3221	0.3208	0.3195	0.3182
Greece	0.3260	0.3259	0.3258	0.3257	0.3256	0.3256	0.3255	0.3254	0.3254	0.3253
Ireland	0.3171	0.3162	0.3154	0.3145	0.3137	0.3129	0.3121	0.3113	0.3105	0.3097
Italy	0.3220	0.3210	0.3200	0.3189	0.3179	0.3170	0.3160	0.3150	0.3141	0.3131
Luxembourg	0.3060	0.3050	0.3041	0.3031	0.3022	0.3013	0.3003	0.2995	0.2986	0.2977
Netherlands	0.3050	0.3042	0.3034	0.3026	0.3018	0.3010	0.3002	0.2994	0.2987	0.2979
Spain	0.3621	0.3596	0.3570	0.3545	0.3520	0.3495	0.3471	0.3447	0.3423	0.3399
UK	0.3733	0.3701	0.3669	0.3638	0.3607	0.3576	0.3546	0.3516	0.3486	0.3457
	Year: 1994									
Denmark	0.3369	0.3353	0.3338	0.3322	0.3307	0.3291	0.3276	0.3261	0.3246	0.3231
France	0.3076	0.3068	0.3059	0.3051	0.3043	0.3035	0.3027	0.3019	0.3011	0.3003
Germany	0.3056	0.3047	0.3038	0.3030	0.3021	0.3013	0.3004	0.2996	0.2988	0.2980
Greece	0.3381	0.3368	0.3354	0.3341	0.3328	0.3315	0.3303	0.3290	0.3278	0.3266
Ireland	0.3308	0.3294	0.3279	0.3265	0.3251	0.3237	0.3223	0.3209	0.3196	0.3182
Italy	0.3239	0.3227	0.3215	0.3202	0.3190	0.3178	0.3166	0.3155	0.3143	0.3131
Luxembourg	0.2905	0.2898	0.2891	0.2884	0.2878	0.2871	0.2864	0.2858	0.2852	0.2845
Netherlands	0.3246	0.3237	0.3228	0.3220	0.3212	0.3203	0.3195	0.3187	0.3179	0.3171
Spain	0.3840	0.3806	0.3772	0.3738	0.3705	0.3672	0.3639	0.3607	0.3575	0.3543
UK	0.3701	0.3670	0.3640	0.3610	0.3581	0.3552	0.3523	0.3495	0.3467	0.3439
	Year: 1999									
Denmark	0.3076	0.3068	0.3059	0.3051	0.3043	0.3035	0.3027	0.3019	0.3012	0.3004
France	0.2996	0.2991	0.2986	0.2981	0.2976	0.2971	0.2966	0.2961	0.2957	0.2952
Germany	0.3188	0.3176	0.3164	0.3153	0.3141	0.3130	0.3119	0.3108	0.3097	0.3087
Greece	0.3667	0.3638	0.3610	0.3581	0.3554	0.3526	0.3499	0.3472	0.3445	0.3419
Ireland	0.3542	0.3519	0.3496	0.3473	0.3450	0.3428	0.3406	0.3384	0.3363	0.3341
Italy	0.3448	0.3430	0.3412	0.3395	0.3378	0.3360	0.3343	0.3327	0.3310	0.3293
Luxembourg	0.3542	0.3515	0.3488	0.3462	0.3436	0.3410	0.3384	0.3359	0.3334	0.3310
Netherlands	0.3245	0.3230	0.3216	0.3201	0.3187	0.3172	0.3158	0.3144	0.3130	0.3117
Spain	0.4109	0.4066	0.4023	0.3981	0.3940	0.3899	0.3858	0.3819	0.3779	0.3740
UK	0.4133	0.4098	0.4064	0.4030	0.3996	0.3963	0.3931	0.3899	0.3867	0.3836
	Year: 2004									
Denmark	0.3290	0.3275	0.3261	0.3246	0.3232	0.3218	0.3204	0.3190	0.3177	0.3163
France	0.3295	0.3280	0.3265	0.3251	0.3237	0.3223	0.3209	0.3195	0.3181	0.3168
Germany	0.3219	0.3207	0.3195	0.3183	0.3171	0.3160	0.3148	0.3137	0.3126	0.3115
Greece	0.3641	0.3613	0.3586	0.3559	0.3532	0.3506	0.3480	0.3454	0.3429	0.3404
Ireland	0.3630	0.3603	0.3577	0.3551	0.3526	0.3500	0.3476	0.3451	0.3427	0.3403
Italy	0.3448	0.3438	0.3429	0.3419	0.3409	0.3400	0.3390	0.3381	0.3372	0.3363
Luxembourg	0.3080	0.3072	0.3064	0.3056	0.3049	0.3041	0.3034	0.3026	0.3019	0.3012
Netherlands	0.3625	0.3602	0.3579	0.3557	0.3534	0.3512	0.3490	0.3469	0.3447	0.3426
Spain	0.3638	0.3617	0.3596	0.3575	0.3555	0.3535	0.3515	0.3495	0.3475	0.3456
UK	0.4379	0.4327	0.4277	0.4227	0.4178	0.4130	0.4082	0.4035	0.3989	0.3944

Table A.12: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.65	0.66	0.67	0.68	0.69	0.7	0.71	0.72	0.73	0.74
	Year: 2009									
Denmark	0.3566	0.3544	0.3523	0.3501	0.3480	0.3459	0.3438	0.3417	0.3397	0.3377
France	0.3082	0.3072	0.3061	0.3051	0.3041	0.3031	0.3021	0.3012	0.3002	0.2992
Germany	0.3380	0.3362	0.3343	0.3325	0.3308	0.3290	0.3273	0.3255	0.3238	0.3221
Greece	0.3490	0.3466	0.3442	0.3419	0.3396	0.3373	0.3351	0.3329	0.3307	0.3285
Ireland	0.3961	0.3925	0.3889	0.3854	0.3819	0.3785	0.3751	0.3718	0.3685	0.3653
Italy	0.3520	0.3506	0.3492	0.3478	0.3464	0.3451	0.3437	0.3424	0.3410	0.3397
Luxembourg	0.3272	0.3255	0.3239	0.3223	0.3207	0.3191	0.3175	0.3160	0.3144	0.3129
Netherlands	0.3269	0.3255	0.3240	0.3225	0.3211	0.3197	0.3182	0.3168	0.3155	0.3141
Spain	0.3646	0.3614	0.3583	0.3552	0.3521	0.3491	0.3461	0.3432	0.3403	0.3374
UK	0.3717	0.3688	0.3659	0.3631	0.3603	0.3576	0.3548	0.3522	0.3495	0.3469
	Year: 2014									
Denmark	0.3366	0.3348	0.3329	0.3312	0.3294	0.3276	0.3259	0.3242	0.3225	0.3208
France	0.3190	0.3178	0.3166	0.3153	0.3141	0.3129	0.3117	0.3106	0.3094	0.3082
Germany	0.3260	0.3243	0.3227	0.3211	0.3195	0.3179	0.3164	0.3149	0.3134	0.3119
Greece	0.3074	0.3064	0.3054	0.3044	0.3035	0.3025	0.3016	0.3006	0.2997	0.2988
Ireland	0.3499	0.3476	0.3453	0.3431	0.3408	0.3386	0.3365	0.3343	0.3322	0.3301
Italy	0.3474	0.3454	0.3434	0.3414	0.3395	0.3375	0.3356	0.3337	0.3318	0.3299
Luxembourg	0.3218	0.3203	0.3188	0.3174	0.3159	0.3145	0.3131	0.3117	0.3103	0.3090
Netherlands	0.3219	0.3205	0.3191	0.3178	0.3164	0.3151	0.3137	0.3124	0.3111	0.3098
Spain	0.3423	0.3404	0.3386	0.3368	0.3350	0.3332	0.3315	0.3297	0.3280	0.3263
UK	0.3934	0.3895	0.3856	0.3818	0.3781	0.3744	0.3707	0.3671	0.3636	0.3601
	Year: 2019									
Denmark	0.3707	0.3682	0.3657	0.3633	0.3608	0.3584	0.3560	0.3536	0.3512	0.3489
France	0.3645	0.3619	0.3593	0.3567	0.3541	0.3516	0.3491	0.3466	0.3441	0.3417
Germany	0.3338	0.3322	0.3307	0.3292	0.3276	0.3261	0.3247	0.3232	0.3218	0.3203
Greece	0.3458	0.3437	0.3416	0.3395	0.3375	0.3355	0.3335	0.3316	0.3296	0.3277
Ireland	0.3887	0.3854	0.3821	0.3789	0.3757	0.3726	0.3695	0.3664	0.3634	0.3605
Italy	0.3606	0.3582	0.3557	0.3533	0.3509	0.3486	0.3462	0.3439	0.3416	0.3394
Luxembourg	0.3291	0.3274	0.3258	0.3242	0.3227	0.3211	0.3196	0.3181	0.3166	0.3151
Netherlands	0.3479	0.3456	0.3433	0.3411	0.3389	0.3367	0.3345	0.3324	0.3302	0.3281
Spain	0.3546	0.3526	0.3507	0.3488	0.3469	0.3451	0.3432	0.3414	0.3396	0.3378
UK	0.3524	0.3501	0.3479	0.3456	0.3434	0.3413	0.3391	0.3370	0.3349	0.3328

Table A.13: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.75	0.76	0.77	0.78	0.79	0.8	0.81	0.82	0.83	0.84
	Year: 1989									
Denmark	0.3378	0.3363	0.3348	0.3333	0.3318	0.3303	0.3289	0.3274	0.3260	0.3245
France	0.2986	0.2982	0.2978	0.2974	0.2971	0.2967	0.2963	0.2960	0.2956	0.2953
Germany	0.3169	0.3156	0.3143	0.3131	0.3119	0.3106	0.3094	0.3082	0.3070	0.3058
Greece	0.3253	0.3252	0.3252	0.3252	0.3251	0.3251	0.3251	0.3251	0.3251	0.3251
Ireland	0.3089	0.3082	0.3074	0.3067	0.3060	0.3052	0.3045	0.3038	0.3031	0.3024
Italy	0.3122	0.3113	0.3104	0.3095	0.3086	0.3078	0.3069	0.3061	0.3052	0.3044
Luxembourg	0.2969	0.2960	0.2952	0.2944	0.2936	0.2928	0.2921	0.2913	0.2906	0.2898
Netherlands	0.2972	0.2964	0.2957	0.2950	0.2943	0.2936	0.2929	0.2922	0.2915	0.2908
Spain	0.3376	0.3353	0.3330	0.3307	0.3285	0.3263	0.3241	0.3219	0.3198	0.3176
UK	0.3428	0.3400	0.3371	0.3344	0.3316	0.3289	0.3262	0.3235	0.3209	0.3183
	Year: 1994									
Denmark	0.3216	0.3202	0.3187	0.3173	0.3159	0.3145	0.3131	0.3117	0.3103	0.3089
France	0.2995	0.2988	0.2980	0.2973	0.2965	0.2958	0.2951	0.2944	0.2937	0.2930
Germany	0.2972	0.2965	0.2957	0.2949	0.2942	0.2935	0.2927	0.2920	0.2913	0.2906
Greece	0.3254	0.3242	0.3231	0.3219	0.3208	0.3197	0.3186	0.3175	0.3164	0.3153
Ireland	0.3169	0.3156	0.3143	0.3131	0.3118	0.3106	0.3093	0.3081	0.3069	0.3057
Italy	0.3120	0.3109	0.3097	0.3086	0.3075	0.3064	0.3053	0.3042	0.3031	0.3020
Luxembourg	0.2839	0.2833	0.2827	0.2821	0.2815	0.2810	0.2804	0.2798	0.2793	0.2787
Netherlands	0.3164	0.3156	0.3148	0.3141	0.3134	0.3126	0.3119	0.3112	0.3105	0.3098
Spain	0.3512	0.3481	0.3451	0.3420	0.3391	0.3361	0.3332	0.3303	0.3274	0.3246
UK	0.3412	0.3385	0.3359	0.3333	0.3307	0.3281	0.3256	0.3231	0.3206	0.3182
	Year: 1999									
Denmark	0.2997	0.2989	0.2982	0.2975	0.2967	0.2960	0.2953	0.2946	0.2940	0.2933
France	0.2947	0.2943	0.2939	0.2934	0.2930	0.2926	0.2921	0.2917	0.2913	0.2909
Germany	0.3077	0.3066	0.3056	0.3046	0.3037	0.3027	0.3018	0.3008	0.2999	0.2990
Greece	0.3393	0.3367	0.3341	0.3316	0.3291	0.3266	0.3242	0.3218	0.3194	0.3170
Ireland	0.3320	0.3300	0.3279	0.3259	0.3239	0.3220	0.3200	0.3181	0.3162	0.3143
Italy	0.3277	0.3261	0.3244	0.3228	0.3212	0.3197	0.3181	0.3166	0.3150	0.3135
Luxembourg	0.3285	0.3261	0.3238	0.3214	0.3191	0.3168	0.3145	0.3123	0.3100	0.3078
Netherlands	0.3103	0.3089	0.3076	0.3063	0.3050	0.3037	0.3024	0.3011	0.2998	0.2986
Spain	0.3702	0.3664	0.3626	0.3589	0.3553	0.3517	0.3481	0.3446	0.3411	0.3377
UK	0.3805	0.3774	0.3744	0.3714	0.3685	0.3656	0.3628	0.3599	0.3572	0.3544
	Year: 2004									
Denmark	0.3149	0.3136	0.3123	0.3110	0.3097	0.3084	0.3071	0.3058	0.3046	0.3033
France	0.3154	0.3141	0.3128	0.3115	0.3102	0.3089	0.3076	0.3064	0.3051	0.3039
Germany	0.3104	0.3093	0.3083	0.3073	0.3062	0.3052	0.3042	0.3033	0.3023	0.3013
Greece	0.3379	0.3355	0.3330	0.3306	0.3282	0.3259	0.3236	0.3213	0.3190	0.3168
Ireland	0.3379	0.3355	0.3332	0.3309	0.3287	0.3264	0.3242	0.3220	0.3199	0.3177
Italy	0.3353	0.3344	0.3335	0.3326	0.3317	0.3309	0.3300	0.3291	0.3282	0.3274
Luxembourg	0.3005	0.2998	0.2991	0.2985	0.2978	0.2972	0.2965	0.2959	0.2953	0.2947
Netherlands	0.3404	0.3383	0.3363	0.3342	0.3322	0.3301	0.3281	0.3261	0.3242	0.3222
Spain	0.3437	0.3418	0.3399	0.3381	0.3362	0.3344	0.3327	0.3309	0.3291	0.3274
UK	0.3899	0.3855	0.3812	0.3770	0.3728	0.3686	0.3646	0.3605	0.3566	0.3527

Table A.14: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.75	0.76	0.77	0.78	0.79	0.8	0.81	0.82	0.83	0.84
	Year: 2009									
Denmark	0.3356	0.3336	0.3317	0.3297	0.3277	0.3258	0.3239	0.3220	0.3201	0.3182
France	0.2983	0.2974	0.2964	0.2955	0.2946	0.2937	0.2928	0.2920	0.2911	0.2902
Germany	0.3205	0.3188	0.3172	0.3156	0.3140	0.3124	0.3109	0.3093	0.3078	0.3063
Greece	0.3264	0.3243	0.3222	0.3201	0.3181	0.3161	0.3141	0.3121	0.3102	0.3083
Ireland	0.3621	0.3590	0.3559	0.3528	0.3498	0.3468	0.3439	0.3410	0.3382	0.3354
Italy	0.3384	0.3371	0.3358	0.3345	0.3332	0.3319	0.3306	0.3294	0.3281	0.3269
Luxembourg	0.3114	0.3099	0.3084	0.3069	0.3055	0.3041	0.3026	0.3012	0.2998	0.2984
Netherlands	0.3127	0.3114	0.3101	0.3088	0.3074	0.3062	0.3049	0.3036	0.3024	0.3011
Spain	0.3345	0.3317	0.3290	0.3262	0.3235	0.3208	0.3182	0.3155	0.3130	0.3104
UK	0.3444	0.3419	0.3394	0.3369	0.3345	0.3321	0.3297	0.3274	0.3251	0.3229
	Year: 2014									
Denmark	0.3191	0.3175	0.3158	0.3142	0.3126	0.3110	0.3094	0.3078	0.3063	0.3047
France	0.3071	0.3059	0.3048	0.3037	0.3026	0.3015	0.3004	0.2993	0.2982	0.2971
Germany	0.3104	0.3089	0.3075	0.3061	0.3047	0.3033	0.3019	0.3005	0.2992	0.2979
Greece	0.2979	0.2970	0.2961	0.2952	0.2944	0.2935	0.2927	0.2919	0.2911	0.2902
Ireland	0.3280	0.3259	0.3239	0.3218	0.3198	0.3179	0.3159	0.3140	0.3120	0.3101
Italy	0.3281	0.3262	0.3244	0.3226	0.3208	0.3190	0.3172	0.3155	0.3137	0.3120
Luxembourg	0.3076	0.3063	0.3050	0.3037	0.3024	0.3011	0.2999	0.2986	0.2974	0.2962
Netherlands	0.3086	0.3073	0.3061	0.3048	0.3036	0.3024	0.3012	0.3000	0.2988	0.2977
Spain	0.3245	0.3229	0.3212	0.3195	0.3179	0.3163	0.3146	0.3130	0.3114	0.3099
UK	0.3566	0.3532	0.3498	0.3465	0.3432	0.3400	0.3368	0.3336	0.3305	0.3274
	Year: 2019									
Denmark	0.3466	0.3443	0.3420	0.3397	0.3375	0.3353	0.3331	0.3309	0.3287	0.3266
France	0.3393	0.3369	0.3346	0.3323	0.3300	0.3277	0.3254	0.3232	0.3210	0.3188
Germany	0.3189	0.3176	0.3162	0.3148	0.3135	0.3122	0.3109	0.3096	0.3083	0.3071
Greece	0.3258	0.3239	0.3221	0.3202	0.3184	0.3166	0.3148	0.3131	0.3113	0.3096
Ireland	0.3575	0.3547	0.3518	0.3490	0.3462	0.3435	0.3408	0.3382	0.3356	0.3330
Italy	0.3371	0.3349	0.3327	0.3305	0.3283	0.3262	0.3240	0.3219	0.3198	0.3177
Luxembourg	0.3136	0.3122	0.3108	0.3094	0.3080	0.3066	0.3053	0.3039	0.3026	0.3013
Netherlands	0.3261	0.3240	0.3220	0.3199	0.3179	0.3160	0.3140	0.3121	0.3101	0.3082
Spain	0.3360	0.3342	0.3325	0.3308	0.3290	0.3273	0.3256	0.3240	0.3223	0.3207
UK	0.3308	0.3287	0.3267	0.3248	0.3228	0.3209	0.3190	0.3171	0.3152	0.3134

Table A.15: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94
	Year: 1989									
Denmark	0.3231	0.3217	0.3203	0.3189	0.3175	0.3161	0.3147	0.3133	0.3120	0.3106
France	0.2950	0.2946	0.2943	0.2940	0.2937	0.2934	0.2931	0.2928	0.2925	0.2922
Germany	0.3047	0.3035	0.3023	0.3012	0.3001	0.2989	0.2978	0.2967	0.2956	0.2945
Greece	0.3251	0.3251	0.3251	0.3251	0.3251	0.3252	0.3252	0.3252	0.3253	0.3253
Ireland	0.3017	0.3011	0.3004	0.2997	0.2991	0.2984	0.2978	0.2972	0.2965	0.2959
Italy	0.3036	0.3028	0.3020	0.3013	0.3005	0.2997	0.2990	0.2983	0.2975	0.2968
Luxembourg	0.2891	0.2884	0.2877	0.2870	0.2864	0.2857	0.2851	0.2844	0.2838	0.2832
Netherlands	0.2901	0.2894	0.2888	0.2881	0.2875	0.2868	0.2862	0.2856	0.2849	0.2843
Spain	0.3155	0.3134	0.3114	0.3093	0.3073	0.3053	0.3033	0.3014	0.2994	0.2975
UK	0.3157	0.3132	0.3107	0.3082	0.3057	0.3033	0.3009	0.2985	0.2962	0.2939
	Year: 1994									
Denmark	0.3076	0.3062	0.3049	0.3036	0.3022	0.3009	0.2996	0.2984	0.2971	0.2958
France	0.2923	0.2916	0.2909	0.2903	0.2896	0.2889	0.2883	0.2876	0.2870	0.2864
Germany	0.2899	0.2893	0.2886	0.2879	0.2873	0.2867	0.2860	0.2854	0.2848	0.2842
Greece	0.3143	0.3132	0.3122	0.3112	0.3102	0.3092	0.3083	0.3073	0.3064	0.3054
Ireland	0.3045	0.3034	0.3022	0.3011	0.3000	0.2988	0.2977	0.2966	0.2956	0.2945
Italy	0.3010	0.2999	0.2989	0.2978	0.2968	0.2958	0.2948	0.2937	0.2927	0.2917
Luxembourg	0.2782	0.2777	0.2771	0.2766	0.2761	0.2756	0.2751	0.2746	0.2741	0.2737
Netherlands	0.3091	0.3085	0.3078	0.3071	0.3065	0.3059	0.3052	0.3046	0.3040	0.3034
Spain	0.3218	0.3190	0.3163	0.3136	0.3109	0.3083	0.3056	0.3030	0.3005	0.2979
UK	0.3158	0.3134	0.3111	0.3088	0.3065	0.3042	0.3020	0.2997	0.2976	0.2954
	Year: 1999									
Denmark	0.2926	0.2919	0.2913	0.2906	0.2900	0.2894	0.2887	0.2881	0.2875	0.2869
France	0.2905	0.2901	0.2897	0.2893	0.2889	0.2885	0.2882	0.2878	0.2874	0.2871
Germany	0.2981	0.2972	0.2964	0.2955	0.2947	0.2939	0.2930	0.2922	0.2915	0.2907
Greece	0.3146	0.3123	0.3100	0.3077	0.3055	0.3032	0.3010	0.2988	0.2966	0.2945
Ireland	0.3125	0.3107	0.3089	0.3071	0.3053	0.3036	0.3018	0.3001	0.2985	0.2968
Italy	0.3120	0.3105	0.3090	0.3075	0.3061	0.3046	0.3032	0.3017	0.3003	0.2989
Luxembourg	0.3056	0.3035	0.3013	0.2992	0.2971	0.2951	0.2930	0.2910	0.2890	0.2870
Netherlands	0.2973	0.2961	0.2949	0.2937	0.2925	0.2913	0.2901	0.2889	0.2878	0.2866
Spain	0.3343	0.3309	0.3276	0.3243	0.3211	0.3179	0.3147	0.3116	0.3085	0.3054
UK	0.3517	0.3490	0.3464	0.3437	0.3412	0.3386	0.3361	0.3336	0.3311	0.3287
	Year: 2004									
Denmark	0.3021	0.3008	0.2996	0.2984	0.2972	0.2960	0.2948	0.2936	0.2925	0.2913
France	0.3026	0.3014	0.3002	0.2990	0.2979	0.2967	0.2955	0.2944	0.2932	0.2921
Germany	0.3004	0.2995	0.2986	0.2976	0.2968	0.2959	0.2950	0.2942	0.2933	0.2925
Greece	0.3145	0.3123	0.3102	0.3080	0.3059	0.3037	0.3017	0.2996	0.2975	0.2955
Ireland	0.3156	0.3135	0.3114	0.3094	0.3074	0.3054	0.3034	0.3014	0.2995	0.2975
Italy	0.3265	0.3257	0.3248	0.3240	0.3231	0.3223	0.3215	0.3207	0.3199	0.3191
Luxembourg	0.2941	0.2935	0.2929	0.2923	0.2918	0.2912	0.2907	0.2901	0.2896	0.2891
Netherlands	0.3203	0.3183	0.3164	0.3145	0.3127	0.3108	0.3090	0.3071	0.3053	0.3035
Spain	0.3257	0.3240	0.3223	0.3207	0.3190	0.3174	0.3158	0.3142	0.3126	0.3111
UK	0.3488	0.3451	0.3413	0.3376	0.3340	0.3304	0.3269	0.3234	0.3200	0.3166

Table A.16: Results of PaF estimator for all values of α - Cont'ed

	— α values —									
	0.85	0.86	0.87	0.88	0.89	0.9	0.91	0.92	0.93	0.94
	Year: 2009									
Denmark	0.3163	0.3145	0.3126	0.3108	0.3090	0.3072	0.3055	0.3037	0.3019	0.3002
France	0.2894	0.2885	0.2877	0.2869	0.2861	0.2853	0.2845	0.2837	0.2829	0.2821
Germany	0.3048	0.3033	0.3018	0.3004	0.2989	0.2975	0.2961	0.2947	0.2933	0.2920
Greece	0.3063	0.3045	0.3026	0.3008	0.2989	0.2971	0.2953	0.2936	0.2918	0.2901
Ireland	0.3326	0.3298	0.3271	0.3245	0.3218	0.3192	0.3167	0.3141	0.3116	0.3091
Italy	0.3257	0.3244	0.3232	0.3220	0.3208	0.3196	0.3184	0.3172	0.3161	0.3149
Luxembourg	0.2971	0.2957	0.2944	0.2930	0.2917	0.2904	0.2891	0.2878	0.2866	0.2853
Netherlands	0.2999	0.2987	0.2975	0.2963	0.2951	0.2939	0.2927	0.2916	0.2904	0.2893
Spain	0.3079	0.3054	0.3029	0.3004	0.2980	0.2956	0.2933	0.2909	0.2886	0.2863
UK	0.3206	0.3184	0.3162	0.3141	0.3120	0.3099	0.3078	0.3057	0.3037	0.3017
	Year: 2014									
Denmark	0.3032	0.3017	0.3002	0.2987	0.2972	0.2958	0.2943	0.2929	0.2914	0.2900
France	0.2961	0.2950	0.2940	0.2930	0.2919	0.2909	0.2899	0.2889	0.2879	0.2869
Germany	0.2966	0.2953	0.2940	0.2927	0.2915	0.2902	0.2890	0.2878	0.2866	0.2854
Greece	0.2895	0.2887	0.2879	0.2871	0.2864	0.2856	0.2849	0.2842	0.2835	0.2827
Ireland	0.3082	0.3064	0.3045	0.3027	0.3009	0.2991	0.2973	0.2955	0.2938	0.2920
Italy	0.3103	0.3086	0.3069	0.3052	0.3036	0.3019	0.3003	0.2987	0.2971	0.2955
Luxembourg	0.2950	0.2938	0.2927	0.2915	0.2904	0.2892	0.2881	0.2870	0.2859	0.2848
Netherlands	0.2965	0.2954	0.2942	0.2931	0.2920	0.2909	0.2898	0.2887	0.2877	0.2866
Spain	0.3083	0.3068	0.3052	0.3037	0.3022	0.3007	0.2992	0.2977	0.2962	0.2948
UK	0.3244	0.3214	0.3184	0.3155	0.3126	0.3097	0.3069	0.3041	0.3014	0.2986
	Year: 2019									
Denmark	0.3244	0.3223	0.3202	0.3182	0.3161	0.3141	0.3120	0.3100	0.3080	0.3060
France	0.3167	0.3145	0.3124	0.3103	0.3083	0.3062	0.3042	0.3022	0.3002	0.2982
Germany	0.3058	0.3046	0.3034	0.3022	0.3010	0.2999	0.2987	0.2976	0.2964	0.2953
Greece	0.3079	0.3062	0.3045	0.3028	0.3012	0.2996	0.2980	0.2964	0.2948	0.2932
Ireland	0.3304	0.3279	0.3254	0.3230	0.3205	0.3182	0.3158	0.3135	0.3112	0.3089
Italy	0.3157	0.3136	0.3116	0.3096	0.3076	0.3056	0.3037	0.3018	0.2998	0.2979
Luxembourg	0.3000	0.2987	0.2974	0.2962	0.2950	0.2937	0.2925	0.2913	0.2901	0.2890
Netherlands	0.3064	0.3045	0.3026	0.3008	0.2990	0.2972	0.2954	0.2937	0.2919	0.2902
Spain	0.3190	0.3174	0.3158	0.3142	0.3126	0.3110	0.3095	0.3079	0.3064	0.3049
UK	0.3116	0.3098	0.3080	0.3063	0.3045	0.3028	0.3011	0.2994	0.2978	0.2961

Table A.17: Results of PaF estimator for all values of α - Cont'ed

	— α values —					
	0.95	0.96	0.97	0.98	0.99	1
			Year: 1989			
Denmark	0.3093	0.3080	0.3067	0.3054	0.3041	0.3028
France	0.2919	0.2916	0.2914	0.2911	0.2909	0.2906
Germany	0.2935	0.2924	0.2913	0.2903	0.2892	0.2882
Greece	0.3254	0.3254	0.3255	0.3255	0.3256	0.3257
Ireland	0.2953	0.2947	0.2941	0.2935	0.2929	0.2924
Italy	0.2961	0.2954	0.2947	0.2940	0.2934	0.2927
Luxembourg	0.2826	0.2820	0.2814	0.2808	0.2802	0.2797
Netherlands	0.2837	0.2831	0.2825	0.2819	0.2813	0.2807
Spain	0.2956	0.2937	0.2918	0.2900	0.2881	0.2863
UK	0.2916	0.2893	0.2870	0.2848	0.2826	0.2804
			Year: 1994			
Denmark	0.2946	0.2933	0.2921	0.2908	0.2896	0.2884
France	0.2857	0.2851	0.2845	0.2839	0.2833	0.2827
Germany	0.2836	0.2830	0.2824	0.2818	0.2812	0.2807
Greece	0.3045	0.3036	0.3027	0.3018	0.3010	0.3001
Ireland	0.2934	0.2924	0.2913	0.2903	0.2893	0.2883
Italy	0.2908	0.2898	0.2888	0.2878	0.2869	0.2859
Luxembourg	0.2732	0.2727	0.2723	0.2718	0.2714	0.2709
Netherlands	0.3028	0.3022	0.3016	0.3010	0.3004	0.2998
Spain	0.2954	0.2929	0.2905	0.2880	0.2856	0.2832
UK	0.2933	0.2911	0.2890	0.2870	0.2849	0.2829
			Year: 1999			
Denmark	0.2863	0.2857	0.2851	0.2845	0.2840	0.2834
France	0.2867	0.2864	0.2860	0.2857	0.2853	0.2850
Germany	0.2899	0.2892	0.2884	0.2877	0.2870	0.2862
Greece	0.2923	0.2902	0.2881	0.2861	0.2840	0.2820
Ireland	0.2951	0.2935	0.2919	0.2903	0.2887	0.2872
Italy	0.2975	0.2961	0.2947	0.2934	0.2920	0.2907
Luxembourg	0.2850	0.2831	0.2812	0.2792	0.2774	0.2755
Netherlands	0.2855	0.2843	0.2832	0.2821	0.2810	0.2799
Spain	0.3024	0.2994	0.2965	0.2935	0.2907	0.2878
UK	0.3263	0.3239	0.3215	0.3192	0.3169	0.3146
			Year: 2004			
Denmark	0.2902	0.2890	0.2879	0.2868	0.2856	0.2845
France	0.2910	0.2899	0.2888	0.2877	0.2866	0.2855
Germany	0.2917	0.2908	0.2900	0.2893	0.2885	0.2877
Greece	0.2935	0.2915	0.2896	0.2876	0.2857	0.2838
Ireland	0.2956	0.2937	0.2919	0.2900	0.2882	0.2864
Italy	0.3183	0.3175	0.3167	0.3159	0.3151	0.3143
Luxembourg	0.2886	0.2880	0.2875	0.2871	0.2866	0.2861
Netherlands	0.3017	0.3000	0.2982	0.2965	0.2947	0.2930
Spain	0.3095	0.3080	0.3065	0.3050	0.3035	0.3020
UK	0.3133	0.3100	0.3068	0.3036	0.3004	0.2973

Table A.18: Results of PaF estimator for all values of α - Cont'ed

	— α values —					
	0.95	0.96	0.97	0.98	0.99	1
	Year: 2009					
Denmark	0.2985	0.2968	0.2951	0.2934	0.2917	0.2901
France	0.2814	0.2806	0.2798	0.2791	0.2784	0.2776
Germany	0.2906	0.2893	0.2880	0.2866	0.2853	0.2840
Greece	0.2884	0.2867	0.2850	0.2834	0.2817	0.2801
Ireland	0.3067	0.3043	0.3019	0.2995	0.2972	0.2949
Italy	0.3138	0.3126	0.3115	0.3103	0.3092	0.3081
Luxembourg	0.2840	0.2828	0.2816	0.2803	0.2791	0.2779
Netherlands	0.2882	0.2871	0.2860	0.2849	0.2838	0.2827
Spain	0.2841	0.2818	0.2796	0.2774	0.2753	0.2731
UK	0.2998	0.2978	0.2959	0.2940	0.2921	0.2903
	Year: 2014					
Denmark	0.2886	0.2872	0.2858	0.2845	0.2831	0.2818
France	0.2859	0.2850	0.2840	0.2830	0.2821	0.2811
Germany	0.2843	0.2831	0.2820	0.2808	0.2797	0.2786
Greece	0.2820	0.2814	0.2807	0.2800	0.2793	0.2787
Ireland	0.2903	0.2886	0.2869	0.2853	0.2836	0.2820
Italy	0.2939	0.2923	0.2908	0.2892	0.2877	0.2861
Luxembourg	0.2838	0.2827	0.2817	0.2806	0.2796	0.2786
Netherlands	0.2856	0.2845	0.2835	0.2825	0.2815	0.2804
Spain	0.2934	0.2919	0.2905	0.2891	0.2877	0.2863
UK	0.2959	0.2933	0.2907	0.2881	0.2855	0.2830
	Year: 2019					
Denmark	0.3041	0.3021	0.3002	0.2983	0.2964	0.2945
France	0.2963	0.2944	0.2925	0.2906	0.2887	0.2869
Germany	0.2942	0.2931	0.2921	0.2910	0.2899	0.2889
Greece	0.2917	0.2901	0.2886	0.2871	0.2856	0.2841
Ireland	0.3066	0.3044	0.3022	0.3001	0.2979	0.2958
Italy	0.2960	0.2942	0.2923	0.2905	0.2886	0.2868
Luxembourg	0.2878	0.2867	0.2855	0.2844	0.2833	0.2822
Netherlands	0.2885	0.2868	0.2851	0.2834	0.2818	0.2802
Spain	0.3034	0.3019	0.3004	0.2989	0.2975	0.2960
UK	0.2945	0.2929	0.2913	0.2898	0.2882	0.2867